11. Water droplets

If a stream of water droplets is directed at a small angle to the surface of water in a container, droplets may bounce off the surface and roll across it before merging with the body of water. In some cases the droplets rest on the surface for a significant length of time. They can even sink before merging. Investigate these phenomena.
PRESENTATION OUTLINE

WATER DROPLETS

‘ROLLING’ DROPLETS

‘BOUNCING’ DROPLETS

‘SINKING’ DROPLETS

DROPLET LIFE-TIME
DESCRIBED PHENOMENA
Investigated substances:
1. Water
2. Water with soap
3. Vegetable oil
4. Ethyl alcohol

Vessels:
1. Flat disk
2. Bowl
3. Large beaker

Others:
1. Syringe
2. Pipette
ROLLING DROPLETS

1. Water with soap
2. Vegetable oil
3. Water
if water molecules inside the droplets have the contact with water molecules in the vessel, the merge occurs

to obtain rolling droplets, there must be the distinct boundary between droplet and water in the vessel
BOUNCING DROPLETS

1., 2., 3. water
4. water with soap
To obtain bounce, there must be big velocity and momentum of droplet (in perpendicular direction)

1. Droplet ‘hits’ the surface of liquid
2. Hit compresses air under droplet
3. Compressed air throws the droplet away - bounce
FILM 500 FPS
THEORETICAL MODEL

Considering bounce as a totally elastic collision:

\[
\frac{m(\sqrt{V_x^2 + V_y^2})^2}{2} + \sigma_{air} S = \frac{m(\sqrt{\alpha V_x^2 + \beta V_y^2})^2}{2} + \sigma_{water} S_1 + \sigma_{air} S_2
\]

Where:
- \( m \) – mass of droplet
- \( V_x, V_y \) – elements of droplet velocity
- \( \sigma_{air}, \sigma_{water} \) – surface tensions
- \( S \) – droplet area

Energy conservation principle for droplet

\[
m\left[ V_x^2 (1 - \alpha) + V_y^2 (1 - \beta) \right] = 2(\sigma_{water} - \sigma_{air}) S_1
\]
We measured $\alpha$ and $\beta$ using film 25 fps.

$$m \left[ V_x^2 (1-\alpha) + V_y^2 (1-\beta) \right] = 2(\sigma_{water} - \sigma_{air}) S_1$$

$\alpha = 0.60 \pm 0.05$  $\beta = 0.20 \pm 0.05$
SINKING DROPLETS
SINKING DROPLETS
1. Formation of the phenomenon

1. Droplet hits the surface of liquid
2. Droplet sinks with the air around it
3. Droplet is closed in the bubble
2. Water droplet as ‘antibubble’
3. How does the antibubble form?

Air bubble is formed from the air closed under the droplet during hitting surface of the water in the vessel.

‘Antibubble’ is a droplet closed in air bubble.
4. Under what conditions does the antibubble occur?

We obtained antibubbles only using water with soap. It is caused by the structure of soap molecules, part of which is hydrophobic and another part is hydrophylllic.
AIR LAYER THICKNESS CALCULATIONS

$F_b$ – buoyant force

$F_{gr}$ – gravity force

$r_1$ – radius of droplet (with air layer)

$r_2$ – radius of droplet (without air layer)

$(r_1 - r_2)$ – thickness of the air layer

$F_b > F_{gr}$

Antibubble moves upwards

From this movement we can calculate thickness of the air layer

$(r_1 - r_2) \sim 10^{-3}$ mm
PRSENTATION OUTLINE

WATER DROPLETS

‘ROLLING’ DROPLETS

‘BOUNCING’ DROPLETS

‘SINKING’ DROPLETS

DROPLET LIFE-TIME
Potential between droplet and water: ca. 3mV
Potential difference in influence

Potential difference between droplet and water in the container makes the phenomena hard to obtain, because it causes attraction of the water particles in droplet and container.

Potential difference

Droplets merge in short time
POTENTIAL DIFFERENCE

No potential between droplet and water

Water in the container
CONCLUSIONS

• The reason for delining discussed phenomena is a thin air layer between droplet and air in the container.

• The air layer is ca. $10^{-3}$ mm thick.

• Presence of soap or other surface-active substances has a big influence on the phenomenon.
1. E. M. Rogers *Fizyka dla dociekliwych* tom 1
2. S. Frisz, A. Timoriewa *Kurs Fizyki* tom 1
3. J. W. Kane, M. M. Sternheim *Fizyka dla przyrodników* tom 2
4. I. W. Sawieliew *Wykłady z fizyki* tom 1
5. Z. K. Kostic *Między zabawą a fizyką*
6. http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html
In liquid there are strong intermolecular interactions:

- In case of molecule allocated inside liquid all the intermolecular forces acting on it undergo neutralization (1.)

- In case of molecule allocated near the surface of there are no intermolecular forces acting on it in upward direction. Hence, downward forces are not neutralized (2.)
Surface tension can be given by the equation:

\[ \alpha = \frac{W}{S} \left[ \frac{J}{m^2} \right] \]

where:
- \( \alpha \) – surface tension
- \( W \) – work done
- \( S \) – change of free surface

Each substance has different \( \alpha \) coefficient:

- Pure water: \( \alpha_w = 7,28 \cdot 10^{-2} \left[ J/m^2 \right] \)
- Water and soap: \( \alpha_s = 4,5 \cdot 10^{-2} \left[ J/m^2 \right] \)
- Vegetable oil: \( \alpha_o = 3,2 \cdot 10^{-2} \left[ J/m^2 \right] \)
- Ethyl alcohol: \( \alpha_a = 2,23 \cdot 10^{-2} \left[ J/m^2 \right] \)
Analysis of droplet’s shape

We consider balance state for spherical surface with surface tension $\alpha$ and radius $r$:

$$2\pi r \alpha = (p_w - p_z) \pi r^2$$

After dividing by $\pi r^2$:

$$\left( p_w - p_z \right) = \frac{2\alpha}{r}$$

Laplace law for spherical surface

The smaller radius of the sphere, the bigger pressure inside it.

- $r$ – radius of the sphere
- $p_w$ – pressure inside sphere
- $p_z$ – pressure outside sphere
THEORETICAL ANALYSIS

Pressure inside droplet versus radius of droplet
MODEL

Two liquids have different values of surface tension coefficient and density

- All forces acting on a bubble are due to surface energy change and different densities of the liquids
- We consider movement of the bubble’s center of mass - all forces acting on the bubble are applied to the centre of mass
- We neglect

\[ F_b = F_g \]
http://www.lsbu.ac.uk/water/
Bubble crosses a flat cracking interface without its deformation

Forces acting on a bubble:
- Buoyant force $F_B(h)$ – changes because of the $\Delta \rho \neq 0$
- Surface tension forces $F_\sigma(h)$ – due to change of surface energy

$$\vec{F}(h) = \vec{F}_B(h) + \vec{F}_\sigma(h)$$

Resultant force
Bubble’s radius
Archimedes’ buoyant force

Buoyant force is a sum of two terms due to upper part and lower part of the bubble:

\[ \vec{F}_B = \vec{F}_u + \vec{F}_l = -\vec{g}(V_u \rho_u + V_l \rho_l) \]

hence:

\[ F_B = \pi g (4 \rho_l R^3 + h^2 \Delta \rho (h - 3R)) / 3 \]

Density:

- the upper liquid \( \rho_u \)
- the bottom liquid \( \rho_l \)

\( \rho_l - \rho_u = \Delta \rho \geq 0 \)
Surface tension forces

Potential energy of the bubble and part of the interface involved equals:

$$E_c = E_u + E_l + E_{in} = \sigma_u S_u + \sigma_l S_l + \sigma_{in} S_{in}$$

By applying geometry to calculate surfaces $S_u, S_l, S_{in}$ we obtain:

$$E_c = \pi(R^2 + h(h - 2R))\sigma_{in} + 2\pi R(h(\sigma_u - \sigma_l) + 2\sigma_l R)$$

Surface tension of:

- the top liquid
- the interface between liquids
- the bottom liquid
In order to find forces acting on the bubble due to surface tension we can find a gradient of expression for surface potential energy:

\[ E_c = \pi(R^2 + h(h - 2R))\sigma_{in} + 2\pi R(h(\sigma_u - \sigma_l) + 2\sigma_l R) \]

hence:

\[ F_\sigma = -dE_c / dh = 2\pi(R(\sigma_l - \sigma_u + \sigma_{in}) - \sigma_{in} h) \]

If \( h < R \):
- \( F_s \) acts downwards
If \( h > R \):
- \( F_s \) acts upwards
Resultant force

Resultant force is a sum of buoyant and surface tension forces:

\[ F_c = 2\pi(R(\sigma_l - \sigma_u + \sigma_{in}) - \sigma_{in}h) + \pi g(4\rho_l R^3 + h^2 \Delta \rho(h - 3R))/3 \]

\( R = 1 \text{ cm} \)

mercury - water
Resultant force

\[ F_c = 2\pi (R(\sigma_l - \sigma_u + \sigma_{in}) - \sigma_{in} h) + \pi g (4\rho_l R^3 + h^2 \Delta \rho (h - 3R))/3 \]

\[ R = 1 \text{ mm} \]

mercury - water
A magnitude of potential energy’s minimum is greater when the bubble is smaller because:

- Surface tension phenomenon has greater impact on bubble’s motion
- Buoyant force is far smaller than surface tension force
- Kinetic energy is smaller than surface free energy
Bubbles at an interface

A bubble will stop flowing out, if bouyant force equals zero (special case $\rho_u=\rho_1$): $F_w = 0$:

$$h_0 = 2\rho g R^3 / 3 + R(\sigma_{in} + \sigma_l - \sigma_u) / \sigma_{in}$$

If $0 < h < 2R$ is satisfied, the bubble will stop flowing out.

The radius of such bubble is equal (for any given $\rho_g$ and $\rho_d$):

$$R_{min} = \sqrt{3(\sigma_{in} + \sigma_u - \sigma_1)/2\rho_u g}$$

If $(\sigma_{in} + \sigma_u - \sigma_1) \leq 0$ any bubble will cross the interface.
The bubble stops at the interface

Water

Liquid honey

$R = 1.5\,\text{mm}$