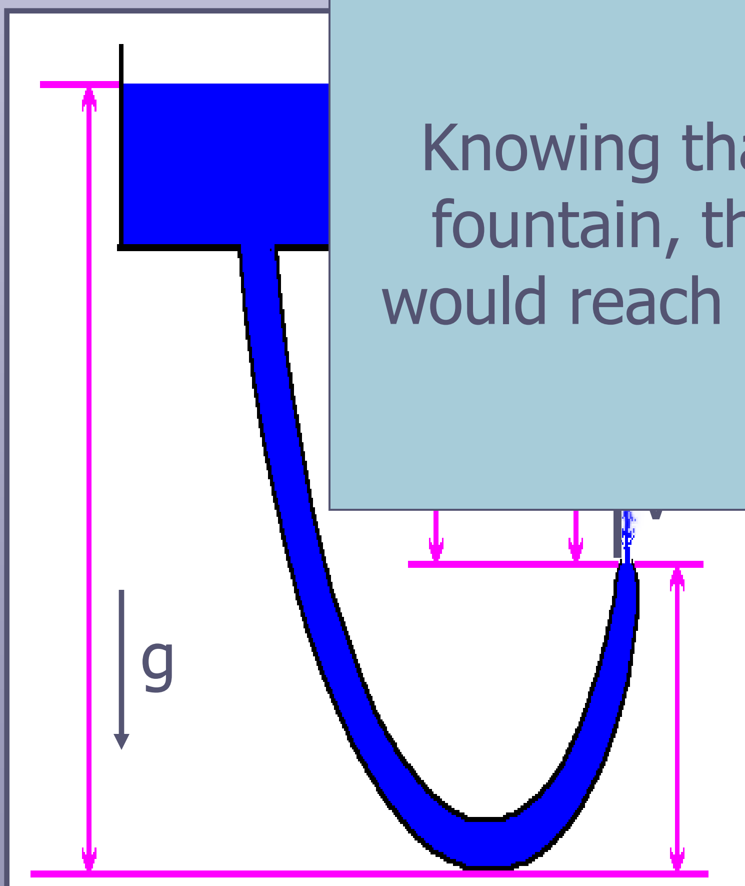
14. Fountain

Construct a fountain with a 1m "head of water". Optimise the other parameters of the fountain to gain the maximum jet height by varying the parameters of the tube and by using different water solutions.

General theory

To calculate what height will the ideal liquid's stream reach with given variables, we may use Bernoulli's equation:



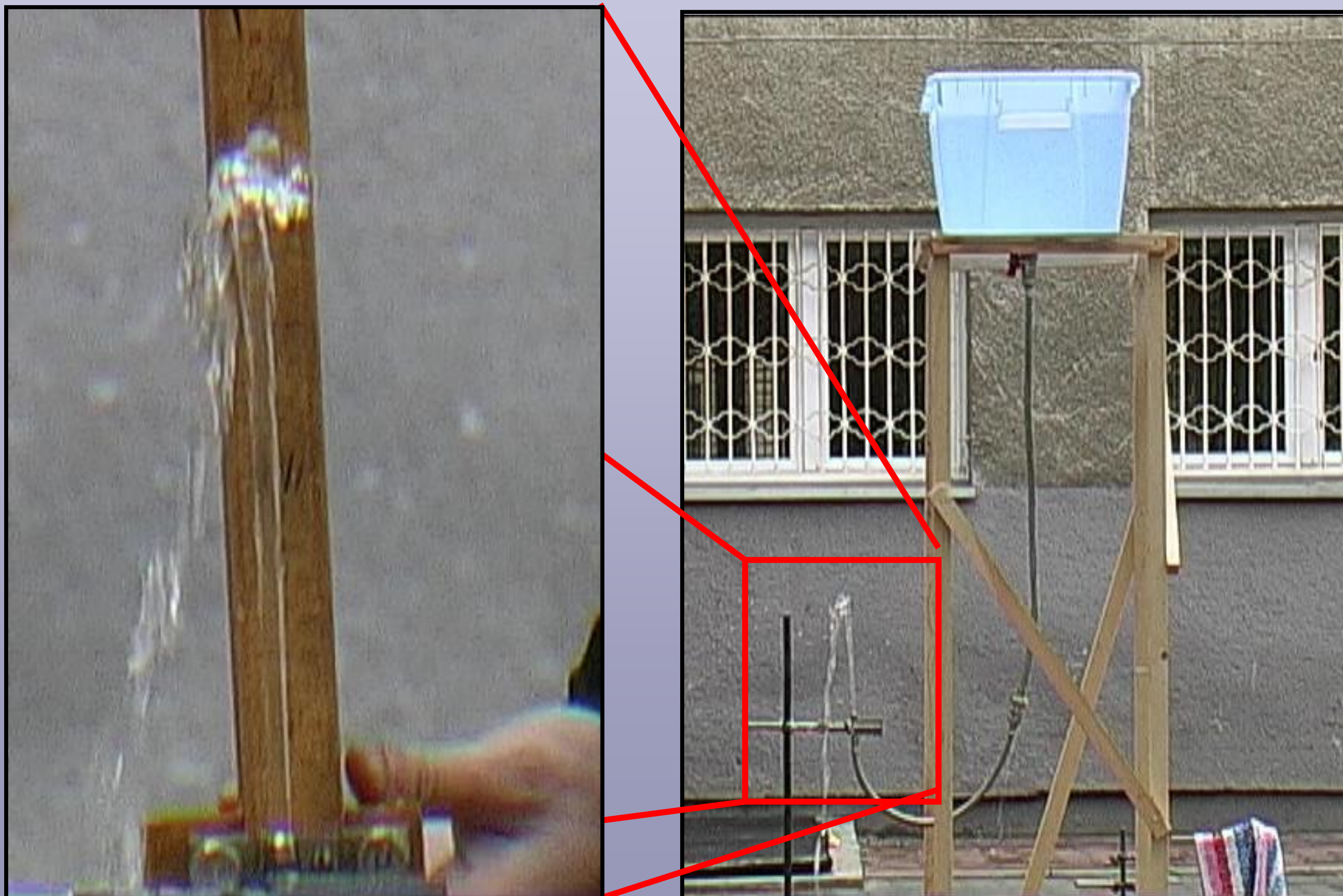
$= const,$

Knowing that, we have constructed a fountain, thinking that the jet height would reach a little less than one meter.

$h = 1m$

Which is an answer to our problem for ideal conditions.

Our first fountain



Why is that theory wrong?

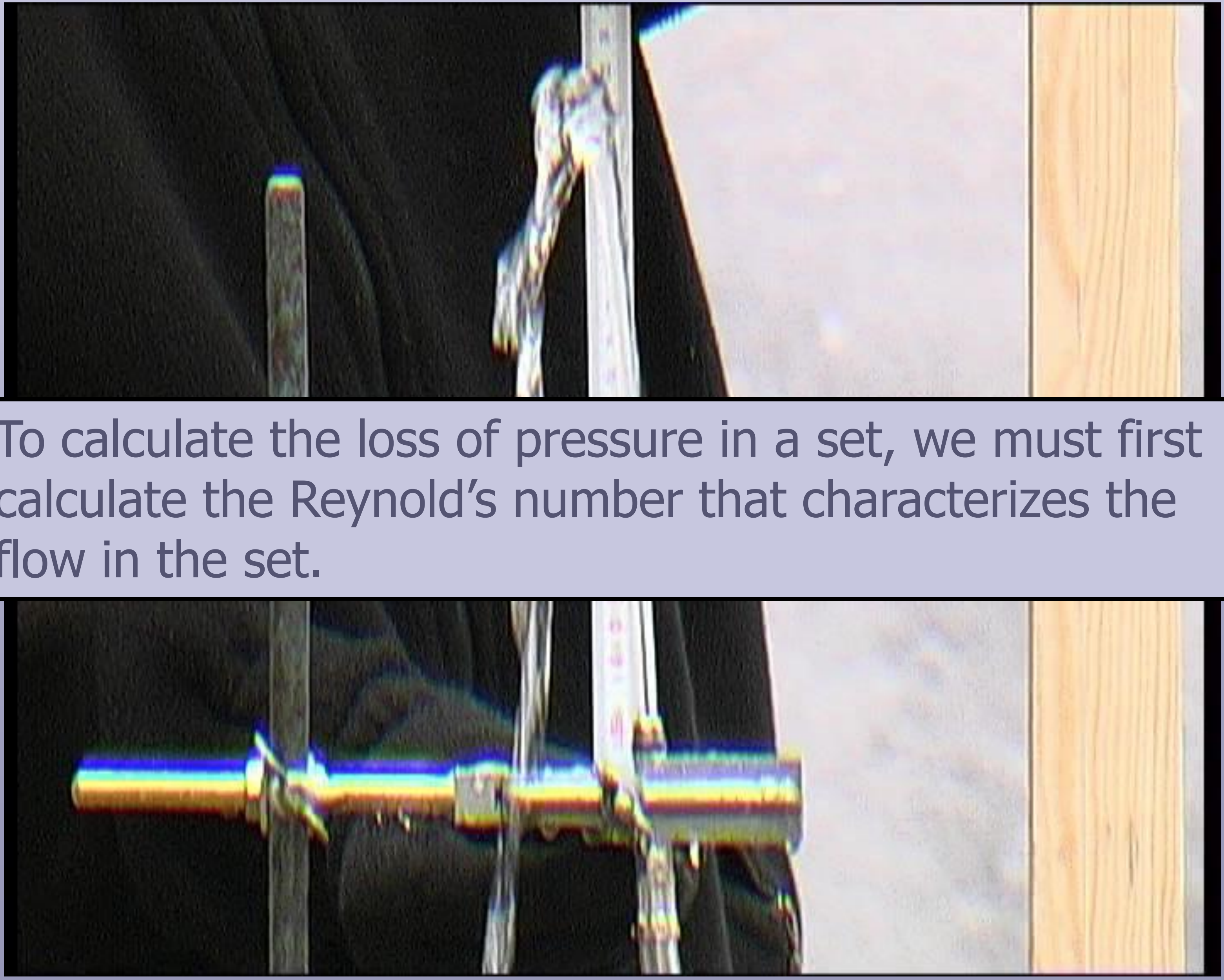
If the liquid in the fountan was ideal, and the resistances equal zero, the speed on the jet would be equal to:

$$V_{\max} \approx 4,4\frac{m}{s} \approx 15,9\frac{km}{h}$$

Those factors have influence on this velocity:

- Linear resitance of flow
- Local resistances of flow
- Air resistance
- Liquids viscosity
- Air's density and pressure
- Jet type
- Type of the flow of water
- Material, that the pipes are made of

Pressure losses



To calculate the loss of pressure in a set, we must first calculate the Reynold's number that characterizes the flow in the set.

Reynold's number

Reynold's number specifies whether the flow is laminar or turbulent.

Reynold's number depends on viscosity coefficient ν, diameter of the flow and liquid's average velocity in the conduit;

$$\mathrm{Re} = \frac{V \cdot l}{\nu} = \frac{\rho \cdot V \cdot l}{\mu}$$

For each fountain the Reynold's number must be calculated separately. It's magnitude should be the smallest possible, because the energy lost by turbulences in the flow is the lowest.

Absolute roughness

Here is a list of the absolute roughness coefficient *k* value for some common materials:

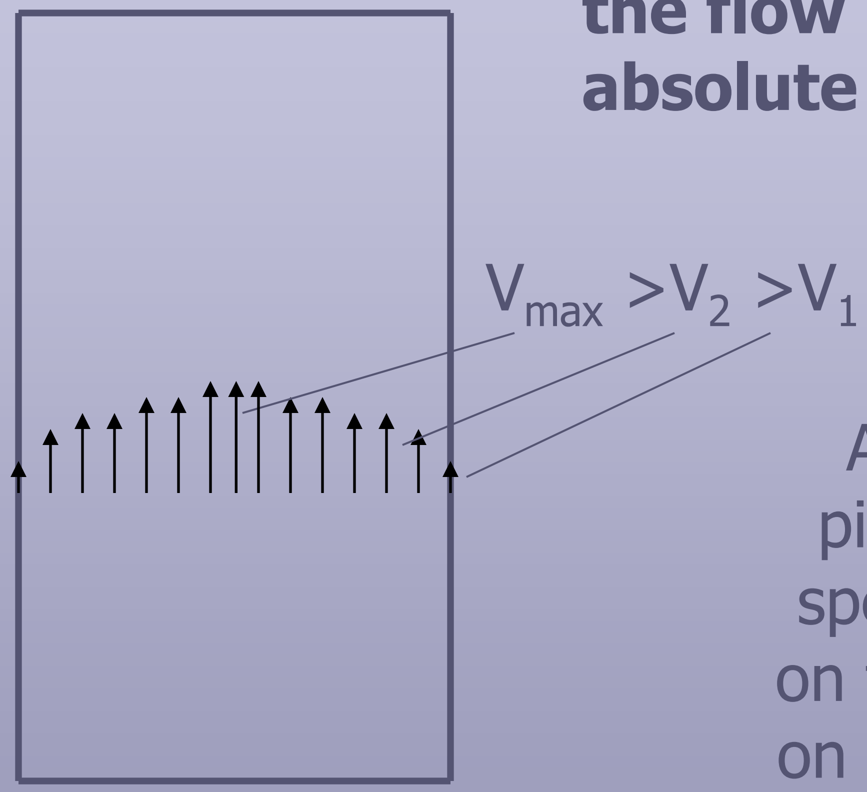
glass	0,0015-0,01
aluminium	0,015-0,06
Steel	0,02-0,10
Corroded steel	0,40
Cast iron	0,25-1,0

$$\varepsilon = \frac{k}{D}$$

***Where epsilon* is the relative roughness, *k* is absolute roughness and D is the flow's diameter**

Relative roughness

The width of the zone in which the flow is slowed depends on the absolute roughness k.



On this picture, the influence of viscosity is omitted.

At some distance from the pipe's surface the flow has a speed which does not depend on the absolute roughness (but on viscosity only). The width of this zone is inversely proportional to relative roughness.

Linear flow resistance

It is the proportion between loss of pressure in the pipe and the specific gravity weight. It is calculated with equation:

$$h_L = \frac{p_1 - p_2}{\gamma} = \lambda \frac{L}{D} \frac{v^2}{2g}$$

where λ is the linear resistance coefficient

Local resistances

Local resistances are calculated from formula:

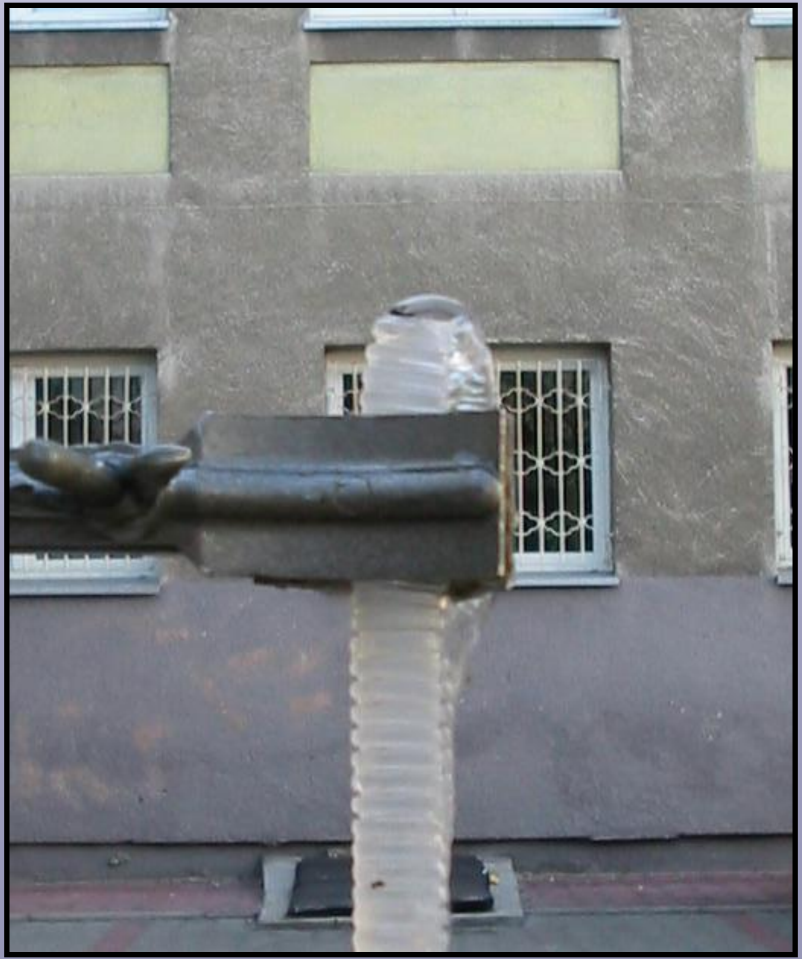
$$h_{lost} = \zeta \frac{V^2}{2g}$$

Where ζ is the local resistances coefficient, and V is an average velocity in the cross-section after the obstacle

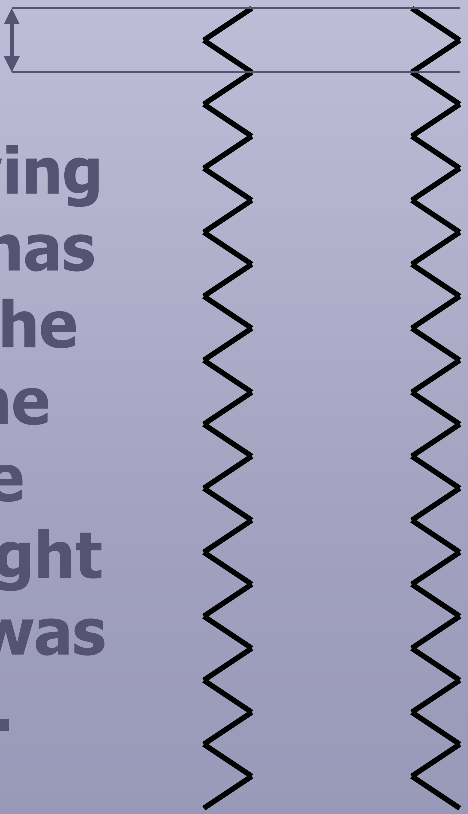
Even smallest increase or decrease of flow's diameter, each valve, bend, etc cause some loss of pressure.

Local resistances

The influence of local resistances is effectively visualized by the height that the stream from one of the fountains we have constructed reached:



Each narrowing in this pipe has decreased the speed of the water. The stream's height in this case was near zero.



What pipe will be the best?

As said before, linear flow resistance are described by equation:

$$h_l = \lambda \frac{L}{D} \frac{V^2}{2g}$$

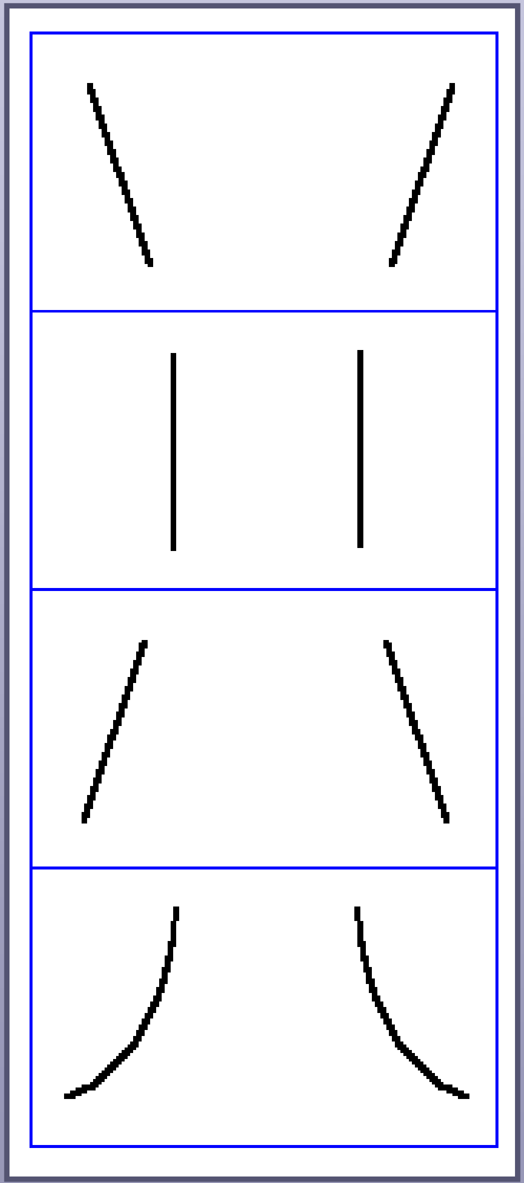
To minimize velocity losses in the flow, we must design such fountain, that length L of the flow would be the smallest, and the diameter D the biggest possible.

$$\frac{L}{D} \approx 1$$

Choosing pipes with lowest roughness, having sizes as above and for low Reynolds numbers, we may calculate:

$\lambda \approx 0.0006$ **so** $h_l \approx 0.0006 \approx 6 \cdot 10^{-4}$

Resitances at waterspout



Conic widening mouthpiece: **turned out to be less effective. Water splashed heavily.**

Cylindric mouthpiece: **It gave various effects, depending on the fountain.**

Conic narrowing mouthpiece: **gave very good results.**

Curved mouthpiece (Weisbach’s jet): **results nearly identical to those from conic narrowing mouthpiece.**

Water solutions

Liquid properties:

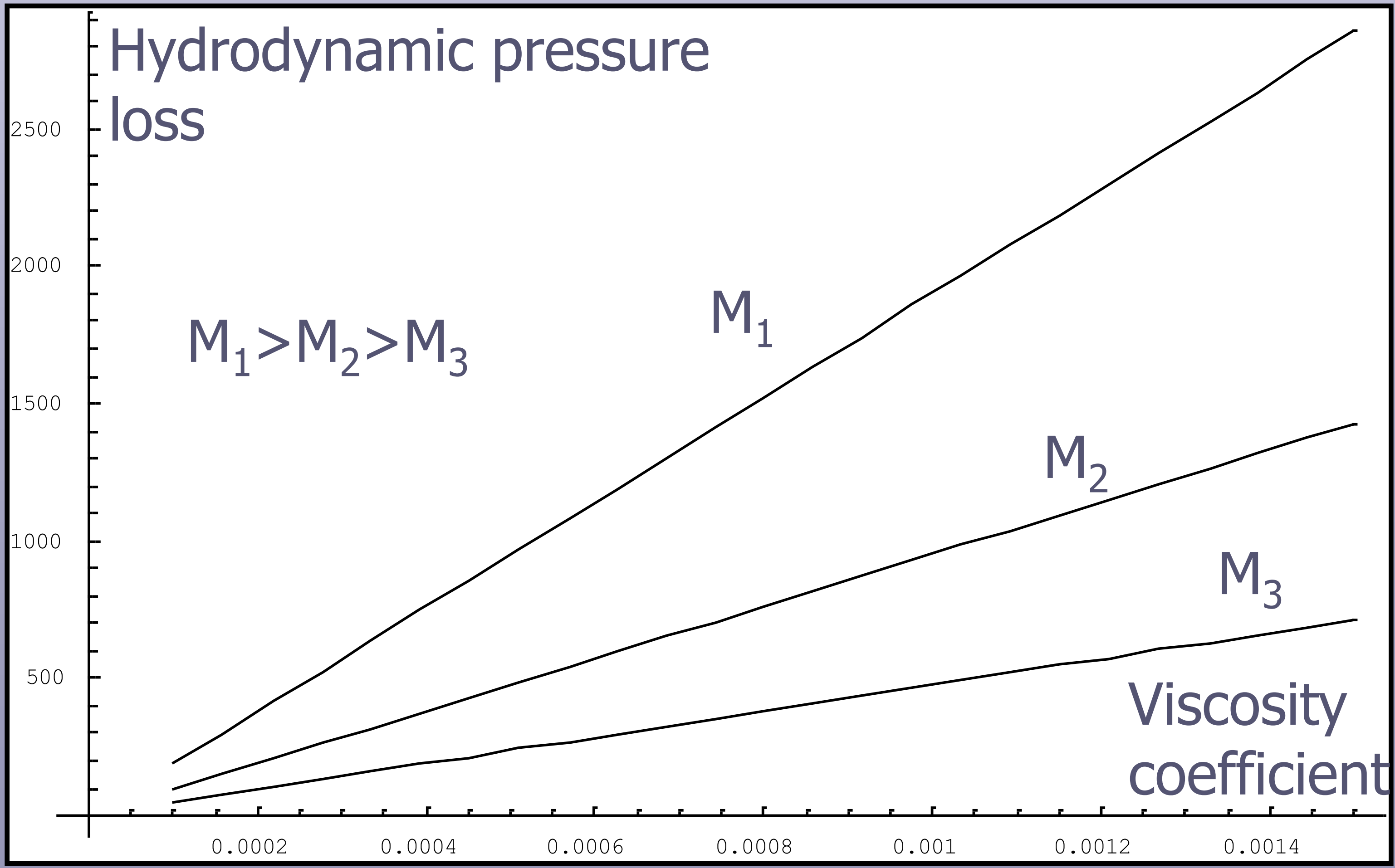
- **density**
- **viscosity**
- **specific heat**
- **compressibility**
- **temperature**
- **surface tension**

We must now find out which of these properties have influence on the height of the liquid's stream.

Viscosity and, indirectly, tempereature, seem to have the biggest influence on the height of the stream.

Liquid's viscosity

From the value of the kinematic viscosity coefficient depends the velocity od the flow lost due to local resistances. The more viscous the liquid is, the higher the resitances are:



The best water solution

To choose the best water solution, we must consider how will it change the water viscosity. Basing on the information from literature, we were able to divide substances into those that:

Lower the viscosity	Increase the viscosity
• soap • ethyl alcohol • fenol	• sugar • salt

To maximize the stream's height, we must minimalize the linear resitances, so the soution's viscosity must be lowest possible.

Our experiments

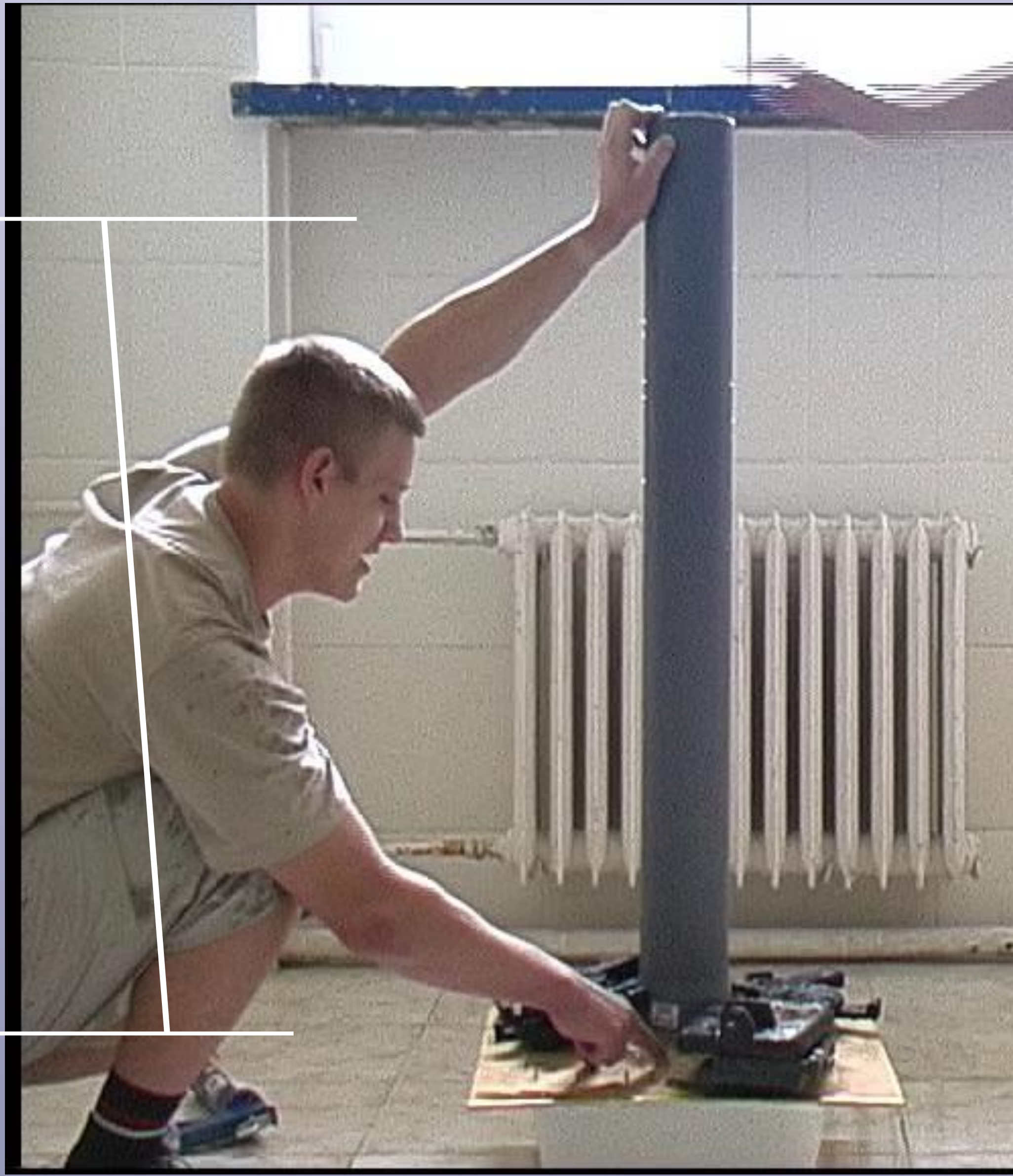
We have tried out three different solutions: water with soap, salt and sugar. Then, we've compared the gathered data with the results for clean water (experiment was carried out on a 0,75 cm diameter conic narrowing mouthpiece)

Clean water	87 cm
Salt solution [1%]	85 cm
Sugar solution [3%]	84 cm
Soap solution	88 cm

The results vary, however, very slightly - they are on the verge of measurement error. However, they seem to confirm our presumptions.

Our experiments

$$h = 0,82m$$



Sugar solution [15%]

Our experiments

$h = 0,78m$

Sugar solution [20%]

Our experiments

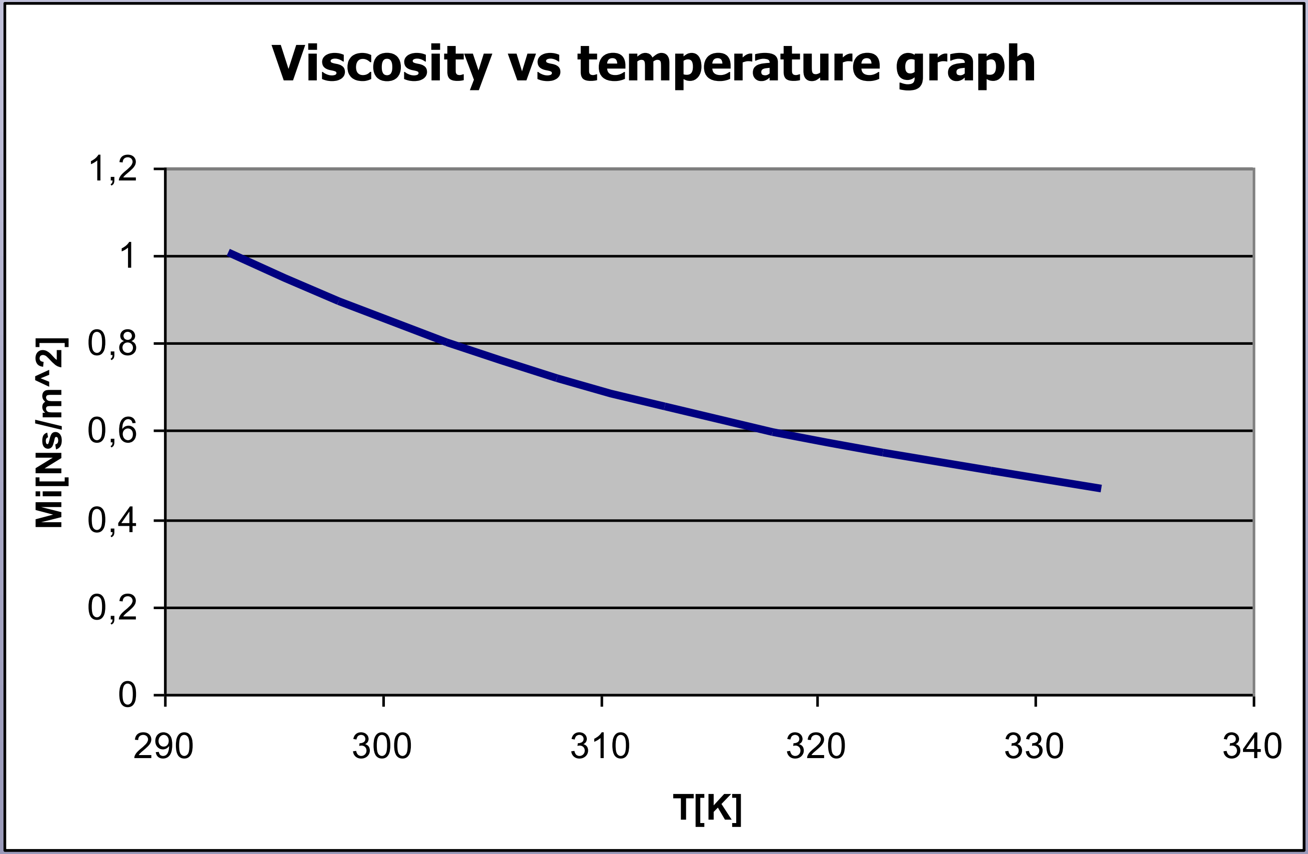
$h = 0,73m$

Sugar solution [25%]

Liquid's temperature influence

As the temperature increases, liquid's viscosity decreases:

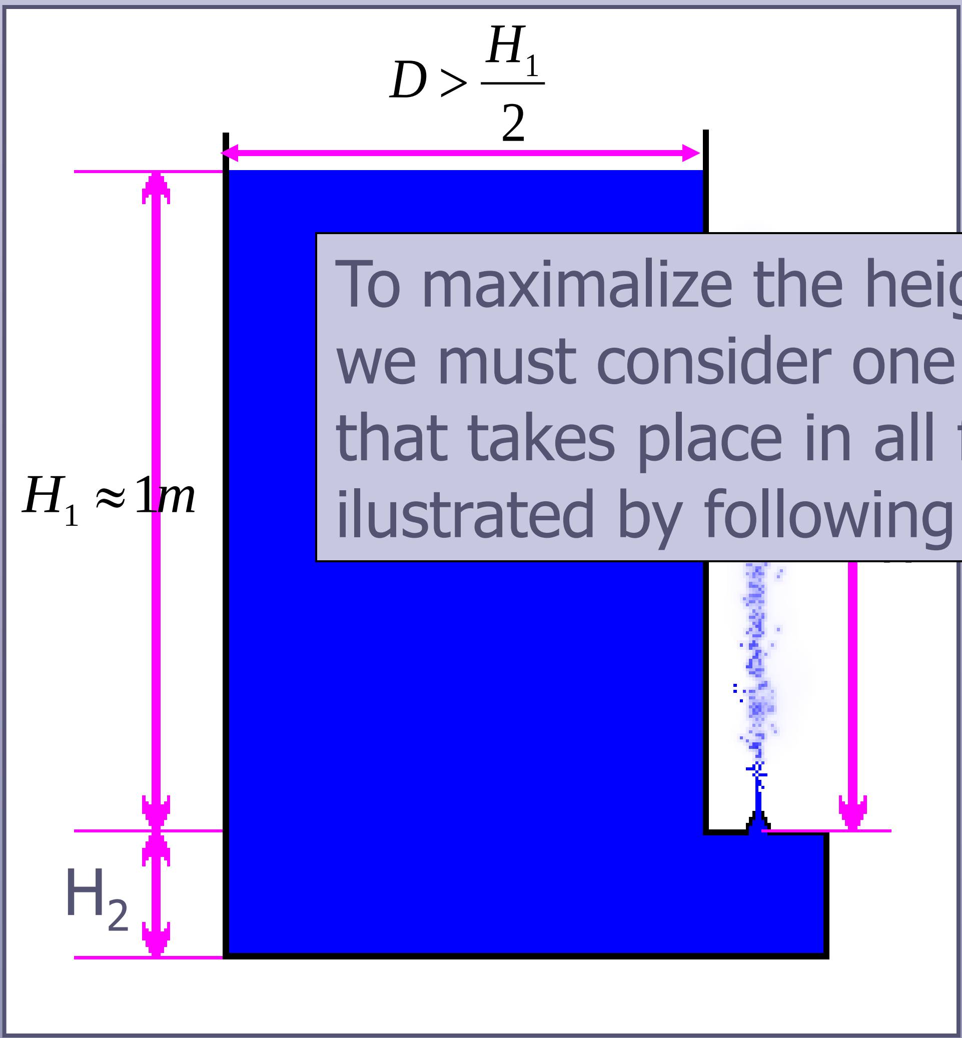
$$\mu = Ae^{\frac{E}{RT}}$$



Our experiments confirmed this theory; hot (ok. 60^{o}C) water gave about 2-3% better results than cold (ok. 10^{o}C).

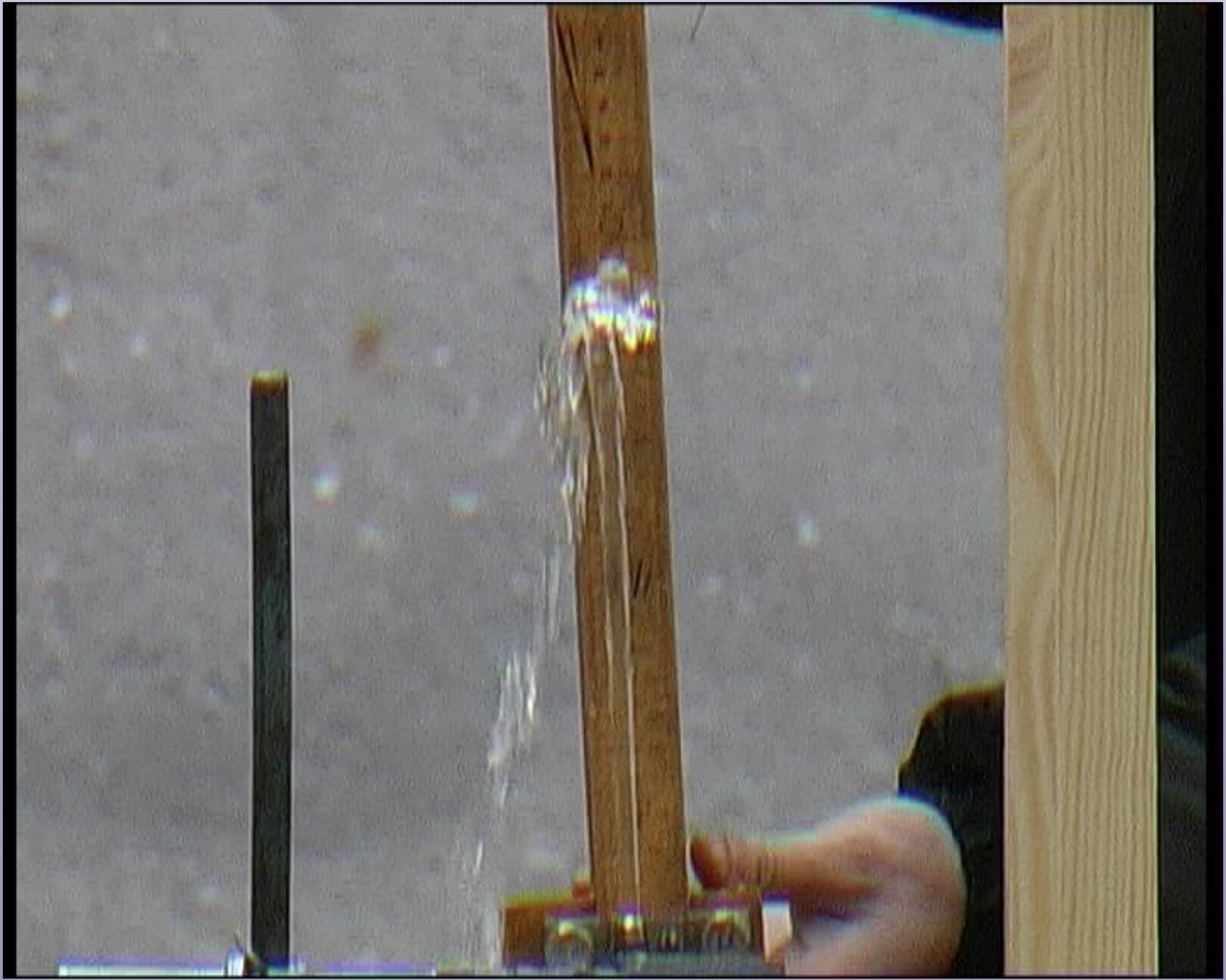
Optimal fountain

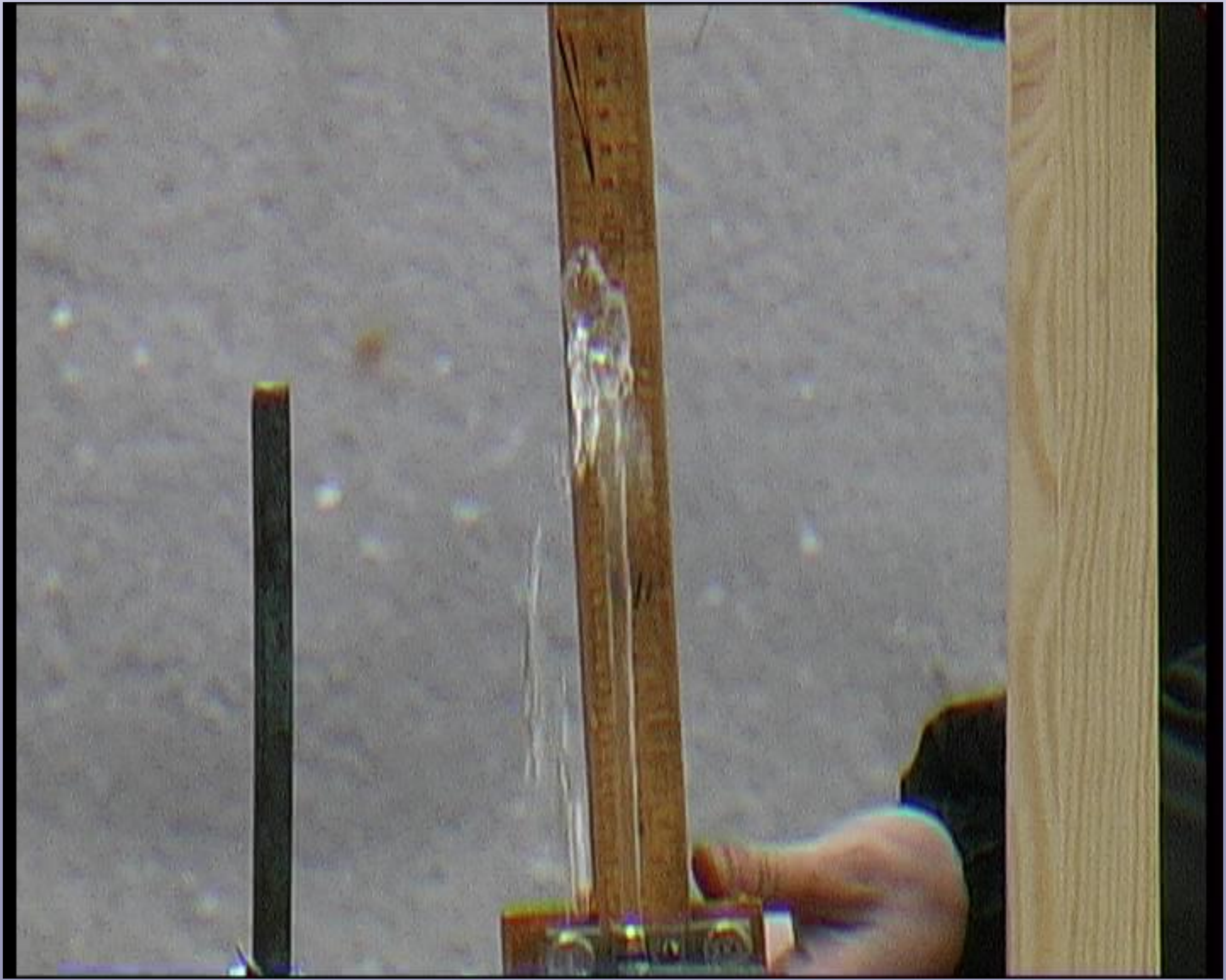
The highest stream height will, in our oppinion, be reached by such fountain:

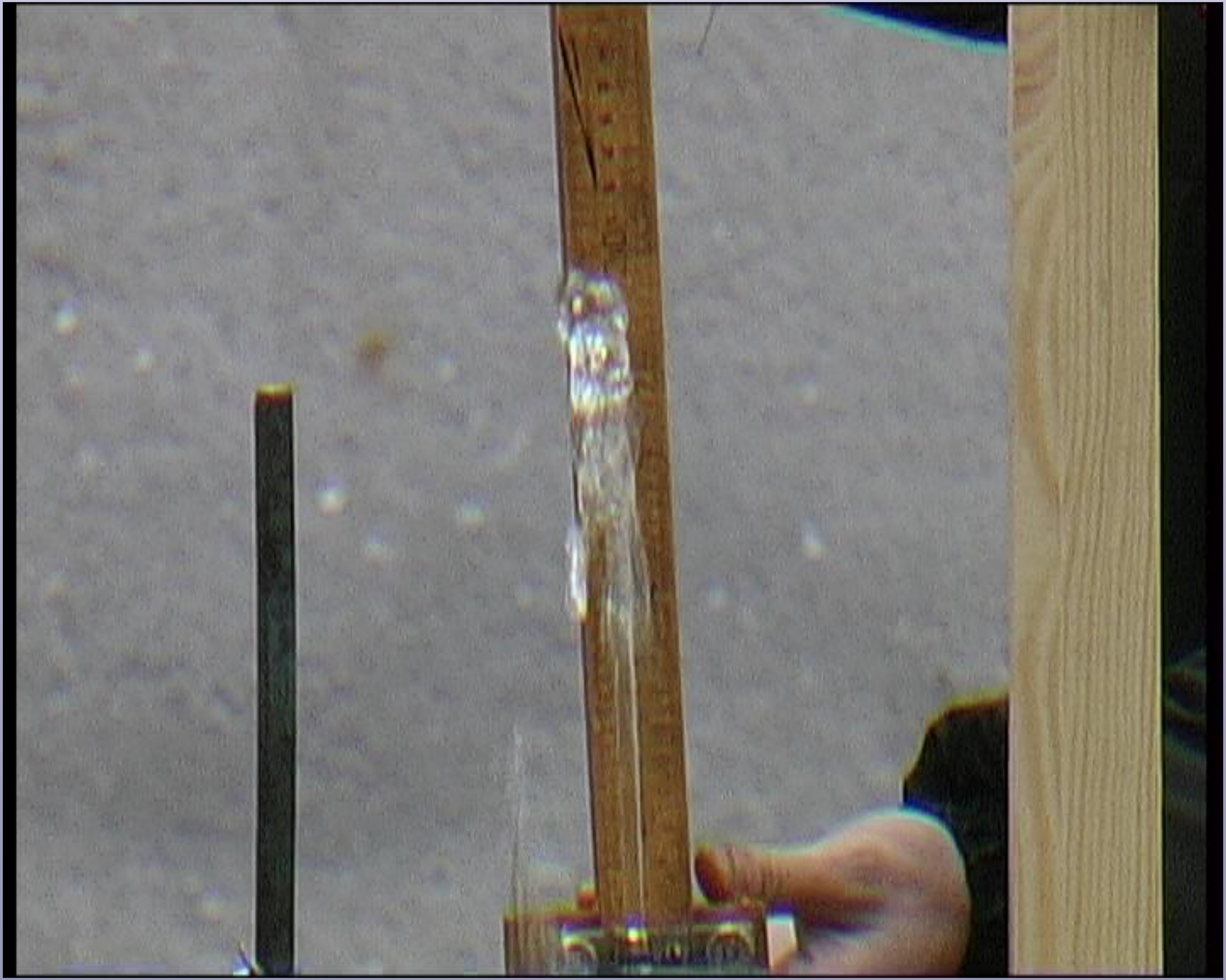


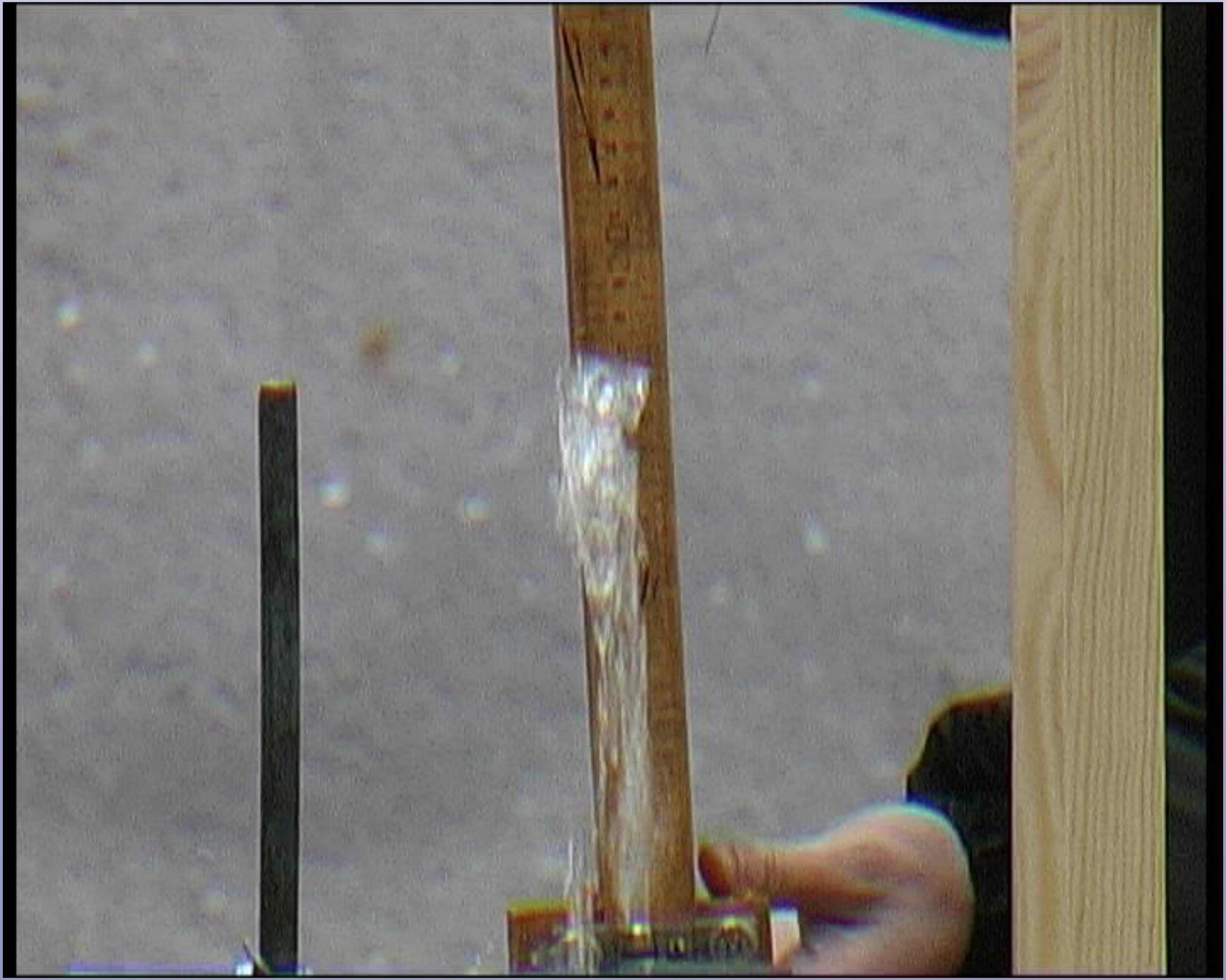
To maximalize the height of the stream, we must consider one more phenomenon that takes place in all fountains. It is ilustrated by following photos.

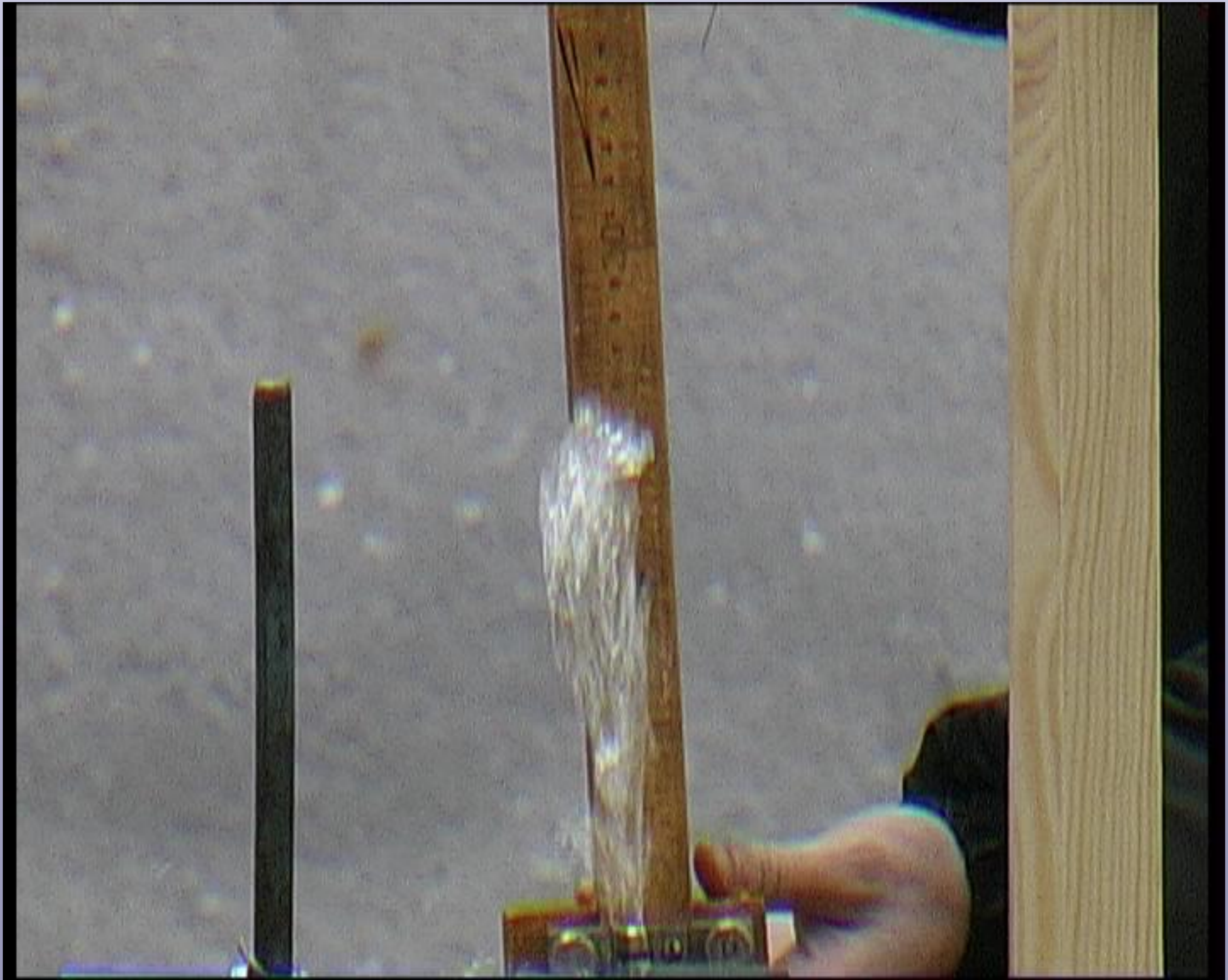
- **Reynolds number for the flow ... ank is very**
- **... izes linear**
- **local resistances exst only at the mouthpiece – everywhere else they are ommitable.**
- **hotest possible soap water solution should be used to minimize viscosity**



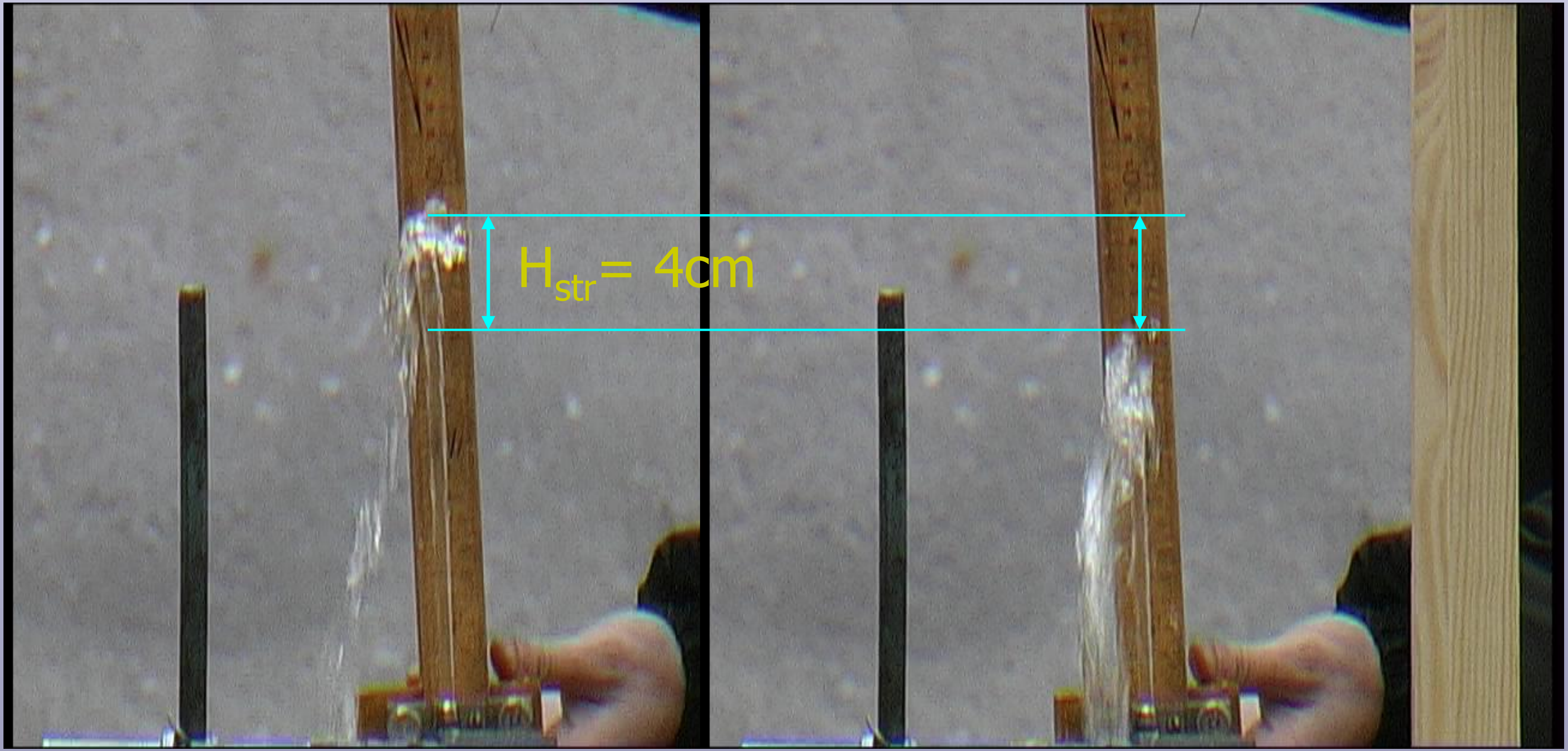








„Falling water”



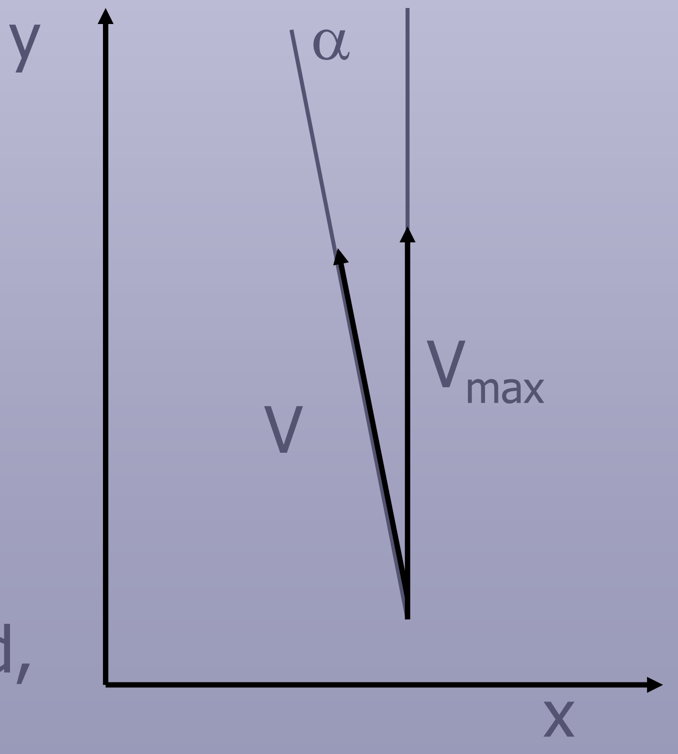
The height lost in this way may reach even 25% maximal height. What can be done to bypass this effect?

„Falling water”

While designing the optimal fountain it is worthy to consider deviating the stream from perpendicularity with gravity field to minimize this effect.

However, it is connected with decreasing the maximal reached height, because the speed of water on vertical axis will be:

$$V = V_{\max} \cos\alpha$$



It needs to be empirically checked, whether it is better to deviate the mouthpiece or not.

Our fountain

Having analyzed the theory, we began constructing our own „optimal" fountain.

We have built a fountain for which the biggest pressure losses are those on mouthpiece, and for which linear and local losses are much smaller, nearly ommitable.

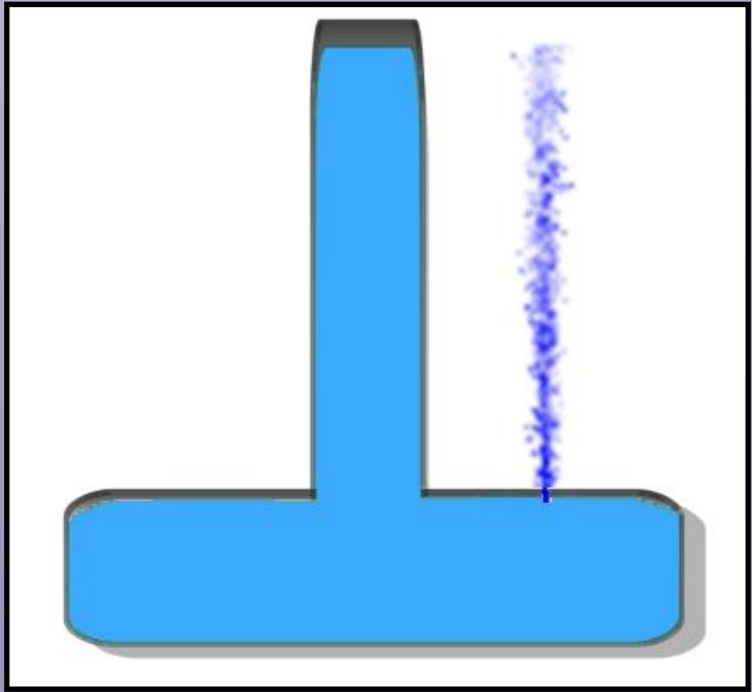
We have used a conic narrowing mouthpiece (Weisbach's jet was too hard to make in school conditions).

We have also decided to deviate the stream from perpendicular of about 4 degrees.

Calculations for our fountain

$\mathrm{Re} < 10$ (for the flow in the tank)

$$V \approx 4{,}15 \frac{m}{s} \approx \frac{95}{100} V_{\max}$$



Our best result

$$h = 0,92m$$

Conclusion

Our fountain’s efficiency reached more than 90%

(92 cm exactly).

There is one more thing to mention: with the problem formuilated this way, it was possible to obtain a height of a stream much greater than one meter by, for example, creating a pressure difference between „head of water” and mouthpiece or by constructing so-called Hero’s fountain.

However, we decided not to consider those matters, and focus on strictly hydrodynamic problems.

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Local resistances

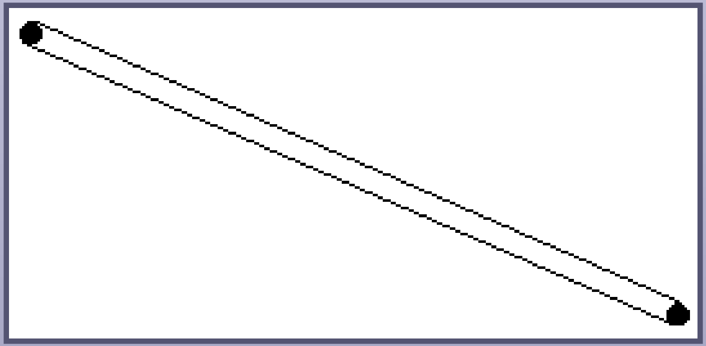
Examplary local resistance coefficients:

Nazwa przeszkody	Współczynnik oporów miejscowych ζ
Wlot z przewodu do zbiornika	$\zeta = 1$
Nagłe zwiększenie przekroju z D_1 do D_2 (tylko dla Re > 3500)	$\zeta = [(\frac{D_1}{D_2})^2 - 1]^2$
Nagłe zmniejszenie przekroju z D_2 do D_1	$\zeta = \frac{1}{2}[1 - (\frac{D_2}{D_1})^2]$
Wodomierz tłoczkowy	$\zeta = 12$
Wodomierz skrętowy	$\zeta = 6$
Kompensator dławikowy	$\zeta \cong 0,2$

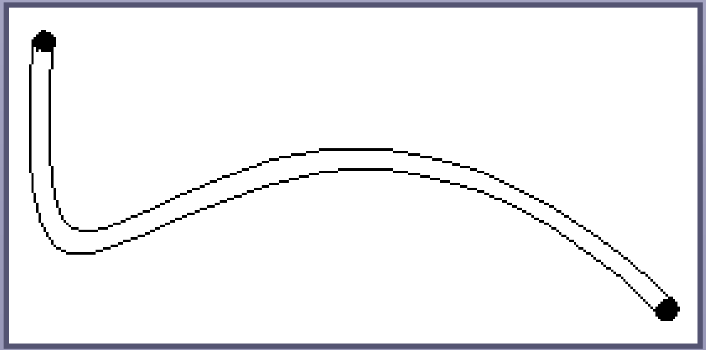
Tablice i wykresy do obliczeń z mechaniki płynów - W. Stefański, K. Wyszk

Pipe's shape

Any pipe may connest two holes. The best connection, from the hydrodynamic point of view is such, that the pipe is ideally straight, because the pressure losses are minimal.



In such pipe only the linear resistance influences the flow.



In such pipe, each narrowing, bend, etc. causes a loss of pressure in the flow.

Resitances at waterspout

Type of hole and mouthpiece	Coefficients			
	throttling φ	Velocity α	discharge μ	Local resitance ξ
Round hole	0,64	0,97	0,62	0,6
Conic widening mouthpiece	1,0	0,45-0,50	0,45-0,50	3-4
Cylindric internal mouthpiece	1,0	0,707	0,707	1,0
Cylindric external mouthpiece	1,0	0,82	0,82	0,5
Conic narrowing mouthpiece	0,98	0,96	0,94	0,09
Curved mouthpiece	1,0	0,98	0,98	0,04

Taken from: ***Tablice i wykresy do obliczeń z mechaniki płynów*** **- W. Stefański, K. Wyszkowski**

Ideal fountain's general assumptions

We aim at a set, in which the influence of all earlier mentioned factors will be optimised for depreciating the flow's resistance and to increase the discharge coefficient:

- smallest possible pipe's length, large diameter and possibly smallest roughness: linear flow resistance → 0;
- lack of narrowings, bends and any other obstacles on water's way: local flow resistances → 0;
- Weischbach's jet instead of usual round hole → discharge coefficient → 1;
- Low viscosity liquid: flow resistances → 0

Pipe types

Each pipe is characterized by few parameters:

- length – L
- internal diameter – D
- material of which it is made and, connected with it, it's absolute roughness – k;

This pipe is, inside, made of rubber

Crimping, plastic pipe – extremely large roughness