7. Bouncing plug

Fill a bath or sink with water and remove the plug. Then place a plastic ball over the plughole. As the water drains the ball starts to oscillate. Investigate the phenomenon.
Presentation of the phenomenon
Experimental setup
The bathtub vortex

Euler liquid – incompressible and inviscid

No viscous shear stress

No torque about central axis

Angular momentum conserved
The bathtub vortex

Conservation of angular momentum for one particle gives:

\[ \omega = \omega_0 \frac{R^2}{r^2} \]

The radial pressure gradient:

\[ \frac{\partial p}{\partial r} = \rho \omega^2 r = \frac{\rho \omega_0^2 R^4}{r^3} \]

Pressure in point A:

\[ p_A = \rho g \left( y_0 + \epsilon \right) \]

The radial pressure gradient is connected with the shape of the surface:

\[ \frac{\partial p}{\partial r} = \rho g \frac{d \epsilon}{dr} \]
The bathtub vortex

Integrating the previous equation, we obtain the dependence $\varepsilon(r)$

$$
\varepsilon(r) = \int \frac{\omega_0^2 R^4}{gr^3} \, dr = \frac{\omega_0^2 R^4}{2gr^2}
$$

$\omega_0 = 1$

$R = 0.5$

$g = 9.81$
Vortex Structure
Ball in a vortex

1. The ball is attracted to the bottom of container.

2. The ball close to the drainhole destroys the vortex structure.

3. Due to this fact the resultant force acts upwards (buoyancy force).

4. The vortex flow appears again.
Measurements

Drainhole diameter:
\[ \varnothing = 25 \text{ mm} \]
\[ \varnothing = 30 \text{ mm} \]

\[ \gamma = 0.4 \]
\[ \gamma = 0.95 \]
\[ \gamma = 0.06 \]
Experimental data

Plughole diameter 2.5 cm

\[ \gamma = 0.4 \]

\[ \gamma = 0.95 \]
Experimental data

Plughole diameter 3.0 cm

\[ \gamma = 0.4 \]

\[ \gamma = 0.06 \]

\[ \gamma = 0.95 \]
Our best result

Ball position

Time [frames]

Green ball, 3.0 cm drainhole
Acceleration dependence on time

Displacement

Time [frames]
1. For different balls (with different diameter to mass ratio) we obtain different character of oscillations.

2. We can determine the acceleration on time dependence for each ball.

3. The main forces, having crucial influence on the phenomenon are buoyancy force and force due to pressure difference caused by vortex flow.

4. The oscillations seem to have irregular run – slower upwards motion, fast falling.

5. The bigger the drainhole, the smaller the oscillations of balls.

6. The character of the oscillations can be qualitatively described when considering pressure distribution.
Acknowledgements

• PhD Tomas Bohr for permission to use his theoretical materials
Literature

• T.E. Faber „Fluid Dynamics for Physicists”, Cambridge University Press 1995
• T. Bohr, „Anatomy of a bathtub vortex”, Fluid Dynamics
Ball in a vortex
Mechanism of the phenomenon
Von Karman vortices
Qualitative analysis

Why is the plug bouncing?
Von Karman vortices
Von Karman vortices form behind a body, that is surrounded by a flowing liquid at a certain Reynolds number range.

**Drag force:**

\[ F_D = c_D A \frac{\rho \nu^2}{2} \]

**Lift force:**

\[ F_L = c_L A \frac{\rho \nu^2}{2} \]