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solution to the problem no. 4
presented by the team of Uzbekistan

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Water streams

A can with three holes in the side-wall at the same height slightly above the bottom is filled with water. The water will escape in three separate streams. By gently touching the streams with a finger they may unite. Investigate the conditions for this to happen.

Abstract

Given conclusion consists of:
- Experimental part based on conditions of the problem.
- Some common sense conclusions helping to find out the most important factors influencing the phenomenon.
- Development of theory on these factors.
- Experimentally based conclusions about the theory results.

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Overview

- Experiments
- Common sense part
- Conclusions

1 Experiments

A set of experiments in joining water jets has been carried out with plastic vessels. We made streams direction perpendicular to vessel's surface, and found out that they can join only if the water level was lower then critical value. We investigated the dependence of this critical height on diameter of holes and distance between them. Experiments with different diameters have shown that most with diameters larger then 4 mm this process is being influenced by holes' roughness and shape. That is why experimental data is shown only for holes of 1 – 4 mm diameters (figure 1). It was also noticed that confluence of streams happens moistening of the vessel's surface between the

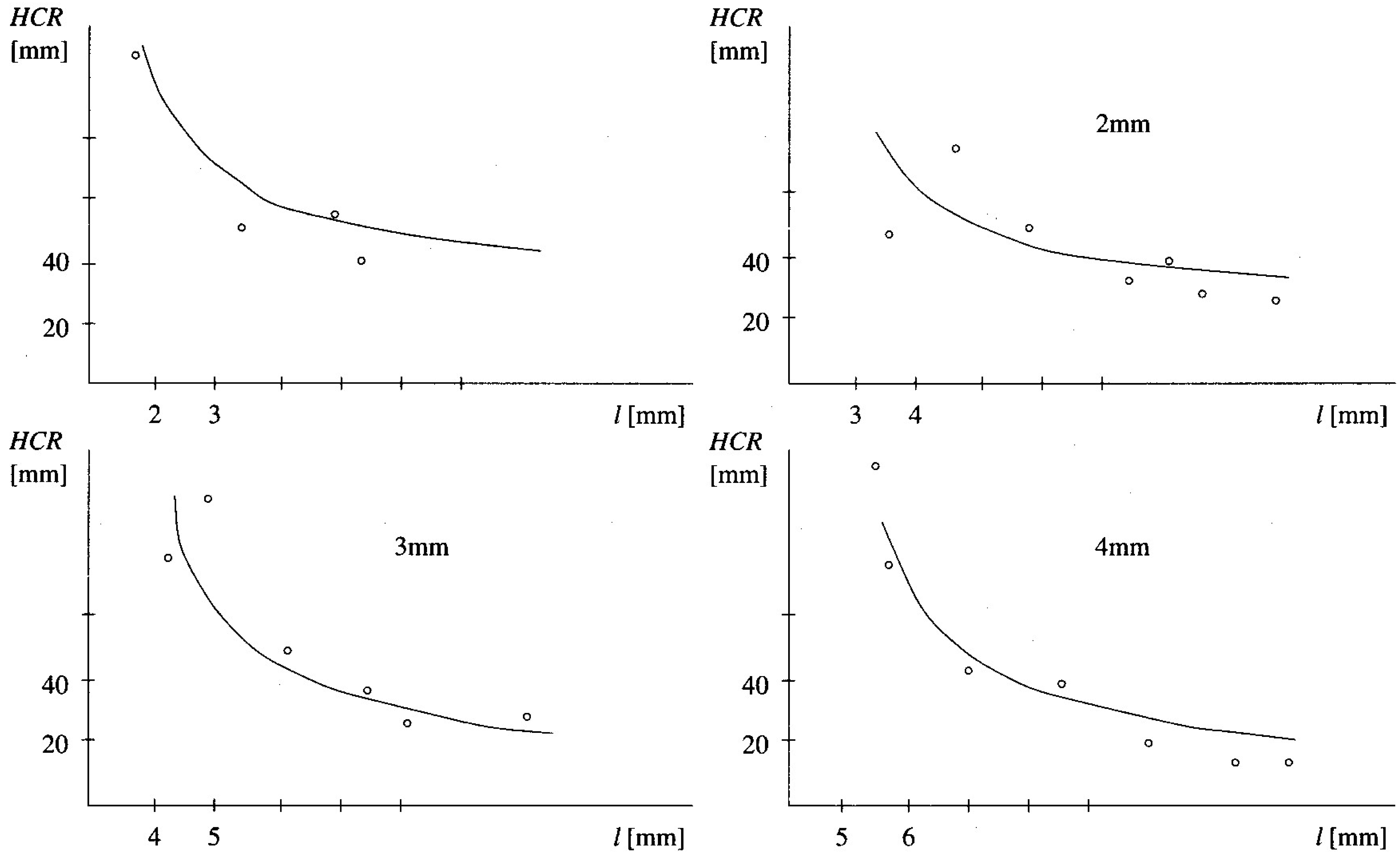

Figure 1: Experimental data for holes of 1 – 4 mm

holes. Because after moistening a thin water film appears and joins streams. Moistening happens with the help of fingers.

2 Common sense part

From common sense we may consider if there is any energy profit in the jets joining. Comparing the total surface energy of three separate streams E and those being joined into one E' and according to law of flow conservation:

$$3S \cdot v \;=\; S' \cdot v \Rightarrow r' = r \cdot \sqrt{3} \tag{1}$$

where
- S - square of area
- r - radius of separated streams
- S', r' - same of the connected stream

We have equation for energy profit $E(pr)$:

$$E(pr) = E - E' = 3 \cdot \pi \cdot a \cdot \sigma \cdot \left(2 - \sqrt{3}\right) \cdot r \tag{2}$$

where
- a - length of separated streams
- σ - coefficient of liquid's surface tension

Experimentally we noticed that at critical height film breaks and streams disjoin. So we may suppose that this energy profit spends on the support of films existence. And there are two possible mechanisms of its destruction. First is probability of film to be broken due to large difference in their directions. And second is probability of film to be broken form the vessel wall due to high pressure produced by jets on film cross-section.

The essence of first process is that pressure of film meniscus P my be balanced by jet pressure P_1, making the film more and more thinner and even break it. From balance of LAPLACE' pressure and liquid's pressure due to vessel curvature we have the following equation:

$$H'_{\max} = \frac{8\sigma r R_c^2}{(l^2 - 4r^2)\,\rho g l^2} \quad (3)$$

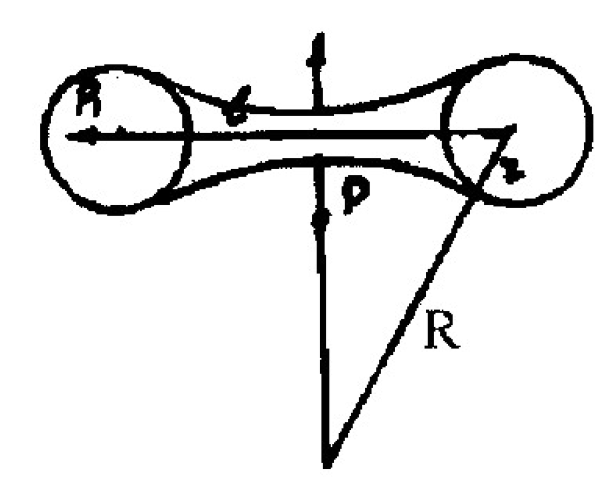

Figure 2: $H'_{\max}$

where
- l - distance between the holes' centres
- ρ - liquid density of water
- R_c - radius of vessel's curvature

This factor is of great importance in the case of small radius of vessel's curvature or small holes radiuses and large distance between them. Let's try to find out the dependence of critical water level on small distances in 2nd item.

Form the balance of pressure acting at films cross-section and surface tension forces we have a equation for maximum height $H_{\max}$:

$$H_{\max} = \frac{\sigma \varphi R_m}{\rho g S} \quad (4)$$

$$\varphi = \arcsin \frac{l}{2(R_m + r)}$$

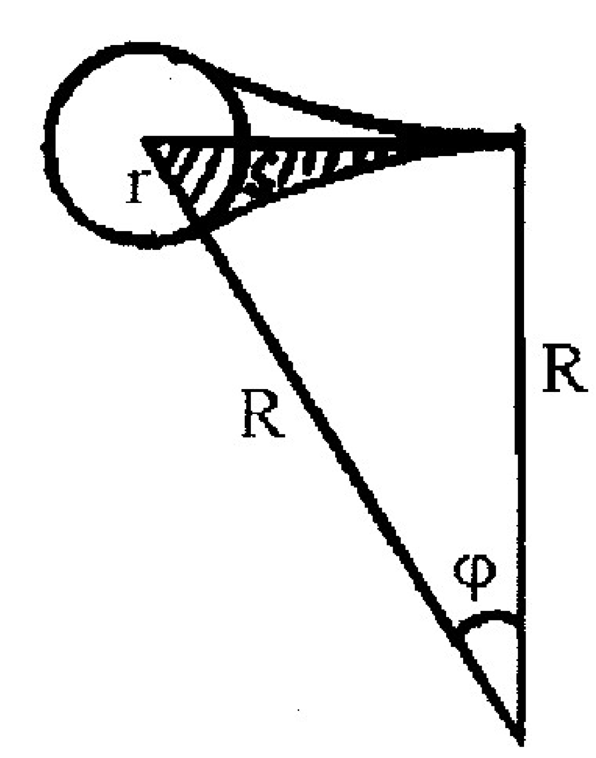

Figure 3: $H_{\max}$

where
- S - cross-section of film meniscus
- R_m - radius of meniscus curvature
- φ - angle between the centres of the hole and the film

This dependence is shown at the graphs (figure 1) by solid line.

3 Conclusions

One can see that this process takes places at rather short distances between the holes, when film's centre is not very thin. So it means that pressure of water flow velocity is of the first importance here. Obviously, to make the process description more detailed velocity change in different parts of the film must be taken into account because the changes pressure and forces balance as well. The cross-section of flowing jet is influenced by the holes' roughness, vessel wall's thickness, shape of the hole. But still our estimations are in agreement with the experimental data.