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solution to the problem no. 2
presented by the team of RussiaII

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Popping body

A body is submerged in water. After release it will pop out of the water.
How does the height of the pop above the water surface depend on the initial conditions (depth and other parameters)?

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1 Introduction

Probably everyone has seen a light body to jump out from under water. But I guess that nobody has been reflecting on a subject as follows: on what and how the height of this jump is dependent. A study of this question has been one of the problems at the 20^{th} Russian Tournament of Young Physicists: A light body, after being immersed below the water surface, is jumping out of water; then how a height of the jump is dependent on various parameters (on the initial immersion depth and so on)?

2 Conditions Parameters

For the beginning, we should understand correctly the conditions of the problem. The latter was stated not for some particular body, but just for any light (not heavy) body in general. But if such a body is of arbitrary shape, it is clear that it will be very difficult to describe mathematically its movement or find an answer with the help of experimentation. Therefore I have taken a sphere as the main object of studies: A sphere is the most simple streamlining geometrical figure with a convenient linear dimension – the radius.

From the conditions of the problem it is also unclear what kind of parameters should be considered. Hence, these parameters are to be found by us.

Water has such properties as a density, viscosity and surface tension. It is impossible to change these properties separately so that the water is still remained to be water, but all these parameters depend on temperature, hence, the height of jumping out also should be temperature dependent. Apart from temperature, it is possible to change an initial position of a ball (depth of submersion) and its features (size and weight).

3 Experiments

The main accent in solving this problem has been made on experimentation, for which an installation including a stand, an aquarium, vacuum pump with a hose and valve was assembled. This construction eliminates initial rotation of a ball and incidental pushes, because the air is very slowly injected into hose, and therefore the ball has no initial acceleration.

For reduction of measurement errors, an electronic filming has been used in the experiments: the height of ball jumping out was measured using the stop-of-ball point on the stopframe of the video recorder. Besides, the video type recorder can be used to define the ball velocity and to study its movement below the water surface.

4 Dependence on water temperature

First experiment was to study the jumping out height dependence on water temperature which varied from 10 to 60°C, and a table tennis ball ($R = 1.85\,\text{cm}$, weight $= 2.15\,\text{g}$) was used as the experimental body. But before the experiment I have tried to assume, how the jumping out height can change with the rise of temperature, and also to derive formulas or other relationships.

5 Calculations – temperature

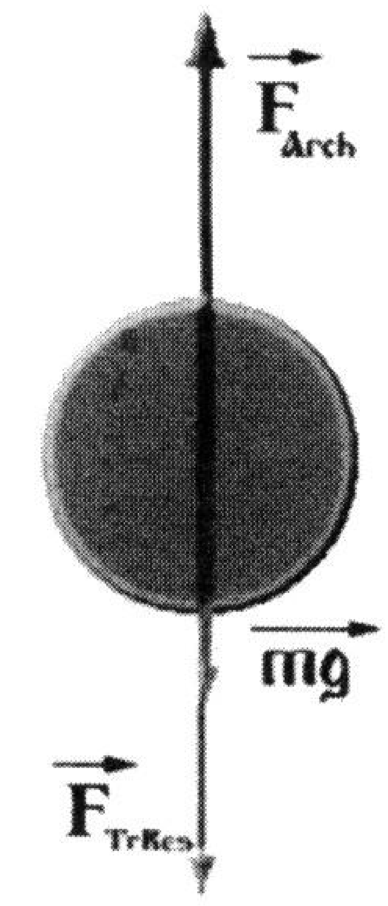
Let's consider a ball under water in rest – it is under influence of two forces: the ARCHIMEDES' buoyancy force and gravity force (a force stipulated by a difference of a pressure inside and outside the tube, compensates their resulting value). While in movement, the ball is also influenced by a third force – that of tractive resistance. In different cases it is different, but as a rule, the forces of tractive resistance in viscous medium are proportional to various powers of velocity. The power is determined by velocity or by such a parameter of solid body-liquid system as the REYNOLDS NUMBER that is defining a ration of inertia forces to the forces of viscous friction (1):

$$Re = \frac{\rho R U}{\eta} \tag{1}$$

where
ρ - density of liquid
R - radius of a ball
U - velocity of the liquids movement
η - dynamic viscosity

If $Re \ll 1$, the force of tractive resistance is proportional to the first power of velocity and is calculated from the STOKES' equation (2):

$$F_r = 6\pi\eta RU \tag{2}$$



But for the ball used in present study, $Re \approx 25000$ and hence to speak about the use of STOKES law is senseless. In some reference books on aerohydrodynamics, the formula was found, where the force of tractive resistance is proportional the the second power of velocity (3):

$$F_r = C_x \frac{\rho S}{2} U^2 \tag{3}$$

where C_x is a factor describing a streamlining of a body (for spheres it is from 0.1 to 0.2) and S is the cross-section.
As no spherical boundary conditions are specified for (3), we shall check up its applicability in such a manner. Let's write the second NEWTON'S LAW (4):

$$mU = \rho V g - mg - C_x \frac{\rho S}{2} U^2 \tag{4}$$

Solving the differential equation, it is possible to derive the dependence of velocity on a time (5):

$$U = \sqrt{\frac{A}{B}} \frac{e^{2\sqrt{AB}t} - 1}{e^{2\sqrt{AB}t} + 1} \tag{5}$$

$$A = \frac{\rho V g - mg}{m} \tag{6}$$

$$B = C_x \frac{\rho S}{2m}$$

As it can be seen from (5):

$$t \to \infty, \qquad U \to \sqrt{\frac{A}{B}}$$

Having experimentally defined a maximum velocity of the ball ($U = 1.5-2\frac{\text{m}}{\text{s}}$), I compared it to the velocity value calculated from the equation ($U = 1.5-2.1\frac{\text{m}}{\text{s}}$). As the values were found to be nearly identical, it is possible to speak about applicability of the equation (3) for solving the given problem.
Now let us define, how temperature can influence on the movement and jumping out of the ball. From the equation (5) it is seen, that from all the properties of liquid, the maximum velocity of a ball depends only on the density. But the density depends on temperature very significantly (table 1), hence it will not influence on $U_{\max}$, and consequently will not influence on a height of jumping out ($H \approx U^2$).

T, °C	10	20	30	40	50	60
ρ, $\frac{\text{kg}}{\text{m}^3}$	0.99973	0.99823	0.99567	0.99224	0.98807	0.98824

Table 1: $\rho(T)$

But as was told before, the viscosity depends on temperature and is, in turn, influenced by the REYNOLDS number (1).
Figure 5 shows a plot of dependence of C_x coefficient on Re. In a logarithmic scale the REYNOLDS number for the given ball lays within the limits from 3 to 4. It is clear from the diagram, that within these limits, C_x is growing with the REYNOLDS NUMBER increase, hence the constant of

T, °C	10	20	30	40	50	60
η, $10^{-6}\frac{\text{kg}}{\text{m s}}$	1307	1004	803	655	551	470

Table 2: $\eta(T)$

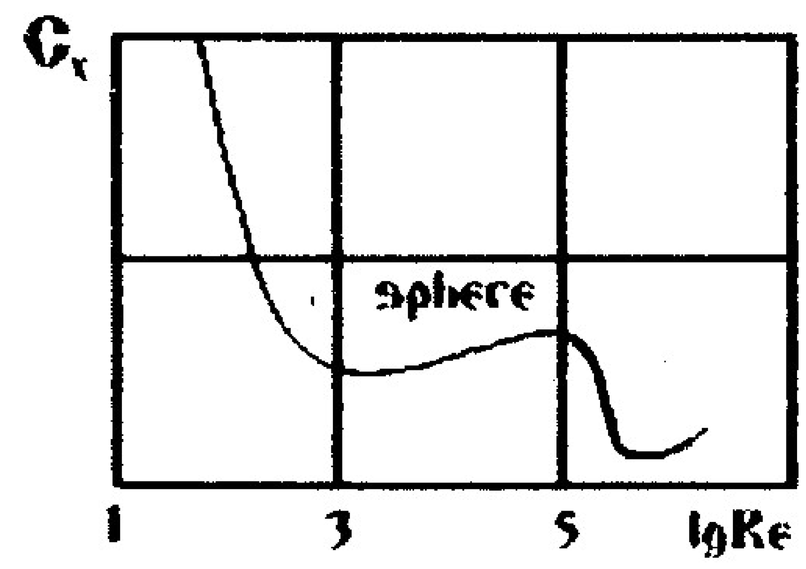


Figure 1: Dependence the streamlining coefficient on logarithm of Reynolds number

proportionality at the tractive resistance is also growing. According to (5) the velocity is inversely proportional to this coefficient, therefore the velocity and, consequently, the height of the ball jumping out should be decreasing with the increase of this coefficient. But besides the viscosity, the coefficient of surface tension is also temperature dependent. On ball going out of water, a part of its kinetic energy is spent to overcome the energy of the surface tension. The higher the temperature, the less is ball energy loss during its escape from the water (see table 3).

T, °C	0	30	60
σ, $10^{-3}\frac{\text{N}}{\text{m}}$	75.6	71.18	66.18

Table 3: $\sigma(T)$

As a consequence, the rise of temperature should be followed by increase of jumping out height. I failed to define theoretically, what is influencing more on the jumping out height: The surface tension or viscosity. Hence the dependence of the jumping out height on temperature can be found only by an experiment.

6 Calculations - energy

Now, let us consider other relationships, and for that we shall write the energy conservation law equation (7):

$$\frac{mU^2}{2} = \int_0^h \rho V g - mg - C_x \frac{\rho S}{2} U^2 \, dh \tag{7}$$

The kinetic energy of a ball at the end of its path is equal to the work of all resulting forces influencing upon it during its displacement. For the determination of this work it would be necessary to take an integral from the sum of all forces. But for the work of tractive resistance force that has appeared to be rather difficult, and secondly, there is more simply way. The tractive resistance force is proportional to the second power of velocity, and, by plotting the time dependence of the second power of velocity (5), it can be seen that on the initial stage, the second power of velocity is proportional to time, and consequently, to the depth of submersion h, and at $t \to \infty$ it does not depend on time. Therefore to find the work function of the tractive resistance force, we shall enter, into its equation, the coefficient α which at small t is equal to 0.5 and at $t \to \infty$. Then (7) will be converted to (8):

$$\frac{mU^2}{2} = \left(\rho V g - mg - C_x \frac{\rho S}{2} U^2\right) h \tag{8}$$

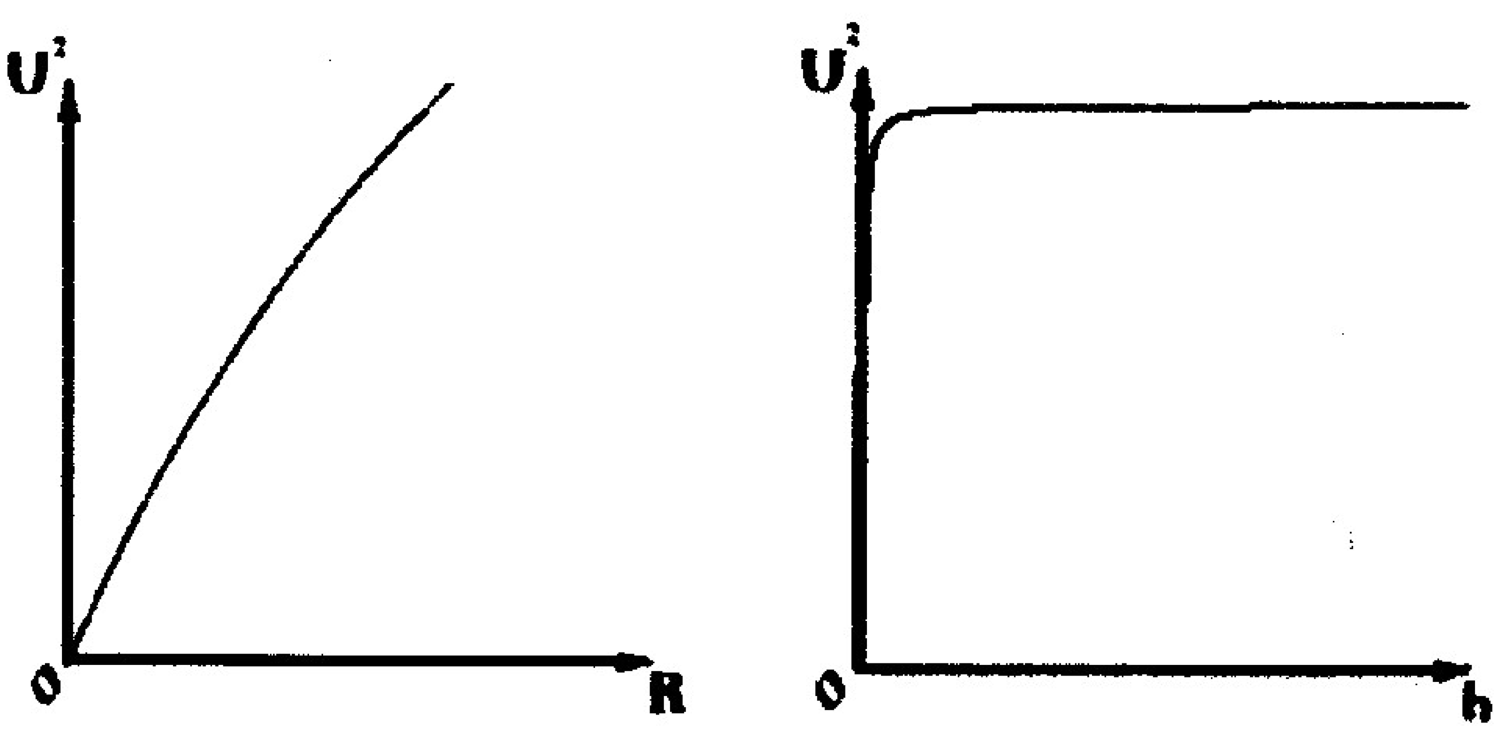


Figure 2: In the beginning; when time tends to infinity

Let's transform (8) to a style of $U^2 = f(h, R, m)$:

$$U^2 = \frac{2gh\left(4\pi\rho R^3 - 3m\right)}{3\left(C_x\pi\alpha\rho R^2 h + m\right)} \tag{9}$$

$$H \approx \frac{2gh\left(4\pi\rho R^3 - 3m\right)}{3\left(C_x\pi\alpha\rho R^2 h + m\right)} \tag{10}$$

7 Comparison with the Experiments

I was not trying to find a proportionality coefficient, as I did not search for numerical values which could be compared with the experimental results, I only wanted to check up general correctness of the theory, and, using (9), I plotted the diagrams in figure 3:

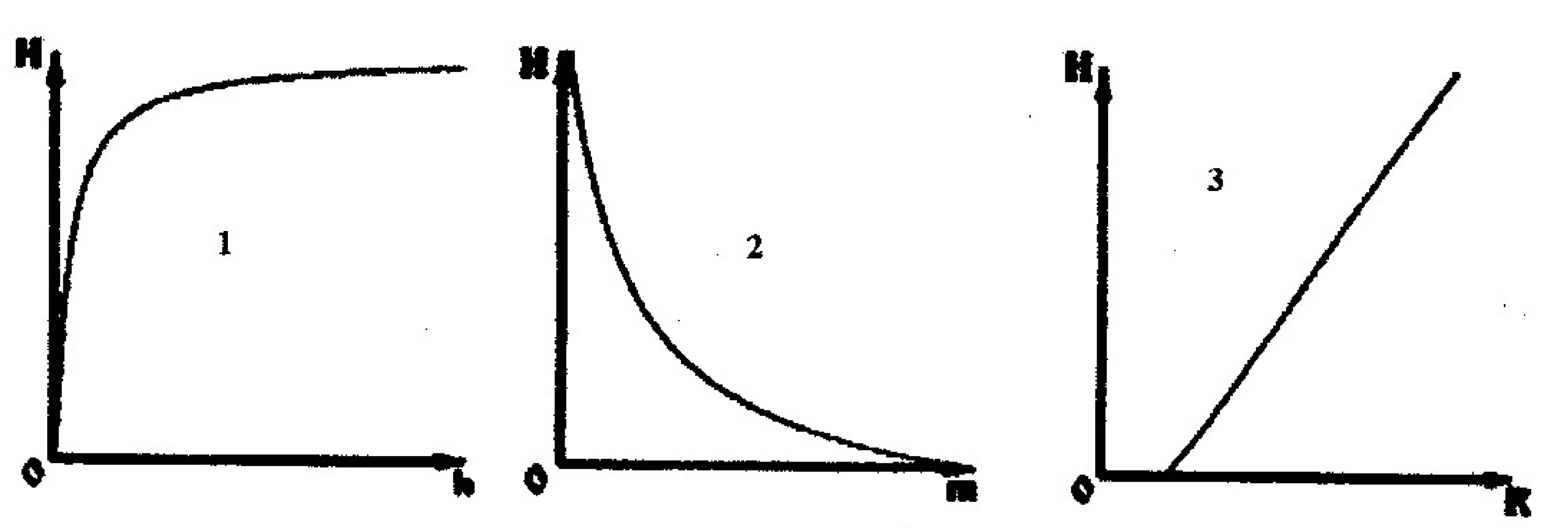


Figure 3: Theoretical plots, $H = f(h)$, $H = f(m)$, $H = f(R)$

After deriving the equations and plotting the diagrams, I have proceeded to experiments, and the first obtained date were for the dependence of a jumping out height on temperature. The results have turned out to be a little bit unexpected (figure 4). In the interval from 10 to 60 ° C, the

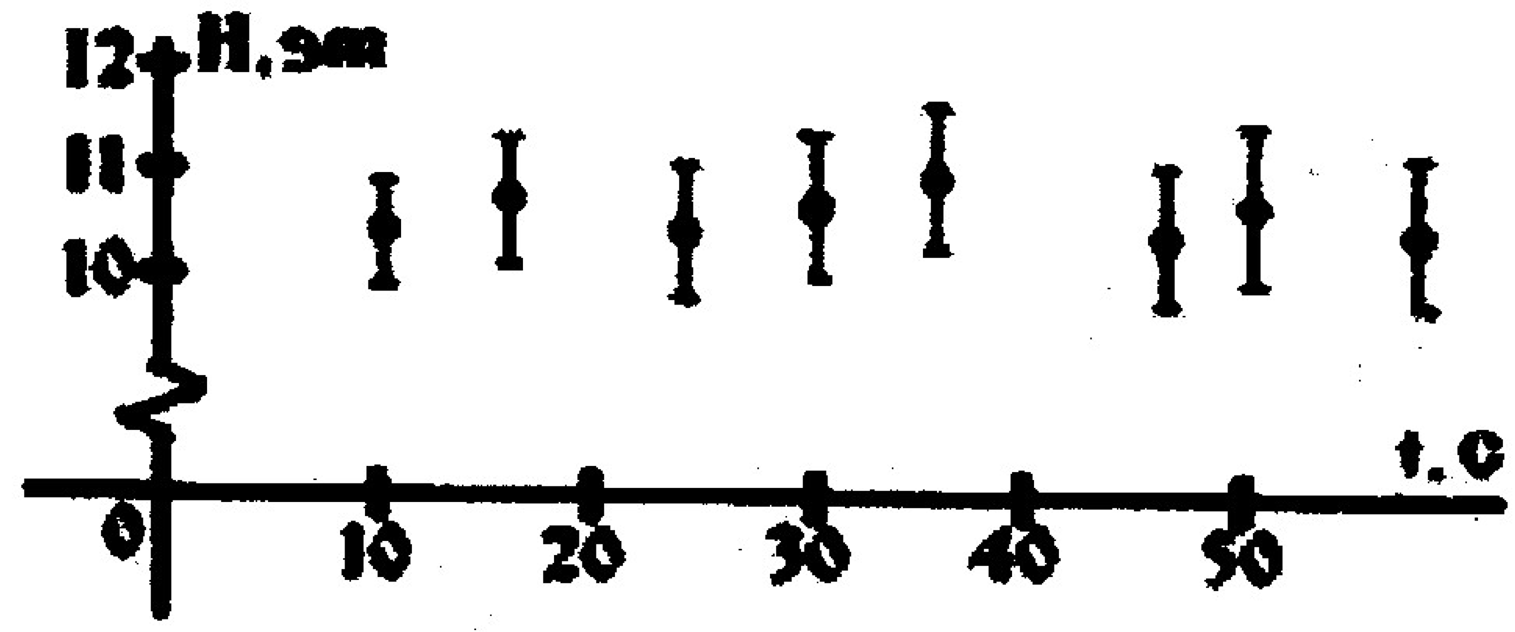


Figure 4: Dependence jumping out height on temperature

jumping out height does not vary within the error limits, therefore on the diagram only points and errors where shown, and it can be explained by the fact that the changes of the two magnitudes (those of viscosity and on surface tension coefficient) are compensating one another.
Experiment further was carried out (see figure 5) with the changing weight of the ball. I could not find bodies of a different weight having identical diameter, and it was necessary to act as follows:

A ball for the table tennis was filled from a syringe with the known volume of water. In spite of the fact that this experiment can not be treated as a correct one, it has give perfect results, confirming earlier derived equation.

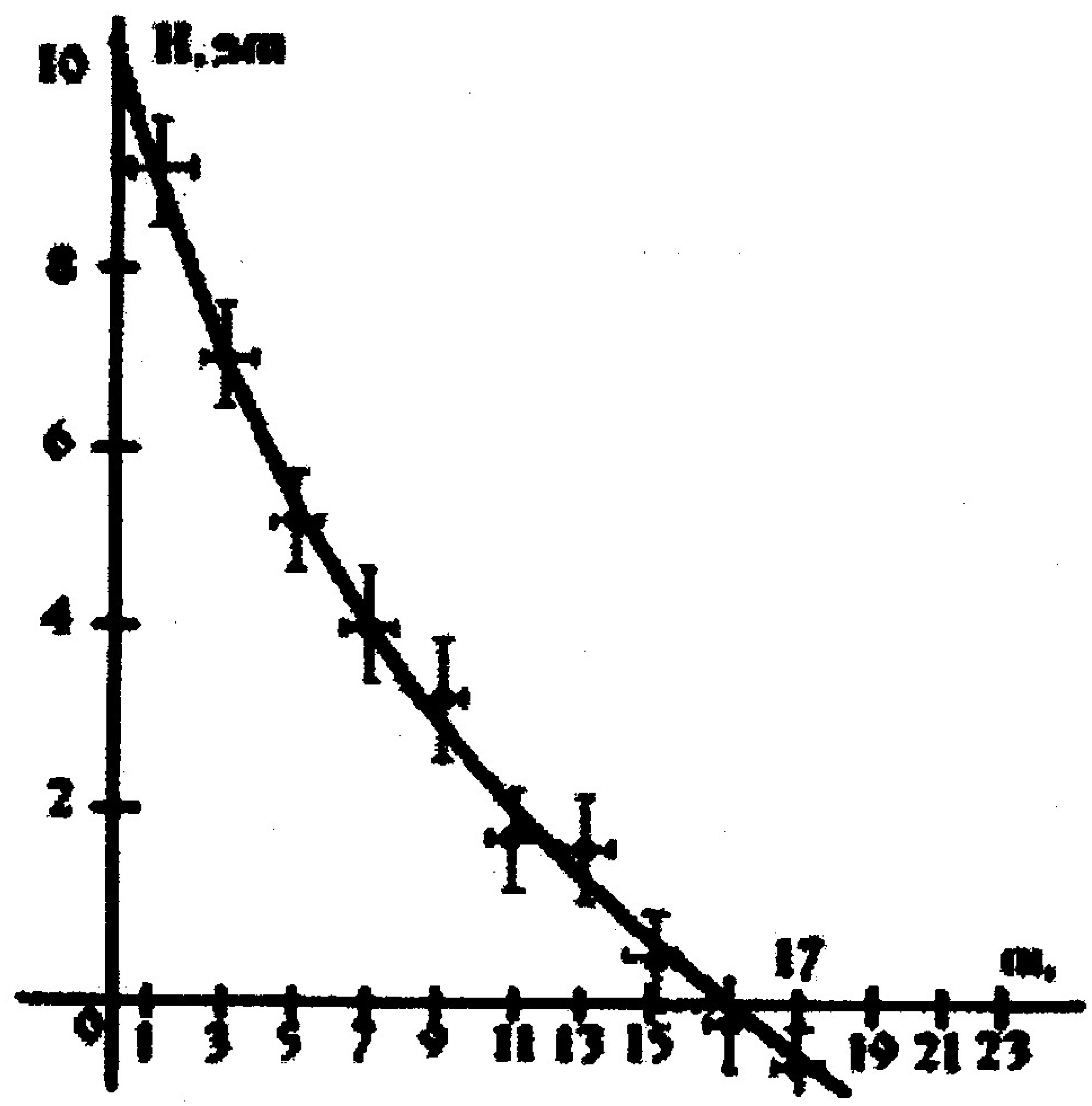


Figure 5: Dependence jumping out height on mass

In the next experiment I have been changing a radius of the ball. But this time it was purely qualitative observation so far as I failed to find balls of identical weight but of different diameter, and I also failed to achieve an approximation to such bodies, therefore the children's small rubber balls were used. If we look at the theoretical plot, it can be seen that with the radius increases, the jumping out height is also increasing practically as a directly proportional magnitude. In the experiment the increase of height was also noticed, but this dependence was not linear, and this is explained just by ans inequality of masses of all balls.
Most interesting dependence was found to be that of a jumping out height on the depth of submersion. The result as the plot is shown in figure 6.

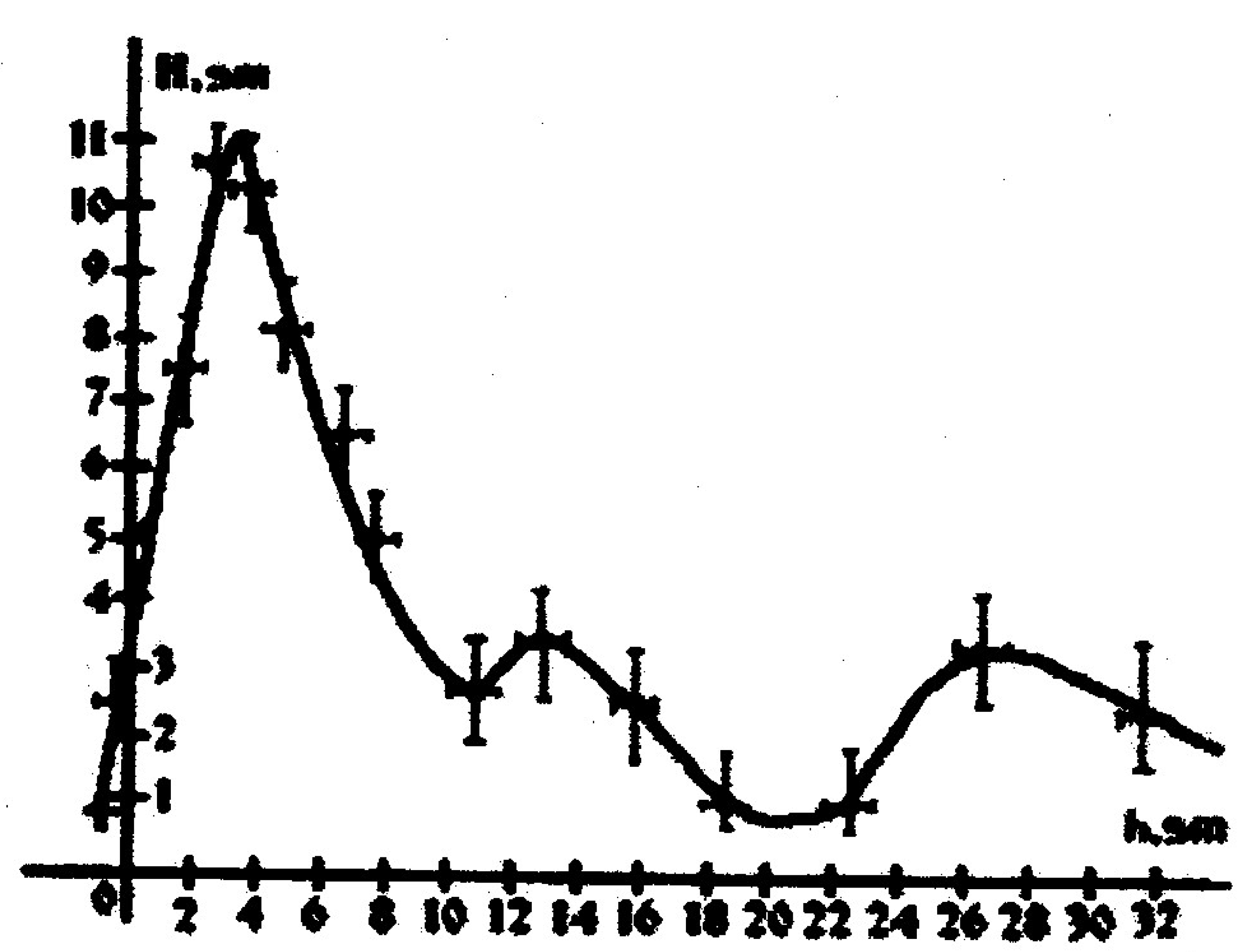


Figure 6: Dependence jumping out height on depth of submersion

Comparing the theoretical and experimental plots, it is possible to tell that only their initial parts are similar. Further the strong distinction is seen. If theoretically the height tends to approach a fixed limit, being augmented with the every next attempt, the experiment displays that, starting from the depth of 3.5 cm, the height begins to decrease. To get an idea concerning a cause of the progress, I have recorded the moment of ball jump out by electronic filming from different sides and has detected, that, starting from the depth of 2 – 3 cm, when the ball rises close to the water surface, on the latter a hill is formed. As a consequence, the hydrostatic pressure increases and a Laplacian stipulated by a curved surface of the liquid appears. Therefore the maxmium velocity, which is dialed already on first 3 – 4 cm, is diminished, and, hence, the maximum height of jumping out is also diminshed. As a result, maximum is formed on the plot.

The further experiments have brought even more unexpected results. At further increase of submersion depth, the height again begins to rise, then to reduce and so on. For study of this phenomenon the whole trajectory of ball displacement was recorded. An approximate picture is shown in the figure. As can be seen there, the ball does not move along a straight line, therefore its velocity vector is not always directed vertically, so at some depth its projection to a vertical axis becomes shorter, and, hence, and the jumping out height also decreases. I have explained the diagram, still it is necessary to explain the reason of the deviation.

As was told before, the Reynolds number for a ball is close to 25,000. At such values of Reynolds number, the current of liquid becomes turbulent and the asymmetrical vortices are formed behind the ball. These vortices at any moment can come off the ball, the latter will receive side impact and will deviate from a rectilinear trajectory. Various types of currents and vortices formed a different Reynolds numbers shown in the figure.

8 Conclusions

In the beginning of solving the problem I have stipulated, that a ball has been chosen as an experimental body for the sake of simple study. But it is not enough for the final solution, therefore some experiments with the order types of bodies have been carried out. In general, no conclusions can be derived from these results because of wide scatter of numerical values, but some common regularities are seen and are fairly well comparable to results of main experiments with spherical bodies, that is the general view of the diagrams is similar. Therefore it is possible to extrapolate data obtained for the spherical bodies to other ones.

It is possible to make the following conclusions from the carried out work: As a result of experiments and the development of some theory, the relationship between the jumping out height of a spherical body out of water and the depth of its submergence, temperature, its weight and radius were clarified. In the course of experiments with the bodies of more complicated shape, it was revealed that is valid for the spherical bodies may be extrapolated to any other bodies, that is the obtained regularities are of general importance.