24th International Young Physicists’ Tournament

POLAND
Place a sheet of paper on a horizontal table and put a cylindrical object (e.g. a pencil) on the paper. Pull the paper out. Observe and investigate the motion of the cylinder until it comes to rest.
Presentation plan

- Theory
- Conclusions
- Experiments

2D system
Presentation plan

- Theory
- Conclusions
- Experiments

2D system

3D system

- Theory
- Simulation
- Experiments

Poland
We assumed that cylinder, table, and sheet of paper are rigid bodies
We investigated motion of cylinder under constant acceleration of the paper: a
### Rigid Body Dynamics

**Newton’s 2nd Law**

\[
\frac{dp}{dt} = F \quad \frac{dL}{dt} = \Gamma
\]

\[
\frac{dV}{dt} = T
\]

\[
0 = R - mg
\]

\[
I \frac{d\omega}{dt} = T \cdot r
\]
Friction Force

Static Friction

\[ F = T < R \mu_1 \]

Dynamic Friction

\[ T = R \mu_1 \]

F – Applied force, R – Reaction force, T – Friction, V – Velocity, \( \mu_1 \) – Coefficient of friction
Static vs. Dynamic Friction

Static Friction

\[ \omega r + V = at \]
\[ T < mg\mu_1 \]

Dynamic Friction

\[ T = mg\mu_1 \]

Critical acceleration of the paper

\[ a_{cr} = g\mu_1 \frac{(I + mr^2)}{I} \]

- \( m \) – Mass of the cylinder
- \( r \) – Radius of the cylinder
- \( I \) – Moment of inertia due to symmetry axis
- \( g \) – Gravitational acceleration
- \( \mu_1 \) – Coefficient of friction
Motion of cylinder

If initial linear and angular velocity is zero

$$\frac{\omega}{V} = \frac{mr}{I}$$

Linear velocity and angular velocity become zero at the same time
2D-Model Conclusions

Static friction

\[ \frac{d_2}{d_1} = \frac{aI^2}{mr^2(mr^2 + I)g\mu_2} \]

Kinetic friction

\[ \frac{d_2}{d_1} = \frac{g\mu_1^2}{(a - g\mu_1)\mu_2} \]
Theory results

\[ \mu_1 = 0.13 \]
\[ \mu_2 = 0.22 \]
\[ \frac{I}{mr^2} = 0.6 \]
Experimental setup

- Bumper
- Pulleys
- String
- Weights

Poland
Different materials used

Paper
\[ \mu_1 = (0.15 \pm 0.02) \]

Fabric
\[ \mu_1 = (0.22 \pm 0.02) \]
Cylinder

Metal cylinder

Plasticine

Bright LED

Micro-processor

Flashing Bright LED
Experiments
Experiments

2 seconds exposure time
Experiments

Poland

Escaping sheet of paper

Moving cylinder

Table
Experiments

Escaping sheet of paper

Moving cylinder

Table
Experiments

Reflection changes

Cylinder is on the paper
Cylinder is on the table
Experimental results - paper
Experimental results - fabric

![Graph showing experimental results against acceleration](image-url)
\[
\frac{\omega}{V} = \frac{mr}{I}
\]

\[
x(\lambda) = \lambda + R \cos \left( \frac{mr}{I} \lambda \right)
\]

\[
y(\lambda) = R \sin \left( \frac{mr}{I} \lambda \right)
\]

\[
mr / I = 56 \left[ \frac{1}{m} \right]
\]
Experimental setup

Blue LED

Green LED
Experiments
Comparison with theory

The ratio between angular and linear velocity is not changed!

Theory \( \left( \frac{mr}{I} = 56 \left[ \frac{1}{m} \right] \right) \)
Reaction force distribution

\[ \Gamma = Td \]

Rotation of the block

\[ \Gamma \neq 0 \]

Escaping base
Reaction force distribution

\[ \Gamma = \int_{0}^{L} (T(x)d)dx + \int_{0}^{L} (R(x)x)dx \]

No detachment from base

[\Gamma = 0]

Escaping base
Reaction forces

Pulling direction
Reaction forces

Reaction force is higher

Reaction force is lower

Pulling direction
Our model assumes two reaction forces in equal distances from the CM.
New frame of reference

\[ \hat{z} \equiv \hat{e}_3 \]

e₁, e₂ – can rotate only in xy-plane

No rotation around e₁ axis:

\[ \omega_1 \equiv 0 \]

\( \omega_1, \omega_2, \omega_3 \) – angular velocity components in \( e_1, e_2, e_3 \)
Equations describing the system in the new frame of reference

\[ m \frac{d\hat{V}}{dt} = r F \]

\[-I_2 \omega_2 \omega_3 = \Gamma_1\]

\[ I_2 \frac{d\omega_2}{dt} = \Gamma_2\]

\[ I_3 \frac{d\omega_3}{dt} = \Gamma_3\]

\(\omega_1, \omega_2, \omega_3\) – angular velocity components in \(e_1, e_2, e_3\)
Equations describing the system in the new frame of reference:

\[
\begin{align*}
\dot{V} &= r F \\
-I_2 \omega_2 \omega_3 &= \Gamma_1 \\
I_2 \frac{d\omega_2}{dt} &= \Gamma_2 \\
I_3 \frac{d\omega_3}{dt} &= \Gamma_3
\end{align*}
\]

CM motion:

No detachment from the surface:

Equations of motion:

components in e_1, e_2, e_3
Simulation – dynamic friction

![Graph showing angle α vs. time (s)](image)
Simulation – dynamic friction
Movie – dynamic friction
Simulation – dynamic friction

Theoretical

Experimental

Cylinder left the paper
Error discussion

- Coefficient of friction is varying over the surface
- Difference between static and dynamic coefficient of friction
- Rolling friction
- Imperfect initial friction
Summary

- Ratio between linear and angular velocity remains the same independent from parameters such as: coefficient of friction or friction type.

- Two reaction force model gives results comparable with experiment. We have built a simulation that allows us to predict cylinders motion.

- Cylinder in 3D situation tends to change its orientation in such a way that it simply rolls on the paper as in 2D situation.
• John R. Taylor “Classical Mechanics”,

Thank you for attention