# $11^{th}$ IYPT '98
# solution to the problem no. 1
# presented by the team of RussiaII

Alekseev Andrey
Алексеев Андрей

**Invent Yourself**

Construct an aeroplane from a sheet of paper (A4, $80 \,{}^{g}/{}_{m^2}$). Make it fly as far and/or as long as possible. Explain why it was impossible to reach a greater distance or a longer time.

**Overview**

- Solution
  - Initial conditions
  - Experimental experiences
  - Experimental measurement of the forces
  - Conditions for a maximum flight
  - Calculation about the forces
- Conclusions

# 1 Solution

## 1.1 Initial conditions

First of all it is necessary to formulate initial conditions of problem, what to take for the flightrange of a model. Contents and duration of flight models was defined by the following way, model was start by hand with certain initial velocity, undoubtedly less, than planning velocity of model. With a time a velocity of a model was installed and model from known height was lower with the constant velocity. We tried to throw a model so, that it fly with the formed velocity above the hurdle by the height 1 metre. Distance from this barriers before the place of boarding we considered for range of flight, but time, for which model passes this distance, for a time of the flight of a model.

## 1.2 Experimental experiences

First, several models of airplanes of different design were made from a sheet of paper so as to see which one flies best of all. For the best model, we studied the dependencies of the lifting force and drag force on the angle of attack, rate of flow, and wing surface area. From these dependencies, the maximum flight range and/or duration were estimated.

## 1.3 Experimental measurement of the forces

In this problem, primary emphasis was on the experiment. To perform the investigations, a wind tunnel was made. Air motion in the tunnel was caused by an air vane with an adjustable speed of revolution of the motor rotor. The air speed was measured with a vane anemometer. The model under test was hanged on a special hanger, which allowed one to change the angle of attack. With the vane switched, the model was subjected to the lifting force and aerodynamic drag force

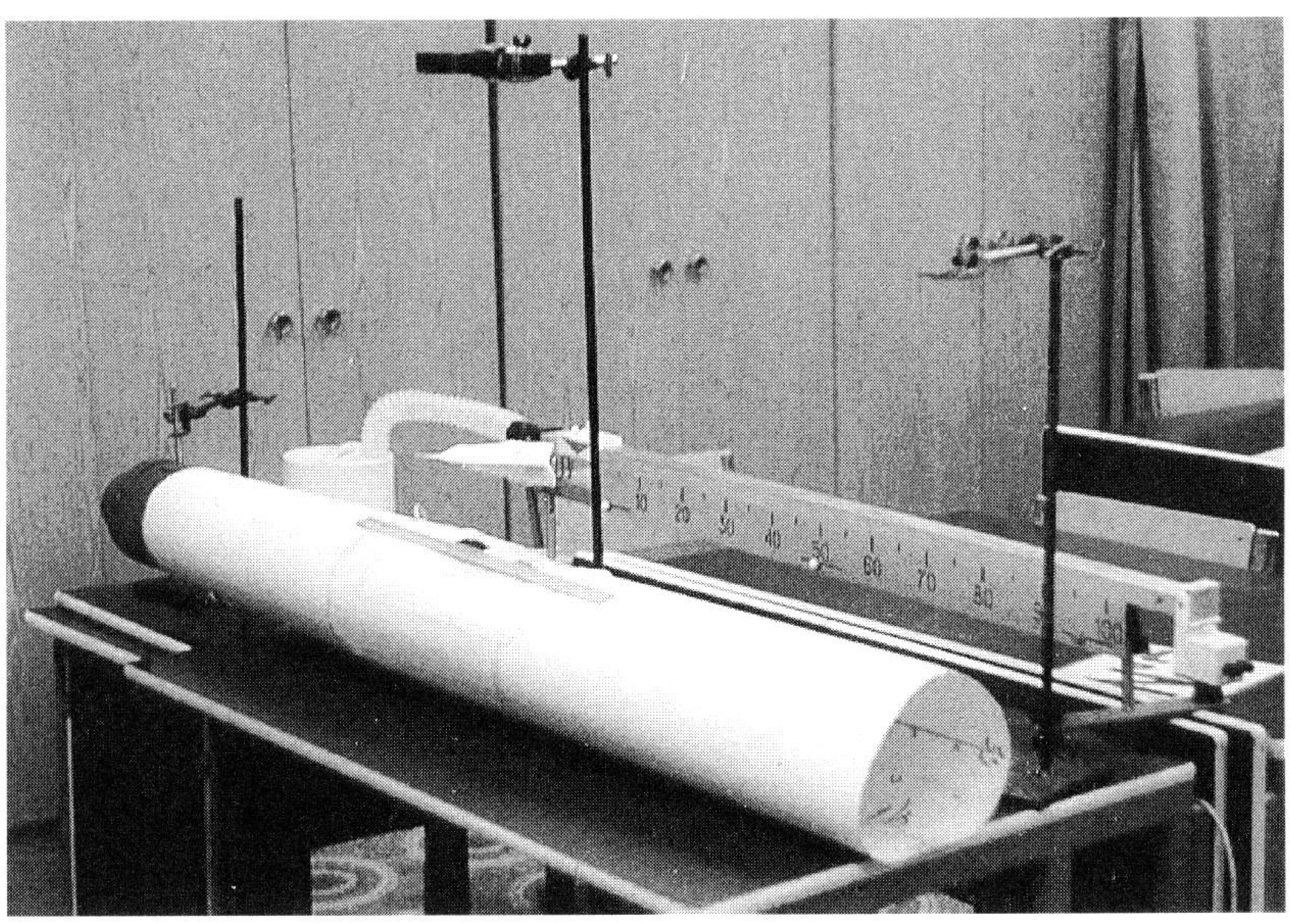

Figure 1: Wind tunnel

and began to deviate from its initial position. The aerodynamic drag force was measured with a millidynamometer by means of a filament attached to the plane tip, whereas the lifting force was determined from the angle of filament deviation from the initial position by the formula

$$Y = mg - Q \tan \alpha$$

From the known formulas for the lifting force and drag force

$$Y = \frac{\rho\vartheta^2}{2} S C_Y$$

and

$$Q = \frac{\rho\vartheta^2}{2} S C_X$$

the lift and drag coefficients were determined for various angles of attack. Then, using the data obtained, we constructed the $C_x$ vs. $C_y$ dependencies (polars), from which the maximum flight range was estimated.

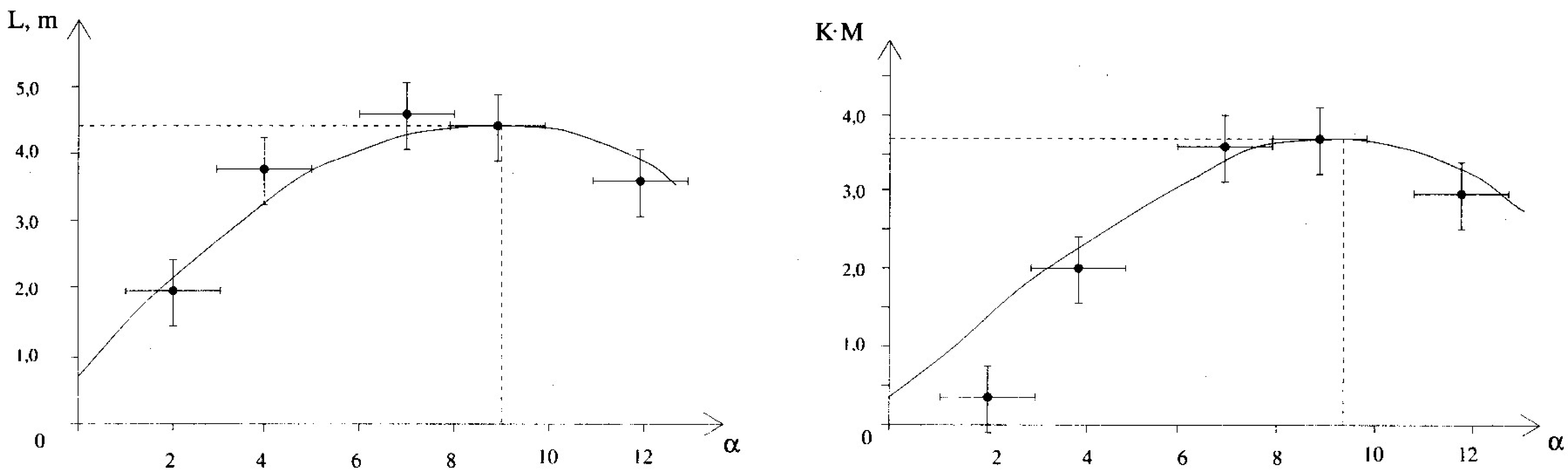


Figure 2: The dependence of the flight range on the angle of attack

| $Y$, mN | $Q$, mN | $\alpha$ | $S$, cm | $t$, s | $L$, m |
|---|---|---|---|---|---|
| 0 | 0,7 | 0 | 240.0 | 1.8 | 5.0 |
| 1.5 | 0.9 | 2 | 204.0 | 1.4 | 4.5 |
| 3.8 | 1.1 | 4 | 168.0 | 1.0 | 3.5 |
| 5.9 | 1.4 | 7 | 5.9 | 1.4 | 7 |
| 8.0 | 2.0 | 9 | 8.0 | 2.0 | 9 |
| 9.0 | 2.6 | 12 | 9.0 | 2.6 | 12 |

Table 1: The dependence of the flight range on the angle of attack

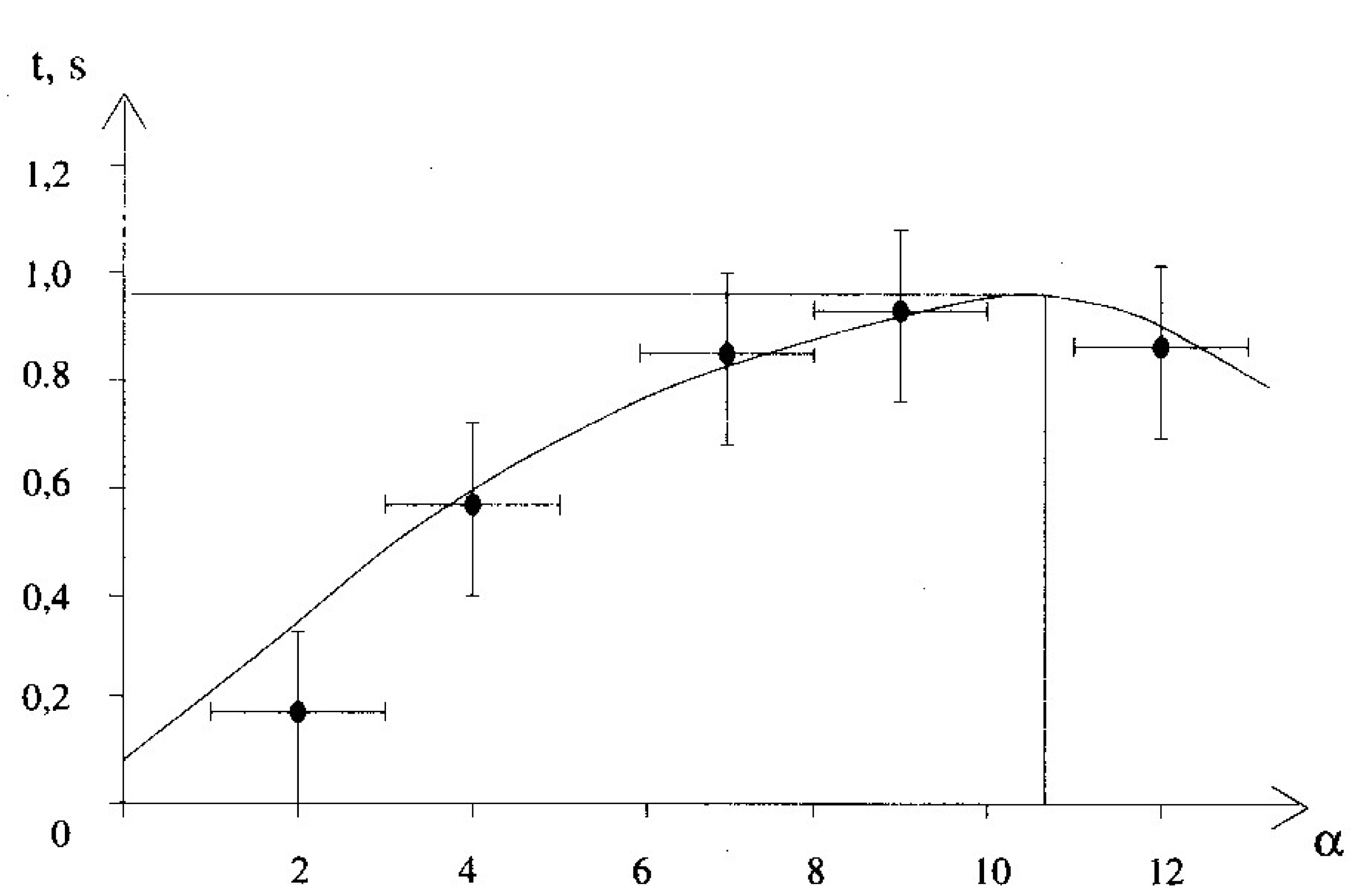


Figure 3: The dependence of the flight duration on the angle of attack

## 1.4 Conditions for a maximum flight

The maximum flight range and/or duration are achieved if two requirements are met.

1. The moments of forces acting on the model should be balanced (balance condition).
2. If the angle of attack of a model changes under the action of some external effect (for example, an atmospheric gust), the model returns to the balancing angle of attack (stability condition).

Thus, the motion of a model must be stationary, that is, the model must glide in a straight-line path with a constant speed. It is known in practice that this is provided if the centre of mass is placed at $\frac{1}{3}$ to $\frac{1}{2}$ from the beginning of the aerodynamic mean chord.

## 1.5 Calculation about the forces

Let us consider forces acting on the plane in the flight. It is subjected to the aerodynamic drag force $R$ and the gravity force $G$. Resolving the former force into two components (along and normally to the direction of motion), we obtain the lifting force $Y$ and drag force $Q$. The gravity force can be resolved into two components in a similar manner.
From the similarity of the triangles ABC and DEF, we have

$$\frac{Y}{Q} = \frac{L}{H} \tag{1}$$

The lifting force-to-drag force ratio is known to be called aerodynamic quality.

$$K = \frac{Y}{Q} = \frac{L}{H} \tag{2}$$

Using an experimental data obtained we constructed the dependence of aerodynamic quality on the angle of attack. Using (2) we can obtain the maximum flight range. But as flight altitude is equal to one meter the flight range is numerically equal aerodynamic quality. From this formula it follows that the flight range can be estimated if the aerodynamic quality and the flight altitude are known. Clearly, the maximum flight range is achieved when $K$ is maximum. The aerodynamic quality was calculated as the ratio of the lift and aerodynamic drag coefficients. Indeed, substituting the lifting and aerodynamic forces into (2) yields

$$K = \frac{C_Y}{C_X} \tag{3}$$

The flight range is obviously dependent on the wing surface area, more strictly, on its elongation. It is known from practice that the maximum range correlates with maximum elongation. In our design, the maximum elongation corresponds to the maximum surface area. The maximum aerodynamic quality provides the maximum flight range but says nothing about flight duration. It was previously assumed that the model moves in a straight-line path with zero acceleration. Hence, $G = R$. Usually, the gliding angle is small, no more than $10-12$ degrees; therefore, one may put $G \cong Y$. Substituting this into the formula for the lifting force, we obtain:

$$G = \frac{\rho\vartheta^2}{2} S C_Y$$

Hence, the flight speed is:

$$\vartheta = \sqrt{\frac{2\frac{G}{s}}{\rho C_Y}} \tag{4}$$

From the similarity of the triangles ABC and DGH, $\frac{Y}{Q} = \frac{\vartheta_x}{\vartheta_y}$. If the angle $\Theta$ is small, $\theta_x \cong \theta$. This means that

$$\vartheta \cong \frac{\vartheta}{K} \tag{5}$$

Substituting (4) gives:

$$\begin{aligned}\vartheta_Y &= \frac{\vartheta}{K} \\ &= \frac{\sqrt{\frac{2\frac{G}{s}}{\rho C_Y}}}{\frac{C_Y}{C_X}} \\ &= \sqrt{\frac{2\frac{G}{s}}{\rho}} \cdot \frac{C_X}{\sqrt{C_Y^3}}\end{aligned}$$

The ratio $\frac{C_X}{\sqrt{C_Y^3}}$ is called a power coefficient and characterises the power developed by the gravity force. The less this coefficient, the smaller the speed of model landing and the longer the flight. The maximum flight duration will be achieved for the maximum airfoil section.

## 2 Conclusions

Using an experimental data we constructed the dependence of flight duration on the angle of attack. Now it only remains for us to adjust the angle of attack of the plane. We have defined maximum range or duration of flight, but what is a maximum range and duration of flight? We consider that maximum range and duration of flight is reached when product of these two values is maximum. Using an experimental data we constructed the dependence of the maximum flight range and duration on the angle of attack. Received data were checked experimentally.
So we made an experiment. From the data obtained from the experiment the maximum flight range and/or duration were estimated. Range and/or duration can not be greater because using other corners of attack range and/or duration are less than maximum.