11th IYPT '98
solution to the problem no. 6
presented by the team of Germany

Katharina Kühn

Mount Everest

Can you see Mount Everest from Darjeeling?

Abstract

The question whether you can see Mt. Everest from Darjeeling has many different aspects. At first, I want to look an the geographical side of the problem. That means in fact whether there is a mountain between Darjeeling and Mount Everest that is so high you cannot see Mount Everest. I want to take into account, that the light might be refracted on its way though the different layers of air. Then I consider the aspects of the problem that are connected with the abilities of the eye, namely the resolving power of the eye and the ability to see contrasts. This leads to the last topic, considerations about the diminuation of contrasts by scattering and a comparison to the actual atmospheric conditions in this area.

Thanks

First of all I would like to thank the layout-team, Martina Schäfer, Jan Theofel and Hendrik Hoeth who are spending a lot of time editing these reports. A lot of thanks also to all the nice people who were so patient to listen to me practising my report and to the whole organising team, especially Mr. Lehn.

Overview

- Geometry
  - Geographical data
  - Obstacles between Mount Everest and Darjeeling
- The refraction of light
- The resolving power of the eye
- Contrasts
- Scattering
  - Rayleigh-scattering
  - Mie-scattering
- The weather in the Himalaya

1 Geometry

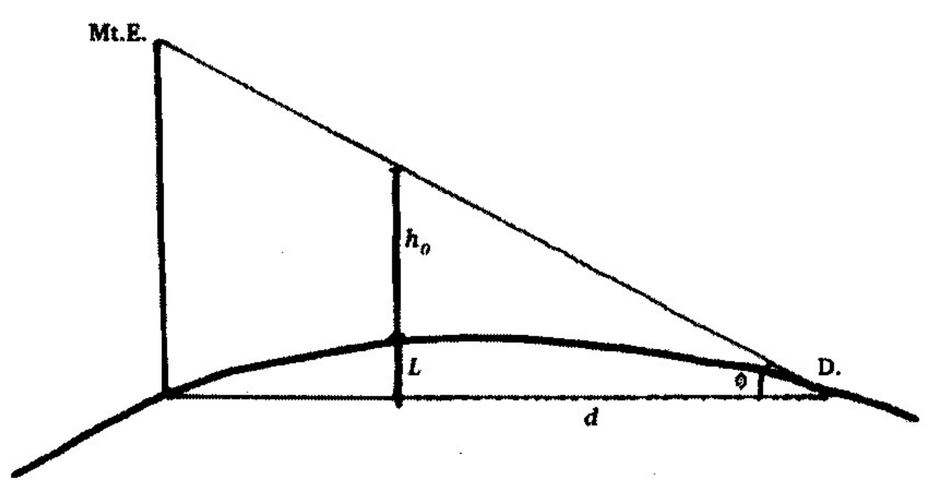
1.1 Geographical data

Darjeeling	Mt. Everest
altitude: 2,185 m	altitude: 8,872 m
latitude: 27°02'N	latitude: 27°58'N
longitude: 88°20'E	longitude: 86°57'E

Therefore the angle Darjeeling – center of the earth – Mt. Everest is 1.54°, the distance is about 170 km.

1.2 Obstacles between Mount Everest and Darjeeling

In order to control whether the mountains between Mt. Everest and Darjeeling are high enough to prevent you from seeing Mt. Everest,



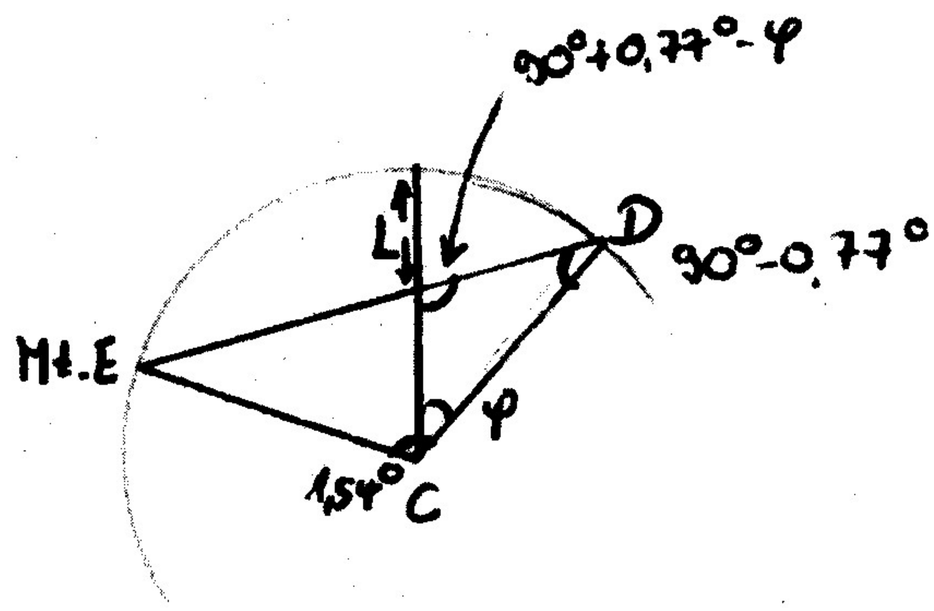
I divide the distance between Mt. Everest and Darjeeling into intervalls of 20 km. For each of these intervalls I calculate the maximum possible hight of a mountain and compare this theoretical maximum hight H with the actual hight of the mountains in this area.
The formula, that gives H, on the one hand has a linear part: The further the mountain in away from Darjeeling, the higher it can be. On the other hand the formula needs a part that takes into account, that the earth is not flat.

$$\begin{aligned} H &= 2{,}185\,\text{km} + h_0 \\ H &= 2{,}185\,\text{km} + \frac{d \cdot 6{,}687\,\text{km}}{170\,\text{km}} - L \end{aligned}$$

according to the trigeometric relations:

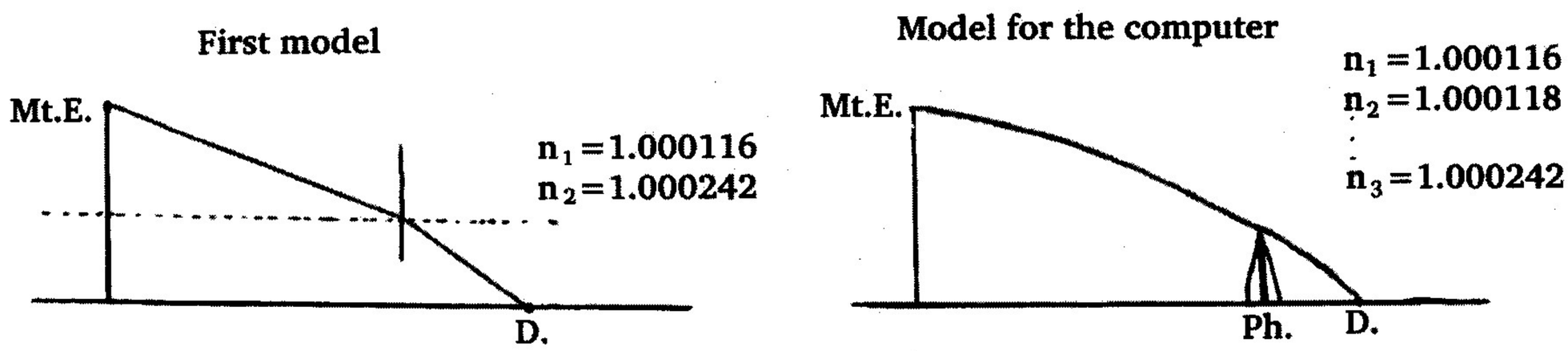
$$\begin{aligned} \frac{\overline{CD} - L}{\overline{CD}} &= \frac{\sin(90° - 0.77°)}{\sin(90° + 0.771° - \phi)} \\ L &= \overline{CD} \cdot \left(1 - \frac{\sin(90° - 0.77°)}{\sin(90° + 0.77° - \phi)}\right) \\ L &= 6{,}370\,\text{km} \cdot \left(1 - \frac{\cos 0.77°}{\cos(0.77° - \phi)}\right) \\ H &= 2{,}185\,\text{km} + \frac{d \cdot 6{,}687\,\text{km}}{170\,\text{km}} - 6{,}370\,\text{km} \cdot \left(1 - \frac{\cos 0.77°}{\cos(0.77° - \phi)}\right) \end{aligned}$$

Now I compare the values, calculated with the formula for different distances from Darjeeling, with the actual hight of the mountains. The result of a close look on the map is, that on the one hand there is a mountain called Phallut, that is hight enough to prevent you from seeing Mt. Everest from the city of Darjeeling, but is not high enough to prevent you from seeing Mt. Everest from a place in the district of Darjeeling, called 'tiger hill'. This fact is also verified by different books.



2 The refraction of light

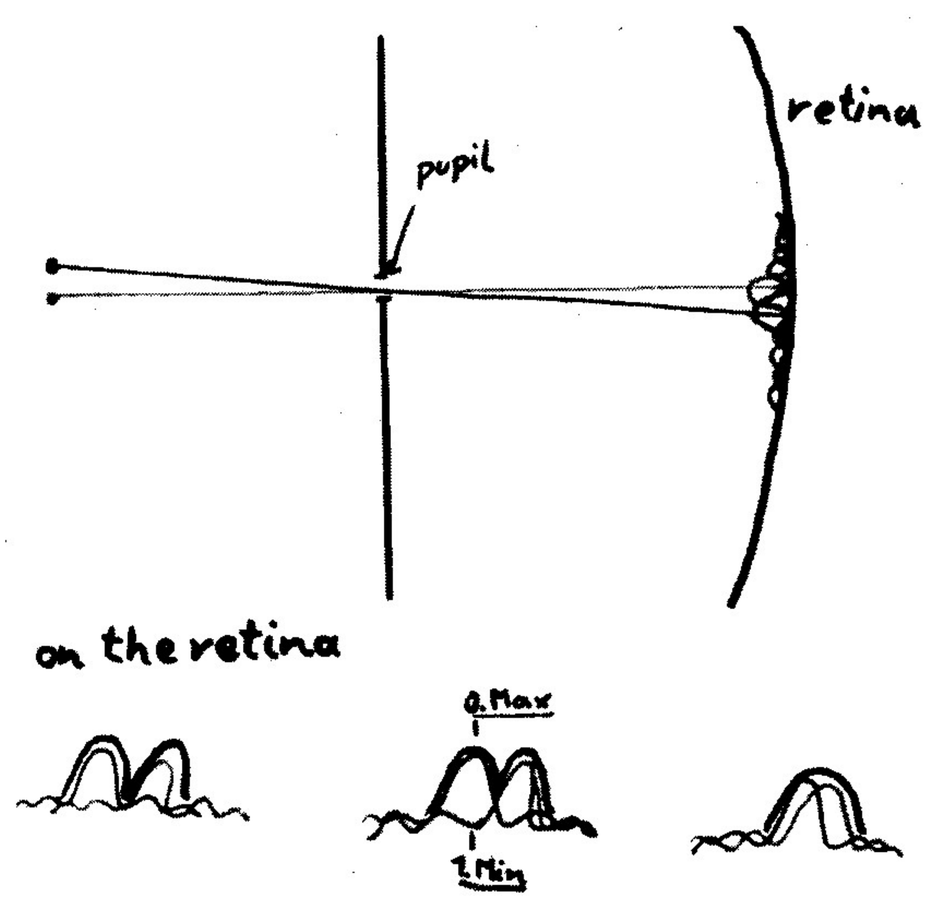
We always considered light to travel in a straight line. The different density of air makes the light bend. Because the density of air is smaller in higher altitudes, the light bends in a curve that



makes it perhaps possible for the light to travel around mountains. The question is, whether the refraction of light has an effect that needs to be considered.
At the first sight, when you calculate with two layers of air, the effect seems to be important. The light reaches Darjeeling 200 m lower than it would reach Darjeeling if the density of air was constant. But if you consider that the light is refracted constantly and that it has to go over the top of Phallut, the effekt is much smaller. A computer program showed that finally the difference end is about 50 m. So the refraction of light helps make Mt. Everst better visible but the effect it is too small to make Mt. Everest visible from the town of Darjeeling.

3 The resolving power of the eye

Since the resolving power of the eye is limited, it is not enough, that one ray of light reaches the eye. You can regard the pupil as a singel slit.
If two points are to be seen as different points, there has to be a minimum between the two main maxima. The smallest possible angle that the human eye is able to resolve is given, when the first order minimum of one of the two light rays is at the same place as the main maximum of the other. The model calculation shows, that in the case of Mt. Everest, this condition is easily fulfilled.
The wavelength λ of the two beams of light is to be 500 nm.
The size d of the pupil is 1 mm.
For a single slit the formula $\sin\alpha = \frac{\lambda}{d}$ gives the angle between the main maximum and the first minimum. This is exactly the minimum angle between the two rays of light, that still allows the eye to see two different points.



$$\sin\alpha = \frac{\lambda}{d} = \frac{500\,\text{nm}}{1\,\text{mm}}$$
$$\alpha = 0.03°$$

The angle between a beam of light coming from the top of Phallut and one coming from the top of Mt. Everest is between 0.5° and 1°. That means that the resolving power of the eye is enough to easily see Mt. Everest.

4 Contrasts

Contrast ct is connected with brightness. Brightness F is defined as energieflow per area.

$$ct = \frac{|F_{\text{surrounding}} - F_{\text{object}}|}{F_{\text{surrounding}}}$$

If we have a totally black object against a black background the contrast ct is 1. The minimum contrast, the human eye can still discern under normal conditions is 2%.
Light is scattered on its way through the atmosphere. The distance d after which $F(d)$ has dropped to $\frac{1}{e}$ of $F(0)$ is called length of extinction L. Since the amount of light lost by scattering is proportional to the amount of light available, the function $F(d)$ is an exponential function.

$$F(d) = F(0) \cdot e^{\frac{-d}{L}}$$

If the light is scattered on its way through the atmosphere, the contrast is diminished.

$$ct(d) \quad = \quad ct(0) \cdot e^{\frac{-d}{L}}$$

In order to find out about the maximum distance a human eye can see, I assume that the object originally was totally black, that means the original contrast was one, and insert the minimum visible contrast 2% for $ct(d)$.

$$\begin{aligned} 0.02 &= e^{\frac{-d_{\text{max}}}{L}} \\ d_{\text{max}} &= 4L \end{aligned}$$

5 Scattering

In order to get a value for L, I take a closer look at the mechanisms of scattering. There is Rayleigh scattering and Mie scattering. Rayleigh scattering is scattering on bodies smaller than the wavelength of light, e.g. the molecules of air. Being a electromagnetic wave, light pushes the charge in the molecules. It induces a Hertzian dipole that scatters the light in all directions.

5.1 Rayleigh-scattering

The strength of a dipole $s = Qd$ is dependent on the material and the strength of the inducing electic field E_1.

$$s = \alpha E_1 \quad \Longrightarrow \quad \alpha = \frac{s}{E_1}$$

Since α is only dependent on the electric properties of the material, it can be expressed as a function of ε_r and the number N of dipoles per volume.

$$\alpha = (\varepsilon_r - 1)\frac{\varepsilon_0}{N}$$

The power P of a hertzian dipole is dependent on the strength of the induced dipole and thus of α. F is again energyflow per area.

$$P = \frac{8\pi^3}{3\lambda^4\varepsilon_0^2}\alpha^2 F$$

So the power of the dipoles per volume is:

$$NP = N\frac{8\pi^3}{3\lambda^4\varepsilon_0^2}\alpha^2 F$$

The power of dipoles per Volume changes the energyflow on its way. So I can write the differential eqation:

$$\frac{dF}{dd} = -NP$$
$$\frac{dF}{dd} = -N\frac{8\pi^3}{3\lambda^4\varepsilon_0^2}\alpha^2 F$$

This leads to the exponential equation:

$$F(d) = F(0)e^{\frac{-d}{L}}$$

with

$$L = \frac{3\,\varepsilon_0^2\,\lambda^4}{8\pi^3 N\alpha^2}$$

For α I can plug in $\frac{(\varepsilon_r-1)\varepsilon_0}{N}$ and it is $L = \frac{N\cdot 3\lambda^4}{8\pi(\varepsilon_r-1)^2 d}$.
Now I have expressed L only with known constants. ε_r of air is 1.00015. I make a table that compares the numbers found with this formula with the numbers found in ROEDEL: *Physik unserer Umwelt*, Springer Verlag.
Comparison of the length of extinction:
(The values from literature are experimental results.)

λ	L (calculated)	L (literature)	$d_{\max} = 4L$ (literature)
400 nm	23 km	22 km	88 km
500 nm	57 km	55 km	220 km
600 nm	118 km	116 km	464 km
800 nm	372 km	350 km	1,400 km

You can see clearly that the amount light is scattered depends heavily on the wavelength. If you look at the medium part of the spectrum (like 500 nm or 600 nm), it should be easily possible to see Mt. Everest even if you consider that the Mt. Everest is not totally black. But Rayleigh-scattering is not the only scattering, there is also Mie-scattering.

5.2 Mie-scattering

Mie-scattering is scattering on bodys bigger than the wavelength of light, e.g. small drops of water or dust. Mie-scattering is less dependent on lambda than Rayleigh-scattering, but since the mechanisms of scattering depend mainly on the size, but also on the material of the body, there are many different kinds of Mie-scattering. Therefore the material about Mie-scattering is mainly experimental material connected with the atmospheric conditions and the weather. Dry and clear weather can minimize Mie-scattering so much that it becomes small compared to Raileigh scattering. When the weather is extremly good, you can see up to 200 km, but if the weather is bad, you can sometimes see only a few kilometers or less.
So we have to look at the weather conditions in the Himalaya.

6 The weather in the Himalaya

In general, the view is clear only in the morning:
Close to the ground the temperature changes much more than in the air. Each day after sunset air moves down from the ridge top to the valley bottom. Clouds and mist settle in the bottom of the valleys, therefore the skies are clear.
In the morning the rising sun warms the air and dissolves the mist. Warm air starts rising up and forms giant clouds. If you want to see high mountains as Mt. Everest you have to look for them early in the morning or else as the day advances, the rising clouds obscure the view.
In summer there is monsoon, the weather is warm and humide. In winter the air has much less humidity and you can often have a clear view.
It is said that spring is cloudier than fall, therefore morning in late fall is probably the best time to see Mt. Everest. At this time the air is extremly dry and clear.

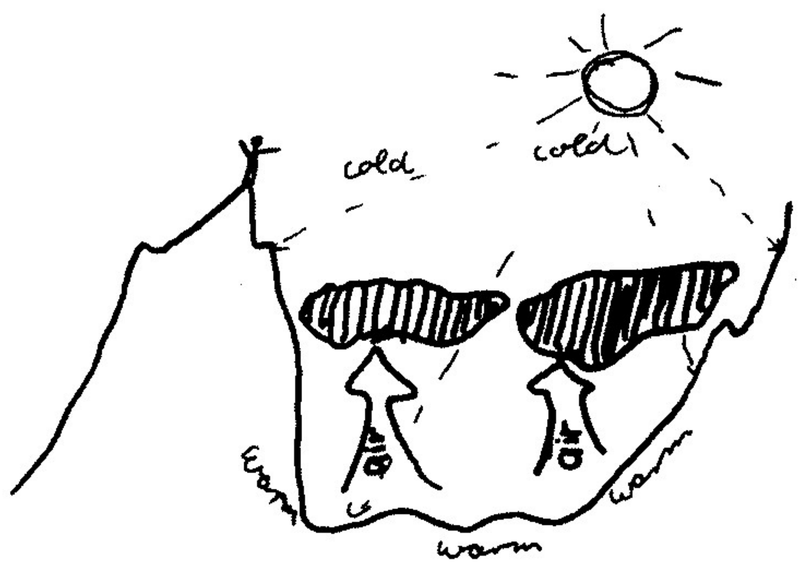


Figure 1: In the morning

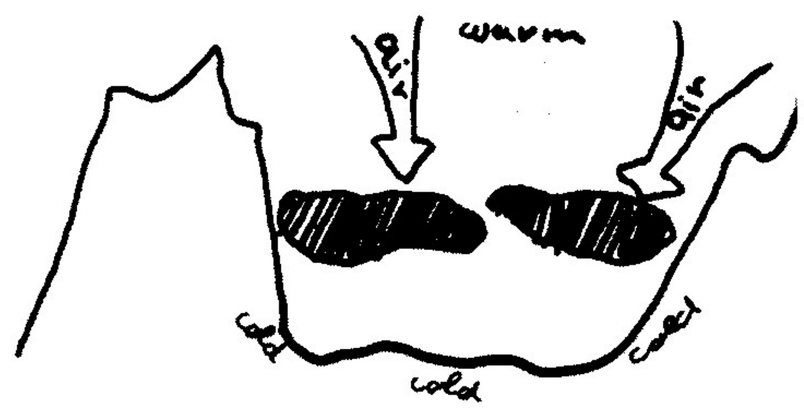


Figure 2: In the evening