24th International Young Physicists’ Tournament

POLAND
A row of dominoes falling in sequence after the first is displaced is a well known phenomenon.
If a row of "dominoes" gradually increases in height, investigate how the energy transfer takes place and determine any limitations to the size of the dominoes.
Observing the falling domino

Typical domino fall is a very rapid process

Recording domino with a high speed camera allows to find main effects associated with the phenomenon.
Characteristic of the phenomenon

Main effects

Dominoes collide inelastically

Smaller corrections

Small separations upon the collisions
Main effects

Dominoes rotate around a fixed axis

Smaller corrections

Small jumps and sliding on the ground
Relevant parameters

- Growth rate
- Sizes and masses of dominoes
- Spacings
- Friction coefficient

Parameters not taken into account: elastic properties of dominoes, friction coefficient of the ground
Equation of motion of a single domino

Between the collisions:

\[ I \varepsilon = \text{Torque of gravity} + \text{Torque of } F_1 + \text{Torque of } F_2 \]
Equation of motion of a single domino

Collision:

\[ I \Delta \omega = \text{Angular impulse } P_1 + \text{Angular impulse } P_2 \]

No influence of gravity – instant collision
Forces between dominoes

Forces between blocks have components parallel and perpendicular to the surface of contact. These components are related by Coulomb’s law.

\[ \frac{F_p}{F_{\perp}} = \mu \]
Dominoes are assumed to collide inelastically, so they are in a continuous contact.
Mathematical model

Sizes and masses
Spacings
Friction coefficient

Set of equations of motion of each individual domino
Geometrical relationship between dominoes

Equation of motion of the leading domino
Mathematical model

- Times of collisions
- Angular velocities of dominoes
- Kinetic energies of dominoes

Numerical evaluation by the fourth-order Runge–Kutta method

Equation of motion of the leading domino
Studied dominoes

Blocks growing in one dimension only

System free parameters: spacings, growth rate
Studied dominoes

Blocks growing in all dimensions with constant aspect ratio

Free parameters: spacings, growth rate

Geometric growth – all dimensions and spacings increasing with the growth rate
Studied dominoes

‘Typical’ domino for comparison

Coefficients of friction for dominoes were measured using an inclined plane

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Recording the times of collisions

Times of collisions for different dominoes with different spacings were found by recording the phenomenon with a 300 FPS camera.
Experimental verification #1

Fitted angular velocity of the first domino

\[ \mu = 0.25 \]
System with no memory

The system very quickly ‘forgets’ the initial velocity

This effect can be utilized to confront the theory and the experiments with no necessity of parameter fitting!

The points were connected to guide the eye

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Theoretical dependance for linear velocity of domino

The range of friction coefficients was determined in a separate experiment.
System with no memory

The theory successfully predicts how the velocity depends on the spacings between dominoes.
Domino as an amplifier

Input energy: potential energy

Output energy: kinetic energy

Input energy is the change of potential energy of the system

Output energy is the sum of the kinetic energies of all N dominoes
For domino amplifier the **output energy** is independent of the **initial energy**.
For domino amplifier the **output energy** is independent of the **initial energy**.
Energy amplification rate

Faster size growth allows faster energy amplification

Simulation results

Size growth multiplicator

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Growth rate limitations

Energy is needed to topple the front domino

Center of mass initially raises

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Growth rate limitations

Unstable – will fall by itself

Potential energy decreases

Potential energy increases

Stable – additional kinetic energy is needed

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Growth rate limitations

What is the maximum growth rate corresponding to an unstable position?

Unstable – will fall by itself

Stable – additional kinetic energy is needed
Growth rate limitations

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Growth rate limitations

Smaller friction allows faster growth
Growth rate limitations

Higher separations allow faster growth for low friction
Growth rate limitations

Optimum separation decreases for higher friction
Summary

• A model of domino fall can be constructed with assumptions of inelastic collisions and fixed axis of rotation

• The model can be verified by observing the velocity of the falling domino

• Both growing and typical domino represent a self-stabilisation behavior

• Faster growth of dominoes allows faster energy amplification. For geometrical growth with given aspect ratio, lower bounds for the growth limits were found.
Literature


• R. Larham, „Validation of a model of the domino effect?“, http://arxiv.org/abs/0803.2898

Where is the energy stored?

First five dominoes have the vast majority of kinetic energy

Right after a collision
System with no memory

The system very quickly ‘forgets’ the initial velocity

The points were connected to guide the eye

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