Explanation

• We considered the granular mixture as a system of two different fluids
• We demonstrate that this approach is adequate
• The denser component sinks to the bottom, in analogy with real fluids
• Larger particle behaviour:
  • If the larger particles have lower density, they will emerge
  • If they are denser, they will sink
Experiment

1. **Intruder ball**

   - Measured quantity: time of emerging/sinking of the intruder
   - Parameters:
     - Frequency
     - Amplitude
     - Ratio of intruder and surrounding balls densities
     - Volume of intruder
**Used intruders**

<table>
<thead>
<tr>
<th>material</th>
<th>mass [g]</th>
<th>density [kg/m³]</th>
<th>diameter [cm]</th>
<th>volume [cm³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>lead</td>
<td>86,3</td>
<td>10550</td>
<td>25</td>
<td>8,2</td>
</tr>
<tr>
<td>wood 1</td>
<td>3,62</td>
<td>864,2</td>
<td>20</td>
<td>4,2</td>
</tr>
<tr>
<td>wood 2</td>
<td>0,42</td>
<td>464</td>
<td>12</td>
<td>0,905</td>
</tr>
<tr>
<td>plastic</td>
<td>0,26</td>
<td>969,8</td>
<td>8</td>
<td>0,268</td>
</tr>
<tr>
<td>styropor 1</td>
<td>1,34</td>
<td>21,7</td>
<td>49</td>
<td>61,6</td>
</tr>
<tr>
<td>styropor 2</td>
<td>0,32</td>
<td>31</td>
<td>27</td>
<td>10,3</td>
</tr>
<tr>
<td>glass 1</td>
<td>14,07</td>
<td>2523,7</td>
<td>22</td>
<td>5,6</td>
</tr>
<tr>
<td>glass 2</td>
<td>5,1</td>
<td>2378</td>
<td>16</td>
<td>2,14</td>
</tr>
</tbody>
</table>
2. Binary mixtures

• Main goal – obtain transition from brazil nut effect to reverse effect

• Parameters:
  • Frequency
  • Volume ratios of the constituents
  • Density ratios of the constituents
  • Shape of constituents
Apparatus

1. Higher amplitude apparatus

- Electric motor
- Shaft
- Eccenter

\[ A = 13 \text{ mm} \]
Measurements

1. Intruder ball

- Conditions for the occurrence of brazil nut effect in our conditions:
  - Frequency above 8.4 Hz
  - Ratio of intruder and surrounding balls densities under ~0.6 at 8.4 Hz
- The emerging balls were:
Styrofoam 2
### Intruder balls - summary

<table>
<thead>
<tr>
<th>material</th>
<th>density ratio to surrounding balls</th>
<th>behaviour</th>
</tr>
</thead>
<tbody>
<tr>
<td>lead</td>
<td>13.19</td>
<td>sinks</td>
</tr>
<tr>
<td>wood 1</td>
<td>1.08</td>
<td>sinks</td>
</tr>
<tr>
<td>wood 2</td>
<td>0.58</td>
<td>floats</td>
</tr>
<tr>
<td>plastic</td>
<td>1.21</td>
<td>sinks</td>
</tr>
<tr>
<td>styropor 1</td>
<td>0.027</td>
<td>floats</td>
</tr>
<tr>
<td>styropor 2</td>
<td>0.039</td>
<td>floats</td>
</tr>
<tr>
<td>glass 1</td>
<td>3.15</td>
<td>sinks</td>
</tr>
<tr>
<td>glass 2</td>
<td>2.97</td>
<td>sinks</td>
</tr>
</tbody>
</table>
2. **Binary mixtures**

- Several mixtures involved

- The reverse effect occurs for heavier constituents of mixture
Theoretical approach

• Several explaining models have been proposed for the brazil nut effect:
  
  
  • Percolation (*T. Rosato et al., Phys. Rev. Lett. 58*)
  
  
  • Air – driven segregation
  
  • We will discuss the fluid model in detail because of its simplicity and experimental proof
The fluid model

Main assumptions:

- The granular system may be regarded as a dense gas with three phases:

1. Solid phase

   The balls are so near that they can’t interchange positions but just oscillate on the same place – analogous to solids
2. Liquid phase
The balls interchange places, but do not move vigorously – analogous to liquids

3. Gaseous phase
The balls move fast and collide at random – analogous to gases
As such, the "granular gas" can be described by a Fermi – Dirac distribution function:

\[
\phi(z) = \frac{1}{1 + e^{\frac{mgd}{T}(z - \eta)}}
\]

- \( \phi(z) \) - Probability of finding a ball at height \( z \)
- \( d \) - ball diameter
- \( m \) - ball mass
- \( z \) - normalized height
- \( T \) - equivalent temperature (function of frequency)
- \( \eta \) - number of layers at 0 deg. Eq.
• This distribution is experimentally substantiated:

Distribution

- experimental data
- theoretical curve

Normalized height (number of row)
• The granular gas density can be defined as

\[ \rho_g = \rho_i \psi \]

\( \rho_g \) – gas density
\( \rho_i \) – density of one ball
\( \psi \) – packing density, defined as \( \psi = \frac{\sum_{i=1}^{N} V_i}{L} \)
\( L \) – the volume taken up by the balls

• In analogy to a fluid we define the buoyancy force:

\[ F_u = V \rho_g \psi g \]

\( V \) – volume of intruder
**Condition for brazil nut effect**

- For intruder balls:
  \[
  \frac{\rho_{\text{int}}}{\rho_i} \geq \overline{\psi}
  \]
  - \(\rho_{\text{int}}\) – intruder density
  - \(\rho_i\) – surrounding ball density
  - \(\psi\) – mean packing density

- For binary mixtures:
  \[
  \frac{\rho_1}{\rho_2} \geq \frac{\overline{\psi_1}}{\overline{\psi_2}}
  \]
  - \(\rho_i\) – densities of mixture constituents
  - \(\psi_i\) – mean packing densities of constituents