Problem 2.

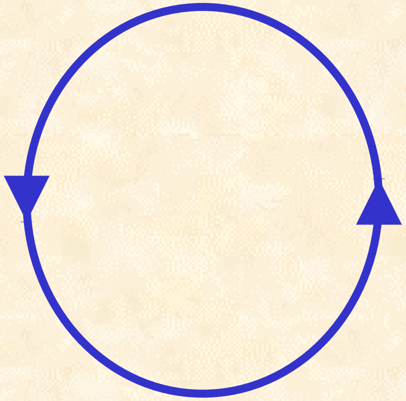
Stubborn ice

Problem

Put a piece of ice (e.g. an ice cube) into a container filled with vegetable oil. Observe its motion and make a quantitative description of its dynamics.

Basic idea

- In warm oil the ice melts making the body of ice adherent water
- Periodical movement occurs due to periodical variation of the density of the body
- The phases of the motion are:



Downward motion

Upward motion

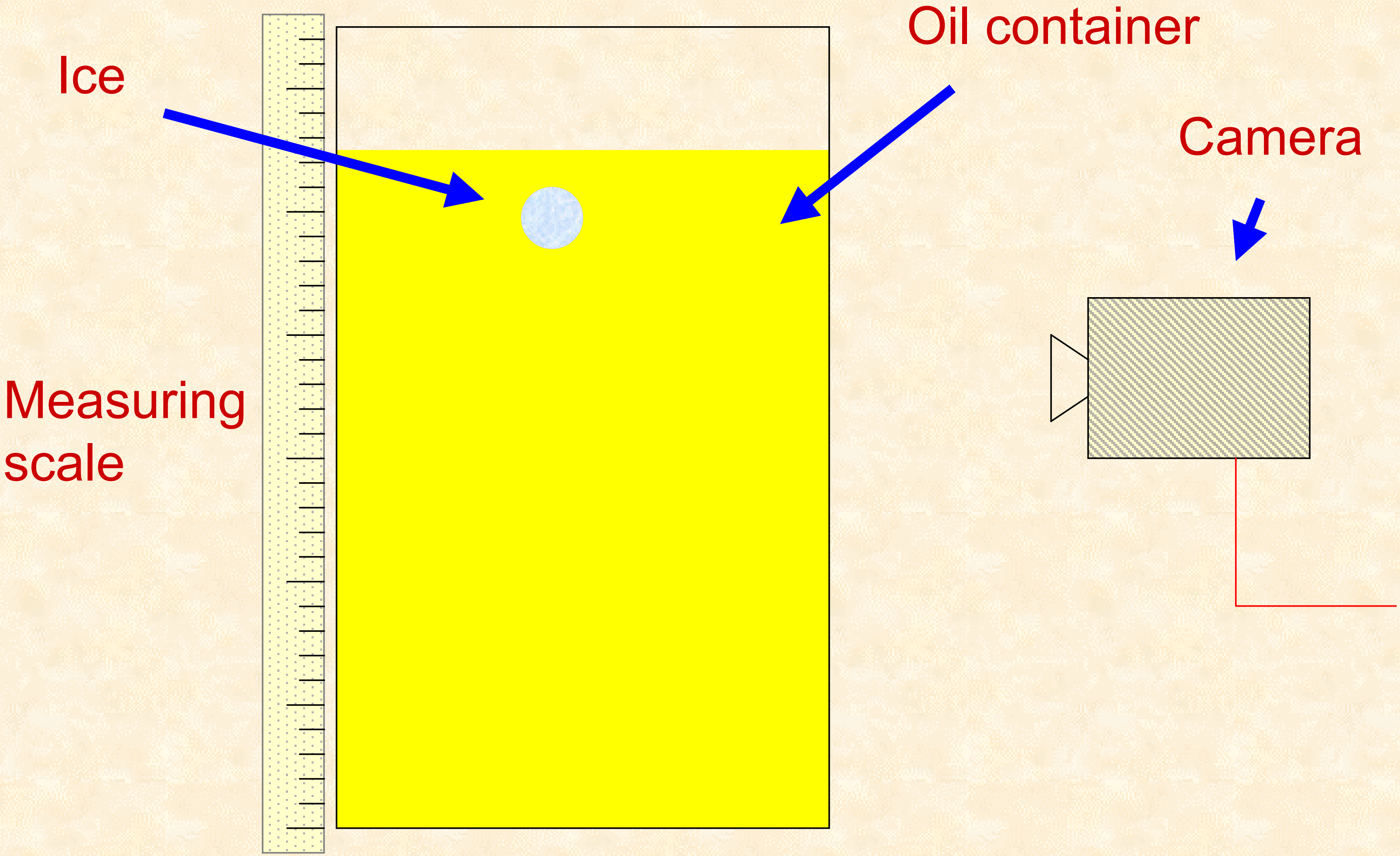
Water drop separation

Experimental approach

Goals:

- Construct an apparatus for performing the measurements
- Determine conditions neccesary for periodical motion to occur
- Find the path of the ice
- Measure the period
- Investigate the formation of water drops

Apparatus



Periodical motion

- **Oscillating condition :**

$$\rho_{ice} < \rho_{oil} < \rho_w$$

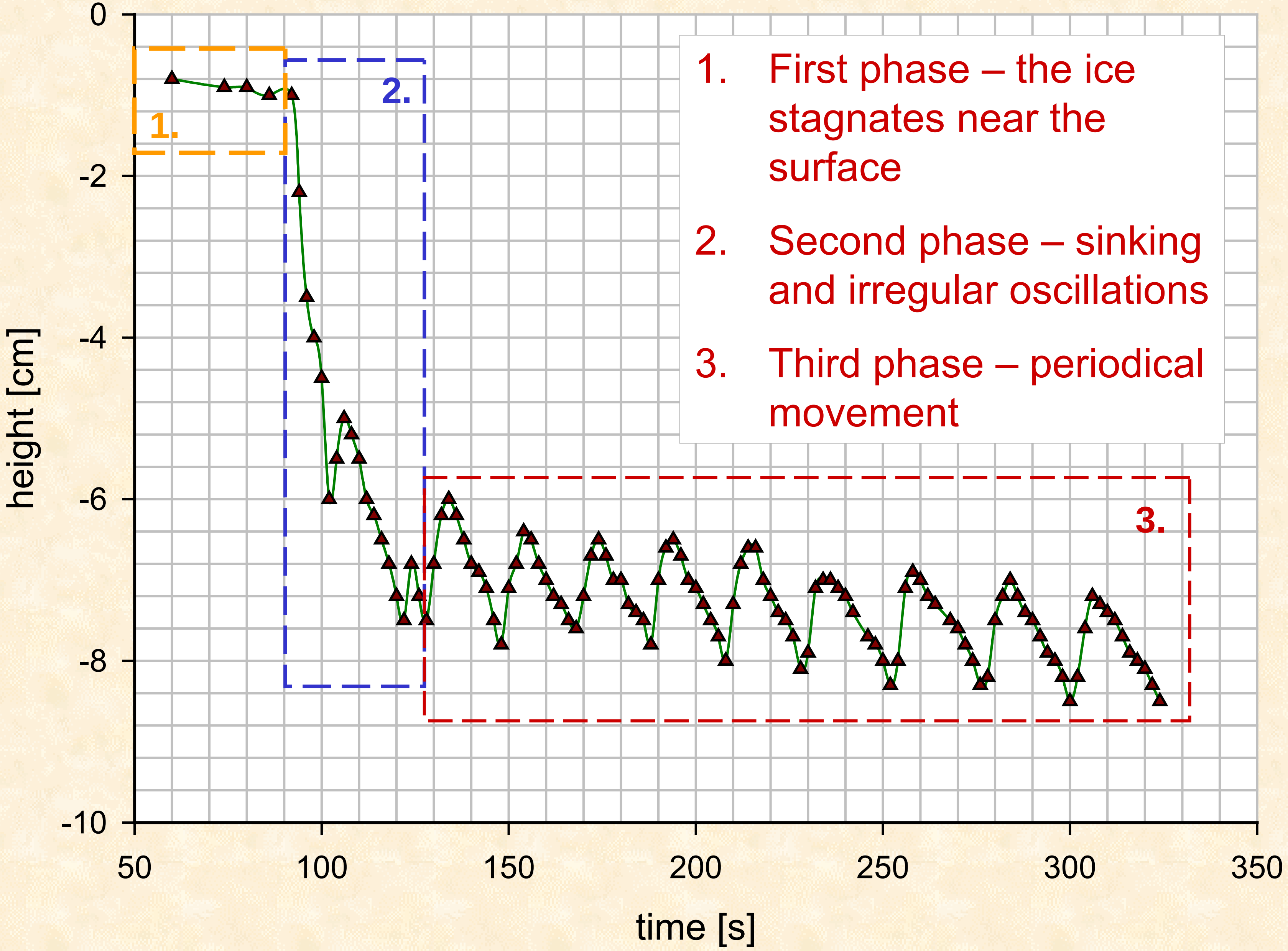
ρ_{ice} – density of ice

ρ_w – density of water

ρ_{oil} – density of oil

- Under certain conditions deviations occur:
  - Wet ice doesn’t oscillate (density is greater
  - If there is air in the ice it will oscillate even in oil less dense (average density is smaller

Periodical motion *cont.*



1. First phase – the ice stagnates near the surface
2. Second phase – sinking and irregular oscillations
3. Third phase – periodical movement

(film)

Periodical motion *cont.*

- The parameters were:
  - Oil viscosity:
  - Initial ice mass: 3 g
  - Oil temperature: 20˚C
- The measured quantities were:
  - Period: ~20 s
  - Maximal depth for one period: reaching from -7.6 to -8.6 cm
  - Minimal depth for one period: reaching from -6 to -7.2 cm

Water drop separation

- The water drop separates from the ice when it gets too heavy to hold to it any longer
- When the drop separates, the ice starts moving upward
- The drop size is always the same:

$$r_{dr} = 3\,\mathrm{mm}$$

Water drop separation *cont.*

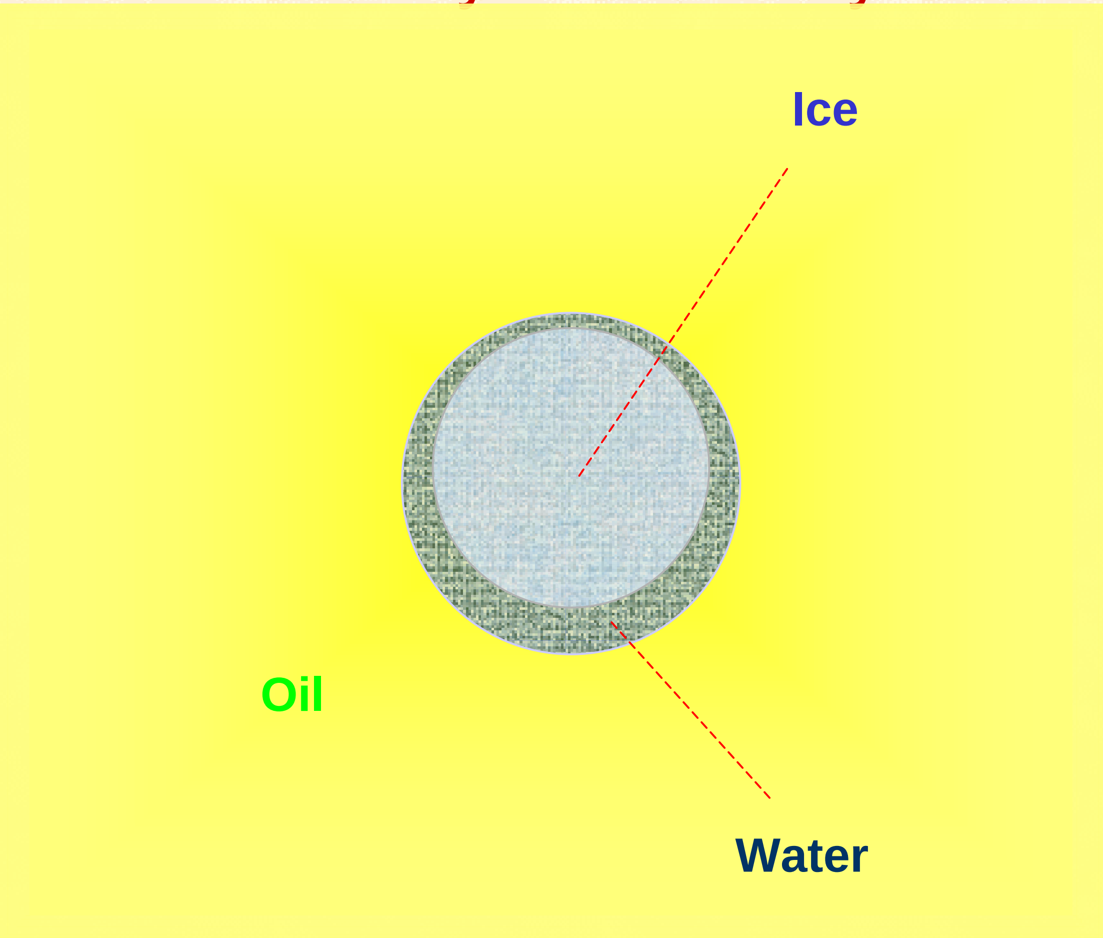
Water drop separation *cont.*

Theoretical approach

Ice melting

- Ice temperature: 0˚C
- Oil temperature: 20˚C
- Heat is transferred to ice

- The ice melts and the produced water adheres to the ice => net density of the body increases



- The water stays with the ice until it gets too heavy and forms a drop

- Linear increase of water mass assumed:

$$m_w = m_{dr} \frac{t}{\tau}$$

m_w – water mass

m_{dr} – water drop mass

τ – period

t- time since melting start

- Sphere volume:

$$V = \frac{M}{\rho_{ice}} - m_w \left(\frac{1}{\rho_{ice}} - \frac{1}{\rho_w} \right)$$

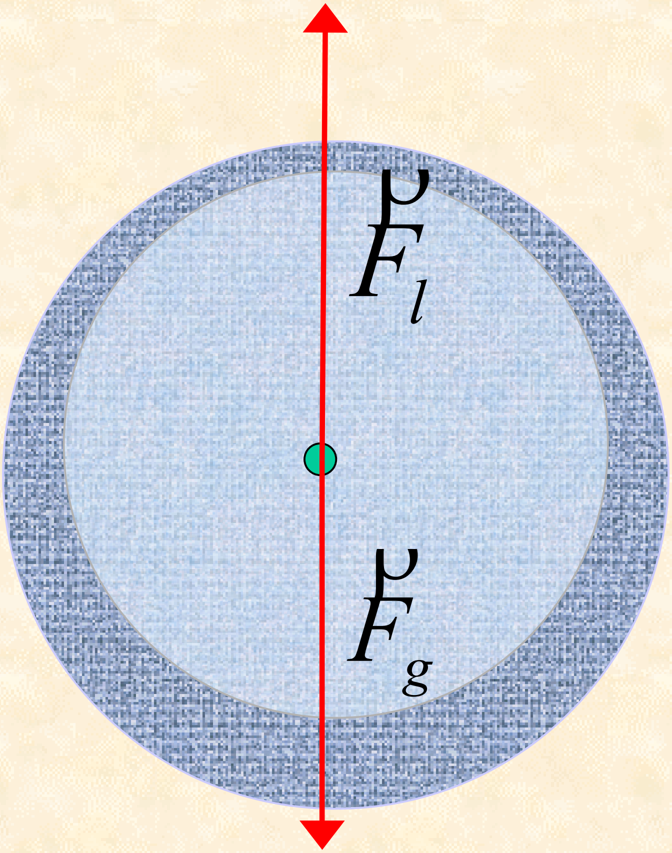
ρ_{ice} – ice density

ρ_{ice} – water density

M – initial ice mass (constant)

- Volume decreases with time
- Volume increase affects the forces on the sphere:

Forces acting on the ice - water sphere – steady state



F_l - lift force: $F_l = V\rho_{oil}g$

V – sphere volume

ρ_{oil} – oil density

g – free fall acceleration

F_g – gravity: $F_g = mg$

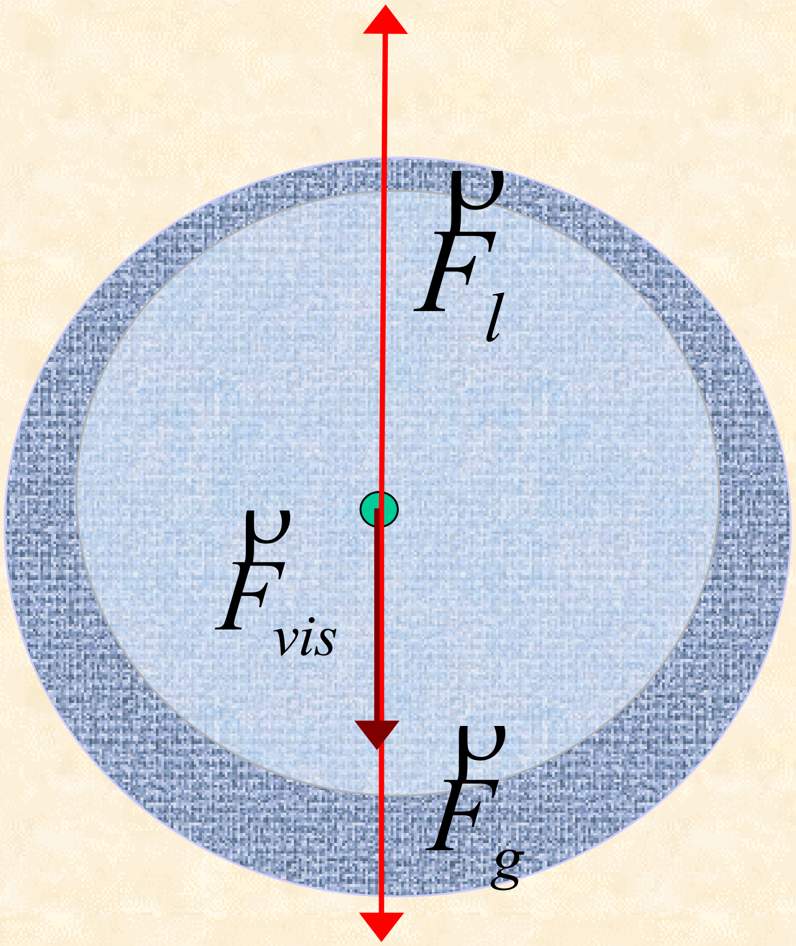
m – sphere mass

Floating condition:

$$F_g \le F_l \quad \Rightarrow \rho_{oil} \ge \bar{\rho}$$

$\bar{\rho}$ - average sphere density

Forces acting on the ice - water sphere – movement



F_l - lift force

F_g – gravity

F_{vis} – viscosity:

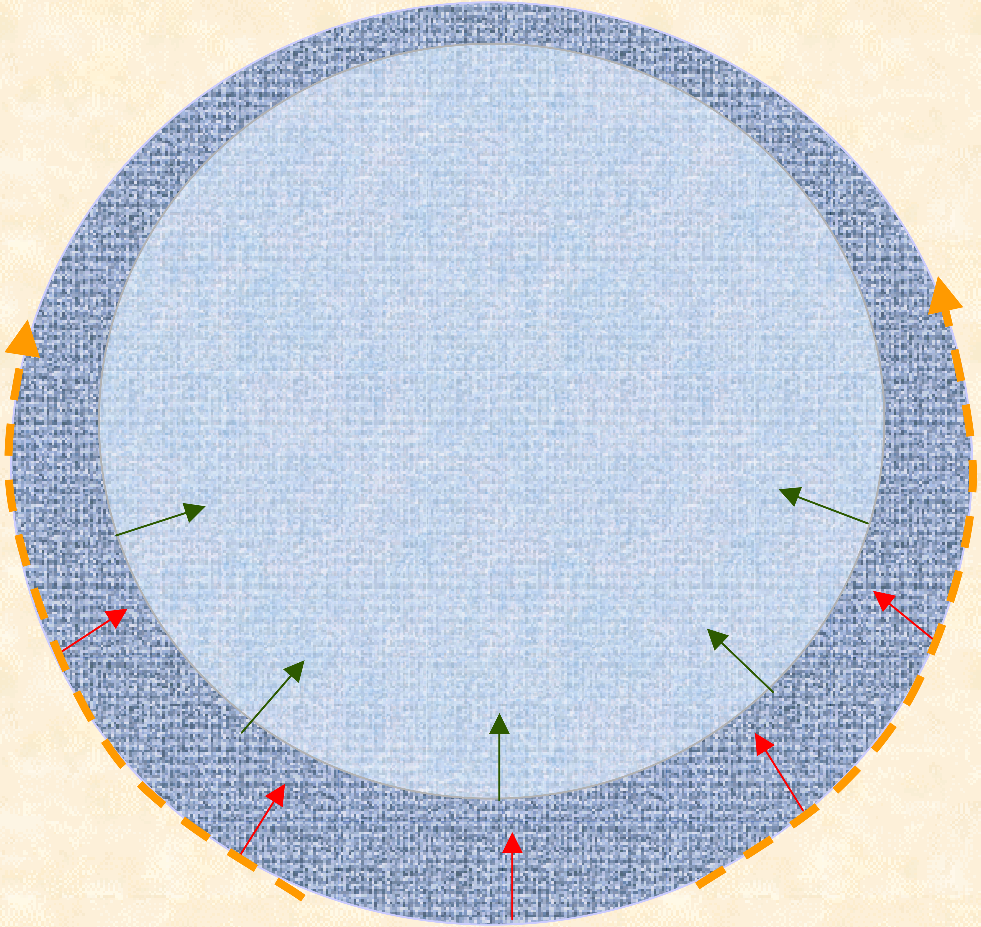
$$F_{vis} = 6R\eta\pi v$$

R – sphere radius

η – oil viscosity

v – sphere velocity

Forces holding ice and water together



→ - surface tension

→ - pressure

→ - adhesion

Equation of motion

- The equation of motion describes one period
- Initial position - minimal height z_0
- Equality of forces assumed :

$$6R\eta\pi v + Mg = V\rho_{oil}g$$

R – radius of ice - water sphere

V – ice – water sphere volume

$$\Rightarrow v = \frac{g}{6R\eta\pi}\left[M\left(\frac{\rho_{oil}}{\rho_{ice}} - 1\right) - m_w\rho_{oil}\left(\frac{1}{\rho_{ice}} - \frac{1}{\rho_w}\right)\right]$$

Conditions for periodical motion

1. Velocity must be directed upward in the beginning of the oscillation:

$$M\left(\frac{\rho_{oil}}{\rho_{ice}}-1\right)\rangle 0 \qquad \Rightarrow \rho_{ice} < \rho_{oil}$$

2. In order to start sinking again, the sphere velocity must have a direction-turning point:

$$M\left(\frac{\rho_{oil}}{\rho_{ice}}-1\right) < m_{dr}\rho_{oil}\left(\frac{1}{\rho_{ice}}-\frac{1}{\rho_{w}}\right) \qquad \Rightarrow \frac{m_{dr}}{M}\rangle \frac{\rho_{w}}{\rho_{oil}}\,\frac{\rho_{oil}-\rho_{ice}}{\rho_{w}-\rho_{ice}}$$

Path of the sphere

- Dependence of immergence depth on time:

$$z(t) = z_0 + \frac{g}{6R\eta\pi}\left[M\left(\frac{\rho_{oil}}{\rho_{ice}} - 1\right)t - m_{dr}\rho_{oil}\left(\frac{1}{\rho_{ice}} - \frac{1}{\rho_w}\right)\frac{t^2}{2\tau}\right]$$

z_0 – initial depth (for one period)

m_{dr} – mass of water drop

τ - period

- Mass of water drop calculated via measured drop radius:

$$m_{dr} = \frac{4}{3} r_{dr}^3 \pi \rho_w$$

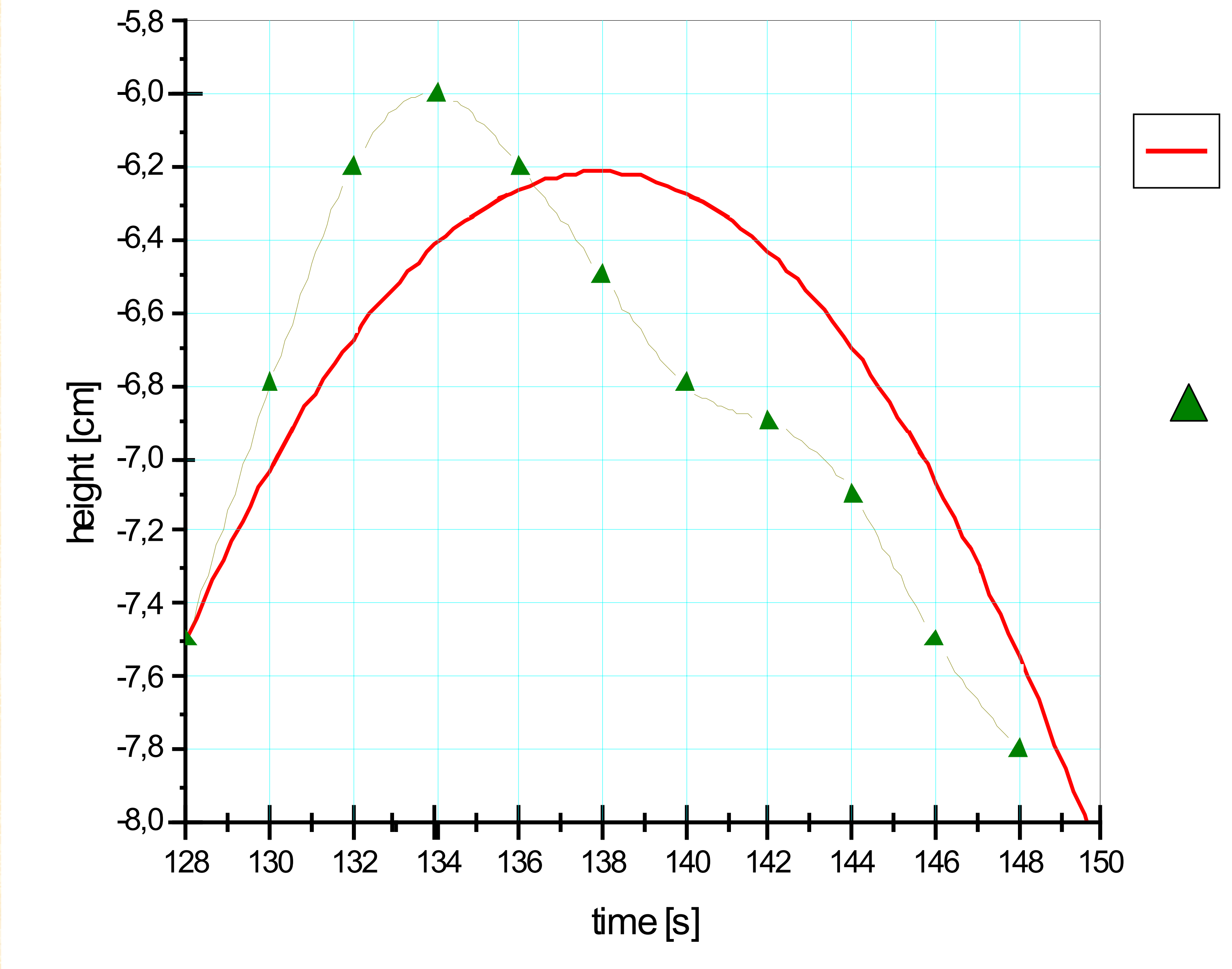
r_{dr} – drop radius

Comparison to experiment

1. Position of height maximum

Parameter	Experimental value	Theoretical value
Time of maximum	6 s	9.3 s
	7 s	
Position of maximum	-6 cm	-6.2 cm
	-6.5 cm	-6.8 cm
	-7 cm	-7.0 cm

2. Comparing the path



- Theoretical curve (path for one period)

- Experimental data (first period - example)

Reasons for disagreement

1. The maximum is shifted
    - The ice moves faster upwards than downwards
    - Reason – hotter oil above ice - smaller viscosity
2. The maximum is higher than predicted
    - Reason – the ice moves faster upwards so it traverses a greater distance

Other disagreements – due to:

- parameter measuring errors
- oil cooling and convection…

Conclusion

- The general condition for periodical movement of ice in oil is

$$\rho_{ice} < \rho_{oil} < \rho_{w}$$

- The ice moves periodically because of its melting and water droplet separation
- The period is approximately constant in time