

11<sup>th</sup> IYPT '98  
solution to the problem no. 7  
presented by the team of Hungary

**Air bubble**

An air bubble rises in a water-filled, vertical tube with inner diameter 3 to 5 mm. How does the velocity of the rising bubble depend on its shape and size?

**Abstract**

On the level of observation, various phenomena concerning rising bubbles in tubes are described. Bubble shape is interpreted through dimensionless constants. Theoretical treatment of small, spherical bubbles is given, as well as the outlines of a possible model for large bubbles (slugs). Finally, further interesting observations are mentioned (such as cylindrical ripples on the surface of slugs), with some reference to theoretical models.

**Thanks**

The members of the hungarian team thank Professors Péter Gnädig and Géza Tichy (Eötvös Loránd University, Budapest Departments of Atomic and Solid State Physics, respectively) for their advice and helpful discussions on the following two problems.

**Overview**

- Shape Regimes
- Small bubbles, low Re Number
- Intermediate Re
- Spherical cap bubbles
- Slugs
- Experiment



## 1 Shape Regimes

With changing size and speed, several distinct shapes are observed: Spherical, dimpled, skirted, wobbling, spherical cap. Shape is determined by the interaction of inertial, viscous and capillary forces. These are expressed by three dimensionless groups:

$$\begin{aligned} Eo &= \frac{g\Delta\rho d^2}{\gamma} \\ Re &= \frac{\rho du_T}{\mu} \\ M &= \frac{g\mu^4\Delta\rho}{\rho^2\gamma^3} \end{aligned} \quad (1)$$

(The first two are the EOTVOS and REYNOLDS numbers, respectively.) The different shape regimes are charted as a function of the above parameters ( $\sim$  *phasediagram*). The chart may be used as a calibration diagram for  $\mu_T$ , since it occurs only in one of the groups. The terminal velocity may also be calculated directly in some spherical cases:

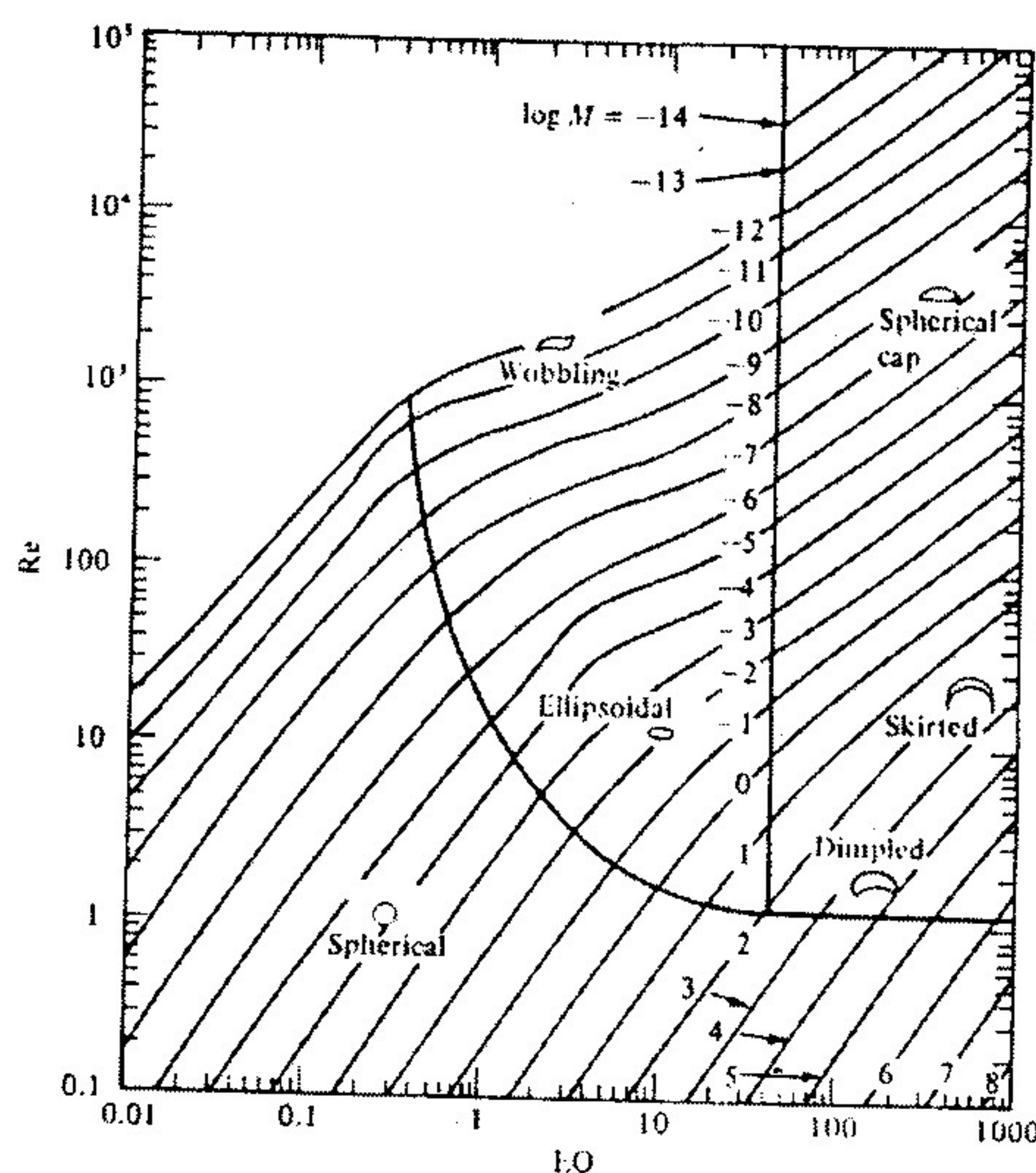


Figure 1: Shape regimes for liquid drops and gas bubbles

## 2 Small bubbles, low Re Number

Solution of flow for a bubbles of  $\mu_1$  in a medium of  $\mu_2$ : HADAMARAD RYBCZINKI (1911),

$$\mu_T = \frac{2}{3} \frac{ga^2\Delta\rho}{\mu} \frac{k+1}{3k+2} \approx \frac{1}{3} \frac{ga^2\Delta\rho}{\mu} \quad (2)$$

where  $k = \frac{\mu_2}{\mu_1}$ .

This applies to infinite media. Wall-effects can be taken into account (HETSRONE AND AL.).

$$F_D \approx 4\pi\mu a \left\langle -u \left[ 1 + \frac{4}{3}\lambda + \frac{16}{9}\lambda^2 \right] \right\rangle \quad (3)$$

where  $\lambda = \frac{a}{R}$  (finite size correction) and  $Re < 1$ .

## 3 Intermediate Re

The STOKES-law has to be corrected:

$$F_D = 12r\pi\eta u$$

## 4 Spherical cap bubbles

(in the regime  $Eo > 40$ ,  $Re > 150$ )



Figure 2: Spherical cap bubbles

$$u_T = \frac{2}{3} \sqrt{\frac{g R_c \Delta \rho}{\rho}}$$

finite size correction:  $u_T^W = u_T 1.13 e^{\frac{d}{a}}$

## 5 Slugs

Above  $\frac{d}{D} = 0.6$ , a bubble must be considered a slug.  $u_T$  is independent of  $H$ ,

$S$  is constant of frequently used correlation  $u_T \sim \sqrt{D}$ .

Using  $\delta = \text{constant}$ :

$$v_b r^2 \pi \rho g H = \mu H \int_{R-\delta}^R 2\pi r \left( \frac{\partial v}{\partial r} \right)^2 dr$$

Assuming a constant velocity profile ( $\frac{\partial v}{\partial r}$  doesn't change along the length of the bubble),  $u_T$  is timely constant.

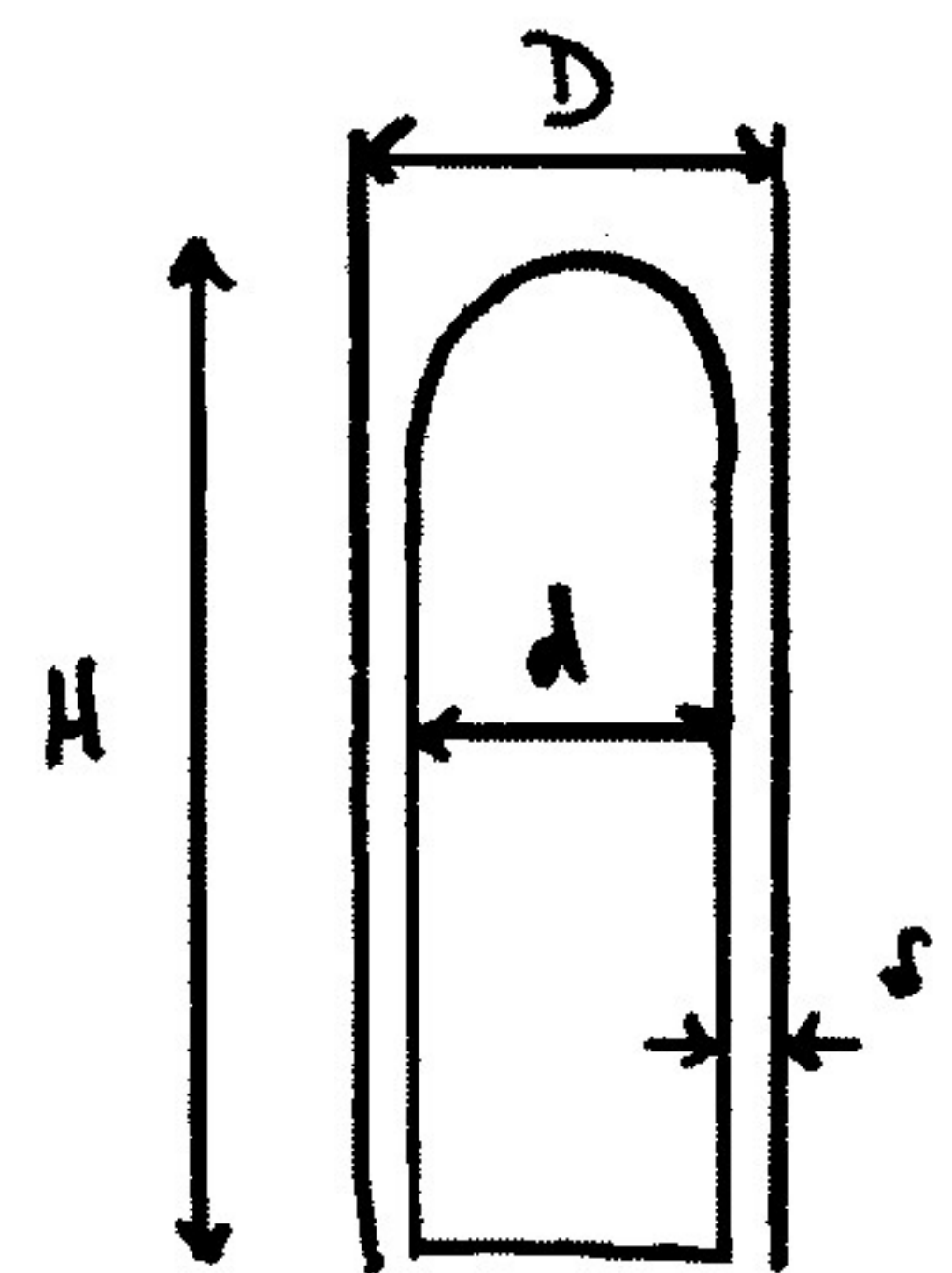


Figure 3: Slugs

## 6 Experiment

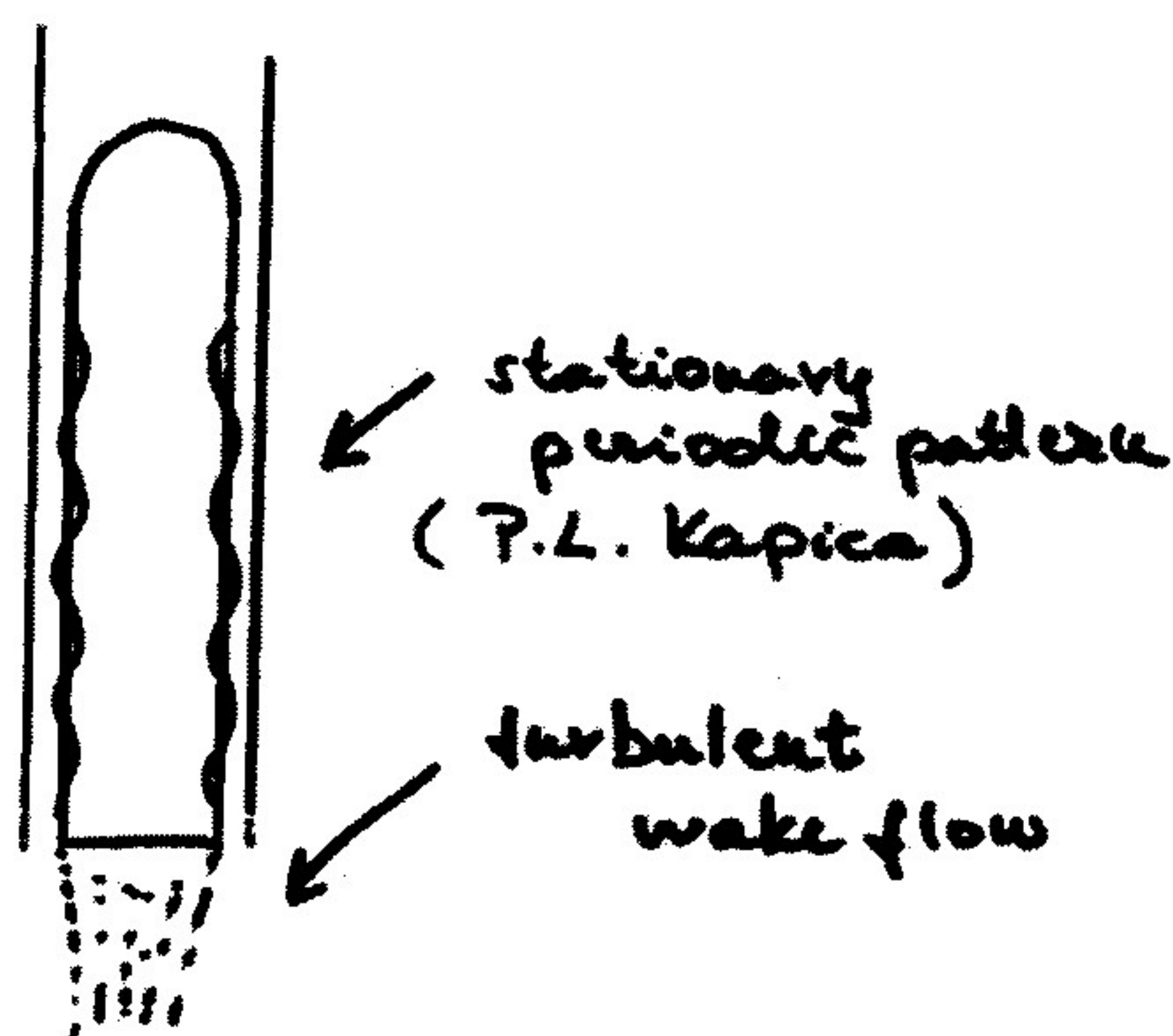


Figure 4: Experiment



# 7: AIR BUBBLE

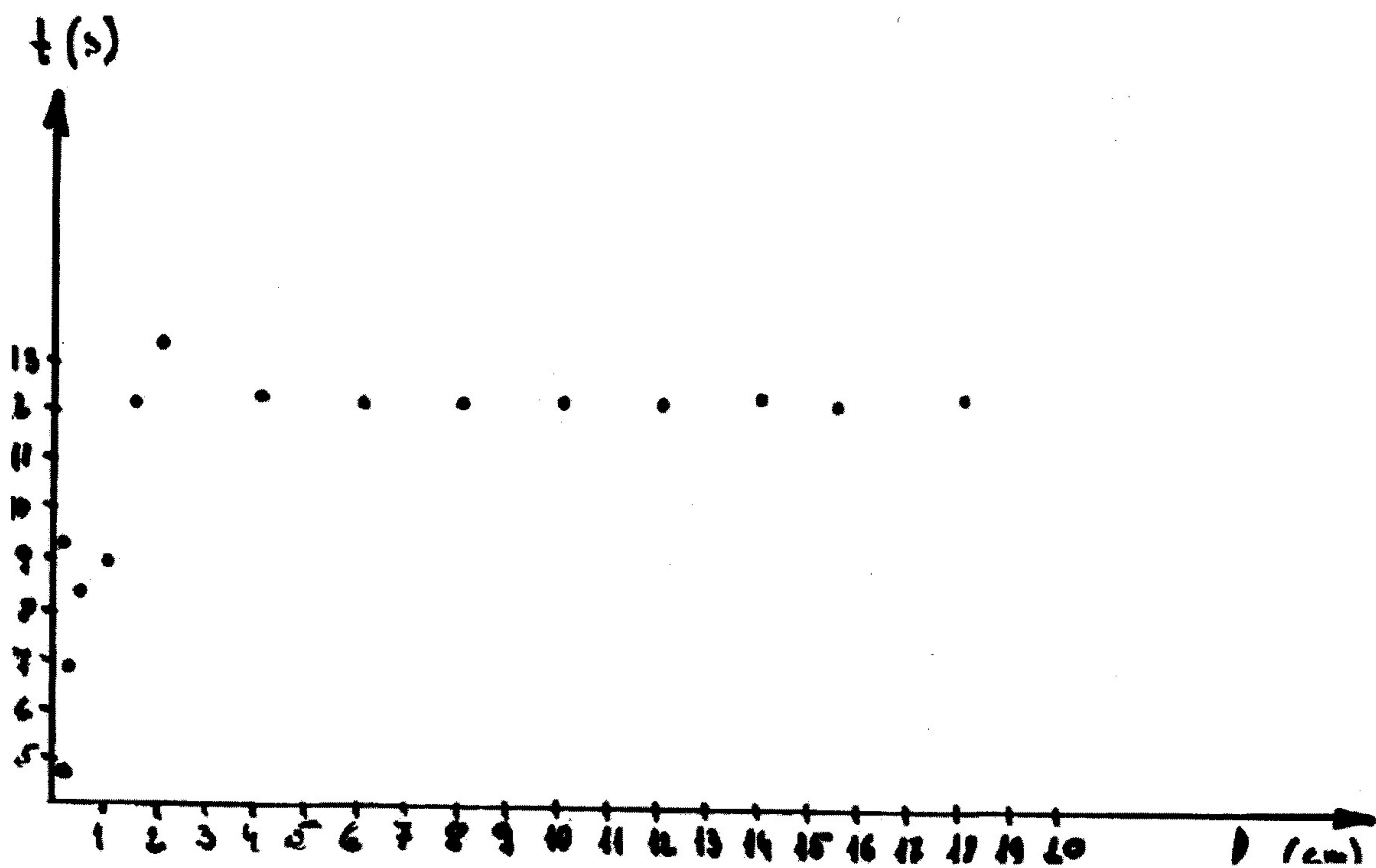
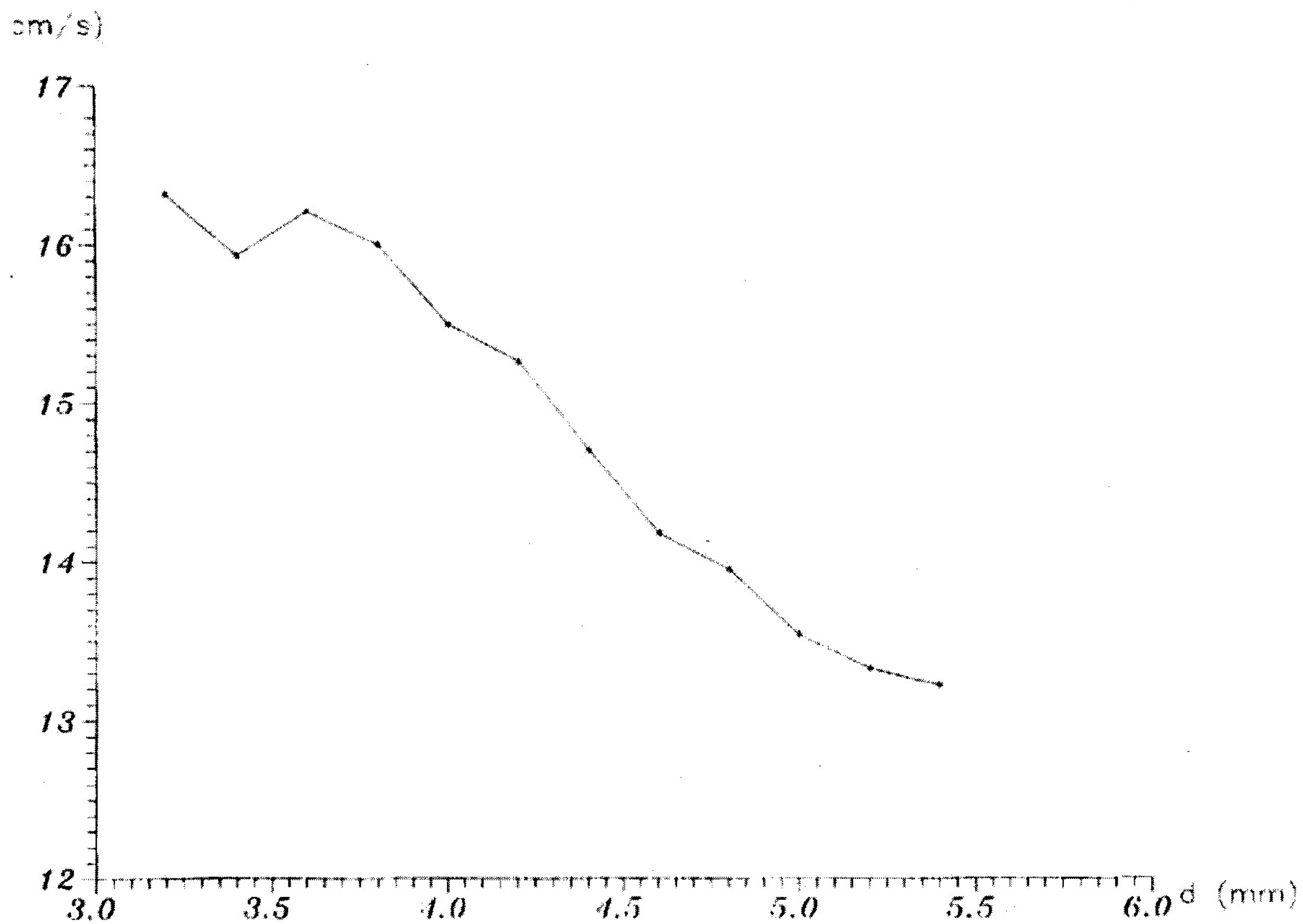


Figure 5: Results