Problem 3.

Electric pendulum

Problem

Use a thread to suspend a ball between plates of a capacitor. When the plates are charged the ball will start to oscilate. What does the period of oscillations depend on?

Short answer

Period of oscilations depends on :

1. Charging voltage
   with increasing voltage the period decreases
2. Lenght of thread
   with increasing thread length the period decreases
3. Ball mass
   with increasing mass the period increases
4. Ball material
   dielectric materials swing slower than conducting ones
5. Distance between plates
   with increasing the distance the period increases

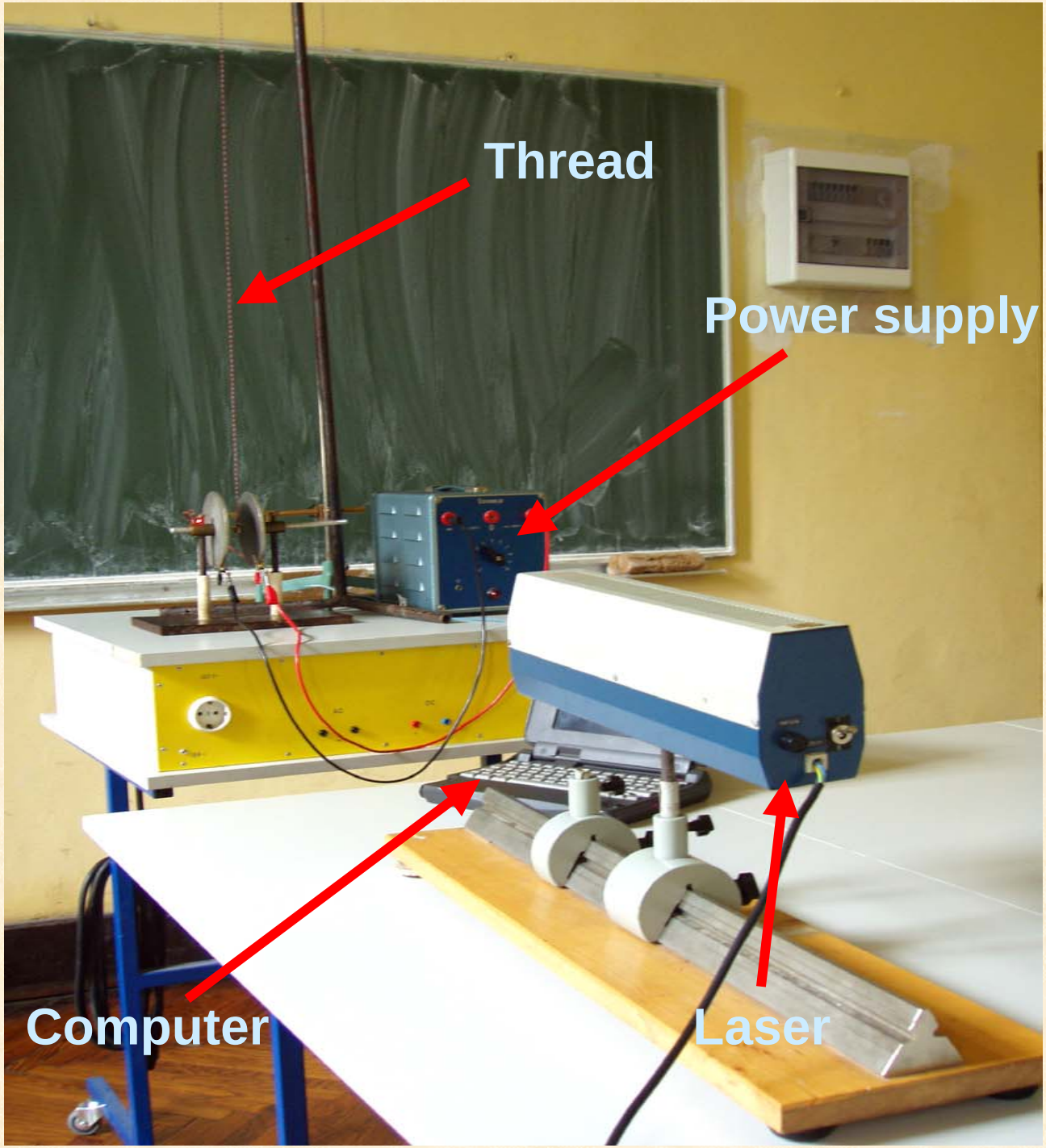
Relevant parameters

- Ball parameters
  - Ball material
  - Ball mass
  - Ball radius
- Other parameters
  - Voltage between plates
  - Distance between plates
  - Thread lenght

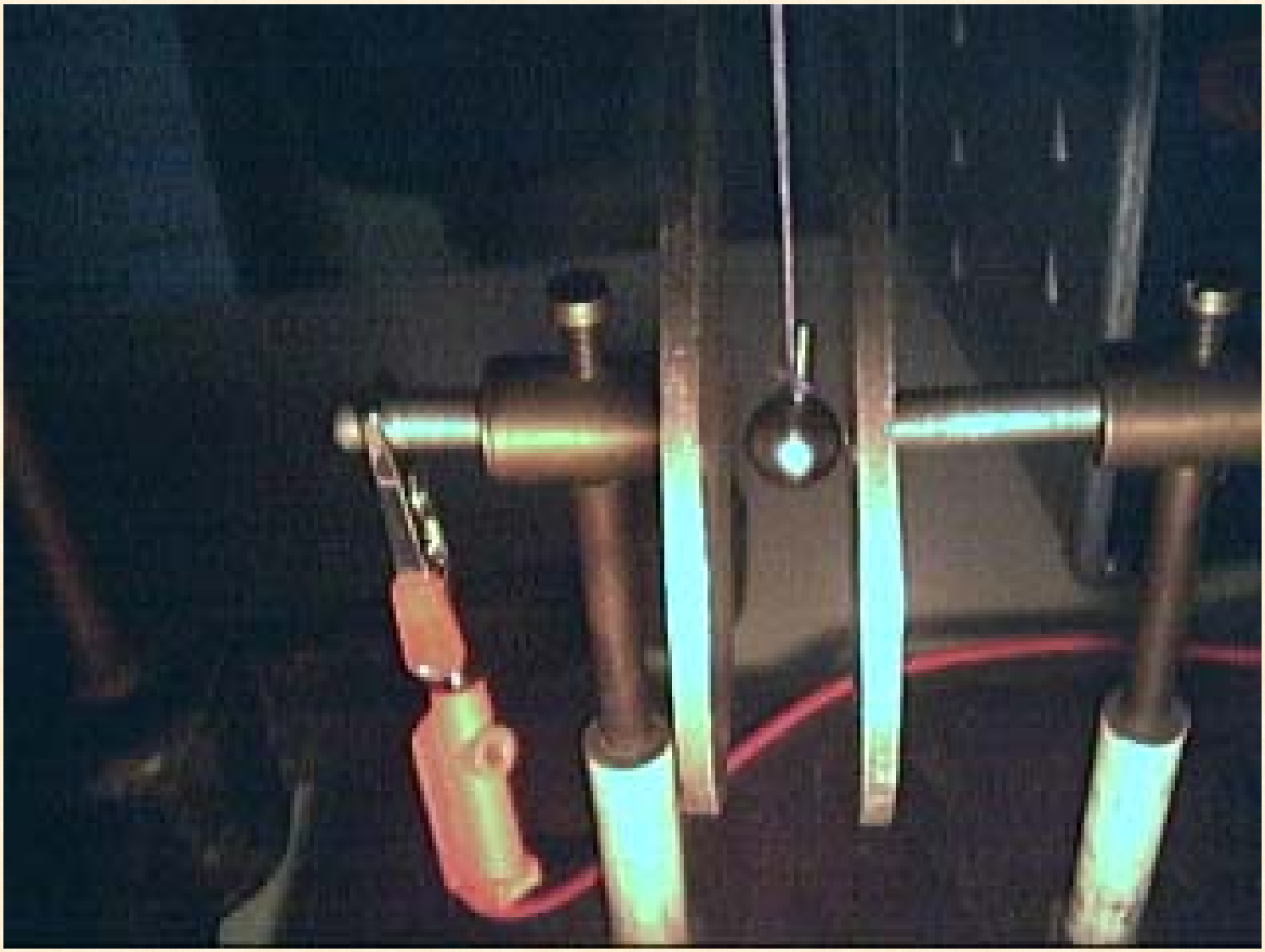
Experiment

- Goals:
  - Apparatus construction
  - Period measurements

Apparatus


Thread
Power supply
Computer
Laser

Apparatus in function:



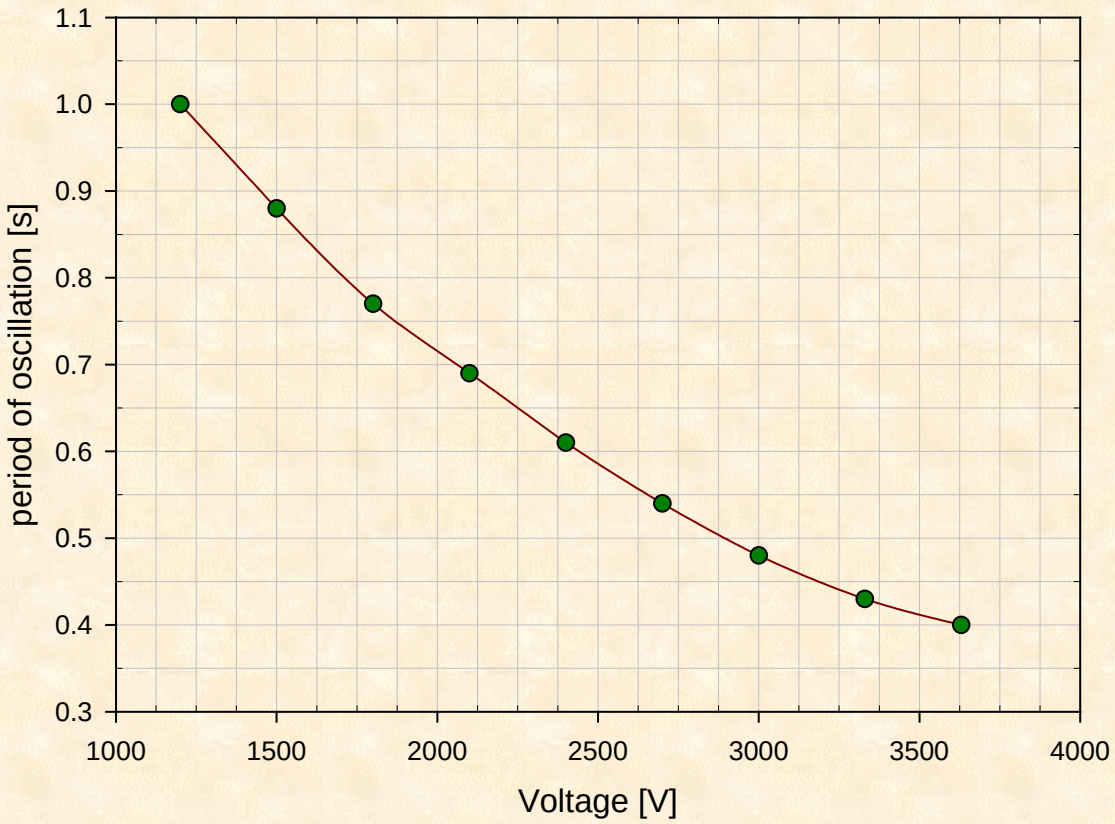
Measuring conditions

- Voltage variation
  - from 1200 to 3600 volts
- Distance between plates
  - for metal balls: 2 – 4 cm
  - for wooden balls: 4 and 5 cm
- Ball radius
  - metal balls: 0.6 and 0.75 cm
  - wooden balls: 0.5 to 1 cm

Measuring results for metal ball

Voltage [v]	Period[s]
1200	1.00
1500	0.88
1800	0.77
2100	0.69
2400	0.61
2700	0.54
3000	0.48
3300	0.43
3600	0.40

Oscillation period



Experiment conditions:

r = 0.75 cm

l = 0.5m d = 2cm

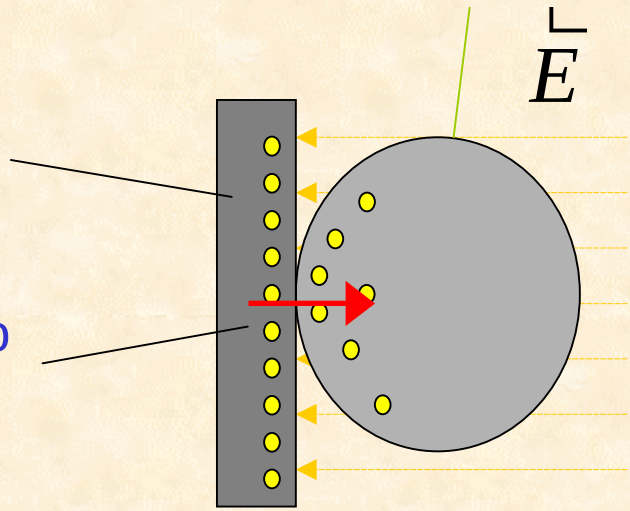
Theoretical approach

Charging of the metal ball

- When in contact with a charged plate, the metal ball will charge until the potentials are equal

Negatively charged plate

Electron current (tending to equalize potentials)



E – electric field

- The charging time is extremely short

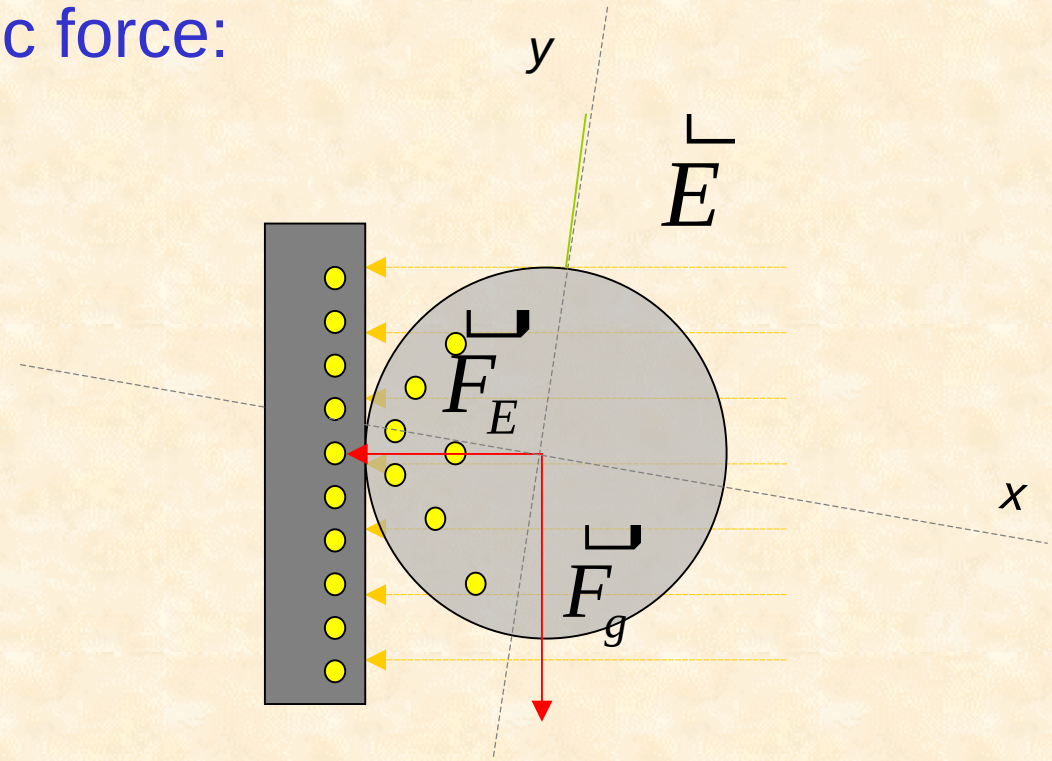
Charging of the metal ball

Charging of the dielectric ball

- The dielectric ball charges much longer
- It doesn't always manage to charge until the potentials are equal
- The ball will keep charging until gravity gets greater than the electrostatic force:

F_E - electrostatic force

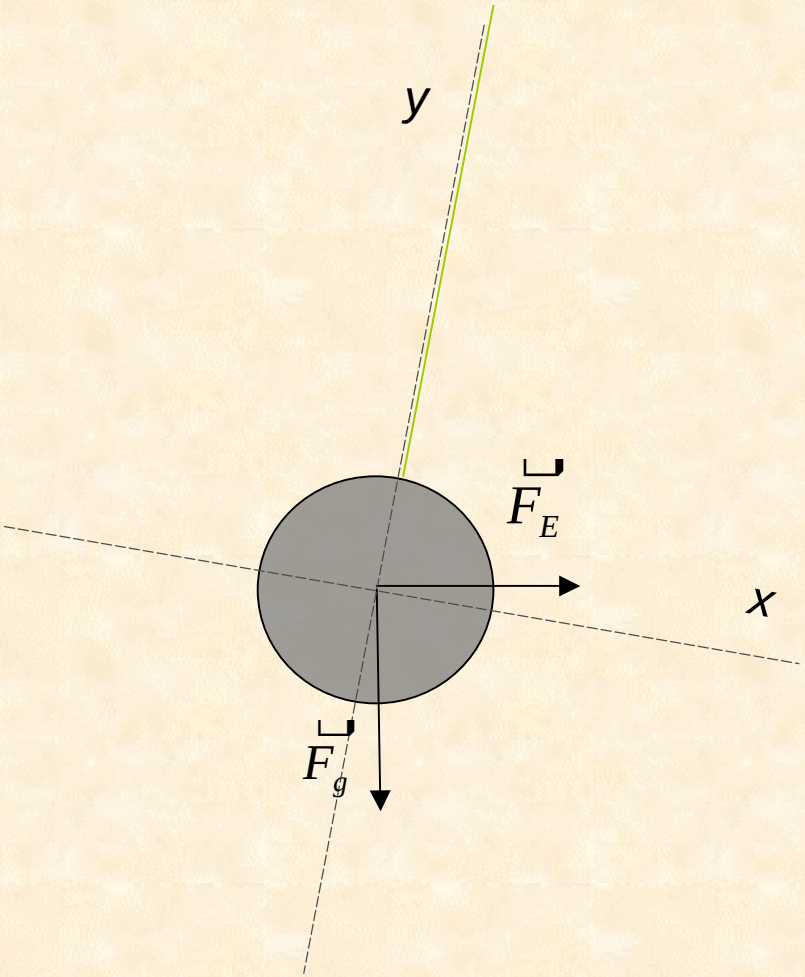
F_g - gravity



Charging of dielectric ball

Motion of the ball

- Forces acting on charged ball moving between the plates:



Electrostatic force:

$$\mathbf{F}_E = q\frac{U}{d}\hat{\mathbf{e}}$$

q – ball charge

U – voltage

d – distance between plates

e – unit vector

Gravity:

$$\mathbf{F}_g = m\mathbf{g}$$

m – ball mass

g – free fall acceleration

- The force affecting motion is (string is unstretchable):

$$F_{mot} = -mg\sin\varphi + q\frac{U}{d}\cos\varphi$$

φ – angle between thread and vertical

- Equation of motion in polar coordinates:

$$ml\frac{d^2\varphi}{dt^2} = -mg\sin\varphi + \frac{Uq}{d}\cos\varphi$$

l – length of thread

- For small angles: $\sin\varphi \approx \varphi$

$$\cos\varphi \approx 1$$

$$\Rightarrow \quad \frac{d^2\varphi}{dt^2} = -\frac{g}{l}\varphi + \frac{Uq}{mld}$$

Solution of the equation of motion

$$\varphi(t) = \left(\frac{qU}{mgd} - \alpha \right) \cos 2\pi\nu t - \frac{qU}{mgd}$$

α – maximum deflection angle (angular amplitude)

υ – frequency

t - time

- For a metal ball the charge *q* is:

$$q = 4\pi\varepsilon_0 rU$$

ε_0 – permittivity of vacuum

r – ball radius

U - voltage

$$\Rightarrow T = 2\sqrt{\frac{l}{g}} \arccos \frac{8\pi\varepsilon_0 rlU^2 - mgd(d-2r)}{8\pi\varepsilon_0 rlU^2 + mgd(d-2r)}$$

T – period of motion

Checking the period expression

1. Asymptotic behaviour

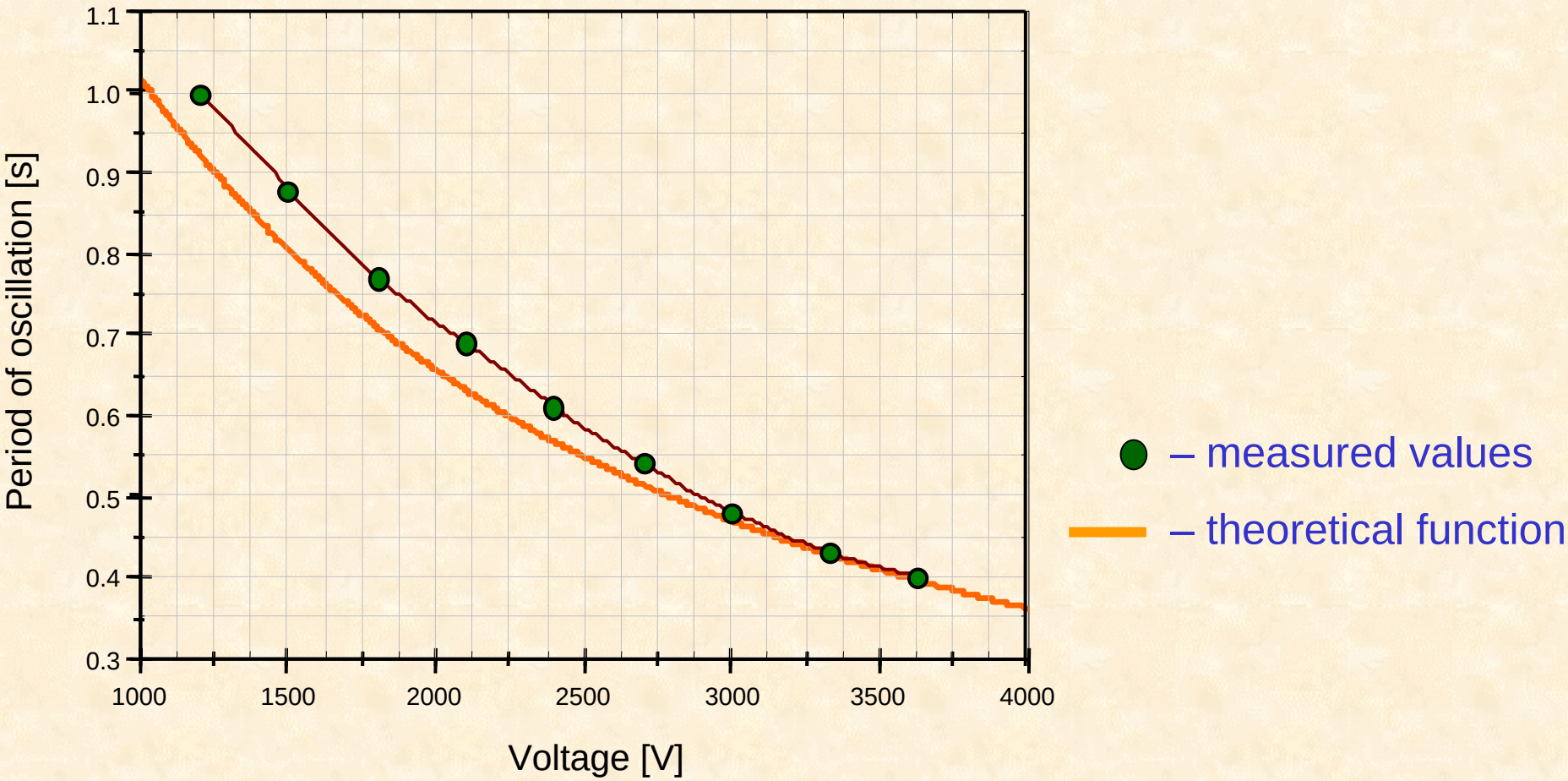
$$\lim_{U \to 0} T = 2\pi\sqrt{\frac{l}{g}}$$

- Period of mathematical pendulum!

$$\lim_{U \to \infty} T = 0$$

- Infinite voltage results in infinite force, so the trajecting time has to tend to zero

2. Correlation with experimental data



- The error is within 10%, which means that our relation describes the real period quite well

Conclusion

- The period of the electrostatic pendulum depends on:
    - Voltage in the capacitor
    - Distance of plates
    - Ball radius
    - Thread length
    - Ball material