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solution to the problem no. 5

presented by the team of RussiaII

Alexandre Mikrioukov

Александр Микрюков

Water jet

If a vertical water jet falls down onto a horizontal plate, standing waves will develop on the surface of the jet. Investigate the dependence of this phenomenon on different parameters.

Overview

- Solution
    - The study of parameters which the phenomenon depends on
    - Finding of the obtained dependencies mathematically
    - Fall of jet on a solid barrier
- Conclusions

1 Solution

Before considering the effect, let's look again at the requirement of the problem. There it is spoken about a horizontal surface, but it is not spoken about what it represents. This surface can be both solid (for example glass plate) and fluid (a surface of water, poured in a cup). As a result of carrying out the tests it becomes clear, that the presence of the effect is not stipulated by a kind of material, to a surface of which the jet falls. Besides surface unnecessarily should be horizontal.

Now let's consider the effect on the example of a jet falling on water surface. The laminar jet streams from a hole of a vertical tube and, freely falling (before the beginning of processes of interruption), hits in a stratum of water, poured in a cup. Because of small pressure it does not form the small cavity on a surface; and right angles in the place of connection are smoothed out under action of surface tension. If the edge of a tube is apart approximately 2 cm from surface of water, it is possible to see a clear pattern on jet surface (see figure 1 and also photos). The pattern is stationary and its shape resembles waves. Because of stability of the pattern it can seem, that the standing waves were formed. But if there are standing waves, the superposition of two coherent waves going towards to each other should take place. Then single variant of real effect can be only following: perturbations (waves) are forming in the falling jet, they reach a surface of water and, being reflected, are superimposed on going towards them. Such pertubations, which at last give a breakage of a jet in drops. But their amplitude is so small that it is impossible to see waves clearly, so they can not create this pattern. There are no other stationary perturbations in laminar jet. So, the oberved pattern is not standing waves, and it can be only running waves. In the place of slope of jet there is a source of disturbing; these pertubations are spread up on surface of a jet as waves, and in each point of a surface the waves have the same velocity, as water has, but it is directed up; i.e. the waves running up with velocity of jet, and relatively the observers the stiffen.

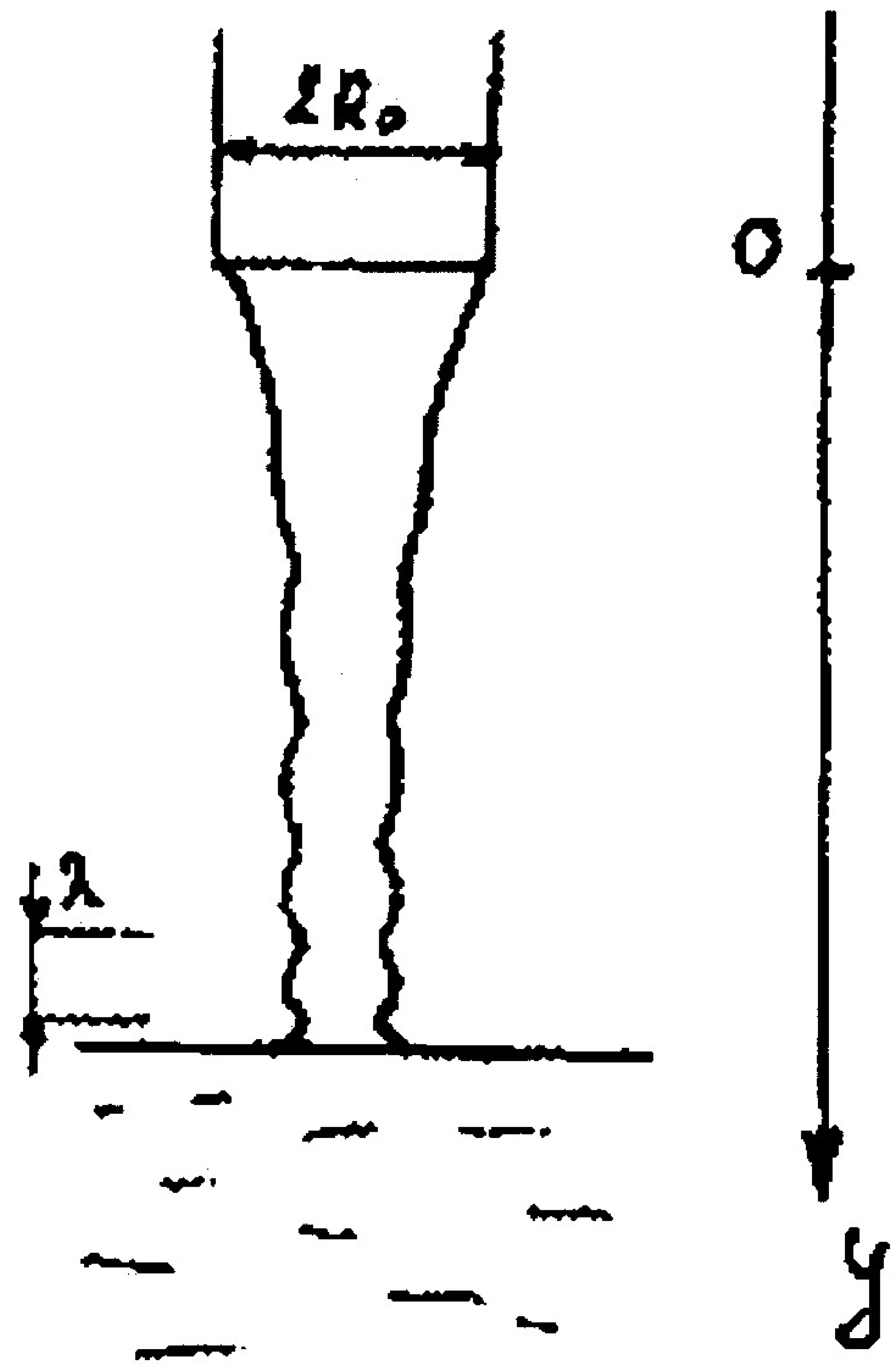

Figure 1: Experiment

1.1 The study of parameters which the phenomenon depends on

What regularities can be seen during a study of the phenomenon? All of them are bound with the length of generated waves.

- The length of waves on the same pattern increases at approach to edge of the tube.
- At increase of distance from the end of the tube to water surface length of the first wave (so also all remaining) decreases. Formed gofer because more compressed downwards, and at some distance ceases to coal the whole jet.
- The same takes place and in case of increase of jet velocity.
- When the diameter of the tube of temperature of water are changed, but values of all remaining parameters are constant, nothing happens.

In order to receive points, at which it would be possible to plot numbered dependencies, the photography of the jet was made (photos are applied to the report). On each photo, if besides the jet there is also a rule on it, it is possible to determine wave length, and also distance from edges of the tube to surface of water. Velocity of water flow from the tube, diameter of the tube and temperature of water are measured during test. For confirmation of the last two expressions (concerning tube diameter and temperature of water) there is no need to plot diagrams, since all is visible already from photos. The diagrams of the first three dependencies are given in the work.

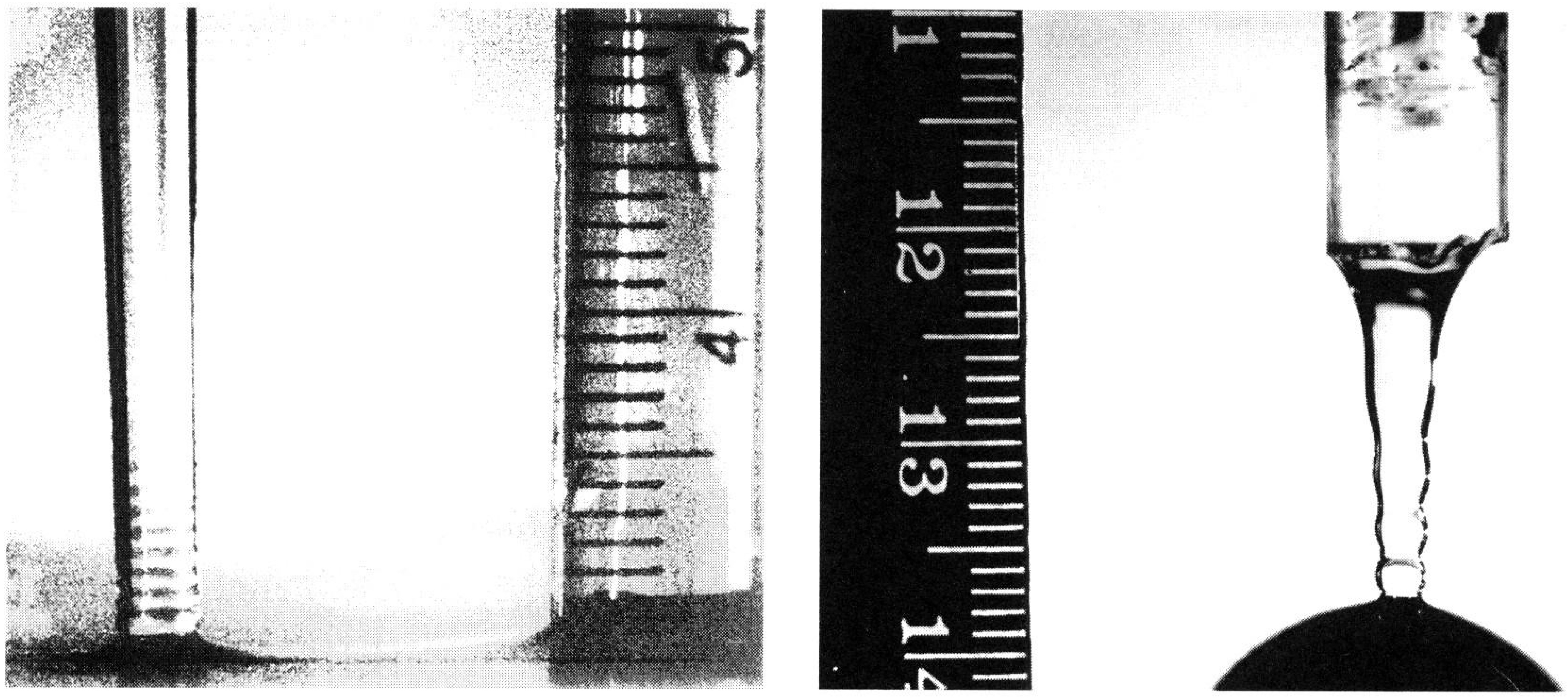

Figure 2: Photos

All of them are hyperbolas.

It is not necessary to be surprised to large errors. The lengths of generated waves do not exceed 4 mm, and they have, as a rule, values not equal to an integer of millimetres, and the measurements, even on photographs were made with millimetre rule. The points marked on the diagrams are experimental; curves on them are deduced partly theoretically, partly empirically (method by which they were found, is illustrated below). It is important that the points locate near curves or lay on them, i.e. the formulas relevant to curves really reflect required dependencies.

1.2 Finding of the obtained dependencies mathematically

In 1935 FRENKEL at study of oscillations of charged horizontal surface of liquid deduced the following formula:

$$\omega^2 = \frac{2\pi}{\rho\lambda} \cdot \left(\rho g + \frac{4\pi^2\sigma}{\lambda^2} - \frac{8\pi^2\alpha}{\lambda}\right) \tag{1}$$

ω = oscillation frequency on surface of fluid (source of oscillations),
λ = wavelength of these oscillations,
α = surface density of charge.

The first term in brackets reflects influence of gravitational forces, second surface tension, and third forces of an electrical origin. This formula can be applied in our case. The gravitational forces, since the surface of the jet is vertical, forces of an electrical origin also will not influence, since the surface of the jet is not charged. Therefore there will be only 2^{nd} term in brackets reflecting activity of surface tension. These forces try to smooth surface perturbations. But in jet one more smoothing factor acts, it is an overpressure under a curved surface, since the jet is as if fitted close by cylindrical film. When on a site of the jet there is a wave, under a hump the pressure is more, than under a cavity and all happens as on the sea; if on water there appears a single wave, because of difference of pressures humps and cavities are levelled. The further stream leaves from edge of tube, the less its radius, the more pressure inside it, the more smoothing factor. If now to get accustomed closer to the third term in brackets in the formula above (1), it is possible to note that α has dimensionality of pressure. When we have substituted pressure instead of α in the formula, we succeeded to select such coefficient at the third term that the curves, relevant to the formula, located close to points of the diagrams. This coefficient did not need a long time for selection. It

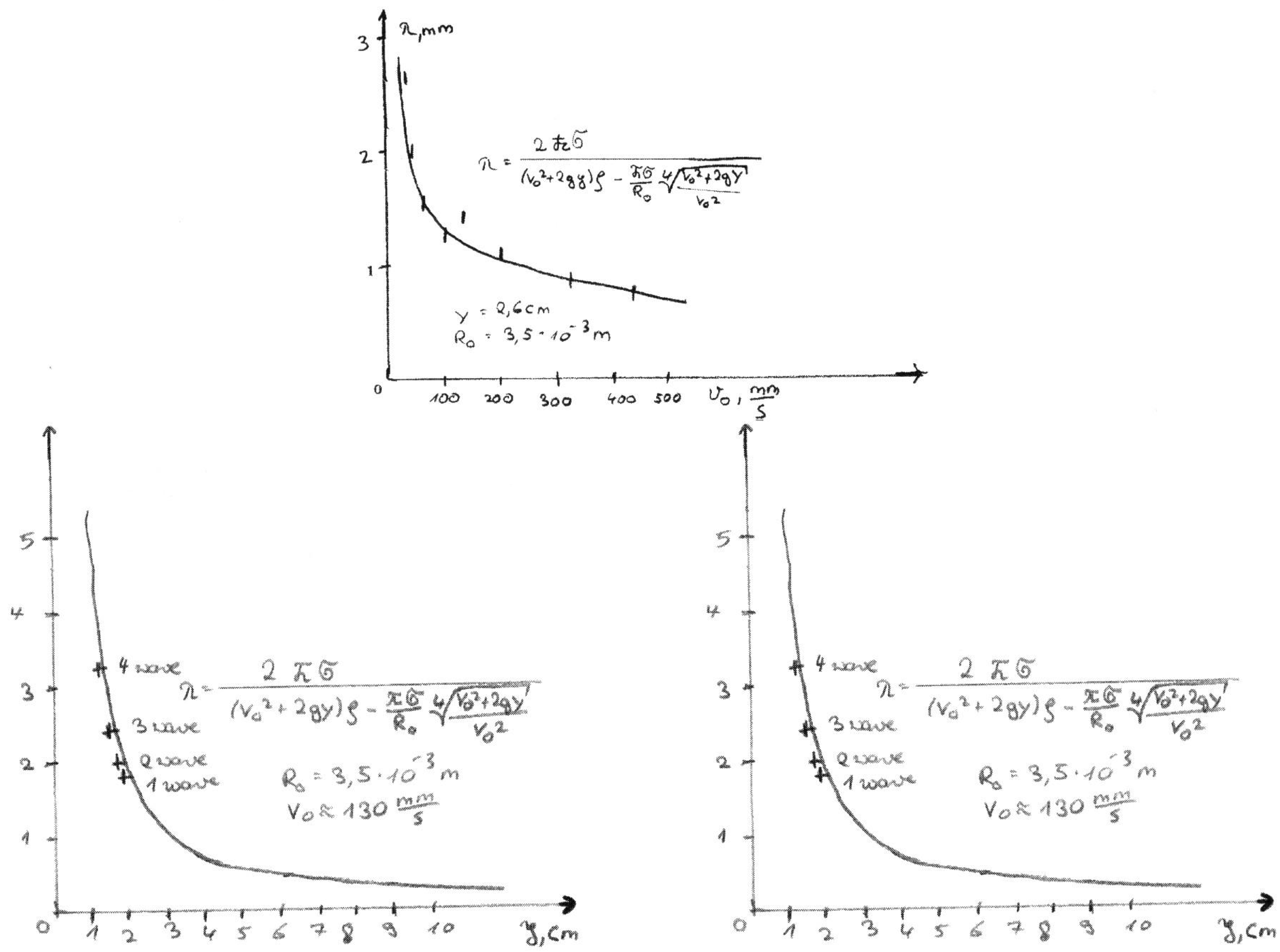

Figure 3: Diagrams

is enough to write 2 instead of numeral 8 before $\pi^2\alpha$:

$$
\begin{aligned}
\omega^2 &= \frac{2\pi}{\rho\lambda}\cdot\left(\frac{4\pi^2\sigma}{\lambda^2}+\frac{2\pi^2 p}{\lambda}\right) \\
\frac{4\pi^2}{\lambda^2}\cdot v^2 &= \frac{8\pi^3\sigma}{\rho\lambda^2}\left(\frac{1}{\lambda}+\frac{1}{2r}\right) \\
v^2 &= \frac{2\pi\sigma}{\rho}\left(\frac{1}{\lambda}+\frac{1}{2r}\right) \\
\lambda &= \frac{2\pi r\sigma}{v^2\rho r-\pi\sigma} = \frac{2\pi\sigma}{\left(v_0^2+2g\gamma\right)\rho-\frac{\pi\sigma}{r}}
\end{aligned}
\tag{2}
$$

From a continuity equation of jet it is possible to express:

$$
\begin{aligned}
\pi R_0^2 v_0 &= \pi R^2 v \\
R_0^4 v_0^2 &= R^2 v^2 \\
r = R &= R_0\sqrt[4]{\frac{v_0^2}{v_0^2+2g\gamma}}
\end{aligned}
$$

Then:

$$
\lambda = \frac{2\pi\sigma}{\left(v_0^2+2g\gamma\right)\rho-\frac{\pi\sigma}{R_0}\sqrt[4]{\frac{v_0^2}{v_0^2+2g\gamma}}} \tag{3}
$$

All curves $\lambda(\gamma)$ and $\lambda(v)$ are derived on the formula (3). From here one can see that the indirect temperature dependence exists (since the surface tension depends on temperature), but it is very feeble. And here dependencies on a diameter are not to be, since finally, all dependencies are reduced to dependence $\lambda(v_0)$. But its less demonstrative.

1.3 Fall of jet on a solid barrier

When the jet with small initial velocity falls on a solid horizontal surface, on a surface because of wetting arises water hump, it is as if that the jet falls not on a solid surface, but again on water. The effect will be observed, but no so clearly because of instability of water surface. Therefore these examinations are not interesting. But if a jet is directed to a glass sphere instead of a plate, the water will flow from it, a jet will fall immediately on a solid surface, and the images will be received even better than on water. In this case all again will be subject to the above described regularities.
This effect reveals rather wide. So, if we touch by the end of a needle a surface of a thick laminar jet leaking from the cock, the same waves will be received on the surface.

2 Conclusions

As a result of examinations we have described all visual regularities of the phenomenon (i.e. dependence of effect on all obvious parameters), therefore it is possible to consider that the problem is carried out.