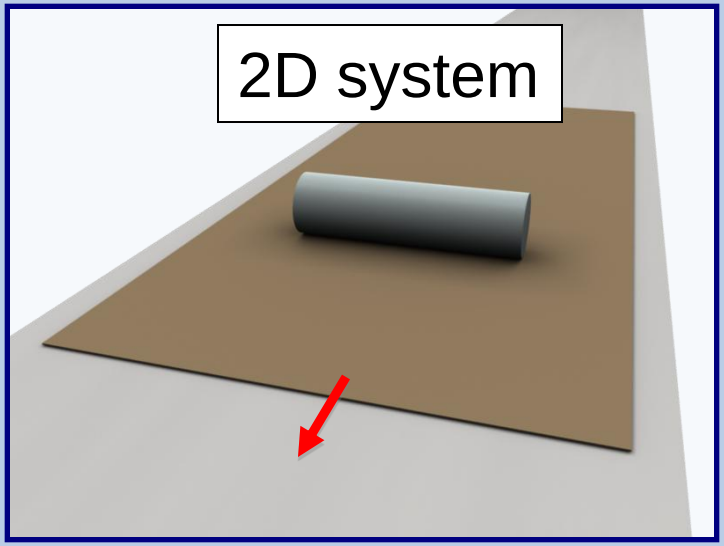
24th International Young Physicists' Tournament

POLAND

14. Moving Cylinder

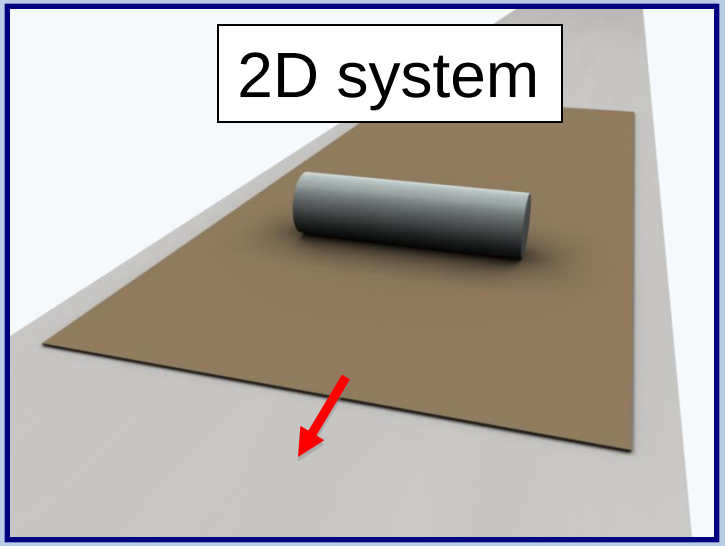
Place a sheet of paper on a horizontal table and put a cylindrical object (e.g. a pencil) on the paper. Pull the paper out. Observe and investigate the motion of the cylinder until it comes to rest.

Presentation plan

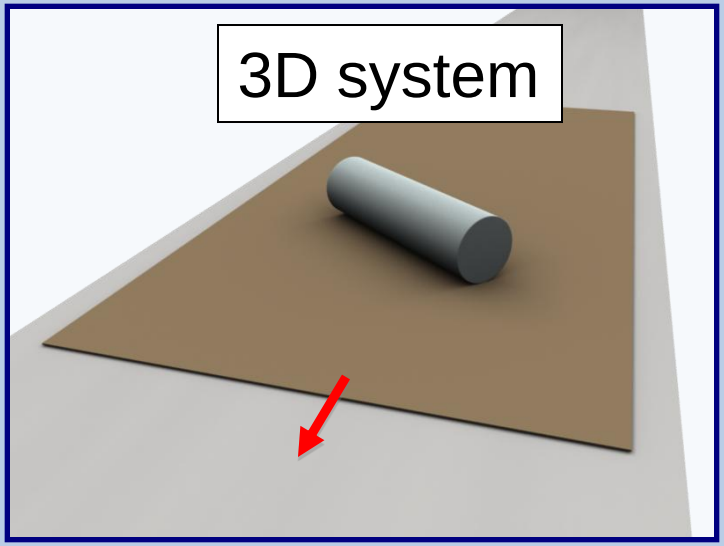


- Theory
- Conclusions
- Experiments

Presentation plan

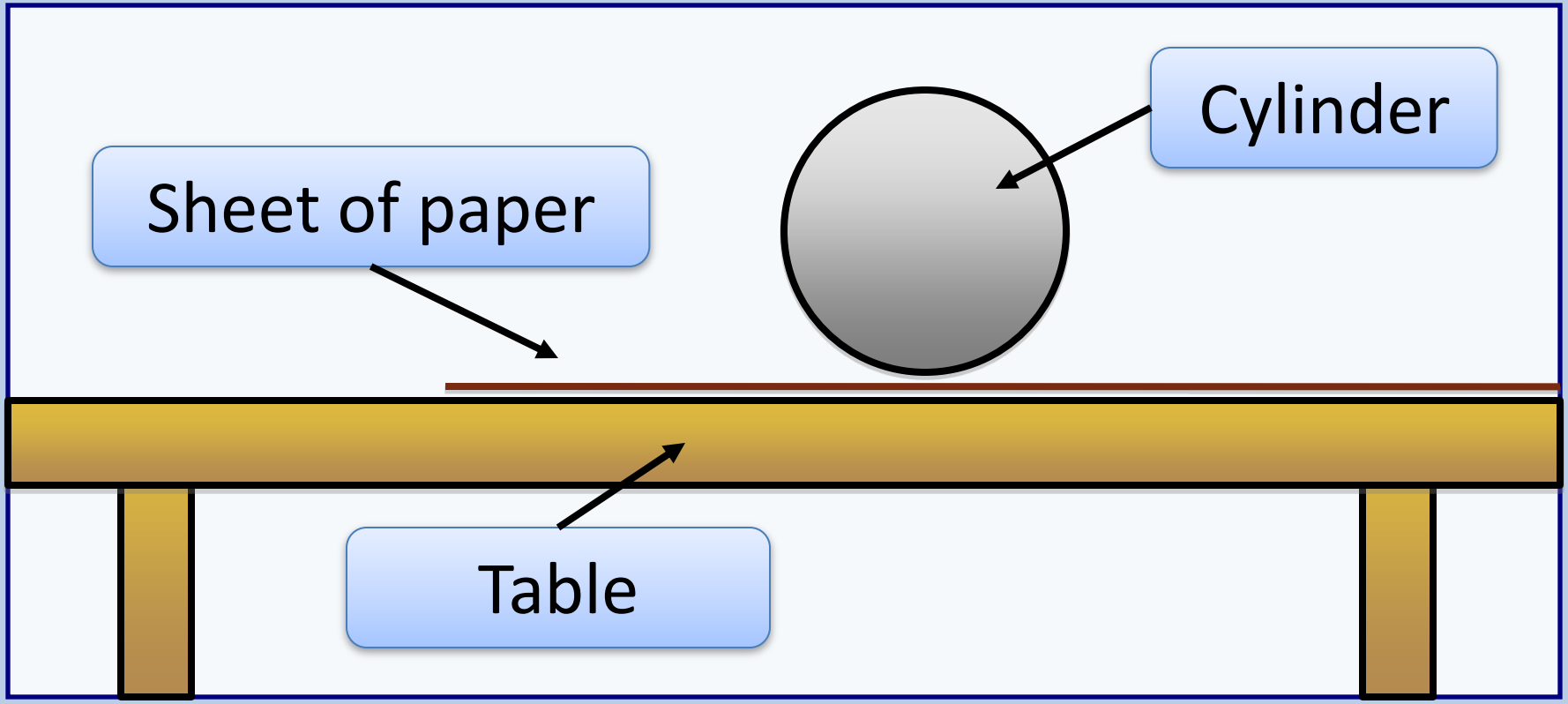


- Theory
- Conclusions
- Experiments



- Theory
- Simulation
- Experiments

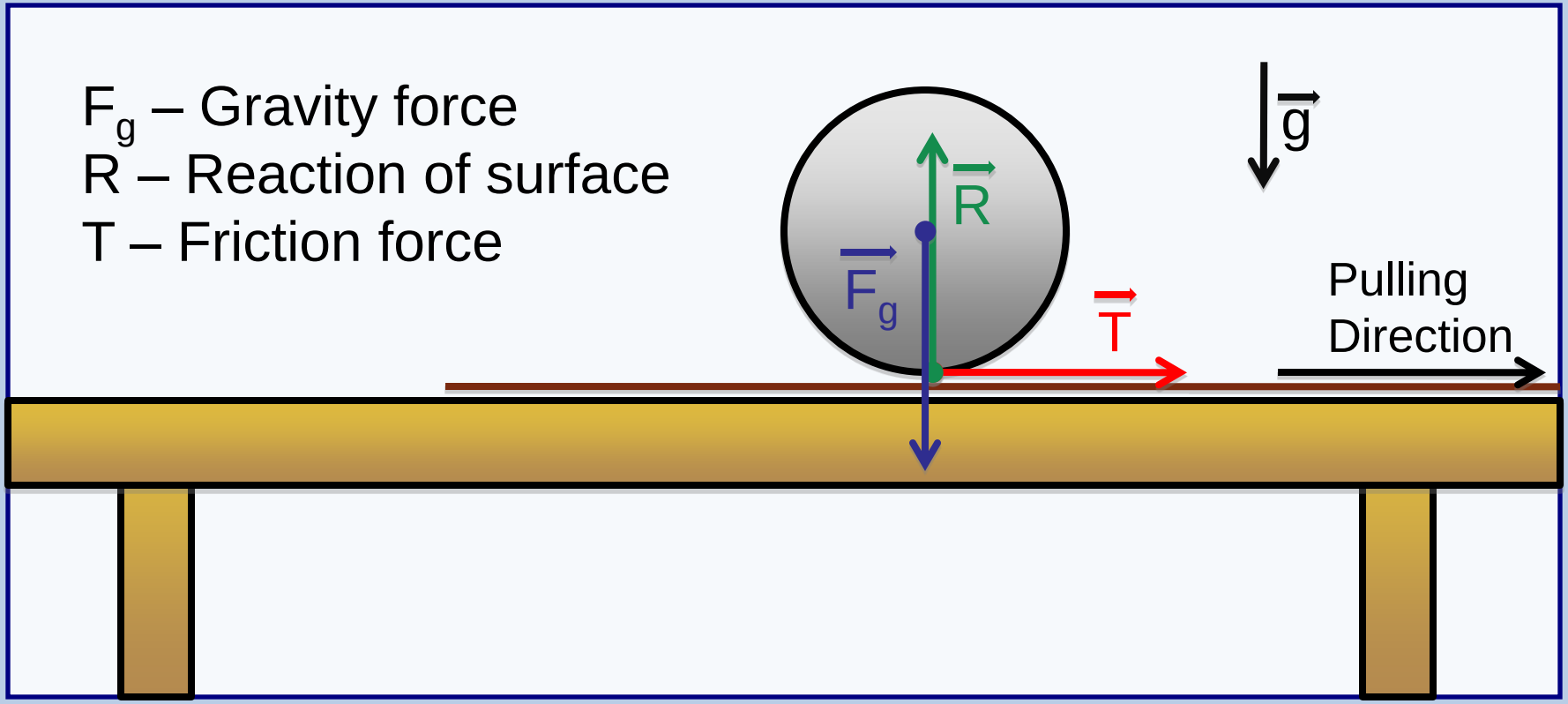
2D - system



We assumed that cylinder, table, and sheet of paper are rigid bodies

2D - system

F_g – Gravity force
R – Reaction of surface
T – Friction force



We investigated motion of cylinder under constant acceleration of the paper: a

Rigid Body Dynamics

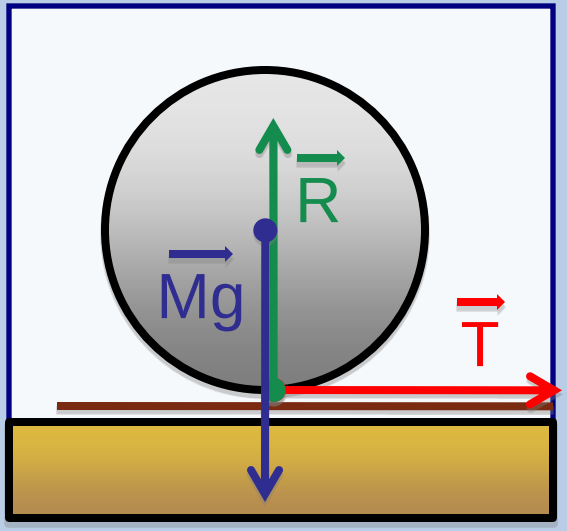
Newton's 2nd Law

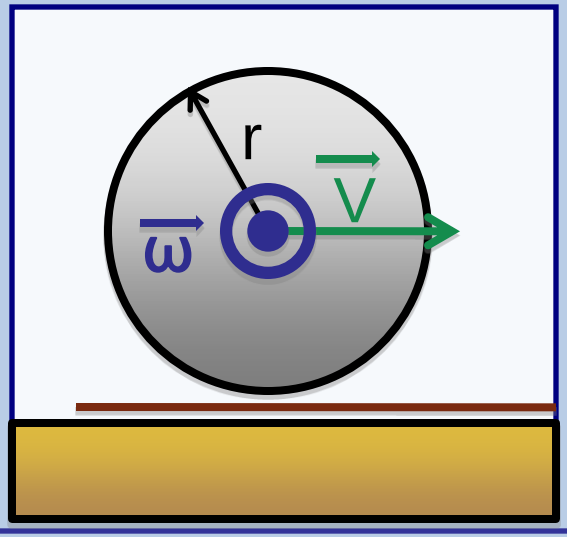
$$\frac{d\vec{p}}{dt} = \vec{F} \qquad \frac{d\vec{L}}{dt} = \vec{\Gamma}$$

$$m\frac{dV}{dt} = T$$

$$0 = R - mg$$

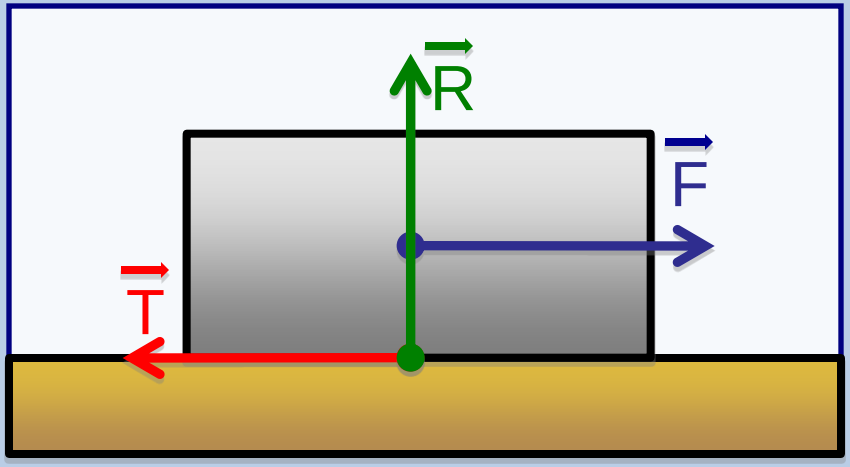
$$I\frac{d\omega}{dt} = T \cdot r$$





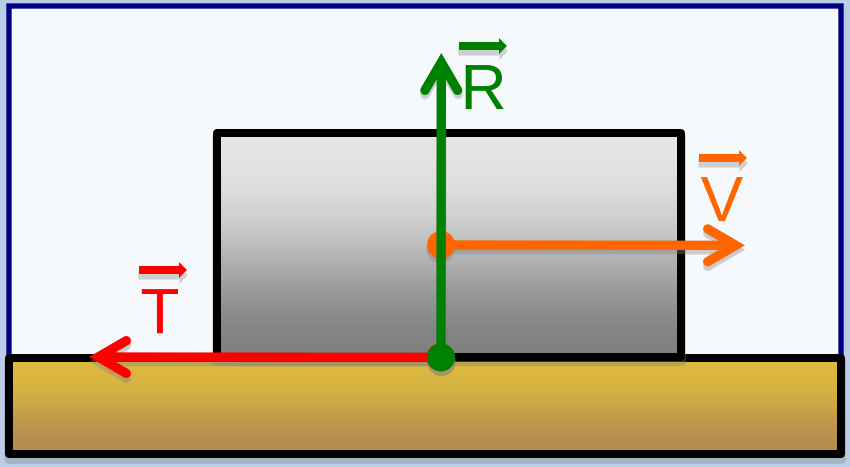
Friction Force

Static Friction



$$F = T < R\mu_1$$

Dynamic Friction



$$T = R\mu_1$$

F – Applied force, R – Reaction force, T – Friction, V – Velocity,
μ_1 – Coefficient of friction

Static vs. Dynamic Friction

Static Friction

$$\omega r + V = at$$

$$T < mg\mu_1$$

Dynamic Friction

$$T = mg\mu_1$$

Critical acceleration of the paper

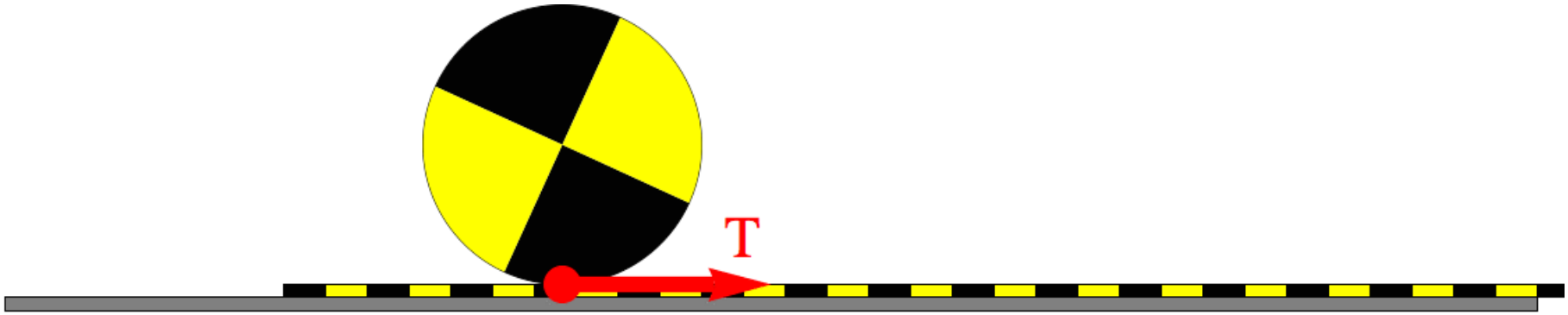
$$a_{cr} = g\mu_1 \frac{(I + mr^2)}{I}$$

m – Mass of the cylinder, r – Radius of the cylinder, I – Moment of inertia due to symmetry axis, g – Gravitational acceleration, μ_1 – Coefficient of friction

Motion of cylinder

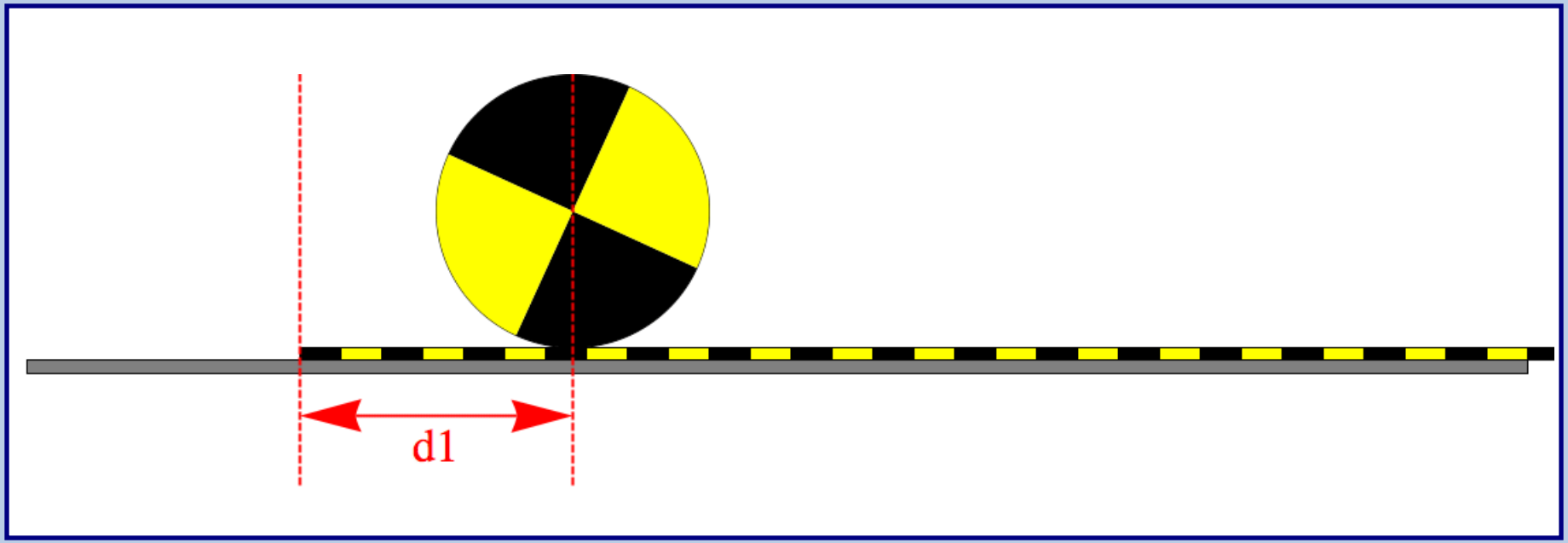
If initial linear and angular velocity is zero

$$\frac{\omega}{V} = \frac{mr}{I}$$



Linear velocity and angular velocity become zero at the same time

2D-Model Conclusions



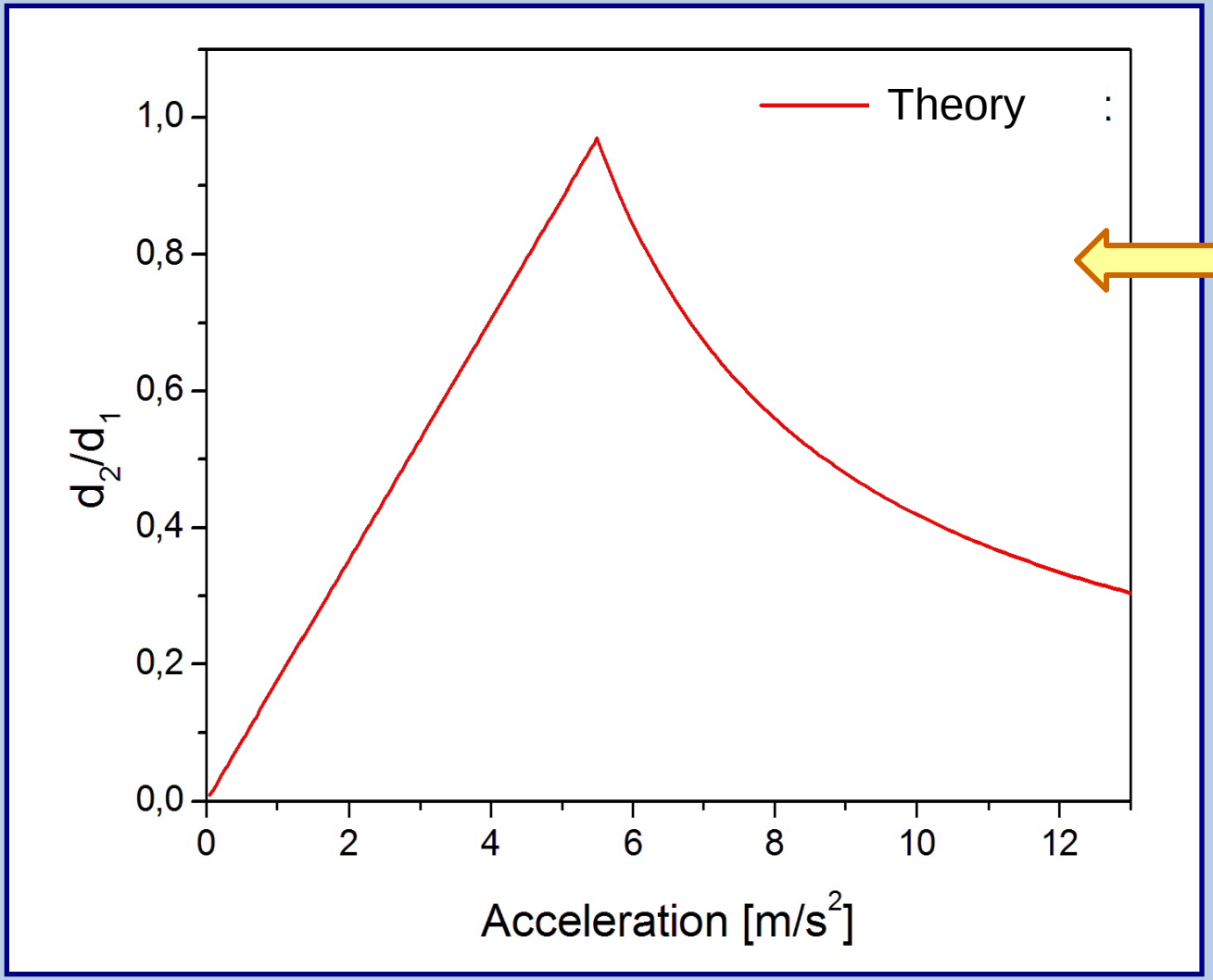
Static friction

$$\frac{d_2}{d_1} = \frac{aI^2}{mr^2\left(mr^2 + I\right)g\mu_2}$$

Kinetic friction

$$\frac{d_2}{d_1} = \frac{g\mu_1^2}{(a - g\mu_1)\mu_2}$$

Theory results

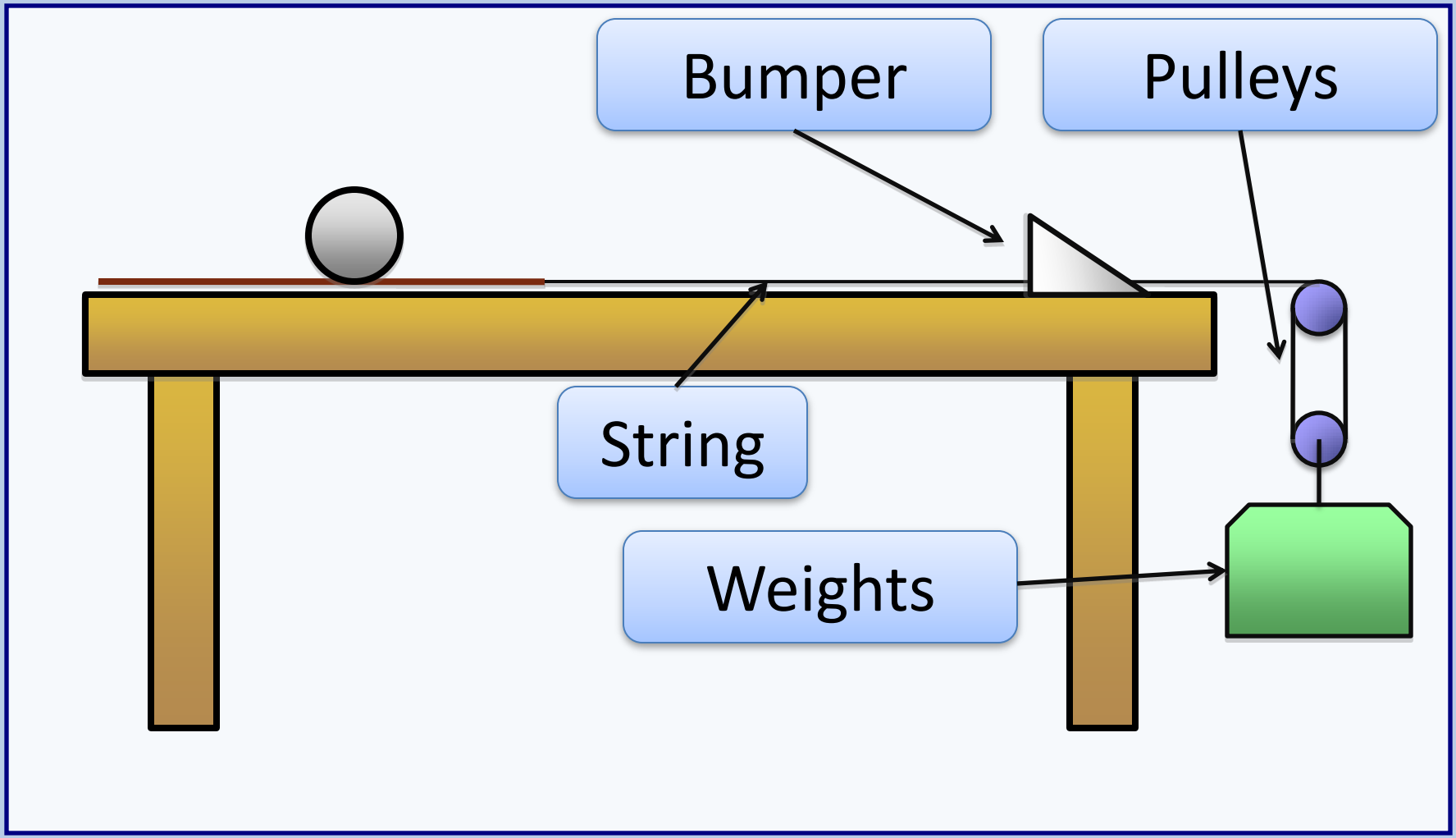


$$\mu_1 = 0.13$$

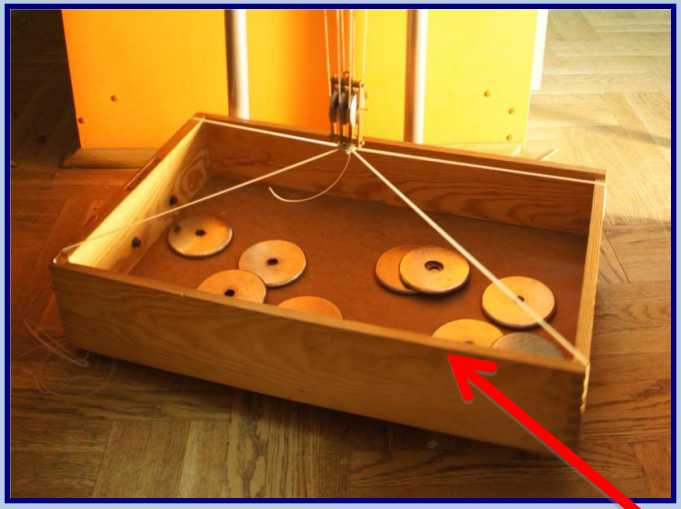
$$\mu_2 = 0.22$$

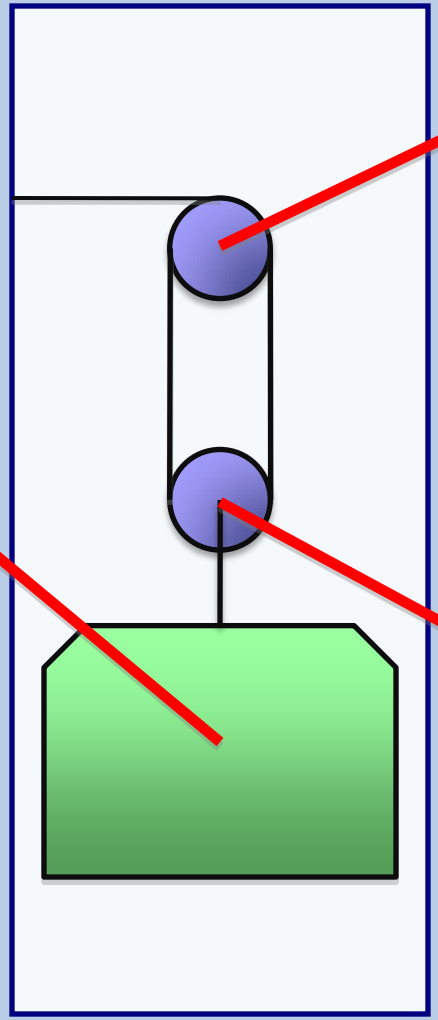
$$\frac{I}{mr^2} = 0.6$$

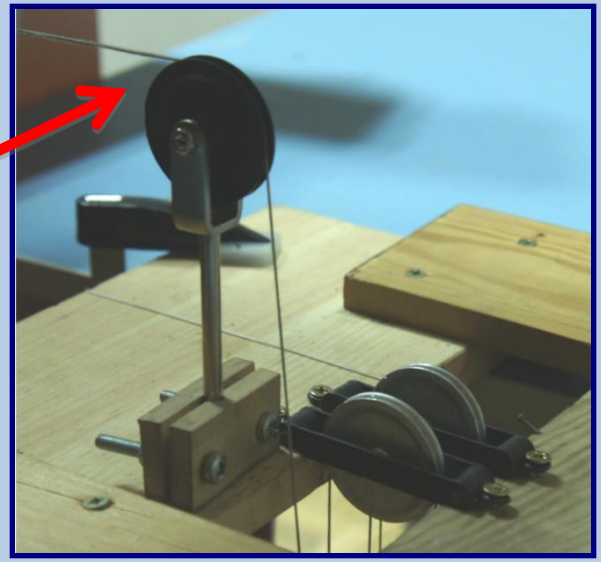
Experimental setup

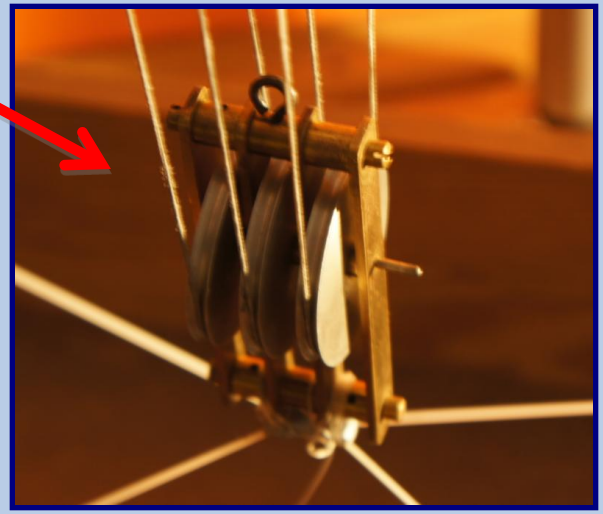


Pulleys and Weights







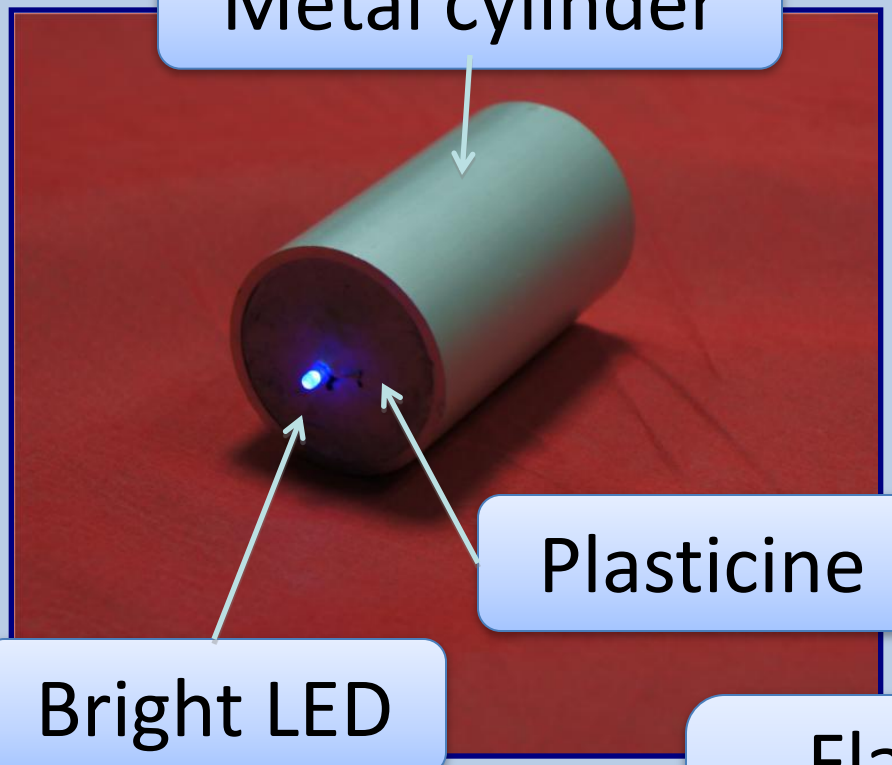


Different materials used

Paper
μ_1=(0.15±0.02)

Fabric
μ_1=(0.22±0.02)

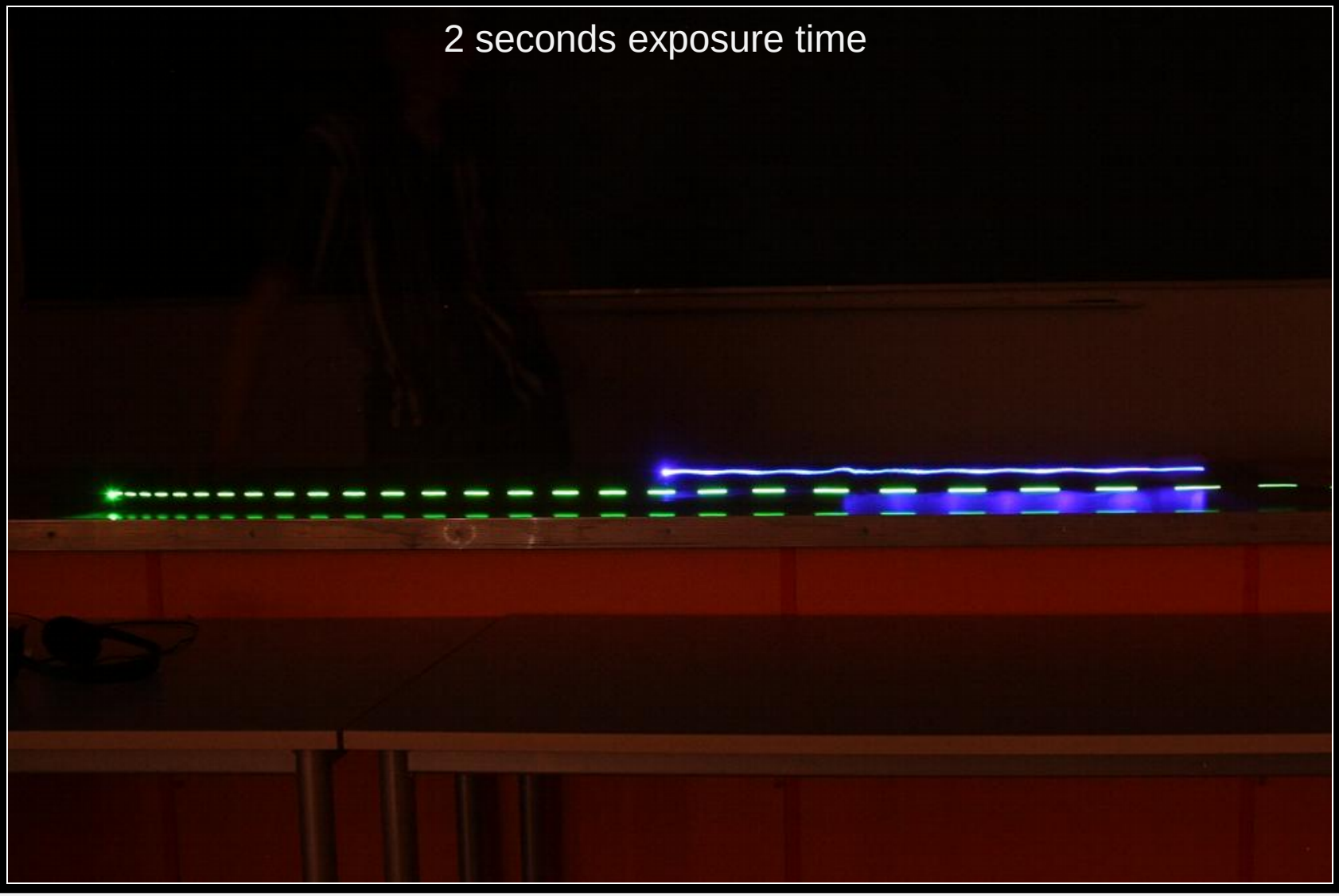
Cylinder


Metal cylinder
Plasticine
Bright LED

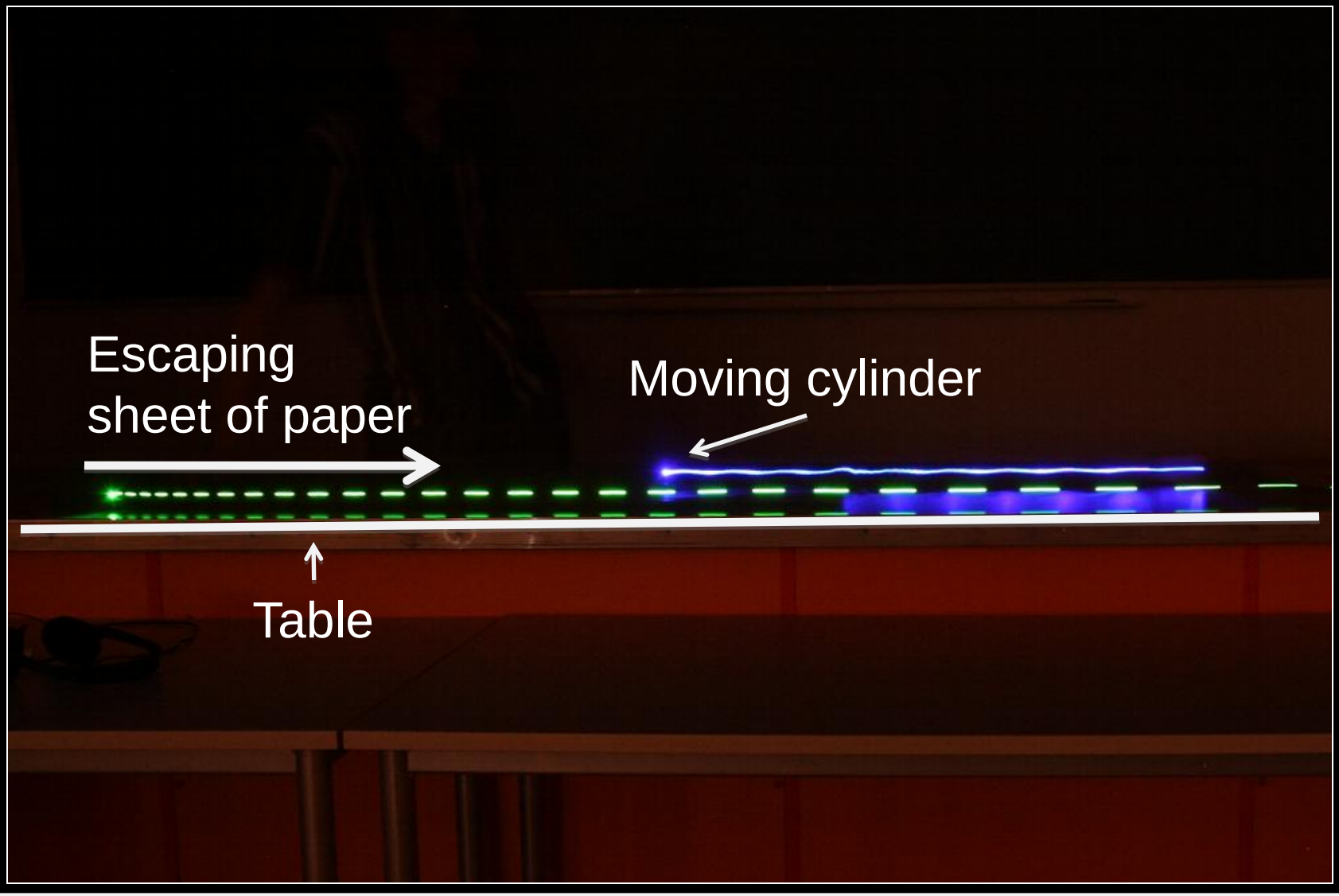

Micro-processor
Flashing
Bright LED

Experiments

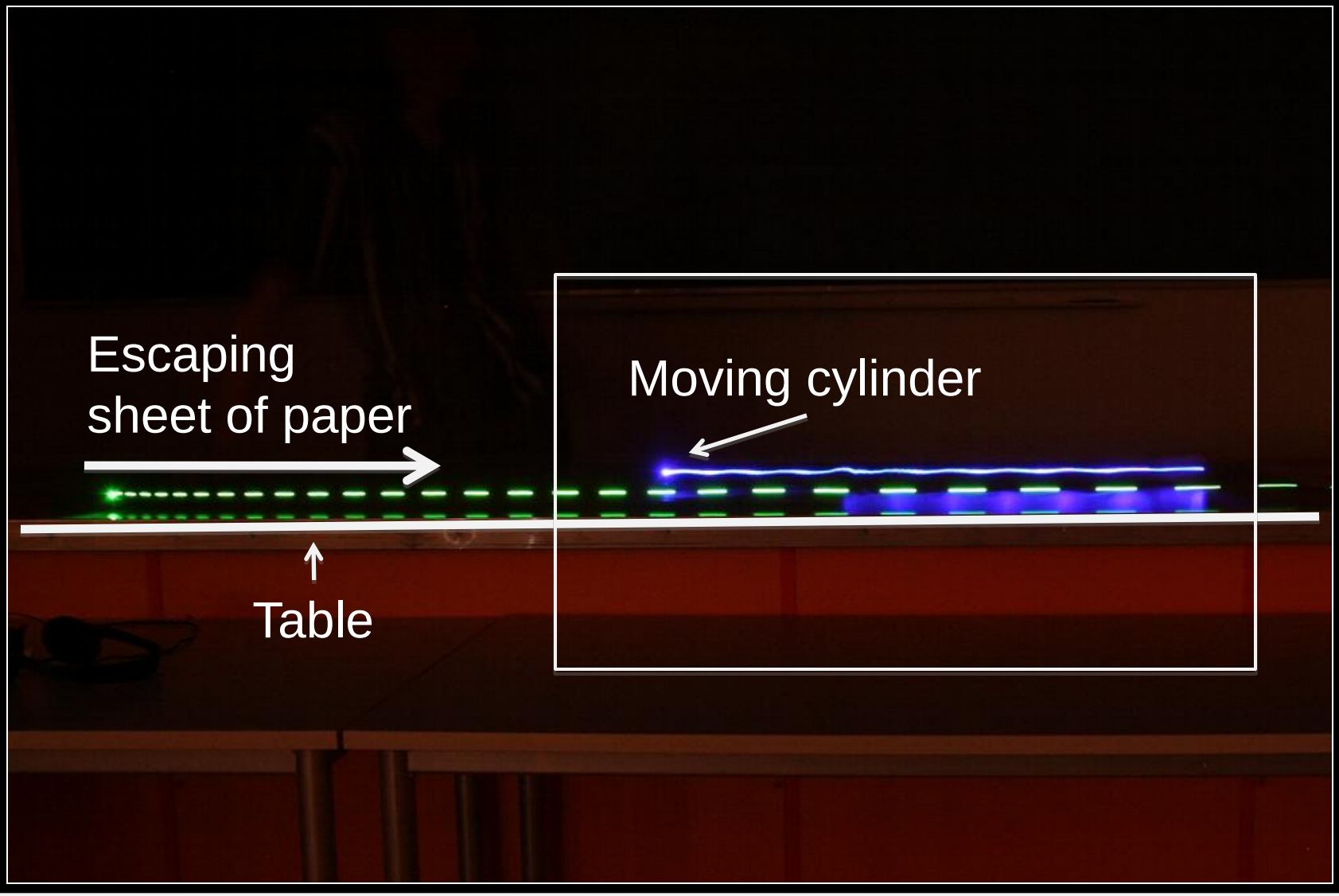
Experiments



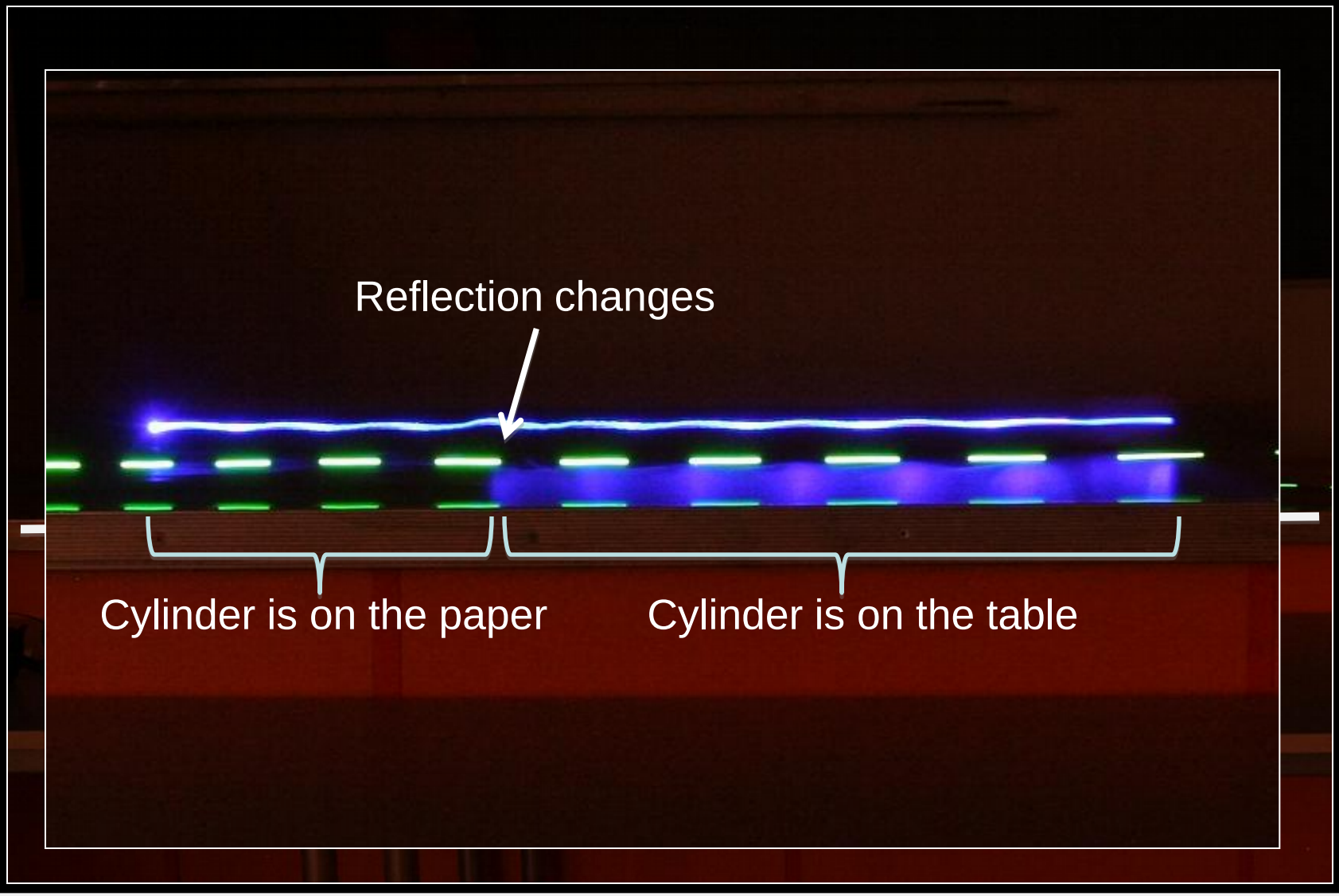
Experiments



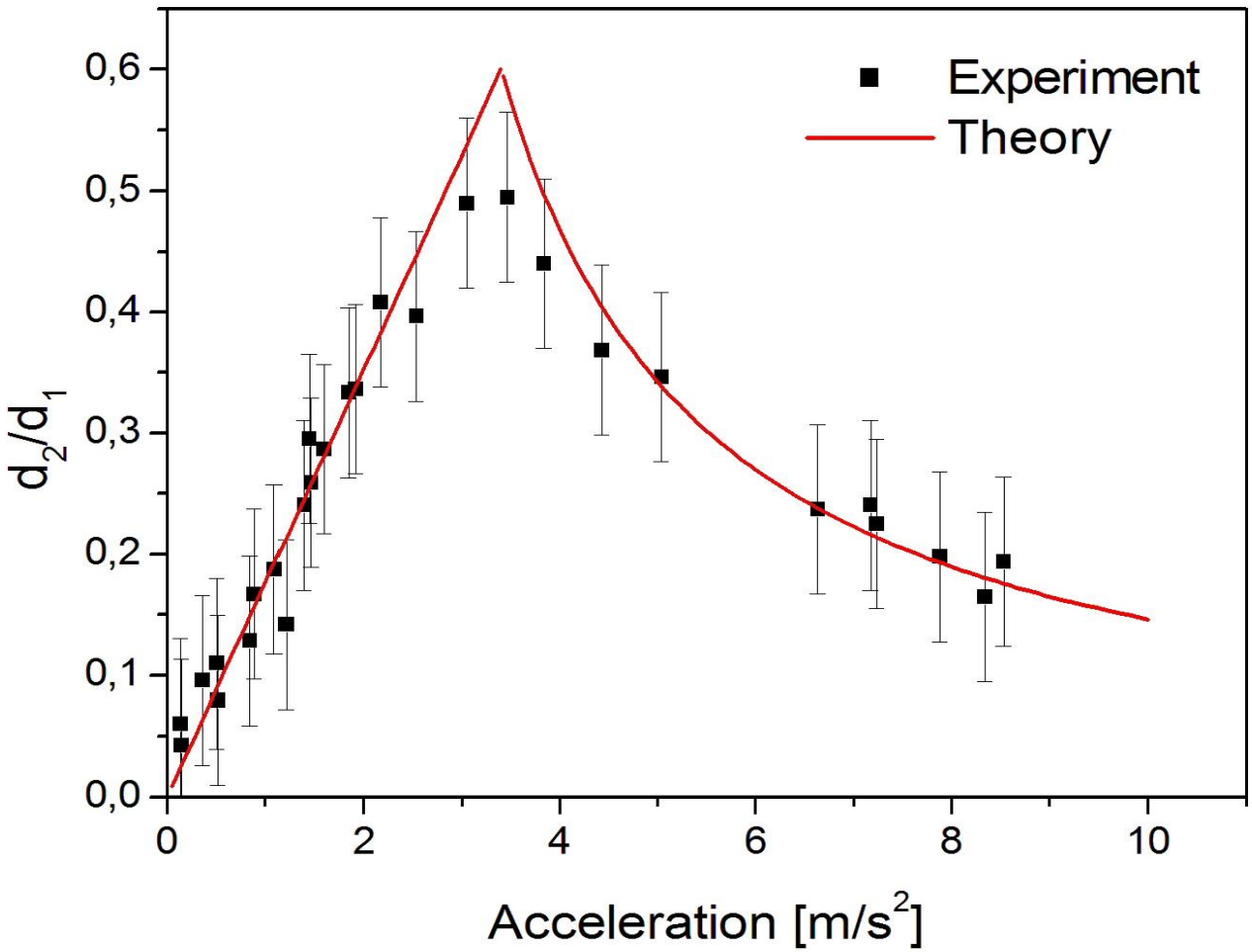
Experiments



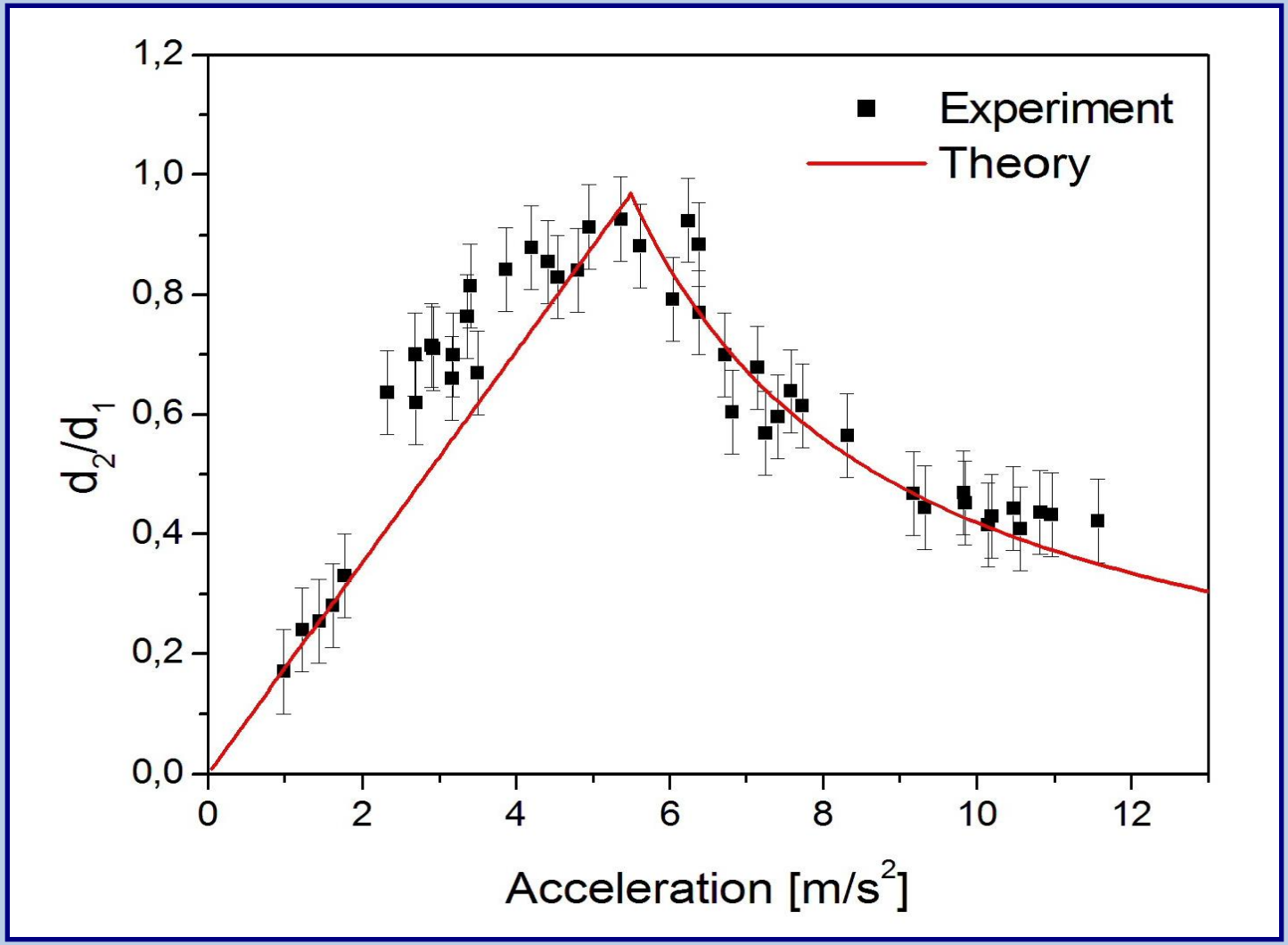
Experiments



Experimental results - paper



Experimental results - fabric

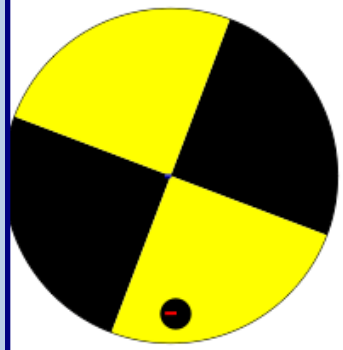


Cycloid

$$\frac{\omega}{V} = \frac{mr}{I}$$

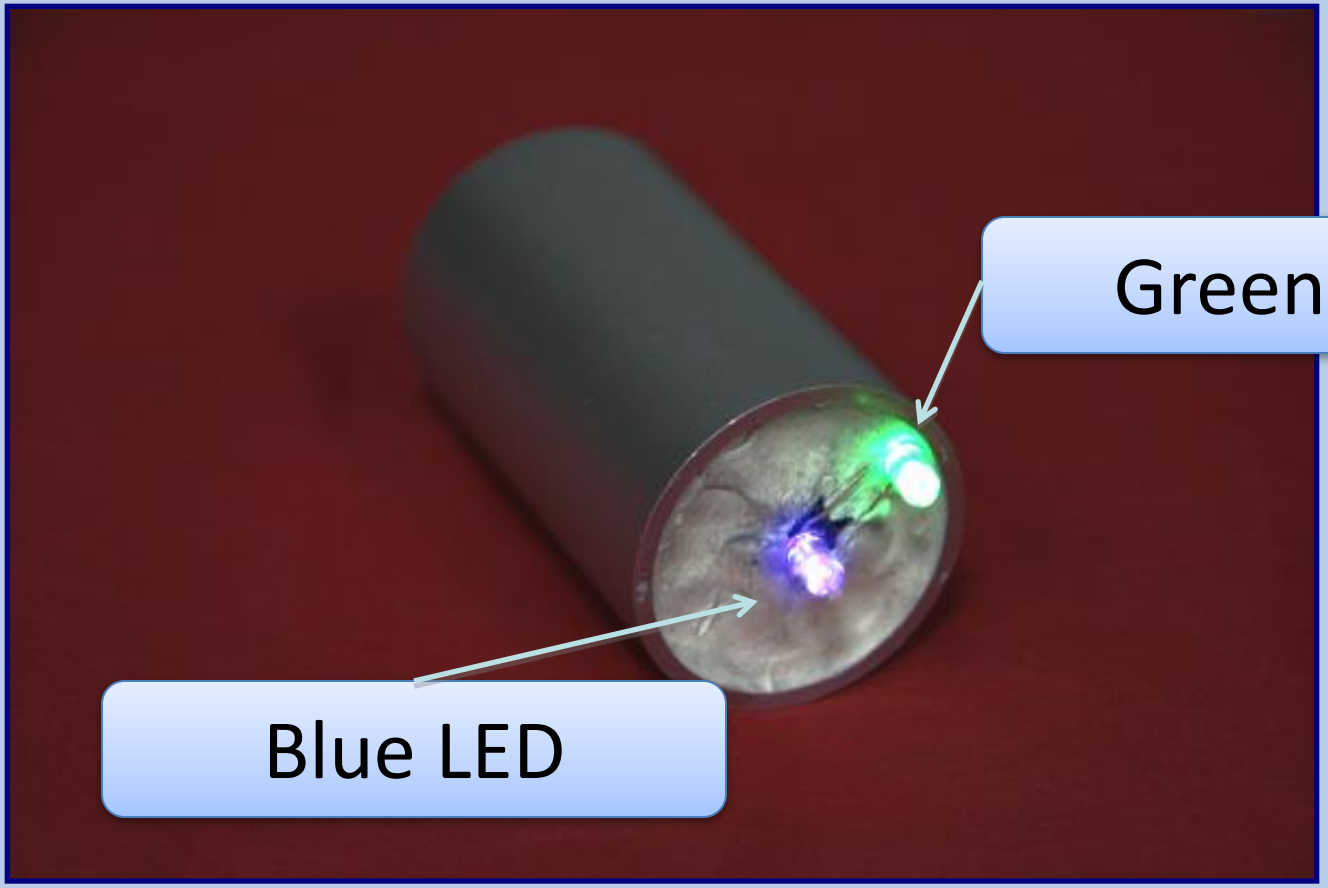
$$x(\lambda) = \lambda + R\cos\left(\frac{mr}{I}\lambda\right)$$

$$y(\lambda) = R\sin\left(\frac{mr}{I}\lambda\right)$$

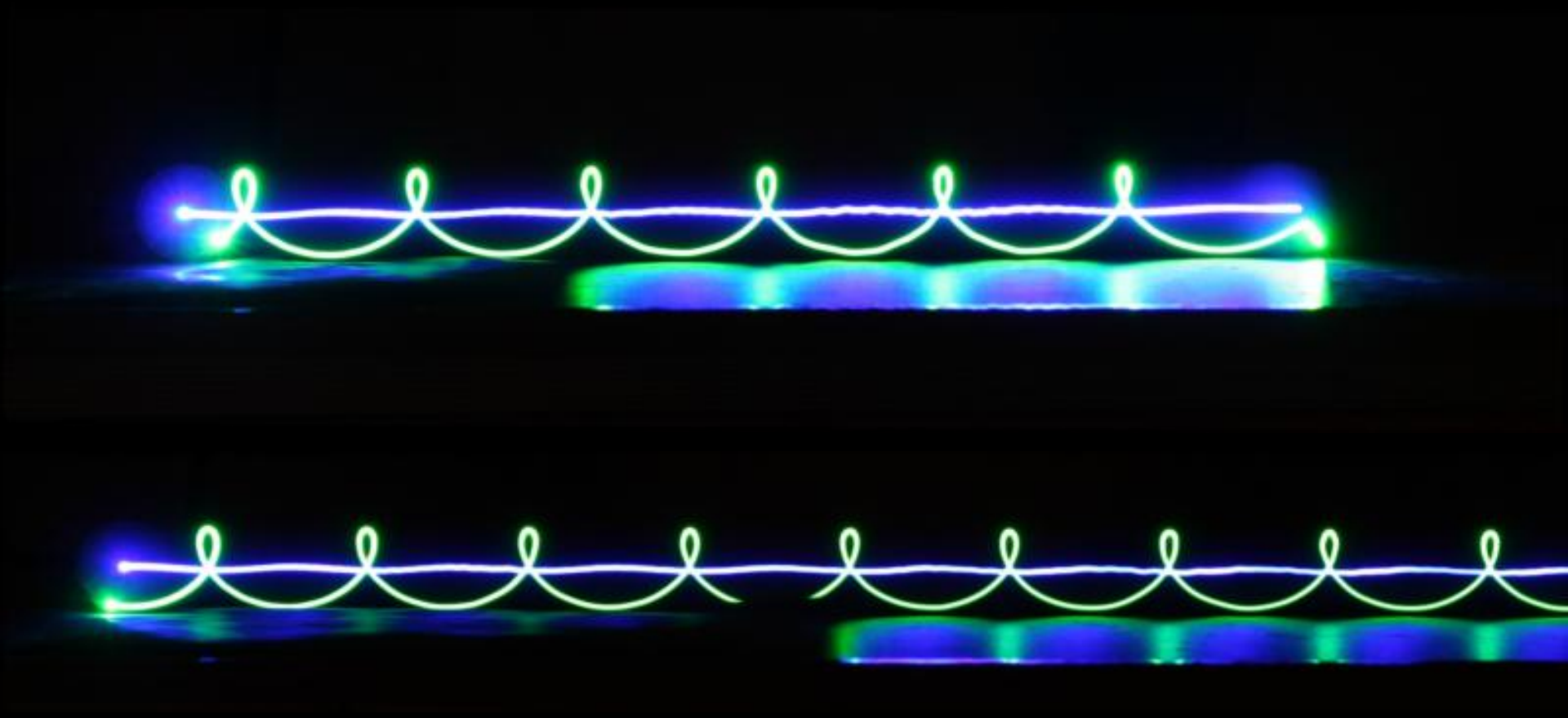


$$mr / I = 56\left[1/m\right]$$

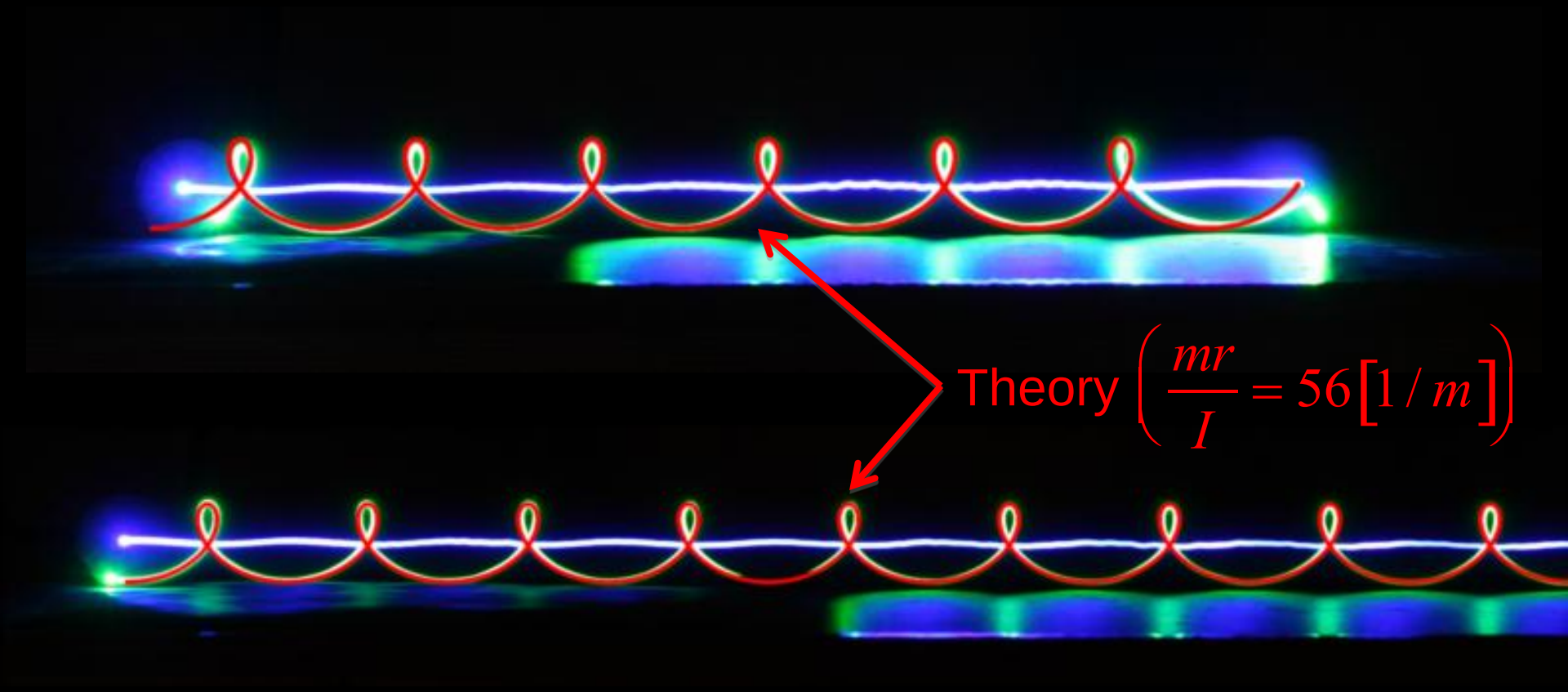
Experimental setup



Experiments

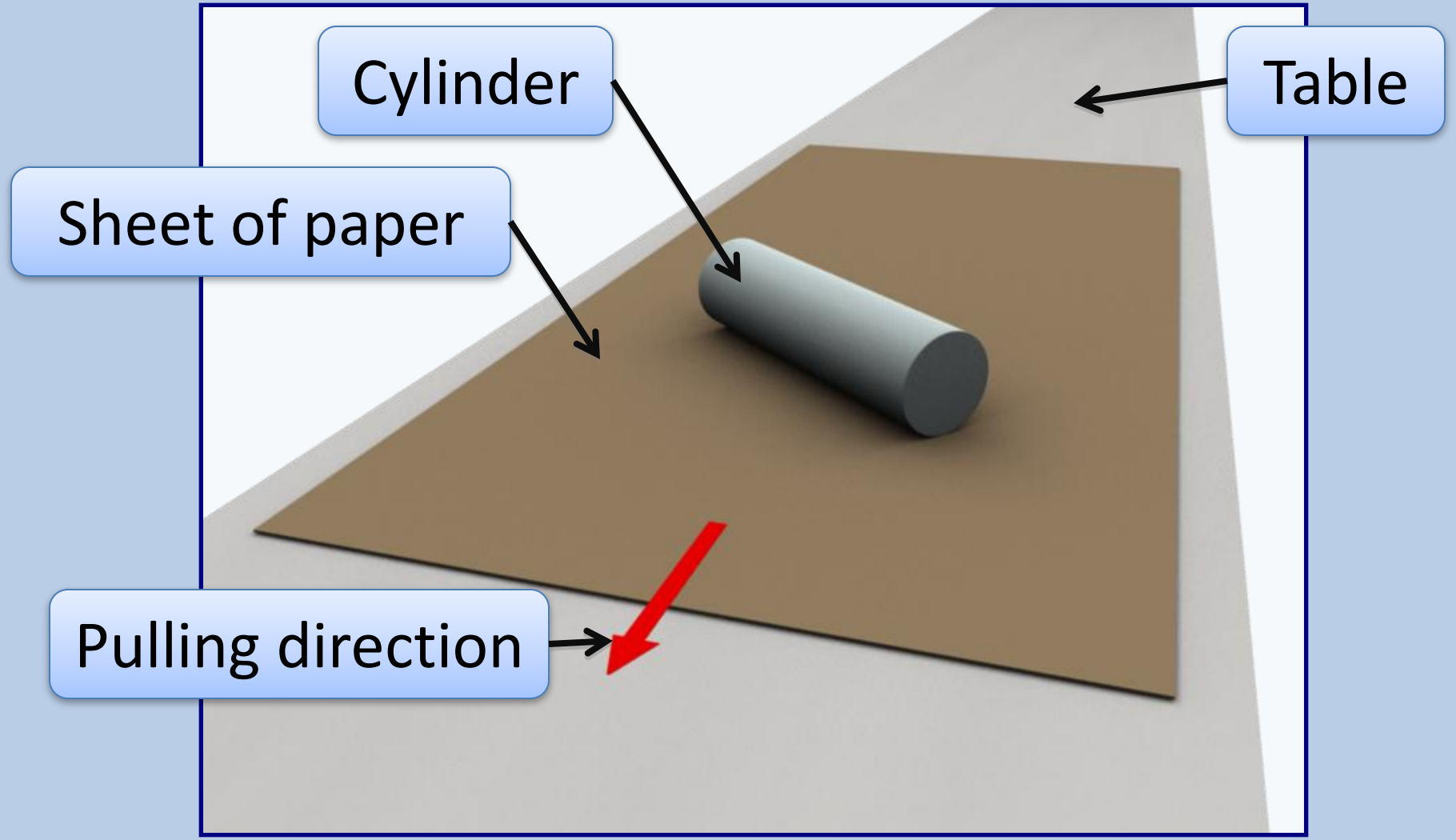


Comparison with theory

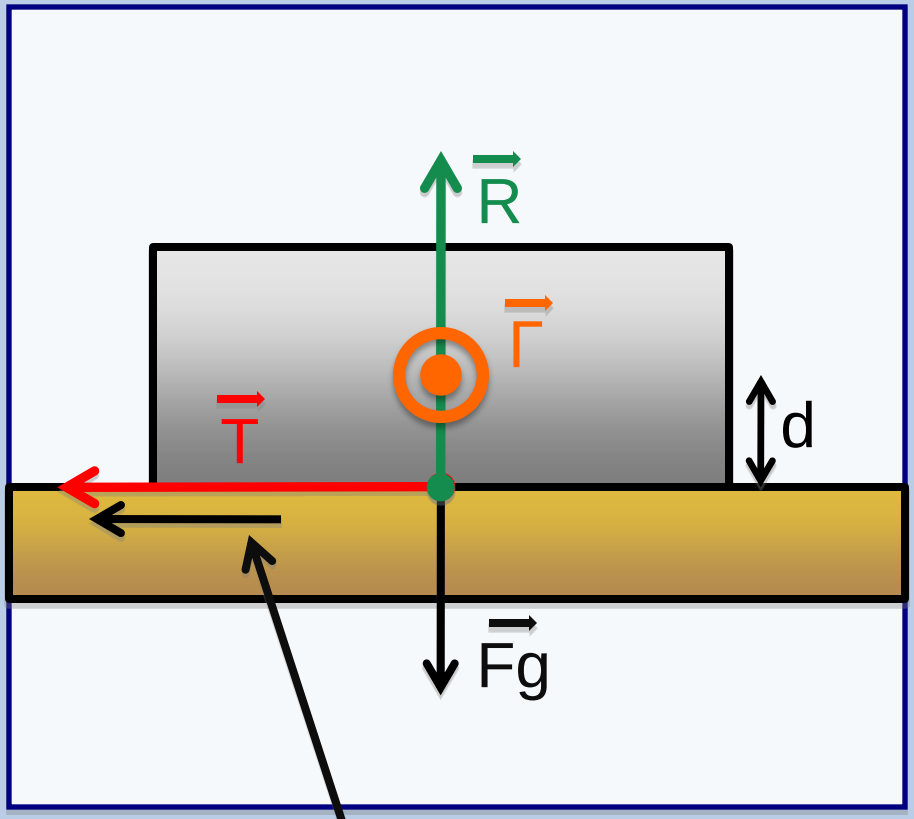


The ratio between angular and linear velocity is not changed!

Full 3D model



Reaction force distribution



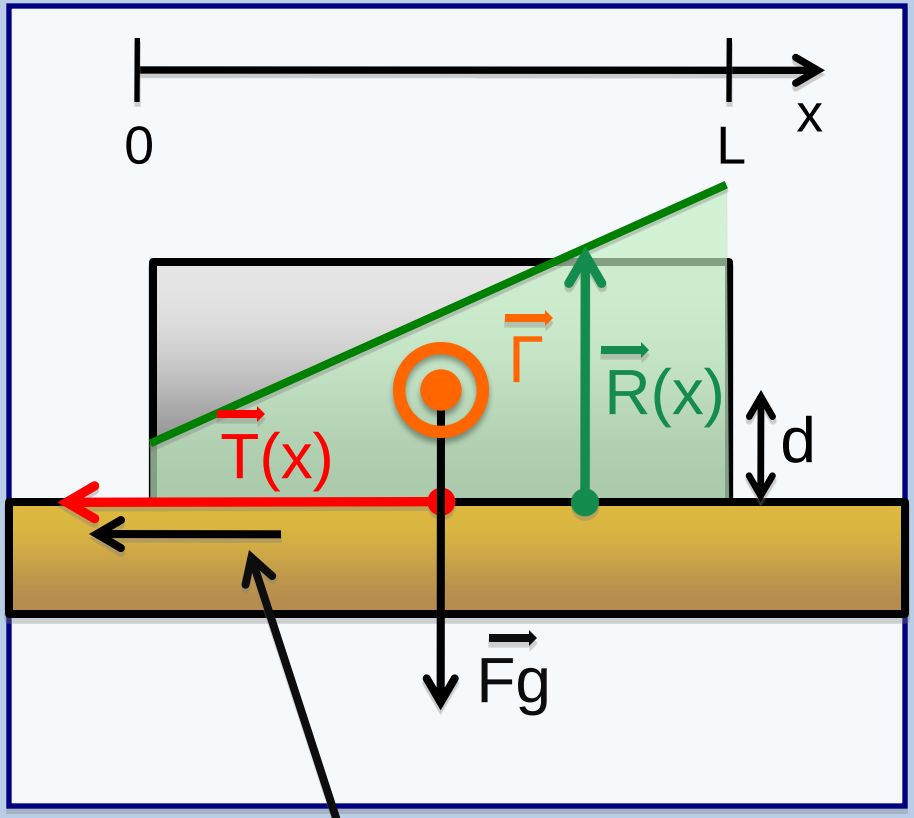
Escaping base

$$\Gamma = Td$$

Rotation of the block

$$\Gamma \neq 0$$

Reaction force distribution



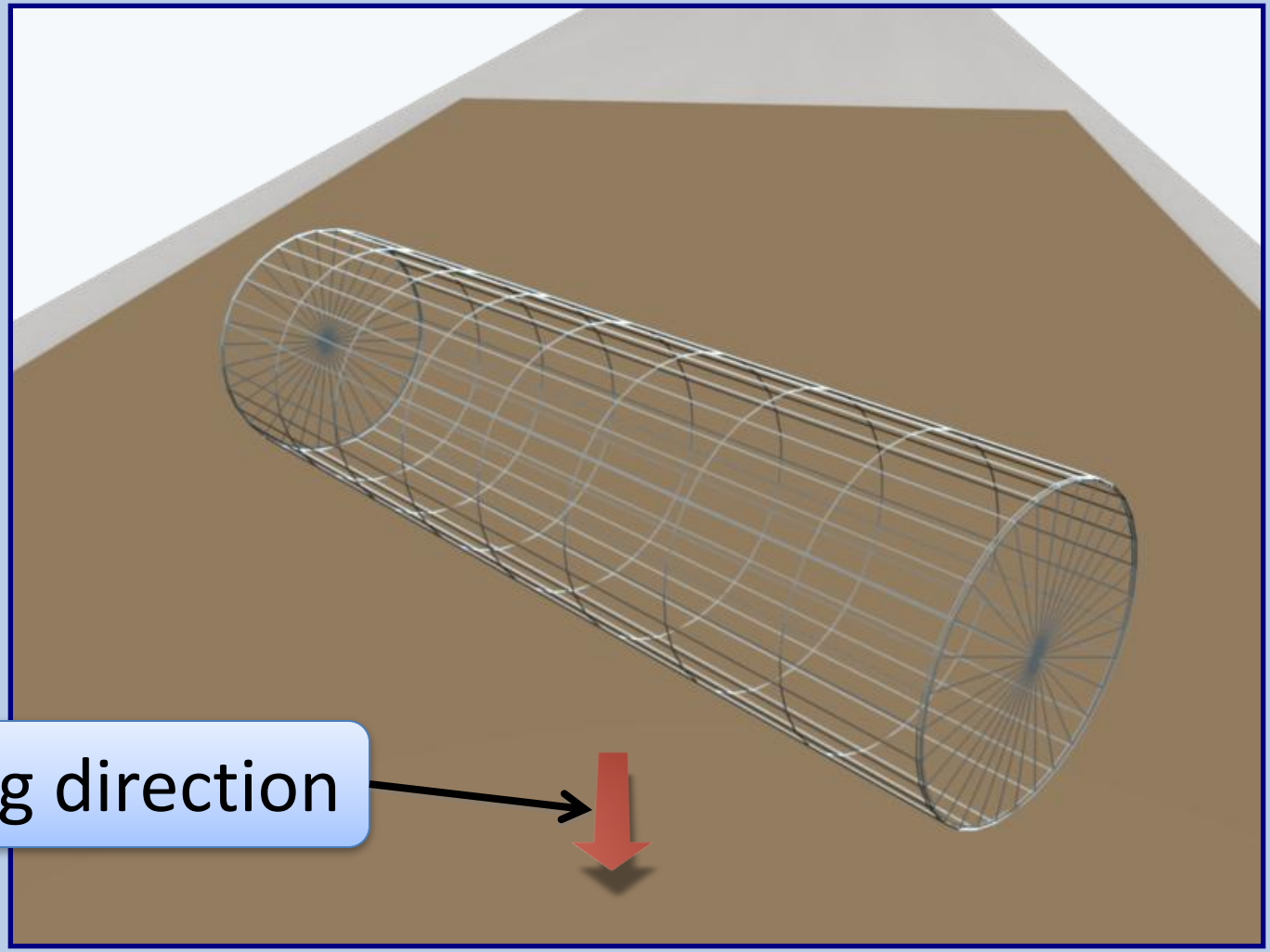
Escaping base

$$\Gamma = \int_0^L \left(T(x)d\right)dx + \int_0^L \left(R(x)x\right)dx$$

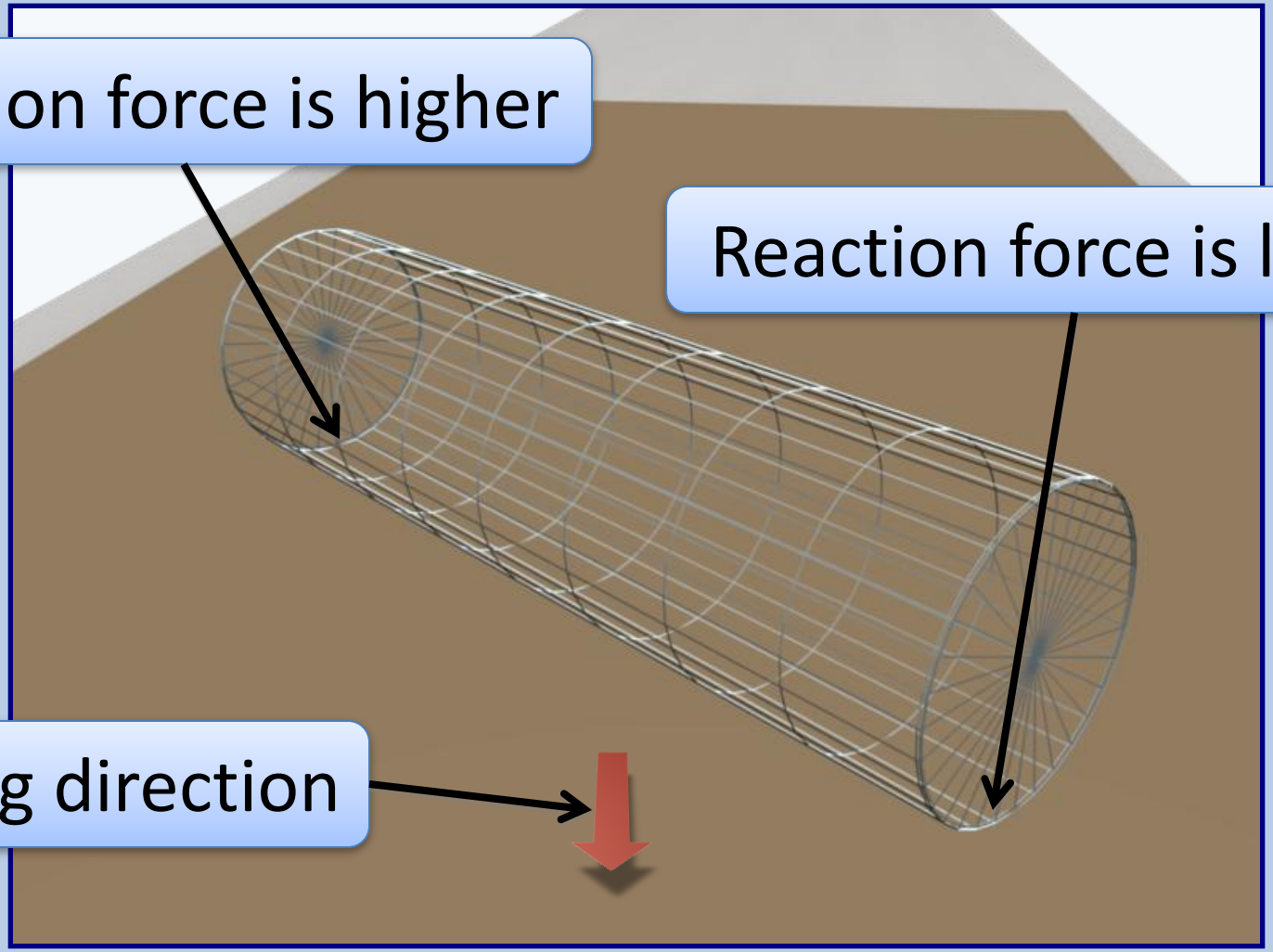
No detachment from base

$$\Gamma = 0$$

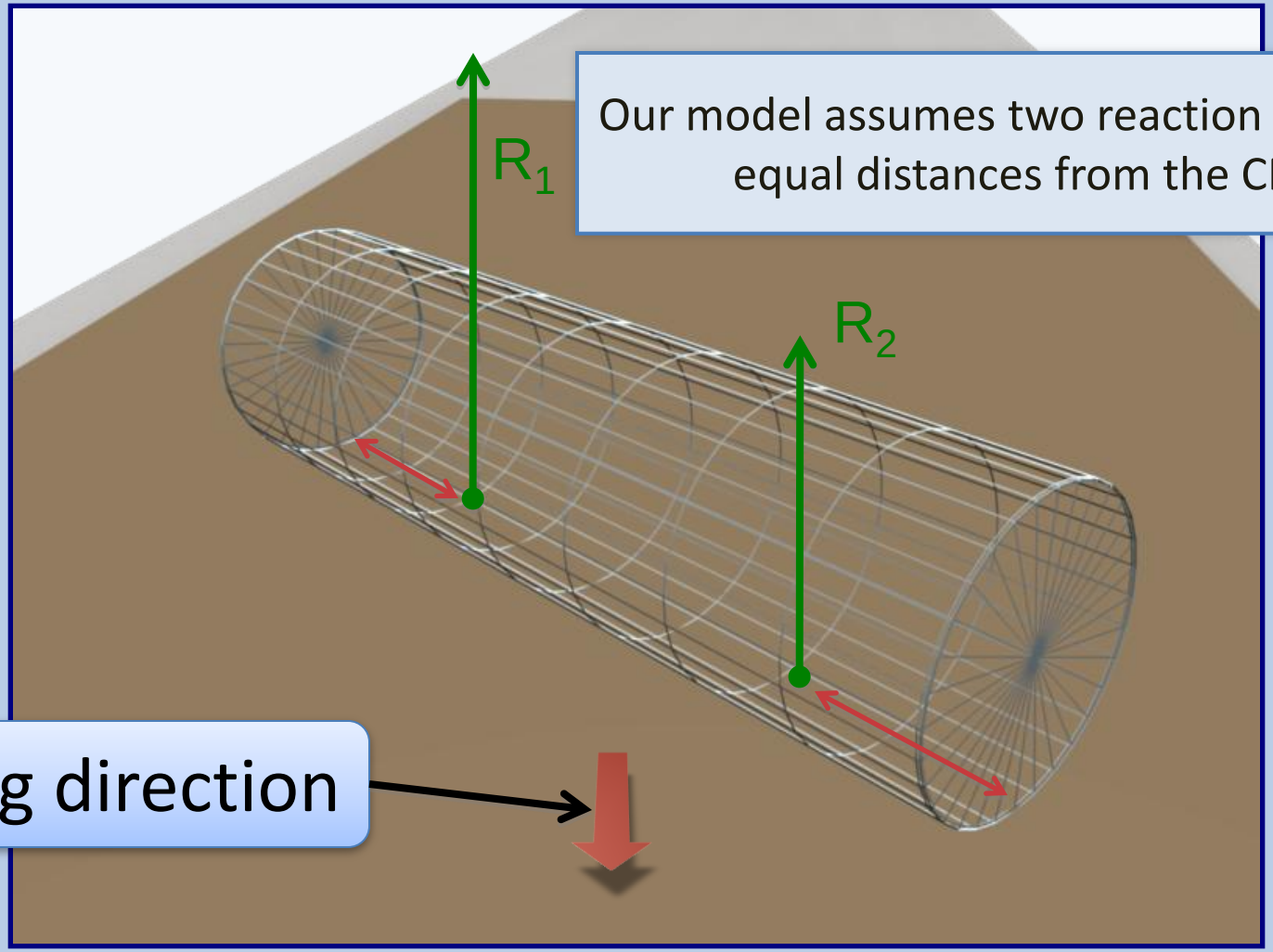
Reaction forces



Reaction forces



Reaction forces



Our model assumes two reaction forces in equal distances from the CM

Pulling direction

Equations

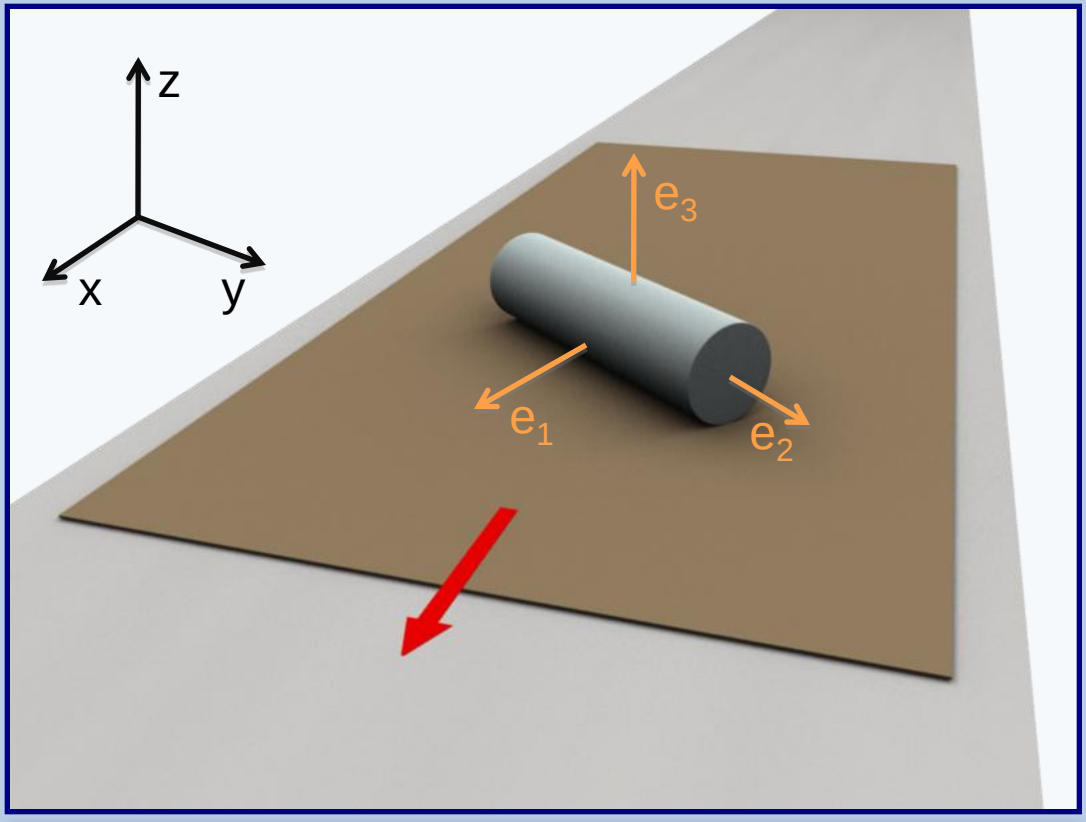
New frame of reference

$$\hat{z} \equiv \hat{e}_3$$

e_1,e_2 – can rotate only in xy-plane

No rotation around e_1 axis:

$$\omega_1 \equiv 0$$



ω_1, ω_2, ω_3 – angular velocity components in e_1, e_2, e_3

Equations

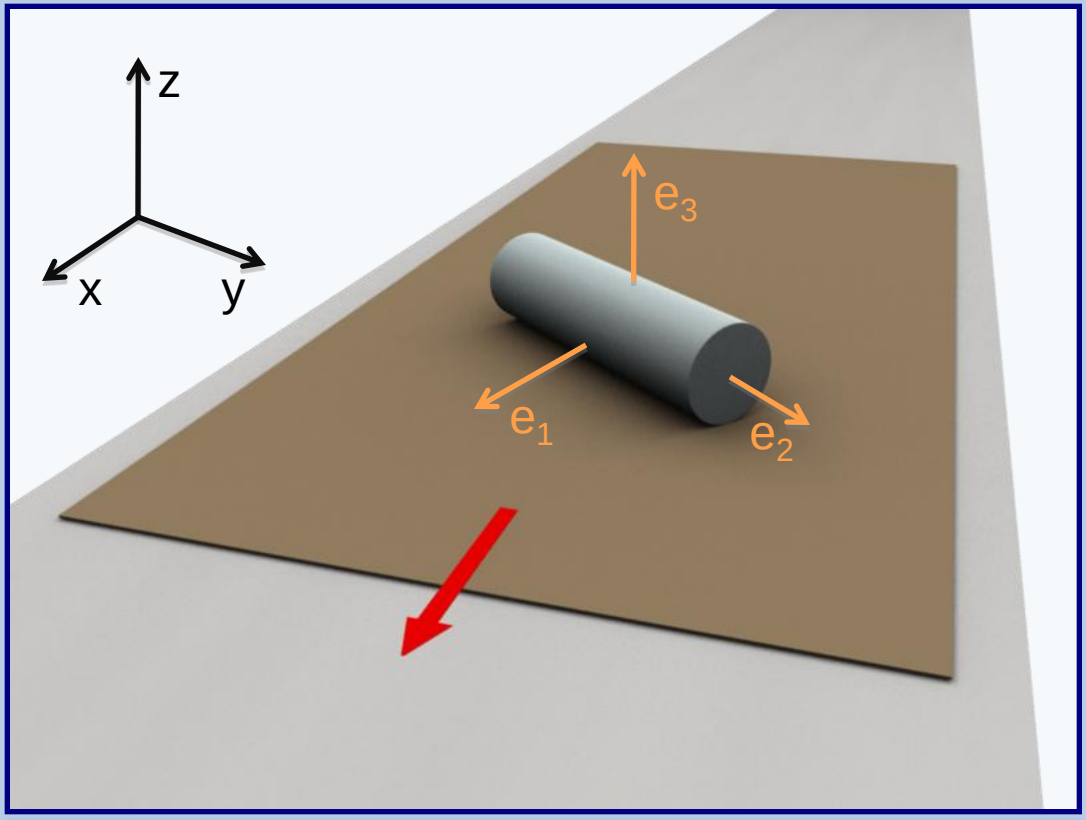
Equations describing the the system in the new frame of reference

$$m\frac{d\vec{V}}{dt} = \vec{F}$$

$$-I_2\omega_2\omega_3 = \Gamma_1$$

$$I_2\frac{d\omega_2}{dt} = \Gamma_2$$

$$I_3\frac{d\omega_3}{dt} = \Gamma_3$$



ω_1, ω_2, ω_3 – angular velocity components in e_1, e_2, e_3

Equations

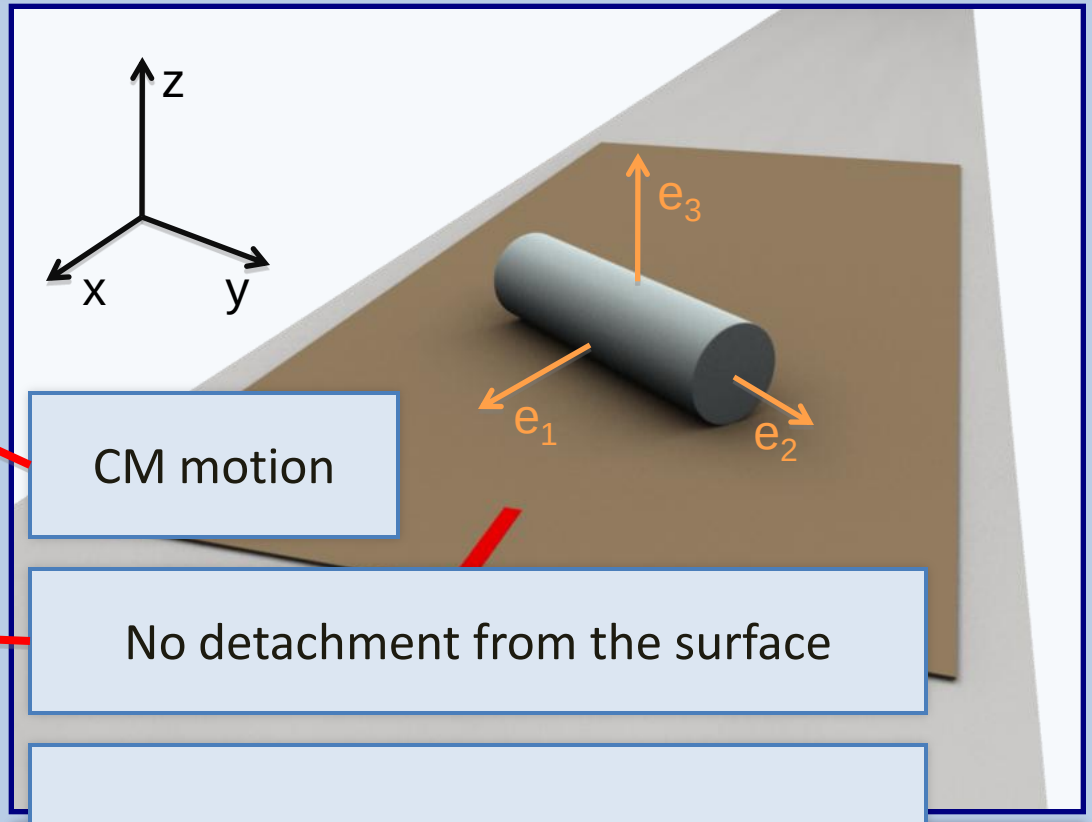
Equations describing the the system in the new frame of reference

$$m\frac{d\vec{V}}{dt} = \vec{F}$$

$$-I_2\omega_2\omega_3 = \Gamma_1$$

$$I_2\frac{d\omega_2}{dt} = \Gamma_2$$

$$I_3\frac{d\omega_3}{dt} = \Gamma_3$$



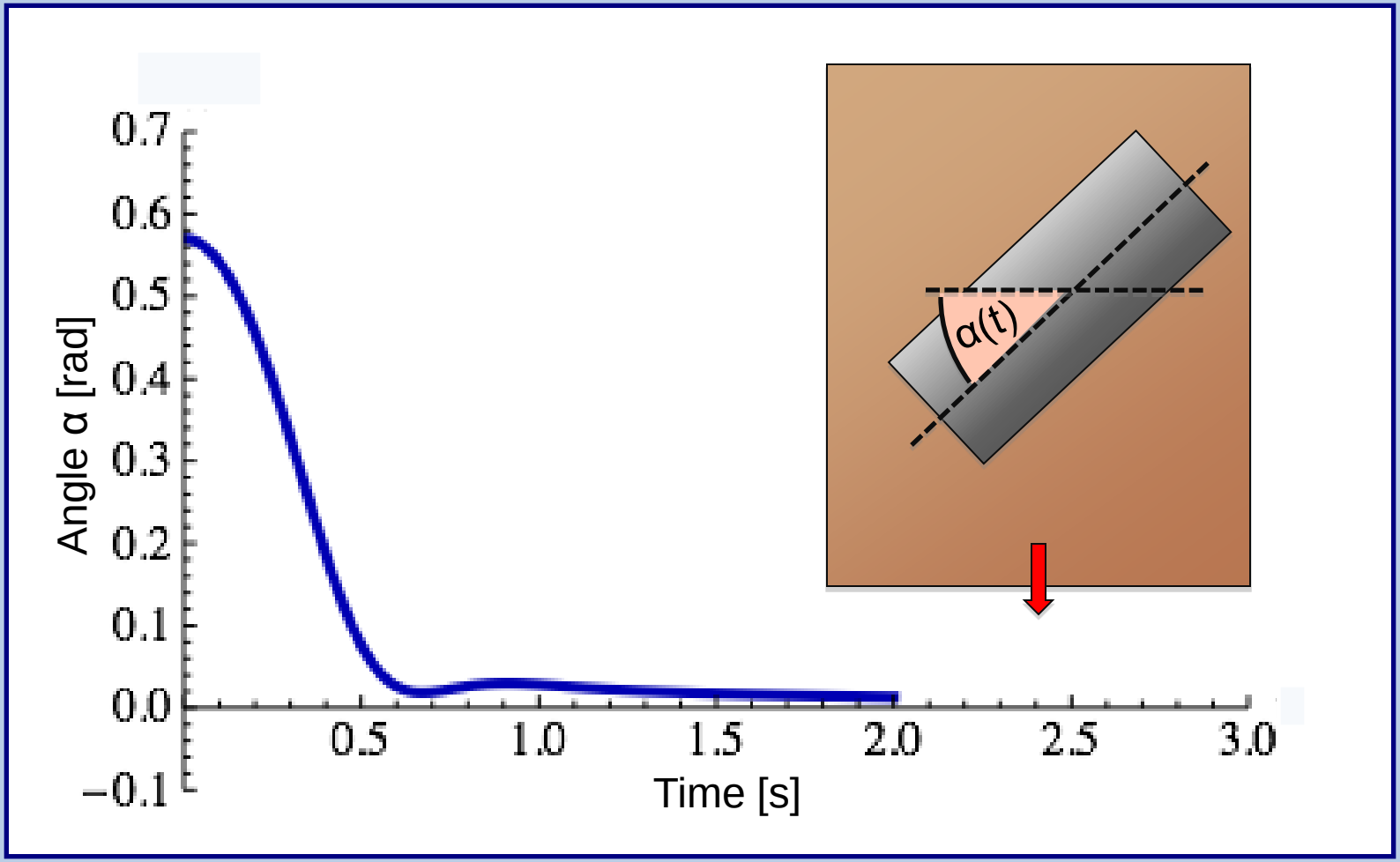
CM motion

No detachment from the surface

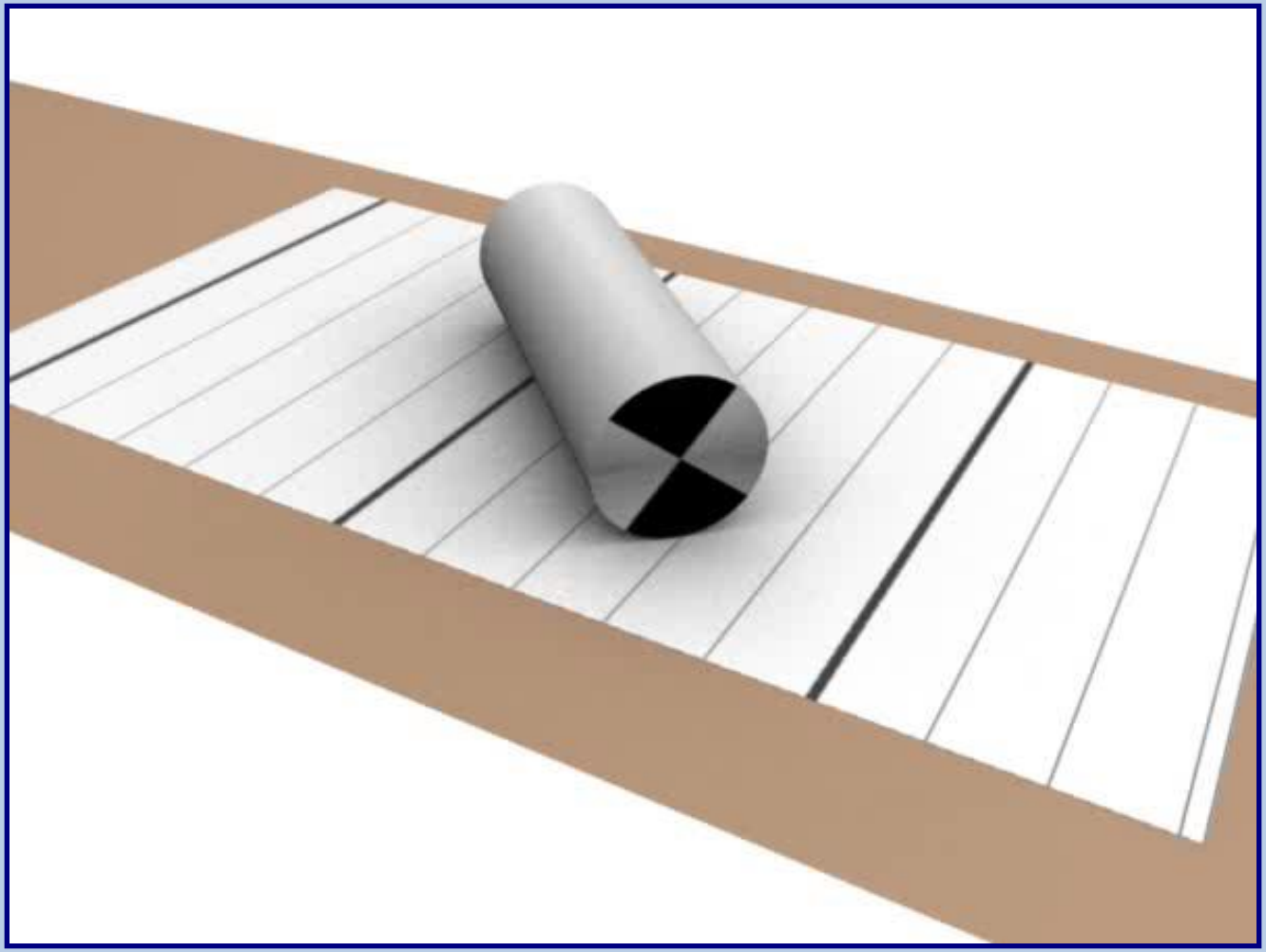
Equations of motion

components in e_1, e_2, e_3

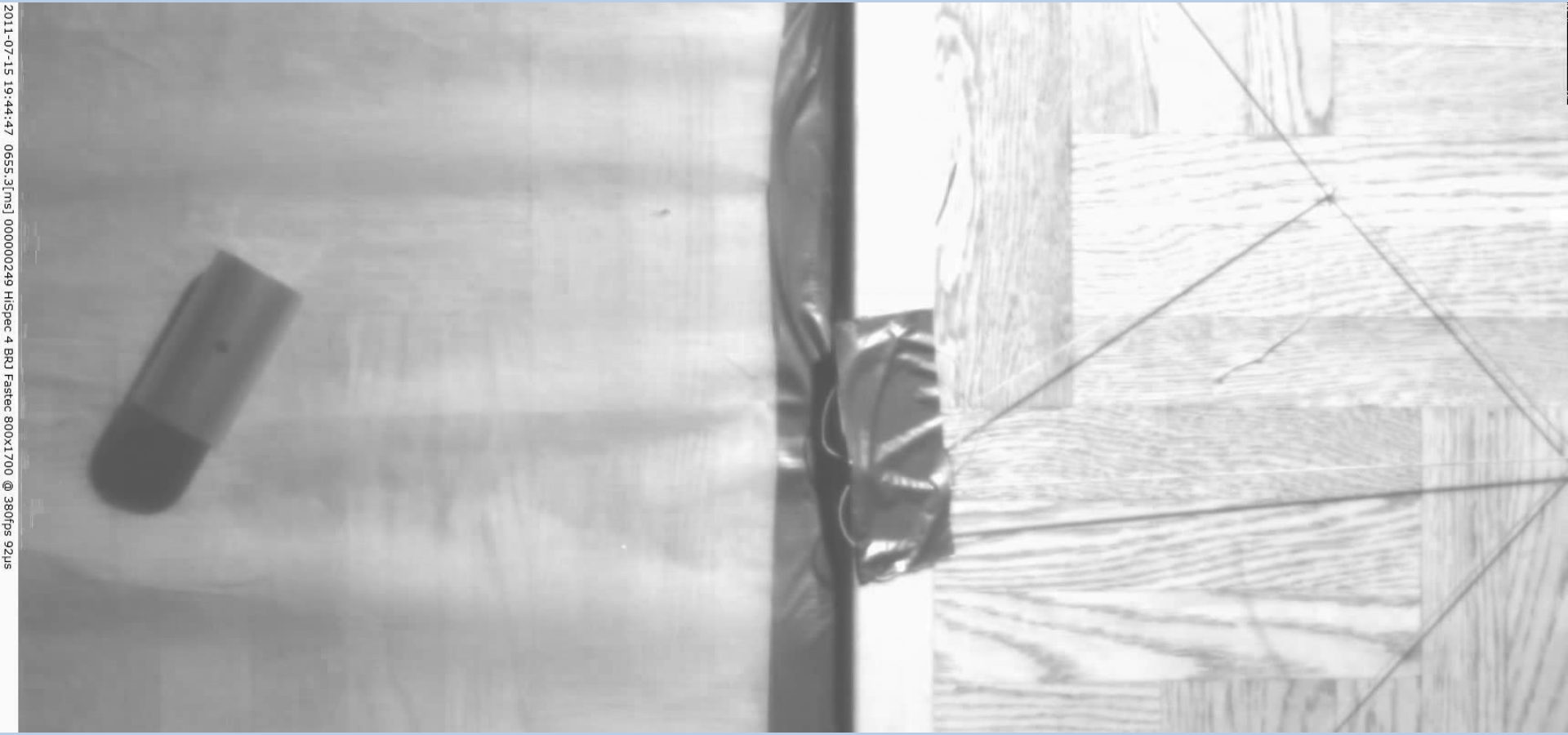
Simulation – dynamic friction



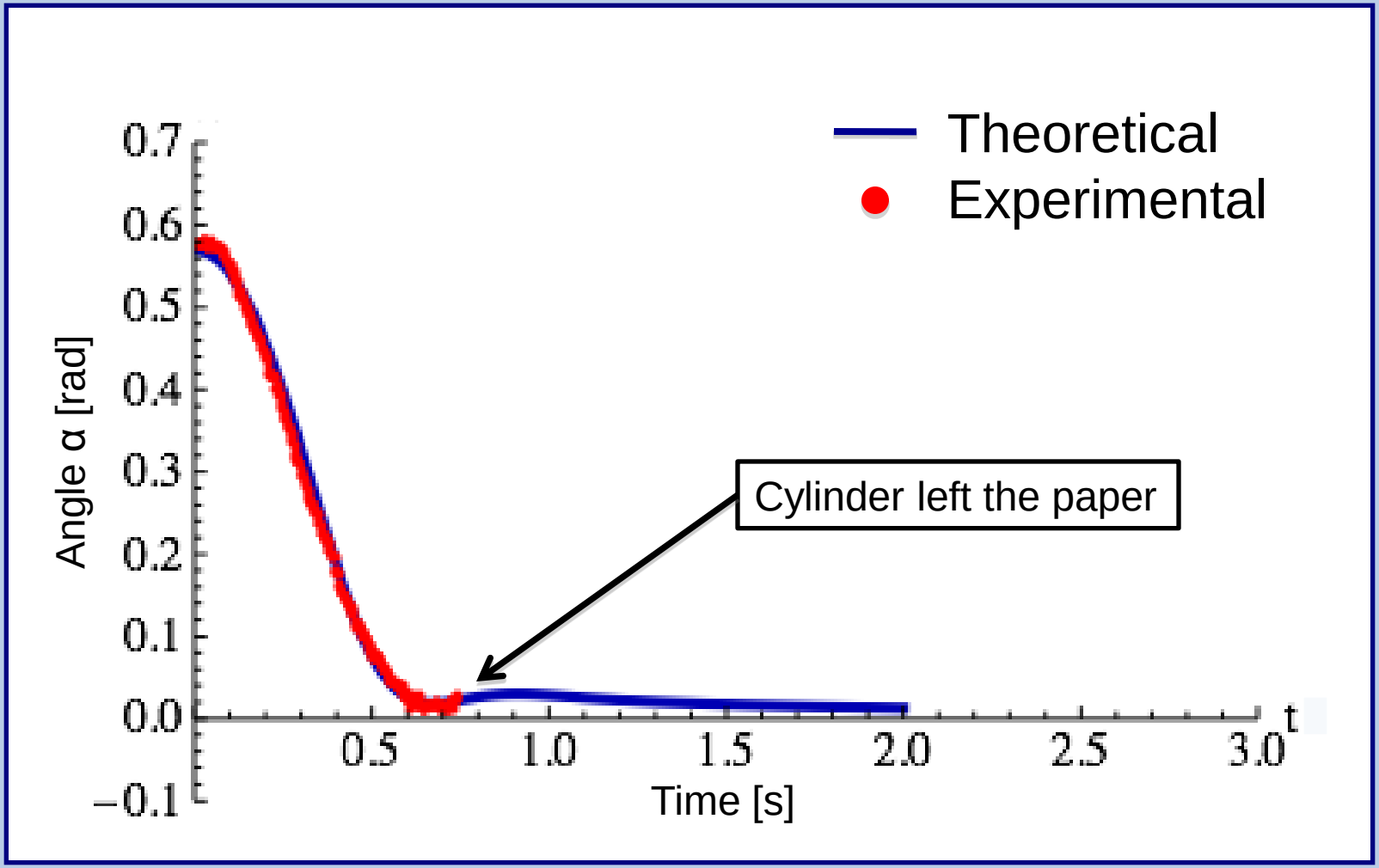
Simulation – dynamic friction



Movie – dynamic friction



Simulation – dynamic friction



Error discussion

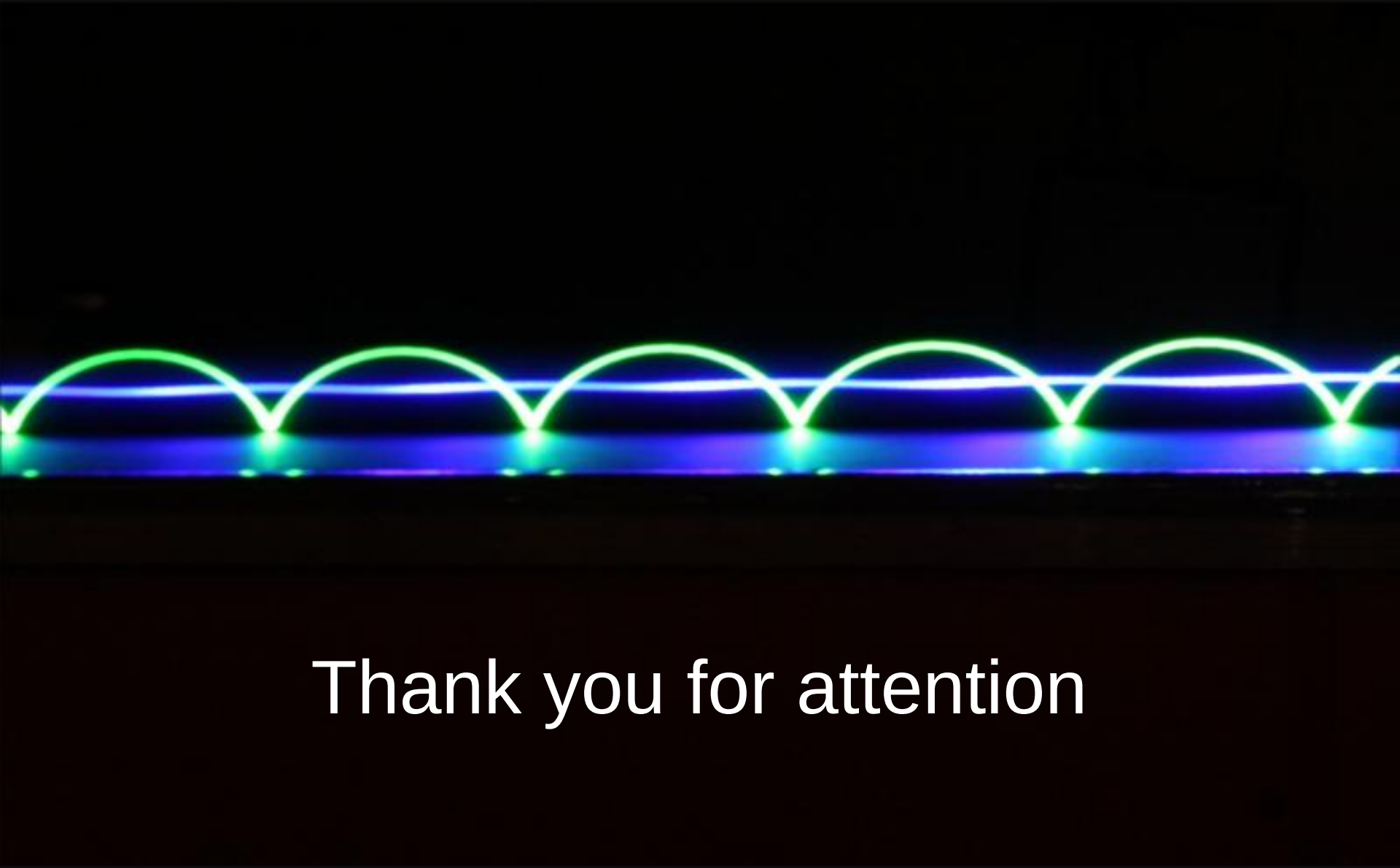
- *Coefficient of friction is varying over the surface*
- *Difference between static and dynamic coefficient of friction*
- *Rolling friction*
- *Imperfect initial friction*

Summary

- *Ratio between linear and angular velocity remains the same independent from parameters such as: coefficient of friction or friction type.*
- *Two reaction force model gives results comparable with experiment. We have built a simulation that allows us to predict cylinders motion*
- *Cylinder in 3D situation tends to change its orientation in such a way that it simply rolls on the paper as in 2D situation.*

Literature

- *John R. Taylor "Classical Mechanics",*
- *H. Goldstein, et al. "Classical Mechanics (3rd Edition)"*



Thank you for attention