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solution to the problem no. 3
presented by the team of RussiaI

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Spinning disc

Investigate and explain the phenomenon of a spinning annular disc as they progress down a straight, cylindrical rod. If the rod is moved upwards at a defined velocity, the disc spins at constant height. Investigate the mechanism.

Overview

- Experiments
- Theory
  - Uniformly running of the disk
  - Rotation
  - Velocity
  - Friction
  - The rod is moved
  - Gravity force

1 Experiments

We carried out a number of experiments and noticed that the disk could uniformly run around the rod and could slip in the plane of the rod, and by slipping there can be the descent with the retardation. Also, we noticed that the disk could move both downward and upward. And we attempted to explain what we had seen.

2 Theory

2.1 Uniformly running of the disk

At first, consider the uniformly running of the disk about the rod. Consider a simple physical model: A thin disk is spinning ideally and rotates about the rod. The angular speed is constant and slips are absent. Clearly, the period of revolution of a disk point, which is contacting the rod, is equal to the period of revolution of the disc's centre of mass about the rod axis. This is evident from the fact that during one revolution of the disc's centre of mass along $R - r$.
(Where R is the ring radius, r is the rod radius.)

2.2 Rotation

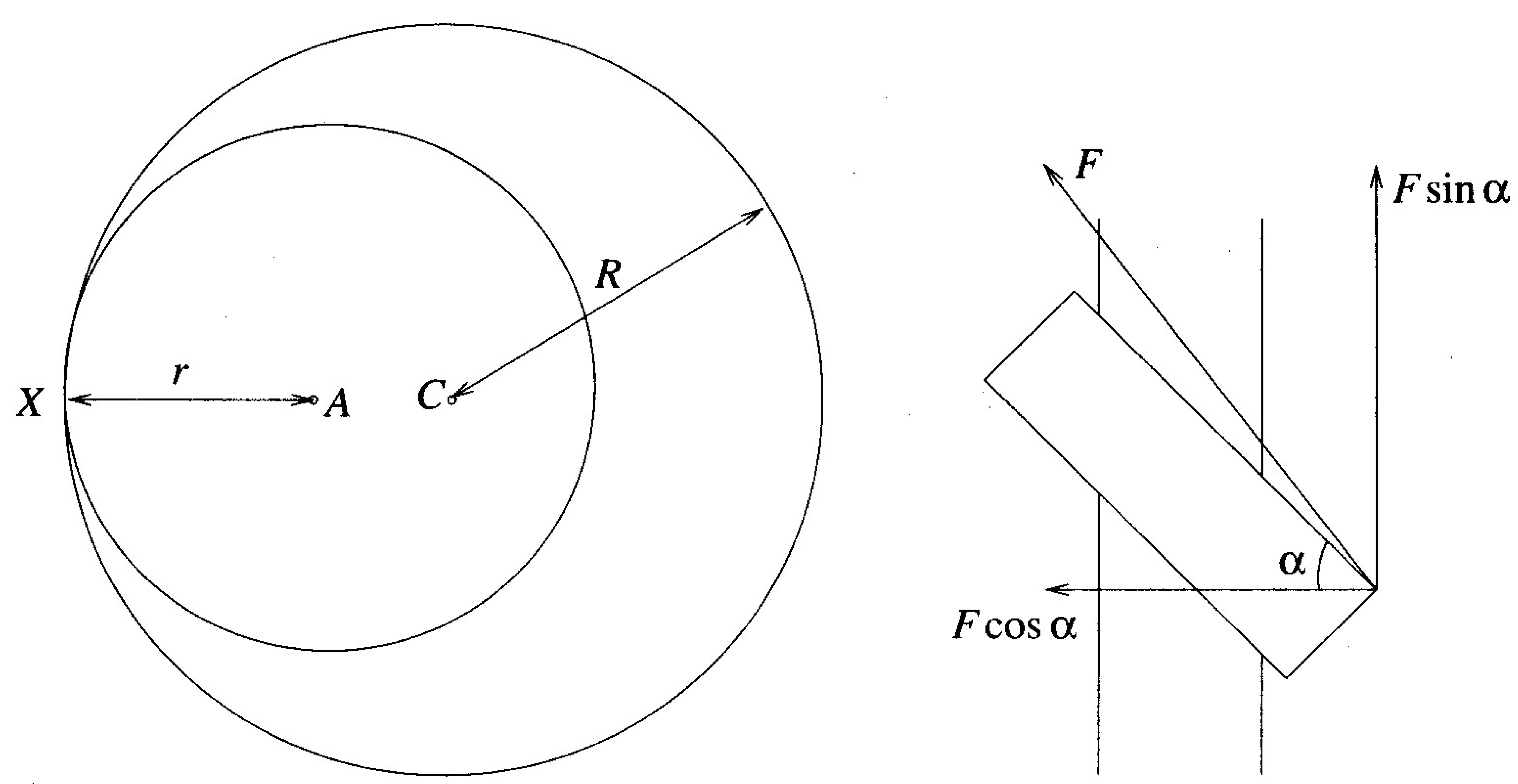

Figure 1: Rotation

The point in contact with the rod performs one revolution, along the radius r. The force that presses the disc against the rod is equal to the product of the centripetal acceleration into the mass of the disc or to the product of the mass into square of the angular speed by the radius difference between the disc and the rod.

$$\begin{aligned} N_A &= N_C \\ v_A &= v_C \\ \frac{\omega_c}{\omega_x} &= \frac{R-r}{r} \\ F &= ma_c = m\omega_c^2(R-r) \end{aligned}$$

The ring's centre of mass is displaced from the axis of rotation (rod axis). That's why the disc will be inclined at some angle relative to the normal of the rod. We can calculate the angle of slope of the disc. Indeed, the sum of projections of all moments of forces that act on the disc, on the direction perpendicular to the disc plane should be equal to zero. So, we see that the sine of the slope angle is the quotient of the gravity acceleration by the product of one square of angular speed into the radius difference between the disc and the rod.

From the experiment, we can see that the disc will move "helically", like a screw, as if screwing itself onto the rod. We found experimentally that the pitch length is the product of the sine of the slope angle into the quotient of the square of the rod radius by the disc radius. Indeed from the right-angled triangle we could find with the help of the PYTHAGOREAN theorem that the pitch length is $R \cdot \sin\alpha$ multiplied by some coefficient. (This coefficient appears because there is the difference between the rod radius and the disc radius.) We can determine this coefficient by the fact, that during one revolution of the ring around the axis of rotation the point of rod, which contacted with the disc in the first moment, makes a number of revolutions which is equal to $\frac{R}{r}$. So this coefficient is $\frac{R}{r}$.

2.3 Velocity

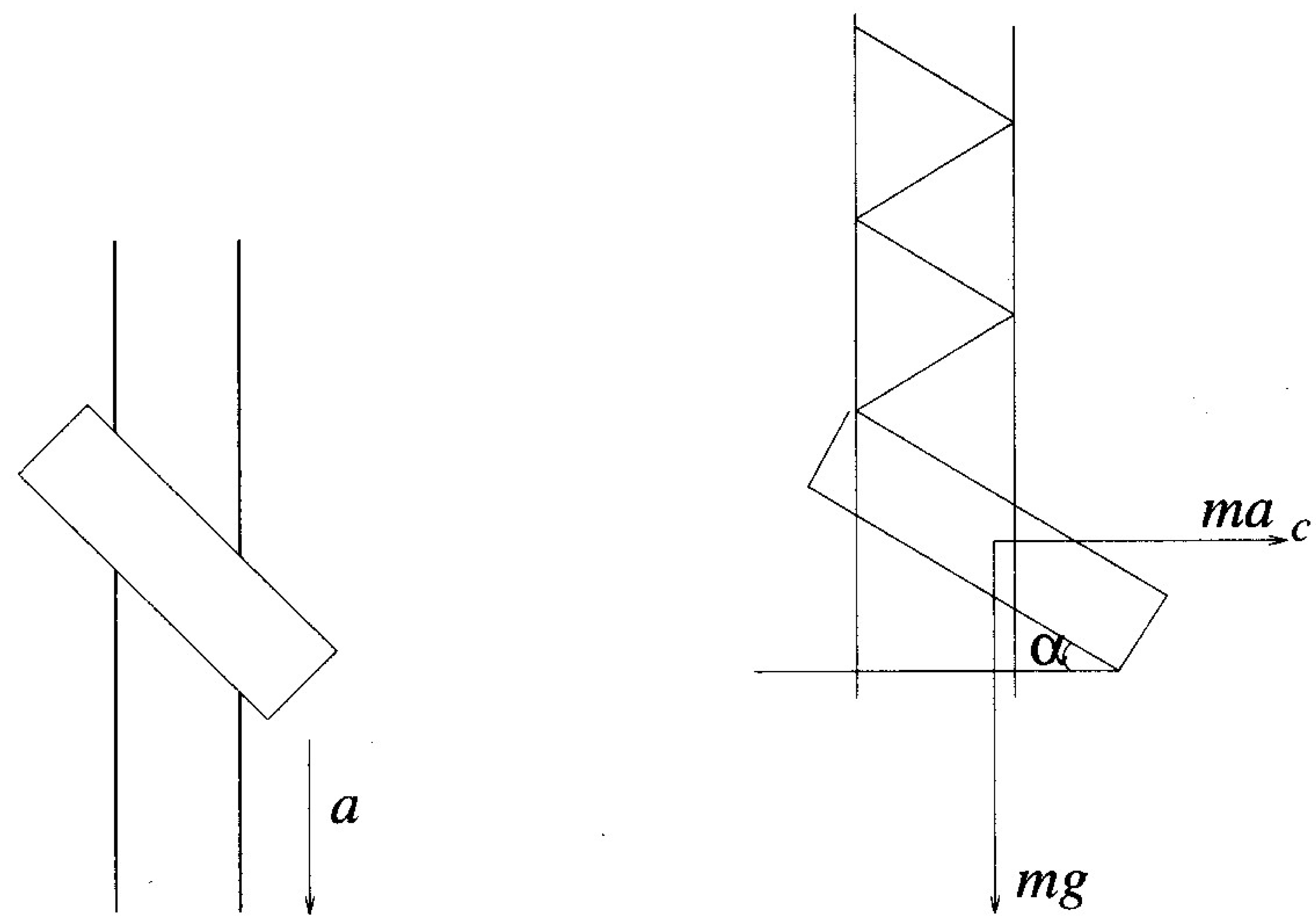

Figure 2: Velocity

$$
\begin{aligned}
a &= g - s\mu\omega^2(R-r) \\
\omega &< \sqrt{\frac{g}{2\mu(R-r)}} \\
\mu &= 0.5 \\
\omega &= 37.5\frac{1}{s} \\
ma_c \sin\alpha &= mg\cos\alpha \\
\cot\alpha &= \frac{a_c}{g} \\
\cot\alpha &= \frac{\omega^2 R\cos\alpha}{g} \\
\sin\alpha &= \frac{g}{\omega^2 R} \\
H &= R\sin\alpha\frac{R}{r} \\
v = \frac{H}{T} &= \frac{H\omega}{2\pi} = \frac{gR}{2\pi r\omega}
\end{aligned}
$$

The velocity of descent we can determine as the quotient of the pitch length by the time of traversal of this length. We can express time as the quotient 2π by the angular speed. Then we substitute so determined time in the formula of descent velocity. After reductions, we get that the velocity of descent is equal to $\frac{gRrk\omega}{2\pi}$.
The descent velocity dependence on time has been obtained experimentally.

(The plot was made by measuring the pitch length and the slope angle and substituting it into the above formula.)
The pitch is around the rod, then we coated the inside of the disc with Indian ink and set the disc running. We got distinct imprint remained on the paper.

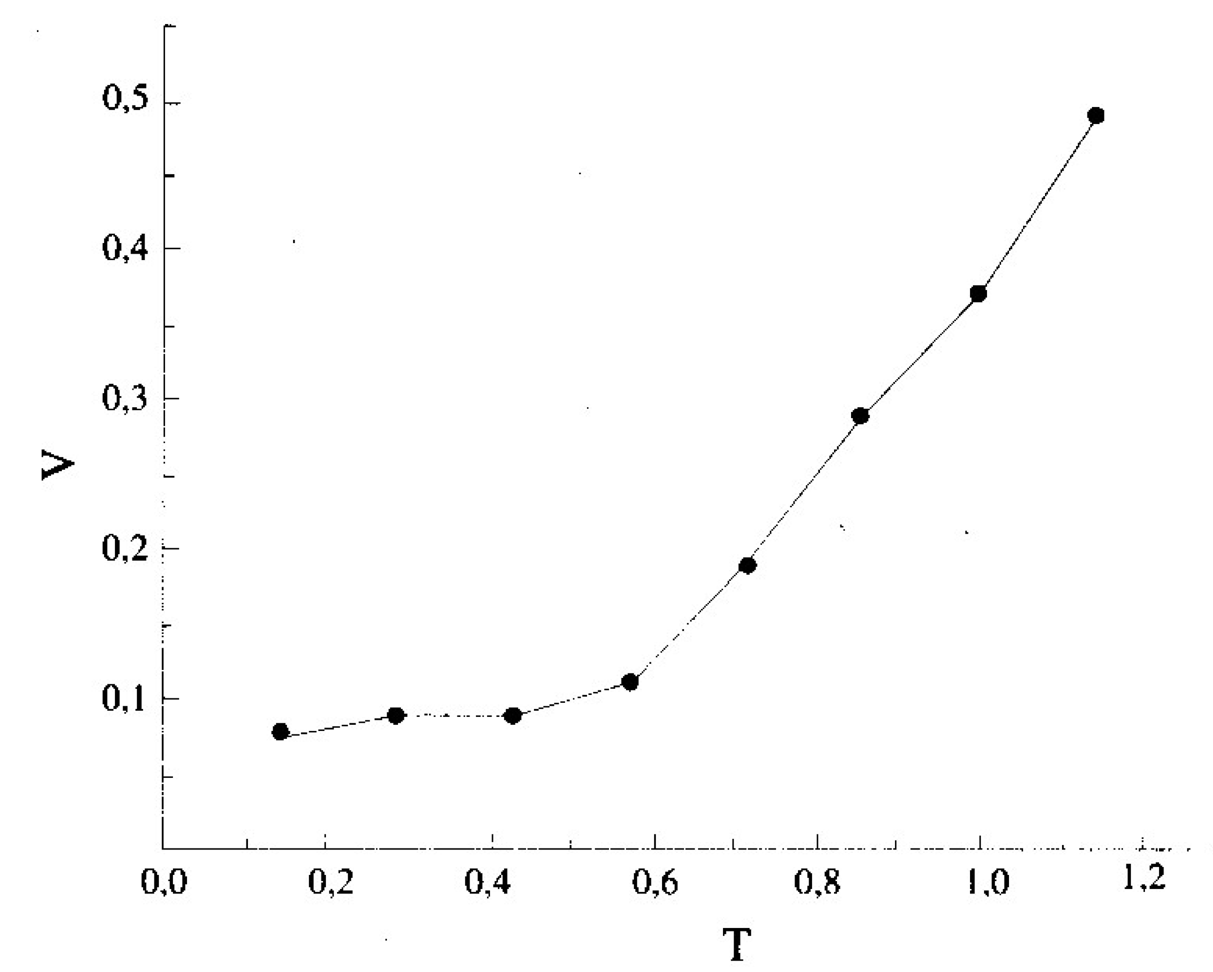

Figure 3: Descent velocity dependence on time

This plot qualitatively agrees theory. Only the gravity force acts in the direction of the movement of the disc. Therefore, the velocity should be depend on time in a linear fashion (on the steady portion of the path) besides, from our formula we can see that the descent velocity is inversely proportional to the angular speed of the disc.
It is evident that the angular velocity decreases during the movement of the disc due to air resistance. And the angular velocity can be so small that the pressing force will be less than the gravity force and the slippage's arise.
The acceleration, which appears of slippage is the difference on the gravity force and the sliding friction force divided by disc mass. The sliding friction force divided by disc mass. The sliding friction we can determine as the product pressing force by the sliding friction coefficient.

2.4 Friction

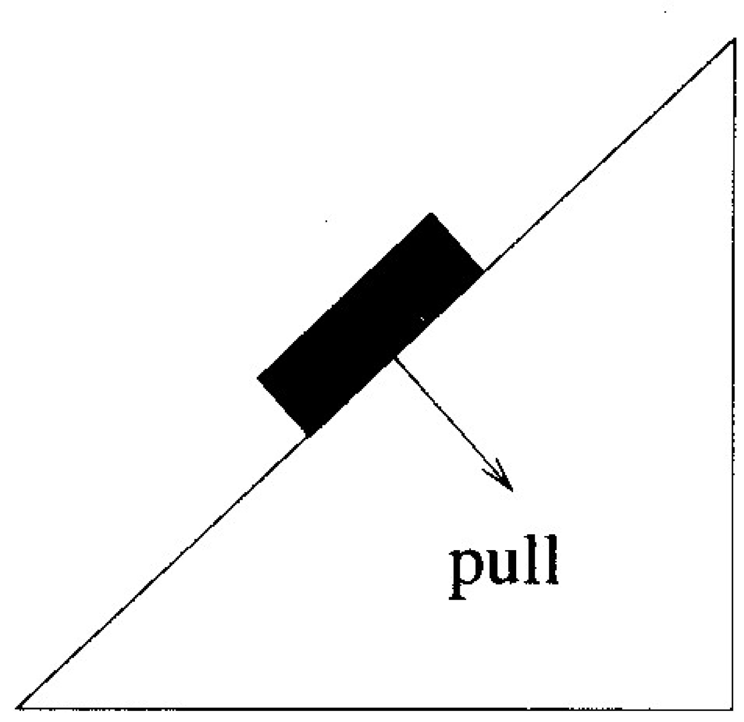

Figure 4: Friction

The sliding friction coefficient was estimated as the tangent of the critical slope angle (the angle at which the disc begins to slip of the rod). But this is the slippage's in horizontal direction. How then can we extend these results to slippage's in the vertical direction.
We carried out following experiment: We put a load on an inclined plane and started pulling that load strictly horizontally. If the slippage's arise in the horizontal direction, they arise in the vertical direction, too. A similar analogy can be drawn for a disc rotating on a rod.

2.5 The rod is moved

Let us pass to the second part of the problem. For this to be done, the disc should uniformly run about the rod. Then by pulling the rod upwards at the same velocity at which the disc descents, we make sure that the relative speed becomes equal to zero. The disc will be on the same level.
As you have seen in the video material, the disc could move upward. If we allow for the fact that the force which set the disc running we applied at a certain angle to the plane of the disc, we can see that the longitudinal component could be greater then transversal component. So the disc can move upward. But what circumstance is responsible for disc's descending with retardation. When the disc slips, in continues to revolve. And any fluctuation of the rod could lead to change of the slope angle (until to opposed in sign angle). It means that the slope angle of axis of rotation will change. And the disc passes in the regime of steady motion. But this position is unstable due to small angular velocity. And the disc will slip against in a short time.

2.6 Gravity force

The following question could arise. The gravity force perfumes the work by the disc descending. Where does the energy disappear. We think that this energy is used to get over the air resistance. Thus we can say that the work of gravity force is equilibrated by the air resistance.