Apparatus

1. thermocouple
2. field measuring apparatus
3. cool end temperature measuring unit
4. hot end temperature measuring unit
5. cooling unit
6. heater

![Diagram of apparatus](image)
1. Thermocouple

- Material: copper – aluminium
- Width of metal strip: 3 cm
- Height of metal strip: 1 cm
- Net length: 17 cm
2. Field measuring apparatus

![Diagram of field measuring apparatus](image)

- Thread
- Magnet
- Coils
- Voltage source
- Needle
- Copper strip
Principle of work

- When the field of the coils equals the thermocouple field the only remaining field acting on the magnet is the Earth field.

Symbols used in the diagram:
- \( B_{tc} \) – thermocouple magnetic field
- \( B_{Hh} \) – magnetic field of the coils
- \( B_E \) – field of the Earth
- \( \mu \) – magnetic moment of the magnet
• The magnet turns in the direction of the resultant field because of its magnetic moment:

\[ \tau = \mu \times B \]

- \( \tau \) – torque
- \( \mu \) – magnetic moment of the magnet
- \( B \) – external magnetic field

• That torque causes rotation of the magnet until its magnetic moment is aligned with the field

• The equality of coils and thermocouple fields enables us to measure the thermocouple field easily
3. Temperature measuring units

**Cool end**
- A thermocouple was used

**Hot end**
- Because of high temperatures it was appropriate to use Pt-Rh thermocouple

- The temperature was calculated from the obtained voltage
4. **Cooler and heater**

- The cooling was accomplished by
  - constant water flow over the cooling rib
  - Liquid nitrogen
• The heater was a Danniel burner

• The temperature range of the whole system was

<table>
<thead>
<tr>
<th>Cool end</th>
<th>Hot end</th>
</tr>
</thead>
<tbody>
<tr>
<td>-150 °C to 50°C</td>
<td>0°C to 600°C</td>
</tr>
</tbody>
</table>
Theoretical approach

Describing the conduction electrons

• Electron gas properties:
  • Great density
  • Momentum conservation (approximately)
  • Free electrons (lattice defects and interactions neglected)

For many metals the free electron model is not valid – introduction of correction parameters
• Mathematical description – Fermi - Dirac energy distribution:

\[
f(u) = \frac{1}{1 + e^{\frac{u-\eta}{kT}}}
\]

- \(f(u)\) – probability of finding an electron with energy \(u\)
- \(\eta\) – highest occupied energy level - “Fermi level”
- \(k\) – Boltzmann constant
- \(T\) – absolute temperature

(\text{I. Supek: Teorijska fizika i struktura meteije, vol. 2})

• The Fermi level is a characteristic of the metal and depends on electron concentration and temperature:

\[
\eta(T) = \eta_0 \left[ 1 - \frac{\pi}{12} \left( \frac{kT}{\eta_0} \right)^2 + \ldots \right]
\]

- \(\eta_0\) – Fermi level at 0 K
Contact potential

• At the join of two different metals a potential difference occurs

• This is due to different electron concentrations in the metals i.e. different Fermi levels

• Concentrations tend to equalize by electron diffusion

• The final potential difference is:

\[ \Delta V = \frac{\eta_1 - \eta_2}{e} \]

\( \Delta V \) – potential difference
\( \eta_i \) – Fermi level of the \( i \) – th metal
\( e \) – charge of the electron
Noneqilibrium
Equilibrium

Shortage of electrons – net charge is positive

Surplus of electrons – net charge is negative
Thompson effect

• In a metal strip with ends at different temperatures a potential difference occurs (Thompson effect) because of different Fermi levels:

Electron current

Nonequilibrium

\[ T, \eta_1 \quad \text{Electron current} \quad T+\Delta T, \eta_2 \]

\[ \eta_1, \eta_2 \] – initial Fermi energies

\[ \eta_{rez} \] – equilibrium Fermi energy
The voltage in the thermocouple is:

\[ dV = S_{AB}dT \]

- \( dV \) – voltage difference
- \( S_{AB} \) - “Seebeck coefficient” of the metal pair

\[ S_{AB} = S_{AB}(T) \quad S_{AB} = S_A - S_B \]

- \( S_A, S_B \) – Seebeck coefficients of single metals (compared to a reference metal)
- \( dT \) – temperature difference of the joins
• The Seebeck coefficients can be determined considering the Thompson effect

\[ S_A = \frac{\pi^2 k^2 T}{6\eta_0 e} \]

- \( k \) – Boltzmann constant
- \( T \) – temperature of the metal
- \( \eta_0 \) – Fermi level at 0 K
- \( e \) – elementary charge

• For correcting the free electron model a correction constant of order 1 has to be introduced

\[ S_A = \frac{\chi \pi^2 k^2 T}{6\eta_0 e} \]

\( \Rightarrow V_{AB} = a(T_B - T_A)^2 + b(T_B - T_A) \)

- \( a, b \) – constants
- \( T_A, T_B \) – junction temperatures
## Seebeck coefficients

<table>
<thead>
<tr>
<th>metal</th>
<th>Fermi - level at 0 K [eV]</th>
<th>measured $S_A$ [$\mu$V/K]</th>
<th>theoretical $S_A$ (free electrons) [$\mu$V/K]</th>
<th>hi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na</td>
<td>3.10</td>
<td>-2.00</td>
<td>7.80</td>
<td>-0.26</td>
</tr>
<tr>
<td>Al</td>
<td>11.60</td>
<td>3.50</td>
<td>2.08</td>
<td>1.68</td>
</tr>
<tr>
<td>K</td>
<td>2.00</td>
<td>-9.00</td>
<td>12.09</td>
<td>-0.74</td>
</tr>
<tr>
<td>Cu</td>
<td>7.00</td>
<td>6.50</td>
<td>3.45</td>
<td>1.88</td>
</tr>
<tr>
<td>Ag</td>
<td>5.50</td>
<td>6.50</td>
<td>4.40</td>
<td>1.48</td>
</tr>
<tr>
<td>Au</td>
<td>5.50</td>
<td>6.50</td>
<td>4.40</td>
<td>1.48</td>
</tr>
</tbody>
</table>

Source for measured data: [www.materials.usask.ca](http://www.materials.usask.ca)
Experimental results

• The field we measured is approximately proportional to the potential difference:

\[ |B_{tc}| = \alpha (T_B - T_A)^2 + \beta (T_B - T_A) \]

\( \alpha, \beta \) – constants

\( B_{tc} \) – thermocouple field

• The constants \( \alpha \) and \( \beta \) have been obtained experimentally but a direct numerical comparison to the theoretical voltage was impossible due to the unknown coefficient of proportionality:
Fit:

\[ \Delta V_{Hh} = 0.0153 \Delta T^2 - 0.0784 \Delta T \]

\( \Delta T \) – junction temperature difference

Field constants:
Conclusion

- Thermocouple voltage (and magnetic field of the couple) will have a maximum if:
  - The constant $a$ in the thermocouple equation is negative
  - The constant $b$ in the thermocouple equation is positive
  - The sign of these constants depends on the metals used in the couple
  - For our Cu – Al couple the function $B(\Delta T)$ showed a minimum and no maximum