Problem 4.

Dusty blot

Problem

Describe and explain the dynamics of the patterns you observe when some dry dust (e.g. coffee powder or flour) is poured onto a water surface. Study the dependence of the observed phenomena on the relevant parameters.

Basic idea

- We constructed device for measuring blot radius in depend on time
- Blot spreading velocity depends on surface tension force
- Blot spreading velocity decrease in time
- Pattrens on the surface are always circles

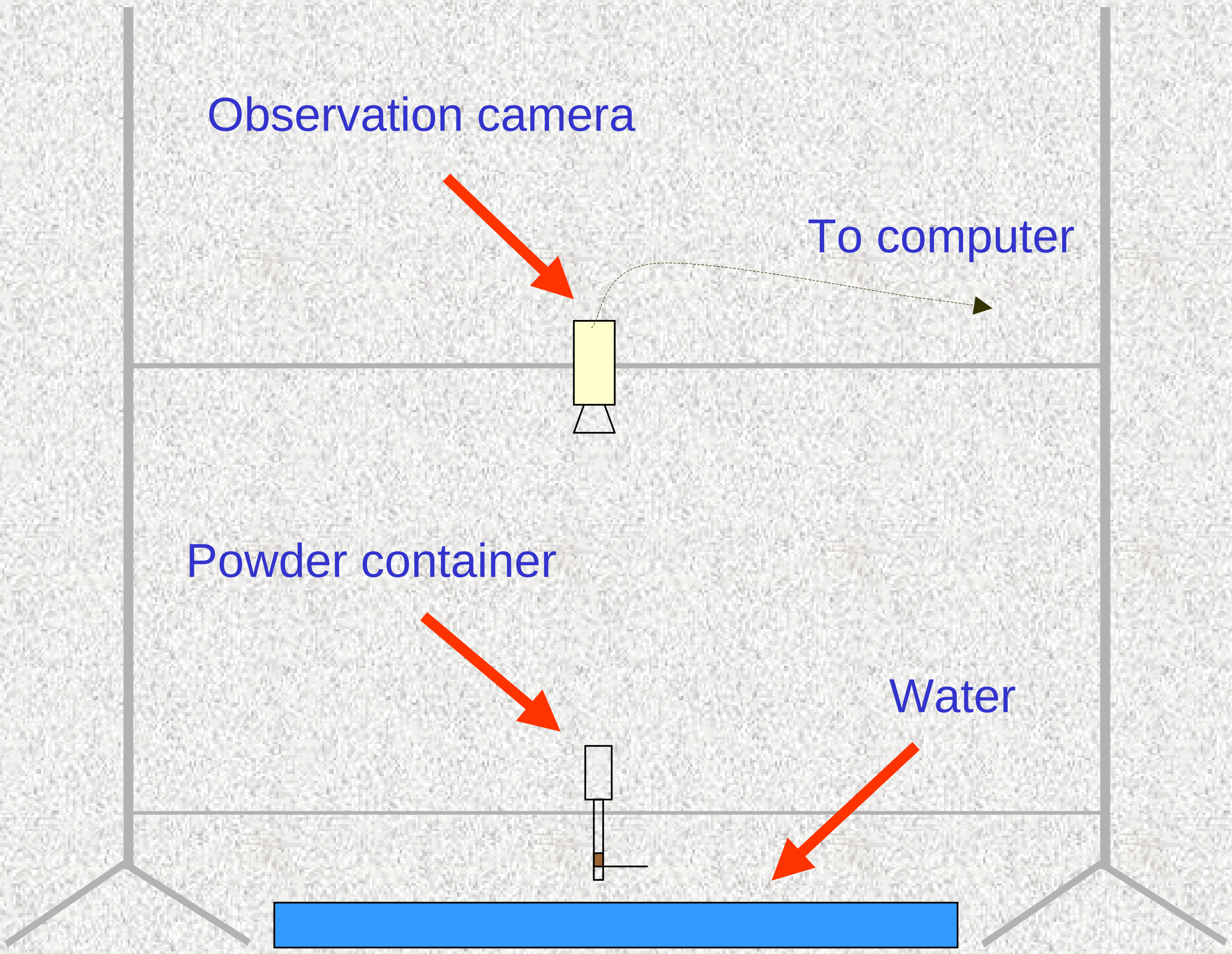
Relevant parameters

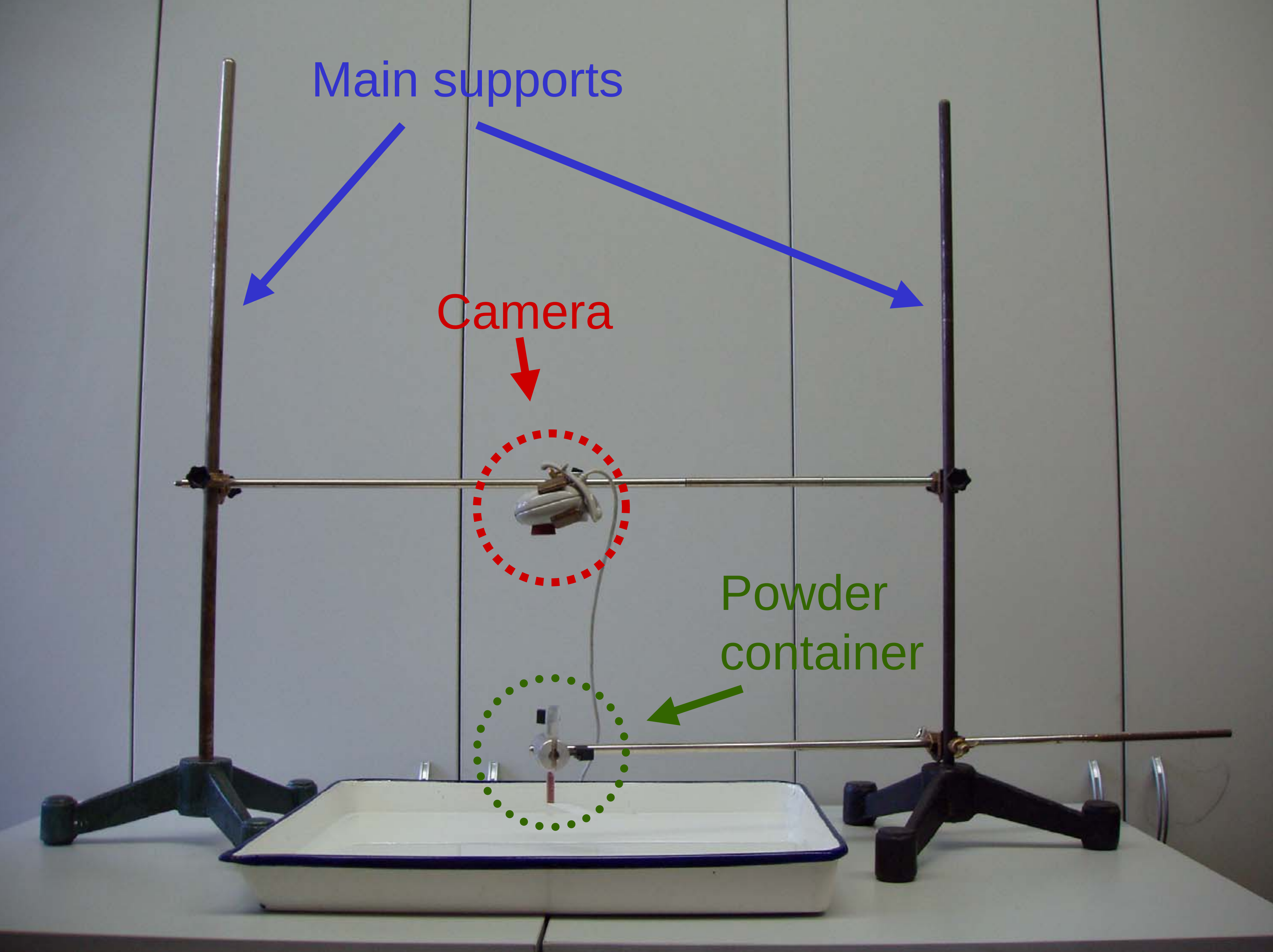
Parameters which influence on spreading velocity :

- Surface tension constant
- Powder mass
- Powder particle dimensions
- Powder solubility

Apparatus

Main parts of apparatus :




Main supports
Camera
Powder
container

Powder container

Powder outlet

Measuring scale

Experimental approach

- In experiment we were strewing diferent powders on water surface
- Powders we used:

Pepper

Cinnamon

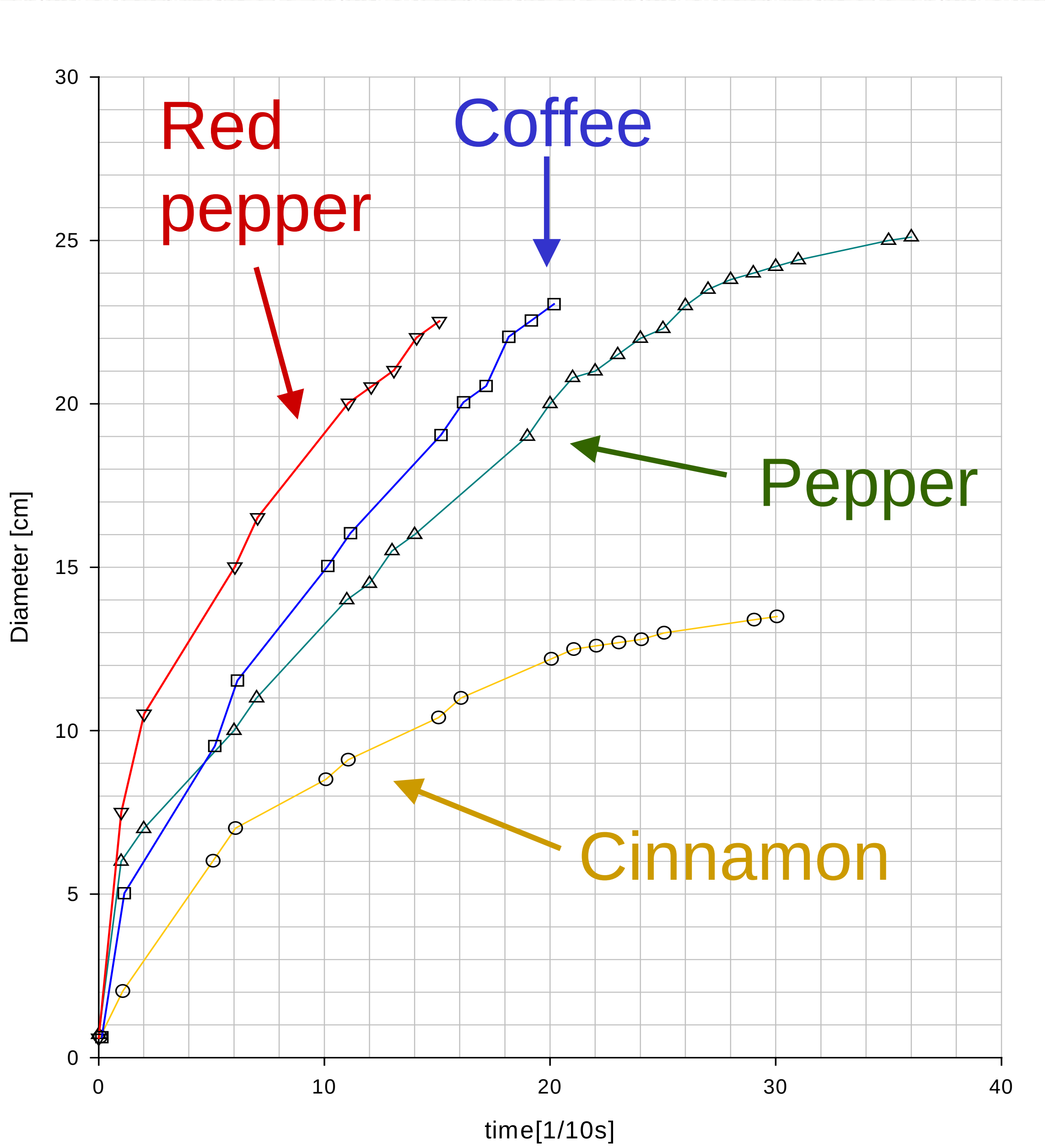
Coffee Red pepper

- We worked with flour (too heavy) and cocoa (too moist)
- Number of clods depends on powder moistness

Clods produced by cohesion

Clods formations of moist powder

Measuring results



Experiment conclusion

Powder	Solubility in water	Particle size
Cinnamon	soluble	large
Pepper	not soluble	large
Coffee	soluble	small
Red pepper	not soluble	small

- Cinnamon spreads slowly
- Paprika spreads fastest

Theoretical approach

Surface tension

- Surface energy change with change of surface:

$$\delta U = \alpha \delta S$$

δU - small energy change

δS - small surface change

α - surface tension constant

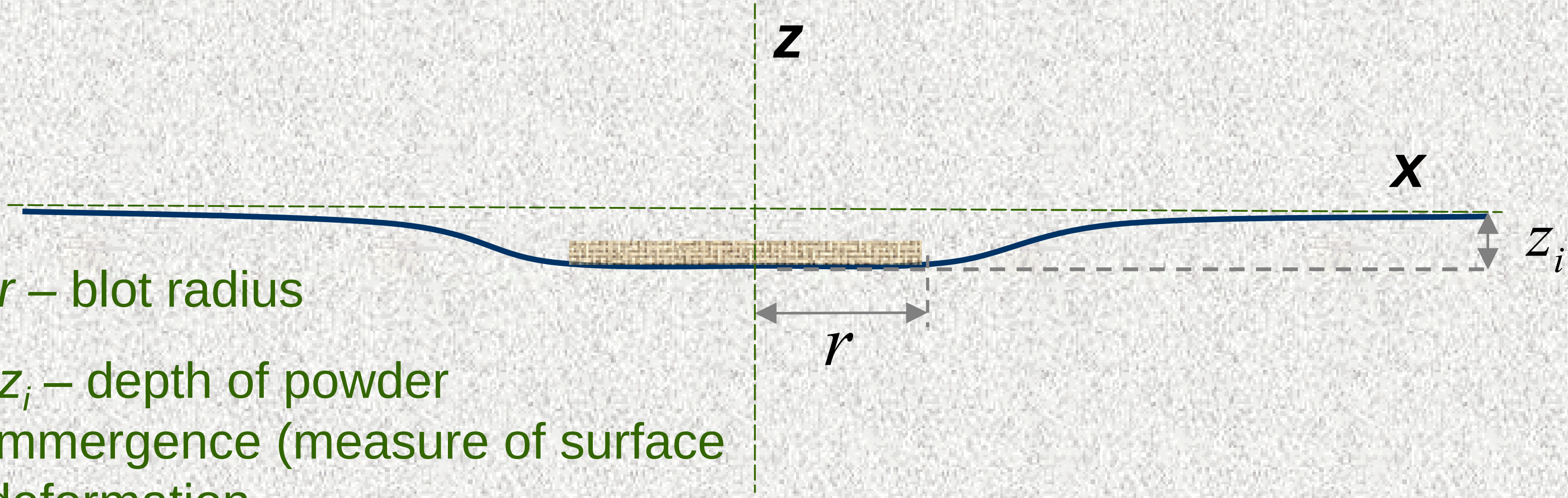
- Force of surface tension:

$$\mathbf{F} = \oint \alpha d\mathbf{s}$$

$\mathbf{s}$ - path

Defining the geometry

- Surface deformation due to powder:



r – blot radius

z_i – depth of powder immergence (measure of surface deformation

- z_i can be obtained via energy conservation:

$$\alpha \delta S = mg \delta z$$

δS – surface deformation

m – powder mass

δz – change in depth

Equation of spreading

- Deformation of surface – approximately:

$$\delta S \approx \delta\left(2r\pi z_i\right)$$

h_0 – initial depth (absolute value)

$$\Rightarrow z_i\left(r\right) = -h_0 \frac{mg - 2r\pi\alpha}{mg - 2r_0\pi\alpha}$$

r_0 – initial blot radius

- Surface tension force on powder blot due to surface deformation:

$$F_t = -\frac{dU}{dr}$$

U – surface energy

r – blot radius (path)

$$\Rightarrow m\frac{d^2r}{dt^2} = \frac{2\pi\alpha h_0}{mg - 2\pi\alpha r_0}\left(4\pi\alpha r - mg\right)$$

Solving equation of spreading

- Initial conditions:
  - Initial blot radius - r_0
  - Initial velocity - zero
- Solution of equation of motion:

$$r(t) = \left(r_0 - \frac{mg}{4\pi\alpha} \right) e^{-2\pi\alpha \sqrt{\frac{2h_0}{mg - 2\pi\alpha r_0}} t} + \frac{mg}{4\pi\alpha}$$

Determining the initial depth

- Initial force equilibrium (pressure not neglected):

$$mg = 2\pi\alpha r_0 - \rho g r_0 \pi h_0 \left(r_0 + 2h_0\right)$$

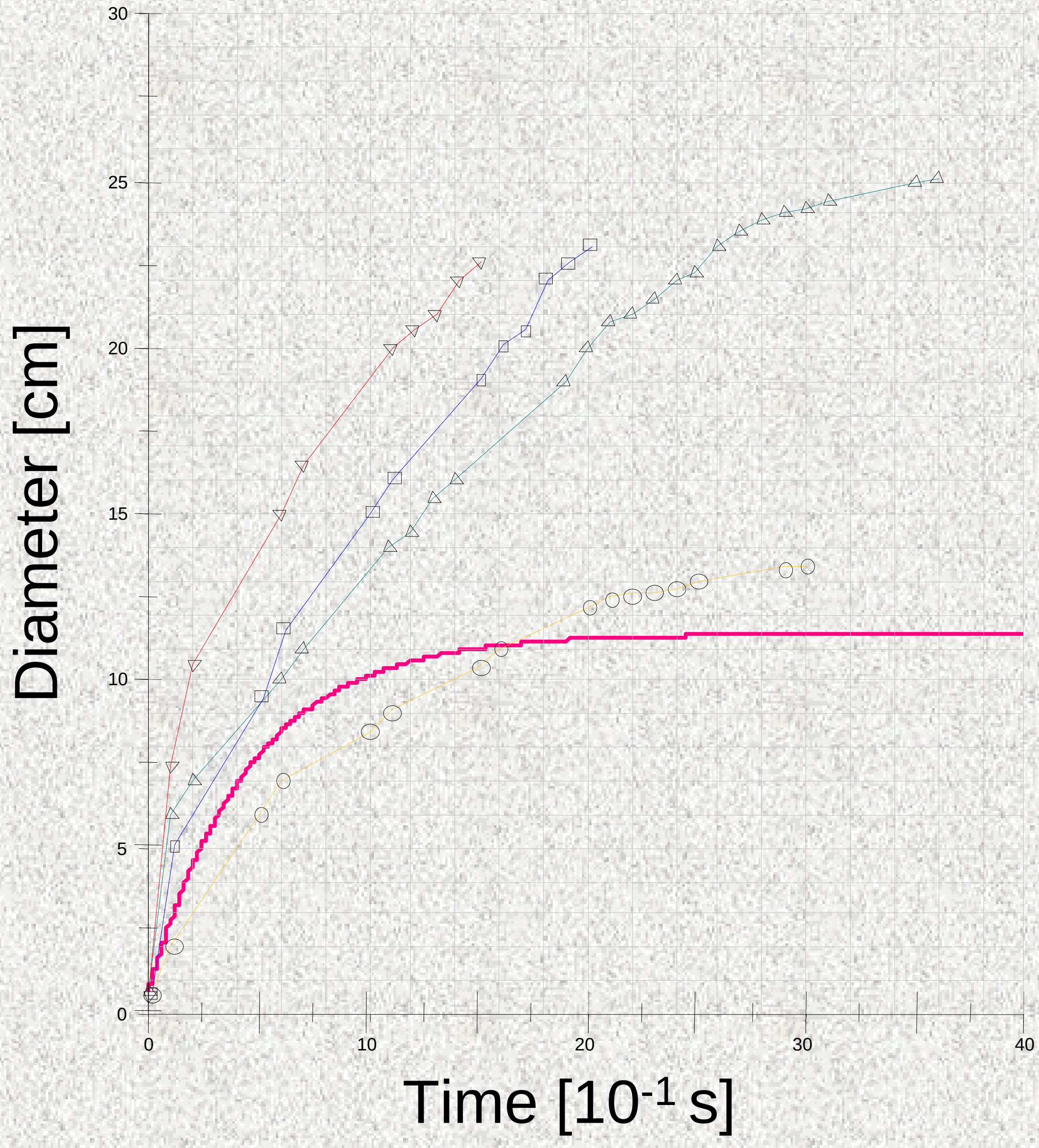
$$\Rightarrow h_0 \approx \frac{8}{\rho r_0^2}\left(\frac{m}{2r_0\pi} - \frac{\alpha}{g}\right)$$

ρ – water density

- Inserting into the equation of motion:

$$r(t) = \left(r_0 - \frac{mg}{4\pi\alpha}\right)e^{-\frac{4\alpha}{r_0}\sqrt{\frac{2\pi}{r_0 g\rho}}t} + \frac{mg}{4\pi\alpha}$$

Comparing with experiment



- Theoretical curve

Parameters:

$$\alpha = 7 \cdot 10^{-2}\ \mathrm{N/m}$$

$$r_0 = 3\ \mathrm{mm}$$

$$m = 0.5\ \mathrm{g}$$

Conclusion

- We constructed device for measuring blot radius in dependence on time
- Blot spreading velocity depends on surface on this parameters:
  - Surface tension constant
  - Powder mass
  - Powder particle dimensions
  - Powder solubility