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solution to the problem no. 2
presented by the team of Ukraine
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Popping body

A body is submerged in water. After release it will pop out of the water.
How does the height of the pop above the water surface depend on the initial conditions (depth and other parameters)?

Abstract

The nonmonotone dependence of the height of a body popping out of the liquid on the depth is investigated. It is shown that if the motion begins far from the liquid surface the height reaches an asymptotic value, which is determined by the geometry of a body. In opposite case of small depth the hydrodynamic resistance force is non stationary and can be neglected. The linear dependence of the height of popping on the depth near the surface is explained by the attached mass effect. The theoretical results are in good agreement with the experimental ones without any fitting parameters.

Thanks

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Overview

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- Ideal liquid
- Height of the popping
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- Conclusions

1 Introduction

In the problem proposed one deals with very complicated system. It is a body moving within a liquid under the gravity force. It is pure hydrodynamic problem. As a rule such problems can not be solved rigorously because of essential non-linearity arisen from the mutual influence of the liquid flow and motion of a body. Moreover the considered problem is assumed to be non stationary. In such a situation we start the solution of the problem with an experiment. We tried to find the dependence of the popping height on the depth of submerging. The experimentally obtained results are shown on figure 1 by crosses.

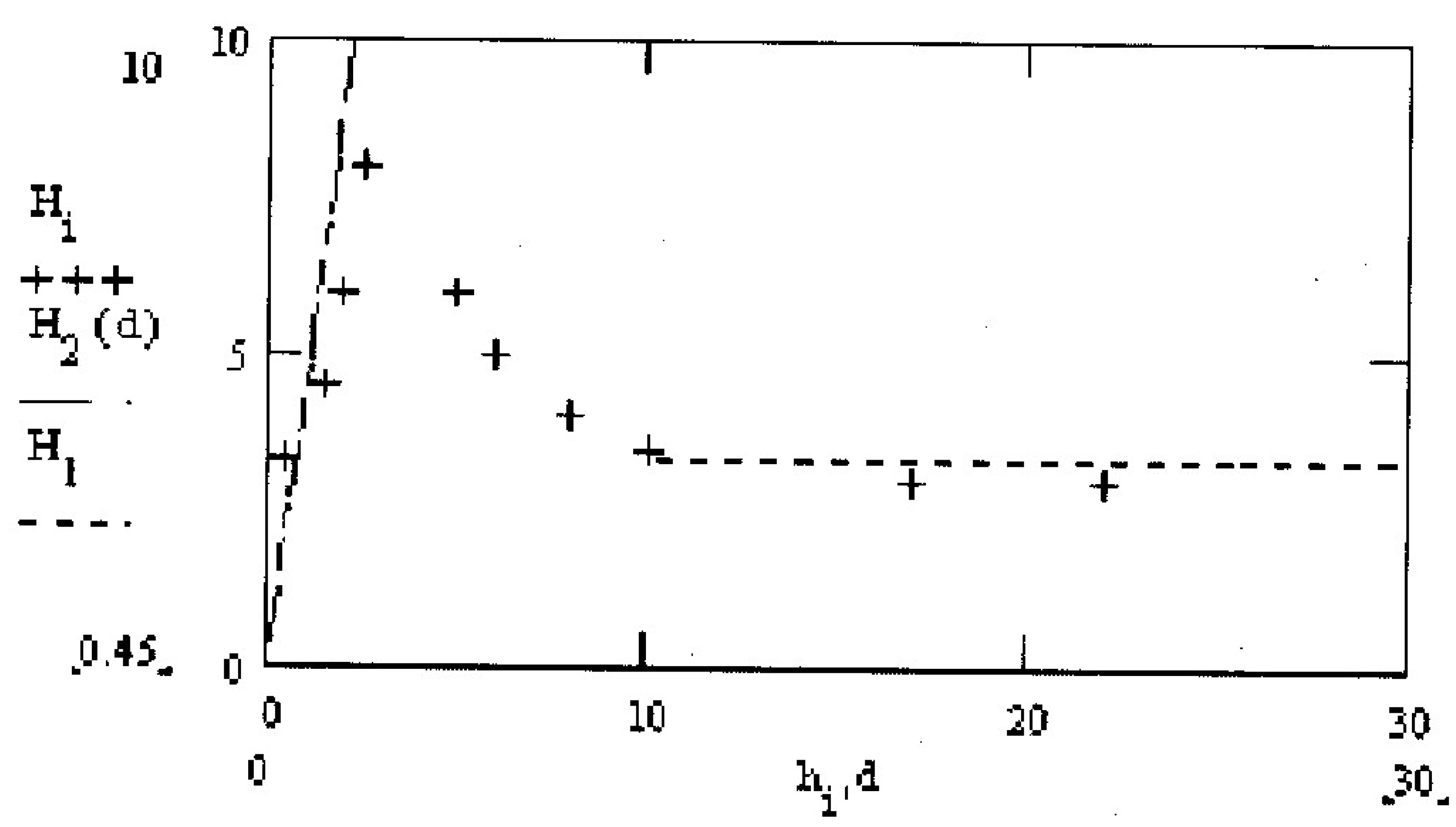

Figure 1: The dependence of the height of popping on the depth of submerging (experimental and theoretical, $R = 0.01\,\mathrm{m}$, $C_x = 0.4$)

This dependence has nonmonotone character although, at first sight, it seems that the deeper one submerges a body, the higher it pops up over the surface.

2 Ideal liquid

Let us explain this dependence theoretically. First to simplify the geometry of the problem we restricted the consideration by spherical shape of a body. To simplify the consideration we assume that the liquid is ideal (viscosity force can be neglected). It is clear that the velocity of the submerging ball in an ideal ($v \to 0$, (v - viscosity coefficient)) fluid approaches stationary value at $t \to \infty$. This velocity can be calculated by the formula:

$$v_\infty = \sqrt{\frac{8\rho R}{3C_x}} \tag{1}$$

where R is the radius of the ball, C_x the hydrodynamic resistance coefficient, for a boddy of spherical shape it can be approximated by the value $C_x = 0.3 - 0.4$, ρ is the density of a liquid. It can be easily obtained by the equality of hydrodynamic resistance and buoyant forces:

$$\rho g V = \frac{1}{2}\rho S v_\infty^2$$

$$S = \pi R^2$$

$$V = \frac{4}{3}\pi R^3$$

3 Height of the popping

The height of popping calculated with the help of formula (1) reads as

$$H_1 = \frac{v_\infty^2}{2\rho} = \frac{4R}{3C_x} \tag{2}$$

For $R = 0.01\,\mathrm{m}$, $\rho = 1000\frac{\mathrm{kg}}{\mathrm{m}^3}$ it is about $3-4\,\mathrm{cm}$. This value is shown on figure 1 by dotted line. This value agrees with experimental one but slightly greater. The difference can be explained by influence of viscosity of the real fluid (water), which decreases the momentum together with pure hydrodynamic resistance.

4 Small submerging

Now let us consider the dependence of height at small (several diameters of a ball or less) depths of submerging. In this case the time of acceleration is small enough and the resistance force is not stationary, because there is not enough time to stabilize the motion. Hence the change of the momentum of the ball caused by this force can be neglected. So we can do not take it into account. Therefore, to get the acceleration we should write down the second NEWTON'S law with buoyant force only:

$$m^* a = F_A \tag{3}$$

Here we take into account the attached mass of the water, which is determined by the following expression:

$$m^* = \frac{1}{2}\rho v^2 \tag{4}$$

where is ρ the density of water and V the volume of the ball. In (3) we neglected the mass of the ball m assuming $m \ll m^*$ (e.g. ping-pong ball), that is why we did not take into account the gravity force of the ball. The buoyant force reads as:

$$F_A = \rho g V \tag{5}$$

From (3) and (5) we get:

$$a = 2g \tag{6}$$

The velocity of the ball can be found immediately as:

$$v = \sqrt{2ad} \tag{7}$$

where d is the depth of submerging.
The height of popping corresponding to (7) is given by simple expression:

$$H_2 = 2d \tag{8}$$

This dependence is shown on figure 1: H_2 on depth d. The agreement is satisfactory at least for small ($d > R$) depth of submerging. The theoretical line goes above the experimental one. Indeed, the formula (8) giving characteristic linear dependence is an upper estimation since we neglected the wave resistance caused by the transmission of the momentum to liquid and thus changing the surface profile. This effect leads to deviation of (8) from the real behavior. The height of popping approaches the maximal value. Then it decreases and soon becomes constant. It is very difficult to describe this crossover interval of depths in rigorous theoretical way as has been said above.

5 Conclusions

Nevertheless, the existence of the depth, which gives the highest popping, follows from the comparison of numerical estimations for (2) and (8). Indeed, for

$$d > d^* = \frac{2R}{3C_x} \approx 2R \tag{9}$$

the inequality

$$H_2(d) > H_1 \tag{10}$$

is fulfilled, therefore there must be the maximum for the depth dependence of the height of popping. The locus of the optimal depth as follows from experiments is about $4R$.