11th IYPT '98
solution to the problem no. 4
presented by the team of Austria

Water streams

A can with three holes in the side-wall at the same height slightly above the bottom is filled with water. The water will escape in three separate streams. By gently touching the streams with a finger they may unite. Investigate the conditions for this to happen.

Abstract

As the initial conditions we considered the diameter of the holes, the distance between them, the diameter of the can and the height of the water. We simplyfied the streams as parallel (a big can), the cross sections as circular and the acceleration of the deflection as constant. The main idea was that the decrease of surface energy compensates additional kinetic energy for deflecting jets. The equation we created out of that corresponded to the experimental results as long as the simplifications of the geometry were valid.

Overview

- Initial conditions
- Mechanism: Surface tension
- Simplify
- Idea
- Rough estimation for the distance where the jets may unite
- Comparison with experiments

1 Initial conditions

- Diameter of the holes
- Distance between the holes
- Diameter of the can
- Height of the water

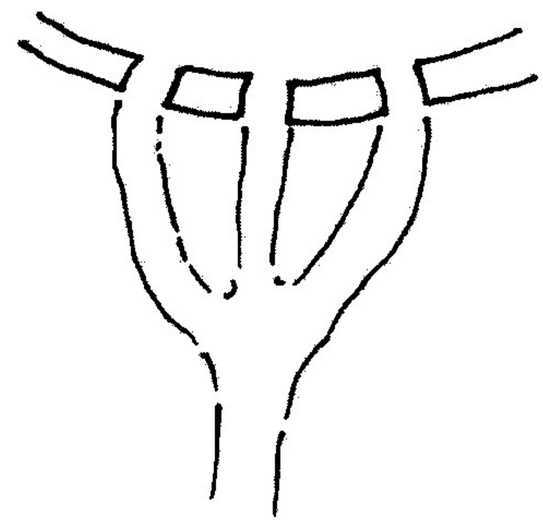


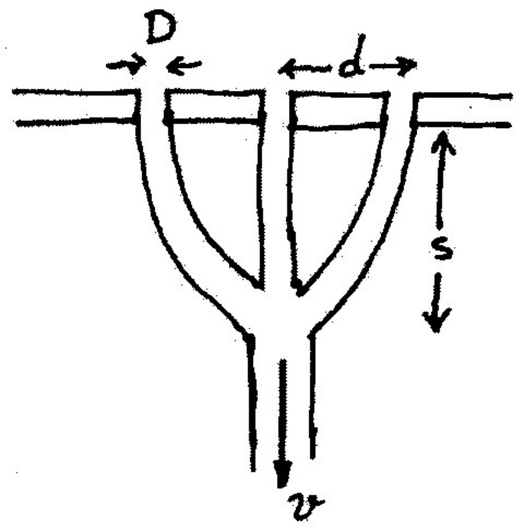
Figure 1: Uniting streams

2 Mechanism: Surface tension

- Surface energy decreased by uniting the streams.
- Too high velocity and/or to large distance between the streams cannot be compensated.

3 Simplify

- Streams parallel (big can)
- Circular cross-section (big can)
- Deflection of jets with constant acceleration



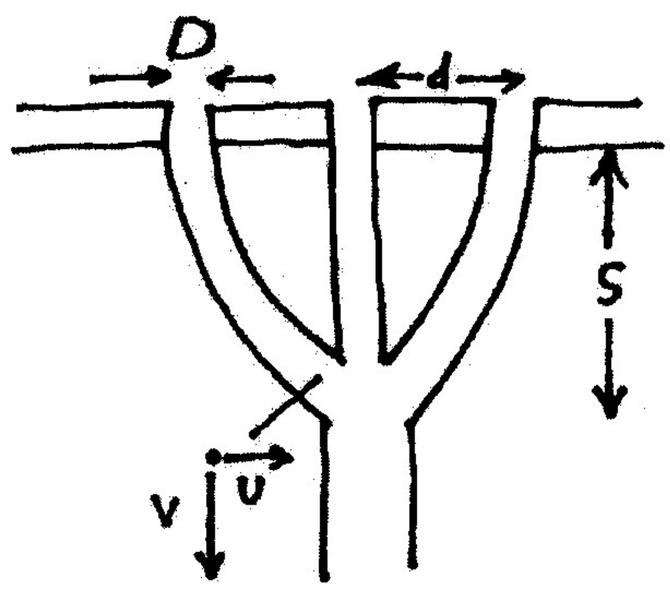
4 Idea

Decrease of surface energy compensates additional kinetic energy for deflecting the jets!

$$\Delta E_{\text{kin}} \leq \Delta E_{\text{surface}}$$

D	=	jet diameter
d	=	hole distance
v	=	jet velocity
l	=	length of separated jet
s	=	distance where jets unite

$$\begin{aligned}
\frac{m \cdot u^2}{2} &\leq \sigma\pi\left(3D - D_{\text{united}}\right) \\
u &\approx \frac{2dv}{s} \\
\left(\frac{D}{2}\right)^2 \pi l\rho \cdot u^2 &\leq \sigma\pi l D\left(3-\sqrt{3}\right) \\
v^2 &\leq \frac{s^2\sigma\left(3-\sqrt{3}\right)}{Dd^2\rho} \\
v^2 &= 2gh
\end{aligned}$$



5 Rough estimation for the distance where the jets may unite

D = hole diameter
d = hole distance
s = length of separated jets
h = height of water

$$s \approx \sqrt{\frac{2ghDd^2\rho}{\sigma(3-\sqrt{3})}}$$

6 Comparison with experiments

Corresponds to experimental results as long as the simplifications of the geometry are valid.
Example:
$h = 5\,\mathrm{cm}, \quad d = 7\,\mathrm{cm}, \quad D = 3\,\mathrm{mm}, \quad s \approx 2\,\mathrm{cm}$