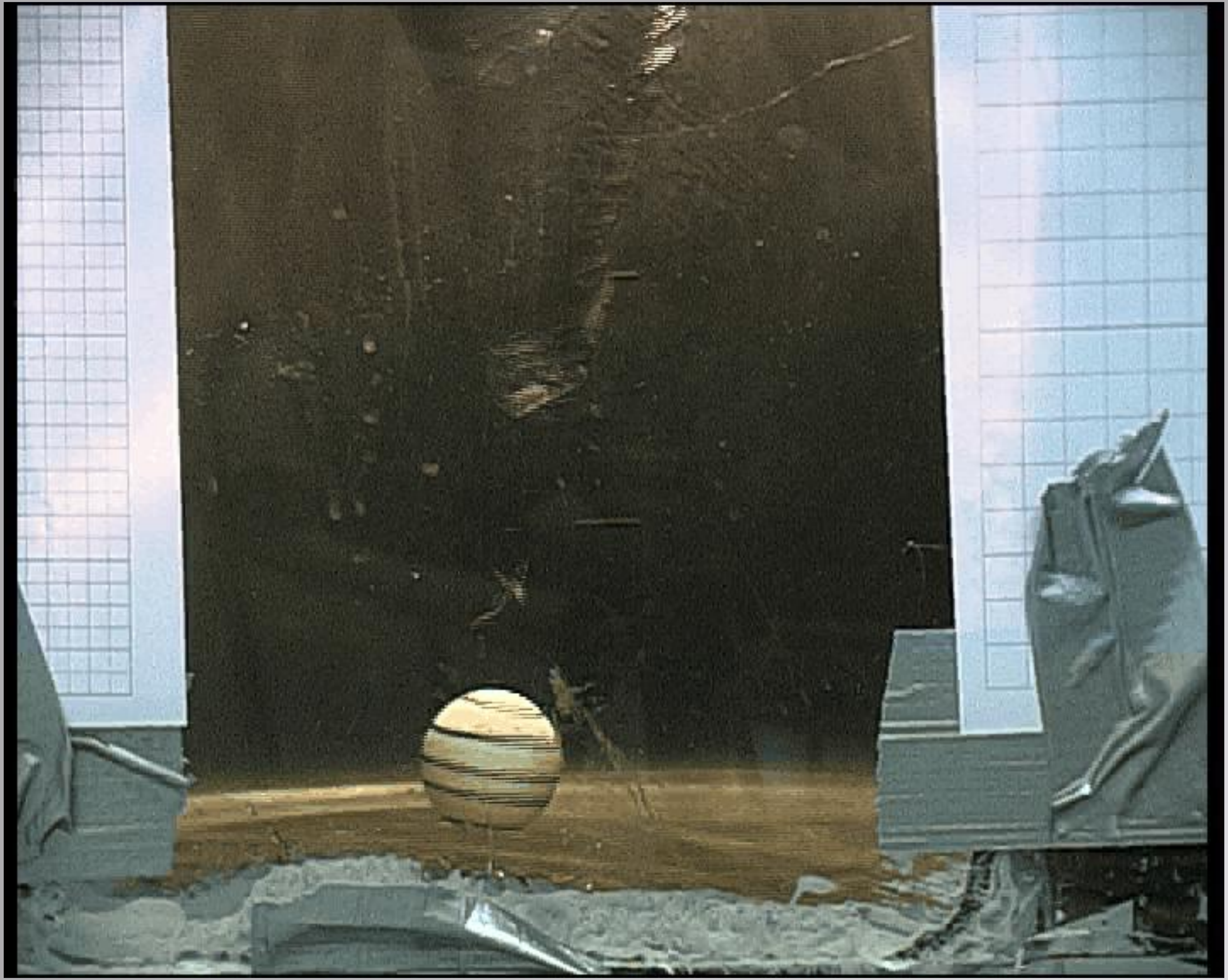
7. Bouncing plug

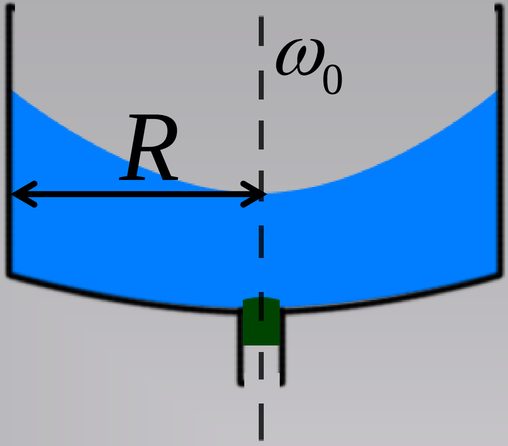
Fill a bath or sink with water and remove the plug. Then place a plastic ball over the plughole. As the water drains the ball starts to oscillate. Investigate the phenomenon.

Presentation of the phenomenon



Experimental setup

The bathtub vortex



Euler liquid – incompressible and inviscid

↓

No viscous shear stress

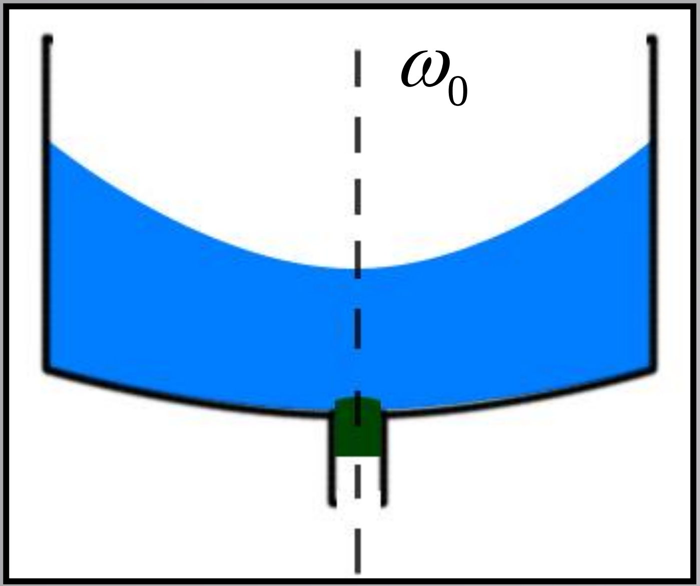
↓

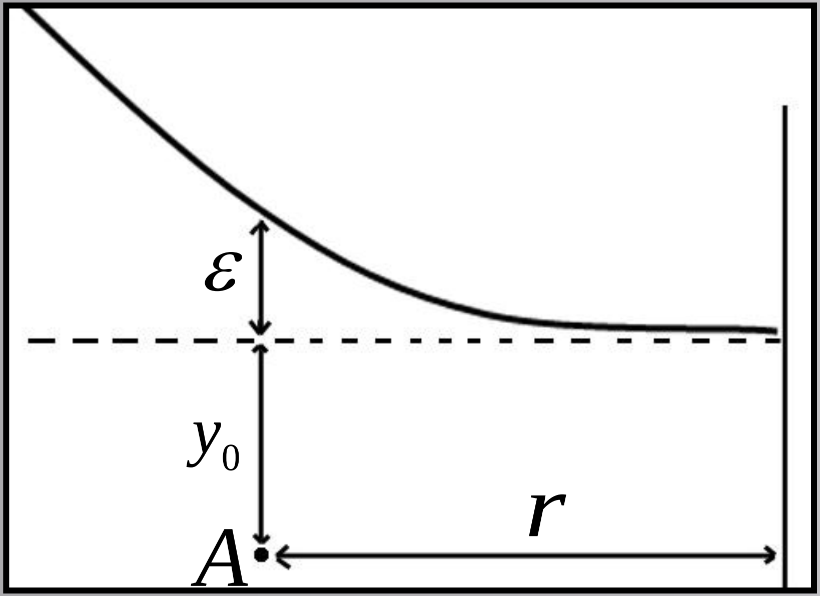
No torque about central axis

↓

Angular momentum conserved

The bathtub vortex





Conservation of angular momentum for one particle gives:

$$\omega = \omega_0 \frac{R^2}{r^2}$$

The radial pressure gradient:

$$\boxed{\frac{\partial p}{\partial r} = \rho \omega^2 r = \frac{\rho \omega_0^2 R^4}{r^3}}$$

Pressure in point A:

$$p_A = \rho g (y_0 + \varepsilon)$$

The radial pressure gradient is connected with the shape of the surface:

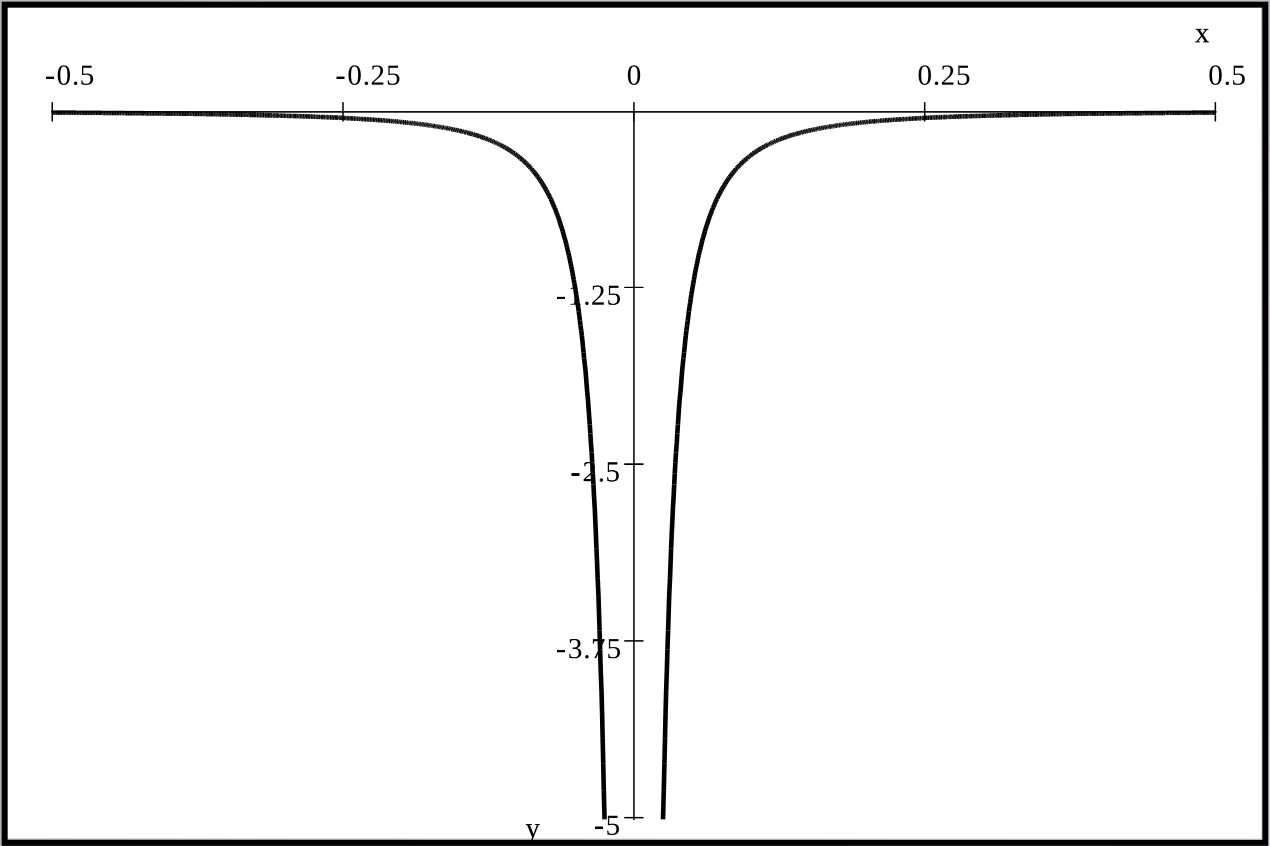
$$\boxed{\frac{\partial p}{\partial r} = \rho g \frac{d\varepsilon}{dr}}$$

The bathtub vortex

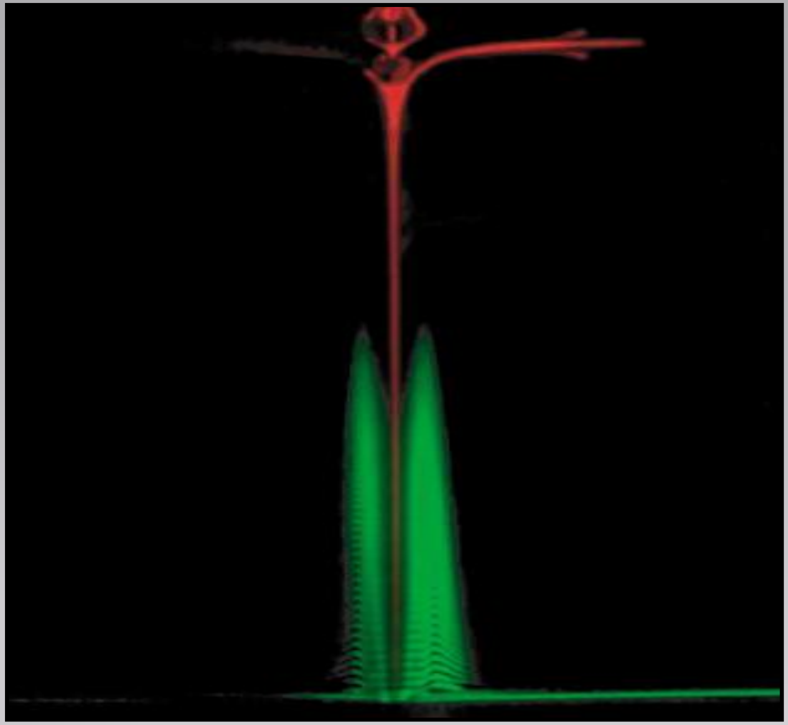
Integrating the previous equation, we obtain the dependence $\varepsilon(r)$

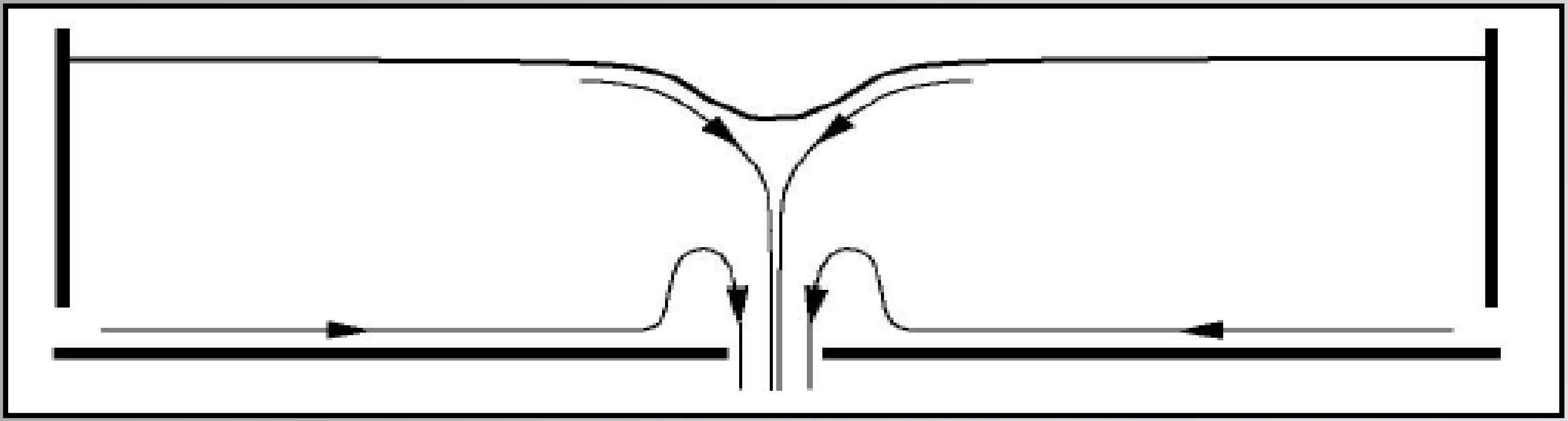
$$\varepsilon(r) = \int \frac{\omega_0^2 R^4}{g r^3} dr = \frac{\omega_0^2 R^4}{2 g r^2}$$

$$\omega_0 = 1$$
$$R = 0.5$$
$$g = 9.81$$

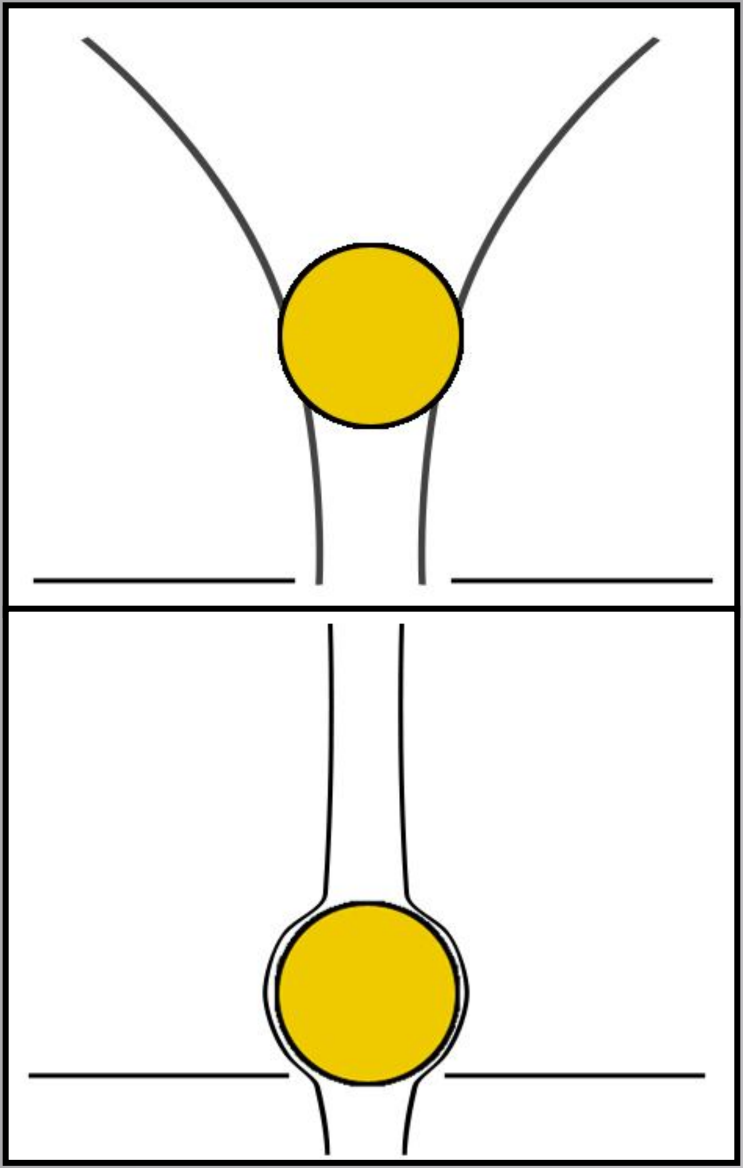


Vortex Structure





Ball in a vortex

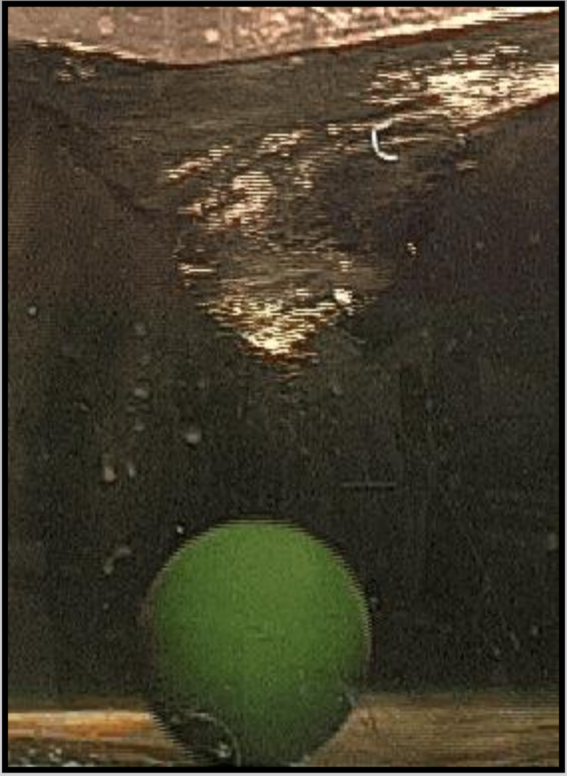


1. **The ball is attracted to the bottom of conatiner.**
2. **The ball close to the drainhole destroys the vortex structure.**
3. **Due to the this fact the resultant force acts upwards (buoyancy force).**
4. **The vortex flow appears again.**

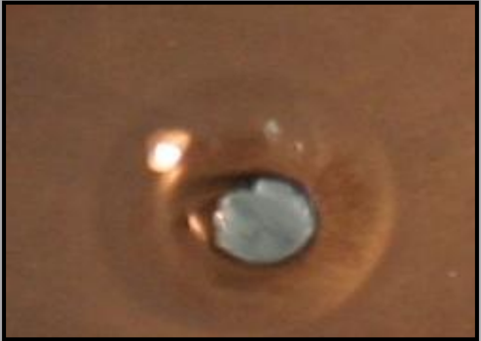
Measurements

4.5 cm
11.1 g
$\gamma = 0.4$

3.7 cm
3.9 g
$\gamma = 0.95$



7.3 cm
117,5 g
$\gamma = 0.06$



Drainhole diameter:

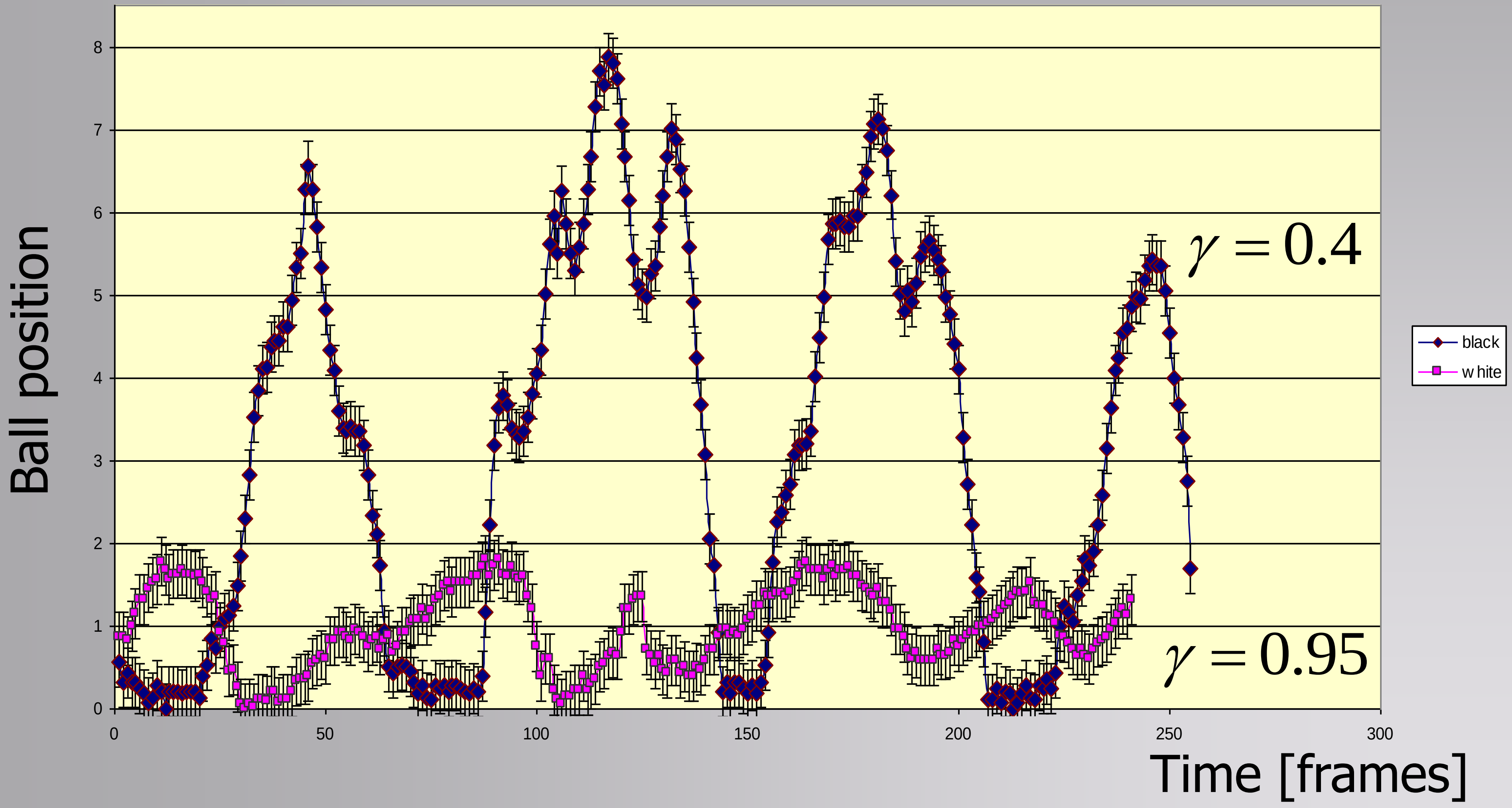
$\varnothing = 25\,mm$

$\varnothing = 30\,mm$

$$\gamma = \frac{d}{m}$$

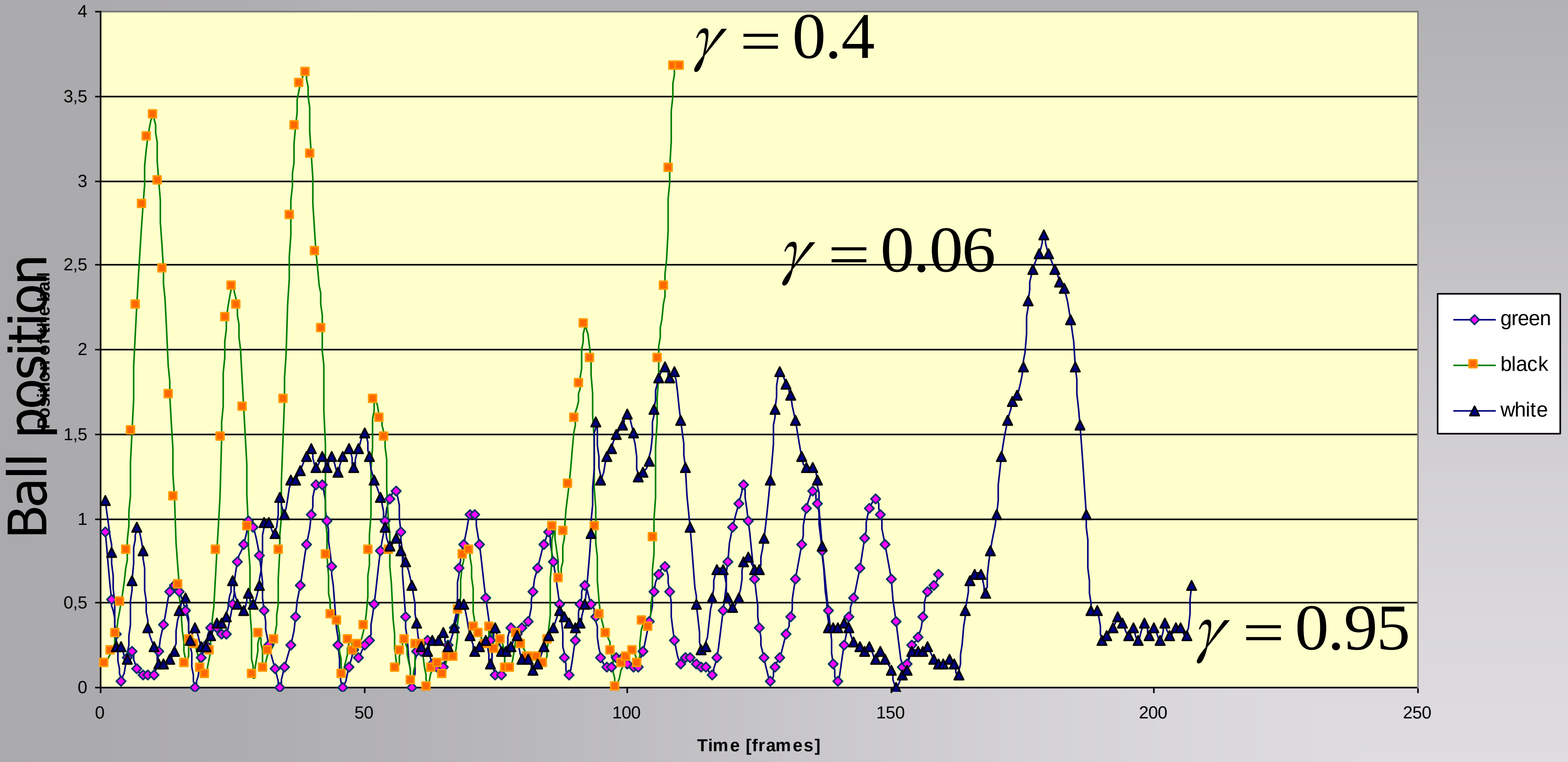
Experimental data

Plughole diameter 2.5 cm



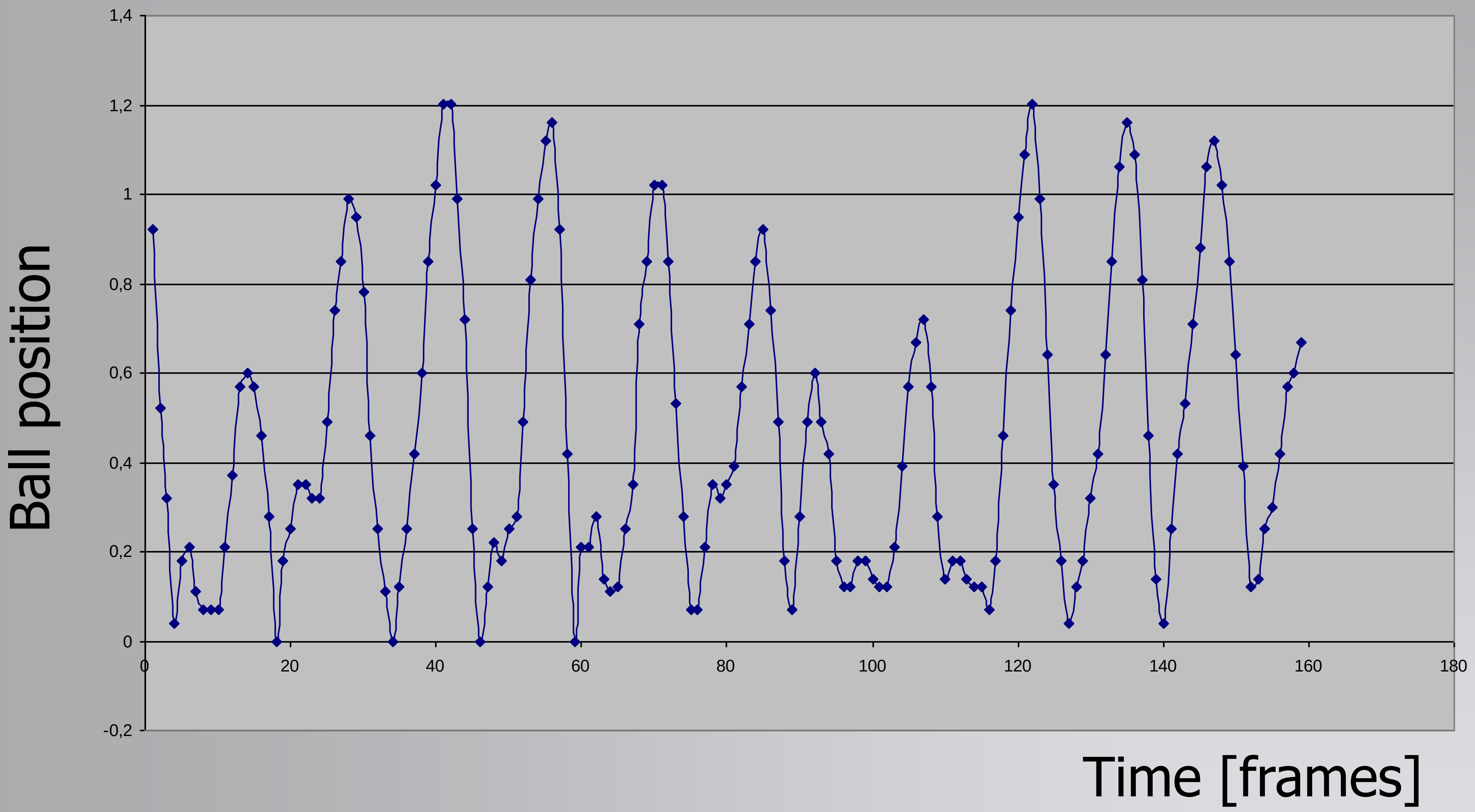
Experimental data

Plughole diameter 3.0 cm



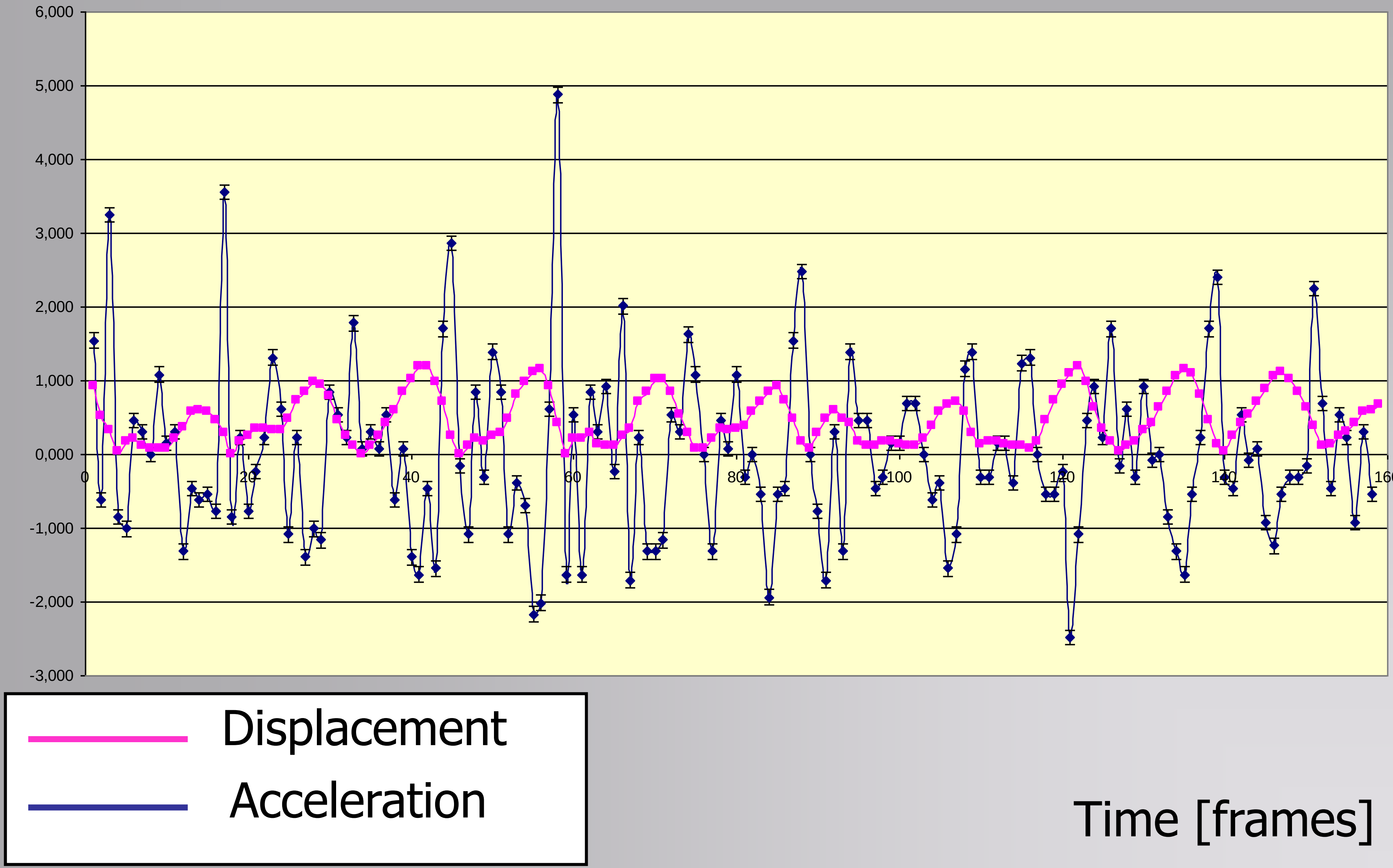
Time [frames]

Our best result



Green ball, 3.0 cm drainhole

Acceleration dependence on time



Data analysis - conclusions

1. For different balls (with different diameter to mass ratio) we obtain different character of oscillations.
2. We can determine the acceleration on time dependence for each ball.
3. The main forces, having crucial influence on the phenomenon are buoyancy force and force due to pressure difference caused by vortex flow.
4. The oscillations seem to have irregular run – slower upwards motion, fast falling.
5. The bigger the drainhole, the smaller the oscillations of balls.
6. The character of the oscillations can be qualitatively described when considering pressure distribution.

Acknowledgements

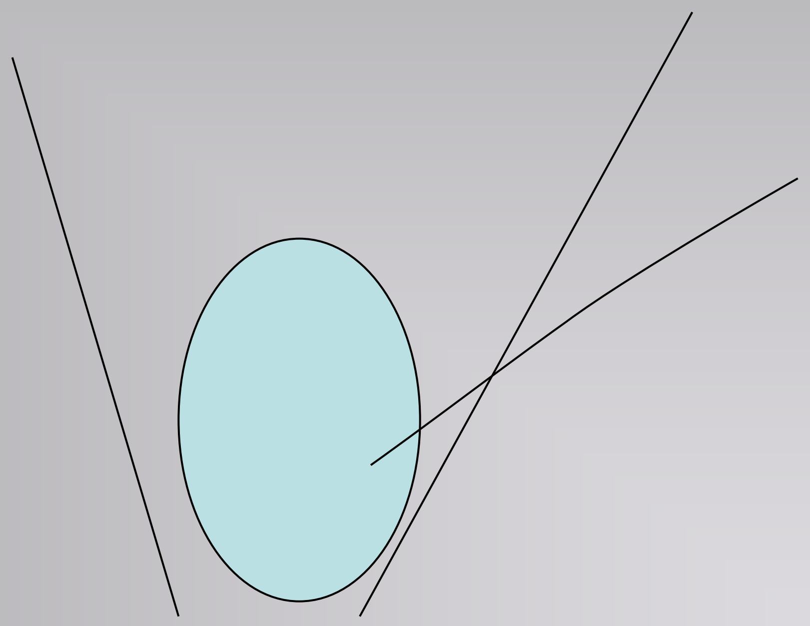
- PhD Tomas Bohr for permission to use his theoretical materials

Literature

- **T.E. Faber „Fluid Dynamics for Physicists", Cambridge University Press 1995**
- **T. Bohr, „Anatomy of a bathtub vortex", Fluid Dynamics**

Ball in a vortex

Mechanism of the phenomenon



Von Karman vortices

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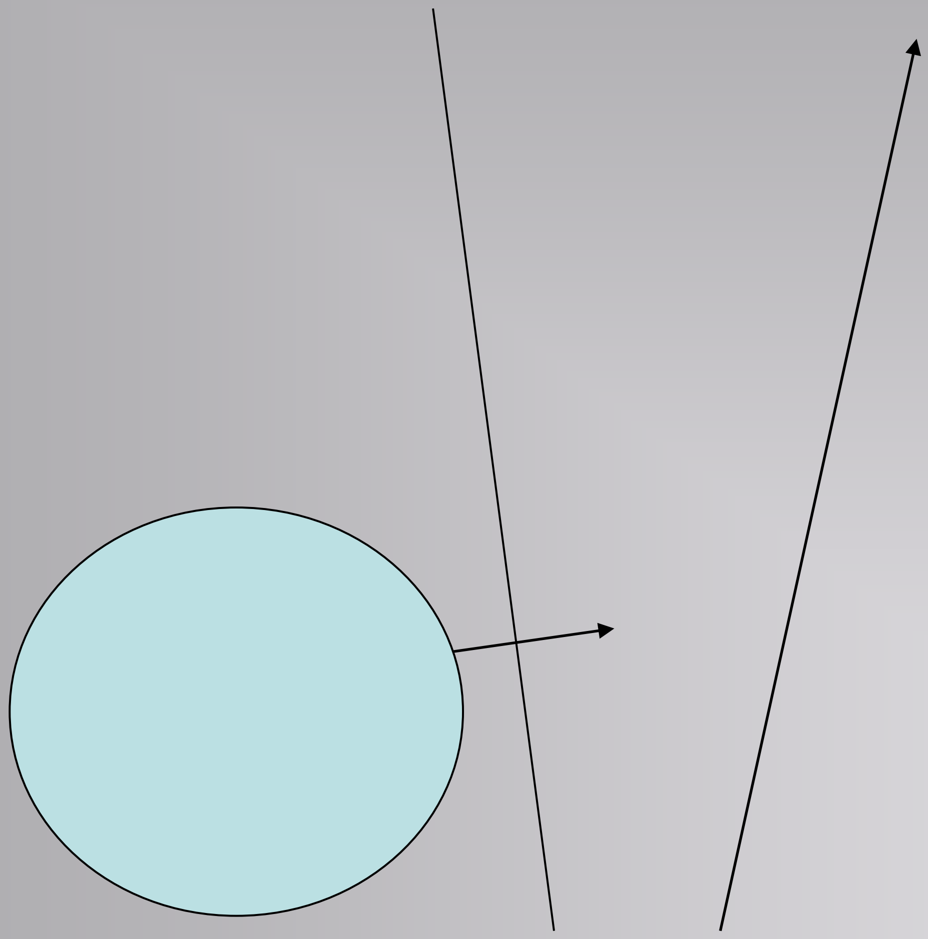
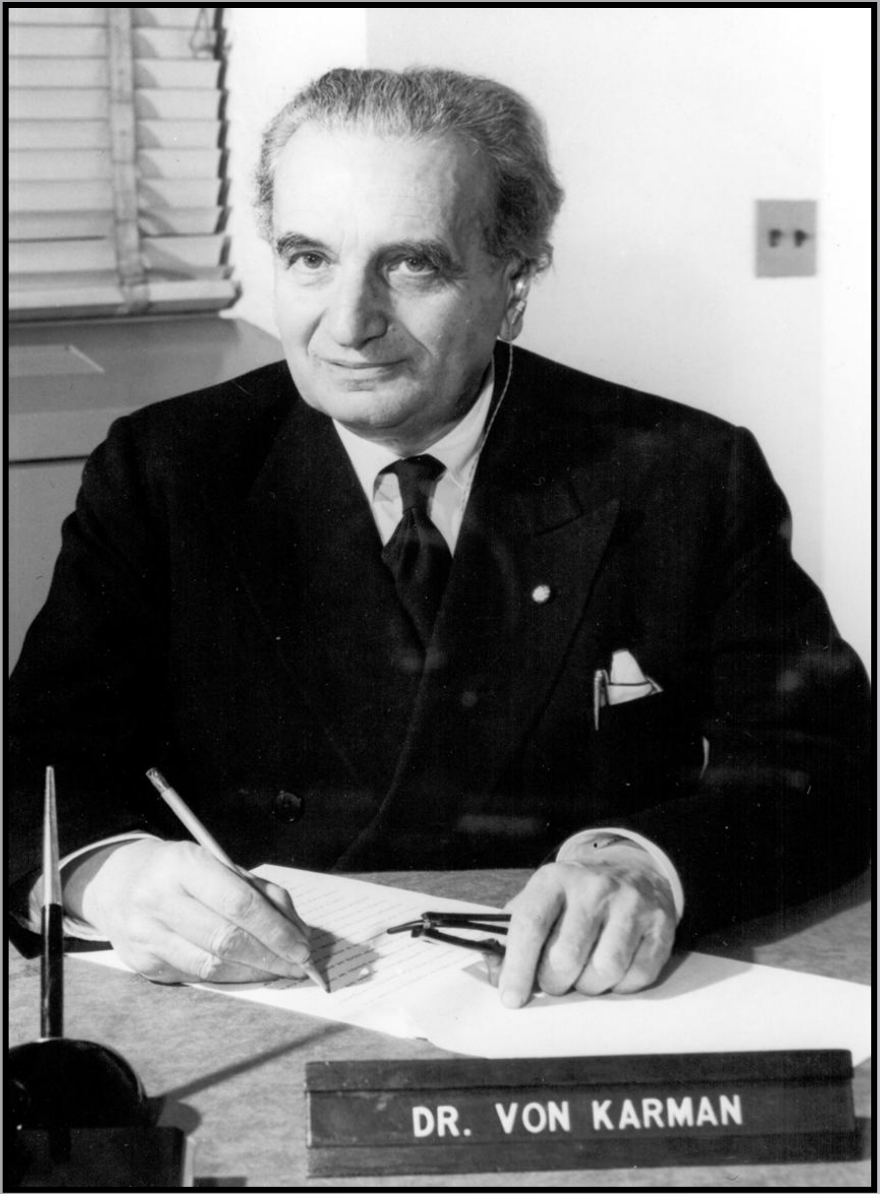


Qualitative analysis

Why is the plug bouncing?

Von Karman vortices



Von Karman vortices dynamics

Non-potential flow

$$\nabla \times \vec{\upsilon} = rot\,\vec{\upsilon} \neq 0$$

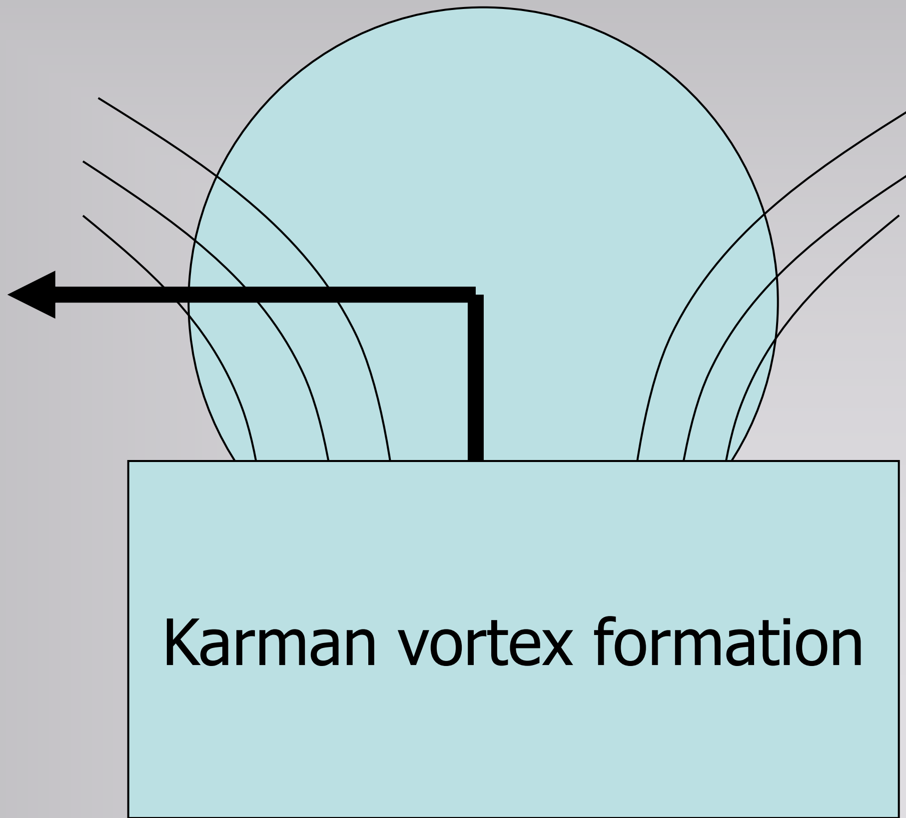
Von Karman vortices form behind a body, that is surrounded by a flowing liquid at a certain Reynolds number range.

Drag force:

$$F_D = c_D A \frac{\rho \upsilon^2}{2}$$

Lift force:

$$F_L = c_L A \frac{\rho \upsilon^2}{2}$$



Karman vortex formation