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solution to the problem no. 8

presented by the team of Ukraine

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Trick

It is known that a glass filled with water and covered with a sheet of paper may be turned upside down without any loss of water. Find the minimum amount of water to perform the trick successfully.

Abstract

It is shown that absolute minimum of the amount of water is determined by the adhesion forces between the molecules. The problem is reformulated as determination of the stable states for the amount of water in glass. It is shown that there exists bifurcation of the stable states for water. It characterizes the height H^* which separates the situations of performing the trick with all amounts of water (from water film to filled glass) and just the definite ones.

Thanks

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Overview

- Experiments
- First model
- Calculations

1 Experiments

At first we carried out the experiments, in which we tried to do this trick both with different amount of water and with different materials. One can ask the question: "Why do we need a piece of paper to perform this trick?"

2 First model

Let us consider the following model. Water in cylindrical glass can be treated as a piston, which can slide without friction. We assume that there is some air between piston and the bottom of the glass. Independently of orientation there is equilibrium position for the piston, if the length of cylinder allows it. In particular, when the glass is bottom up, piston could be hold inside the cylinder. The same effect occurs if we take water instead of piston. But in reality there are some fluctuations on the water surface (oscillations of atmospheric pressure, mechanical perturbations). As a result there are waves on the water surface. This surface is not stable under small fluctuations. Therefore water falls down. So we see that paper is used to stabilize the surface of water. In place of paper we also tried to use other materials, in particular a piece of transparency. Any of such material we will call "bottom".

3 Calculations

Having carried out our experiments we saw that trick could be done with different glasses. We will consider cylindrical glass. There were some questions: "How should we turn glass over? Should we hold the paper by hand, or we shouldn't?" We had tried to do this trick both with paper, which was held by a hand, and without doing this. Obviously, it is easier to perform this trick in a first way than in the other. When we hold paper we damp impulse which water has when it falls. But this trick may be also performed without paper to be held by hands. We successfully did the trick under different values of amount of water we need. The main condition here is the damping off the acceleration. We did this trick under very small amount of water. In this case the performance of this trick depends only on mass of paper. We can say that the water amount must be enough for water film to occur. This film will hold the paper mass not greater than following:

$$m = 2\pi \frac{\sigma R}{g} \tag{1}$$

Here σ is the coefficient of surface tension, R is the radius of glass, g the gravitational acceleration. This trivial estimation follows from the balance of gravity and surface tension forces. The only thing, which is necessary, is the formation of stable water film for the force of surface tension to occur (see figure 1). The amount of water, which is sufficient for the formation of that film, can be

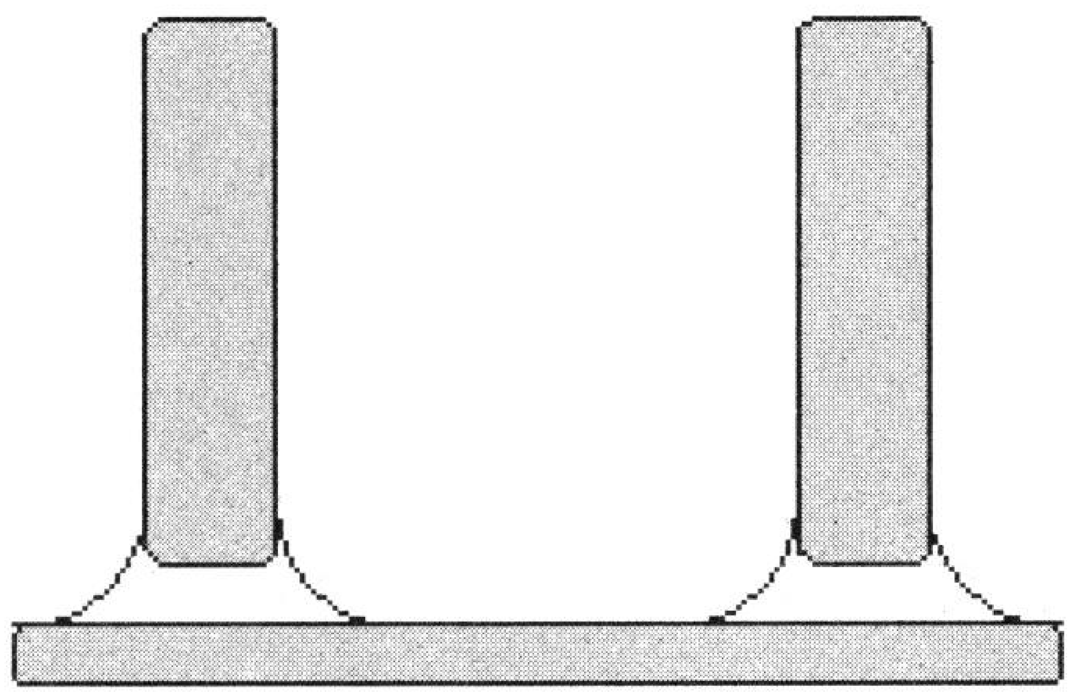


Figure 1: Set-up

treated as absolute minimum of water quantity thus given the answer of the problem. Indeed in this case the for infinitesimal mass of bottom the only gravity force which acts against the surface tension is the weight of water layer itself. Here the adhesion forces between molecules of water and glass determine the mass of such layer.

But we believe that it is not the case of interest. The trick, although easily performed with full glass of water, is more difficult to do with unfilled glass. It is easy to explain as following. When the glass is filled the atmospheric pressure acts only against hydrostatic pressure. In the second case the difference of air pressures acts against hydrostatic pressure.
Thus we come to the following statement of the problem. Let us find the necessary conditions to perform this trick. At first sight the answer is trivial – the hydrostatic height:

$$H_{\max} = \frac{P_0}{\rho g} \tag{2}$$

But the increase in water height leads to the increase of vertical bottom shift. When this happen, the bottom can rotate and move in other directions. At definite vertical shift the instability of surface in a form of bubble may occur. This effect equalizes the air pressure and then everything will fall down. To find this value of bottom shift, which bounds the height of water, we propose the following approach. We can write the equality of pressures, which acts near the water film:

$$\Delta P = \rho g h, \quad \Delta P = P_0 \frac{\Delta}{H - h + \Delta} \tag{3}$$

Here H is the height of glass. The last relation follows from the effect of expanding the air in a glass turned upside down when the bottom shifts on Δ. From (3) we get:

$$\Delta = \frac{\rho g h (H - h)}{P_0 - \rho g h} \tag{4}$$

Obviously Δ should not exceed some value Δ_0, which can be estimated by the capillary radius:

$$\Delta_0 = \sqrt{\frac{2\sigma}{\rho g}} \tag{5}$$

$$\Delta = \frac{\rho g h (H - h)}{P_0 - \rho g h} \leq \Delta_0 \tag{6}$$

Analysis of this relation gives the following results. We obtained a graph, which shows the dependence of bottom deviation on amount of water in the glass. You can see that in dependence on H and h we have different cases. They are:

$$\Delta < \Delta_0, \forall\, h \in (0, H)$$
$$\Delta < \Delta_0, \forall\, h \in (0, h_1) \cup (h_2, H)$$

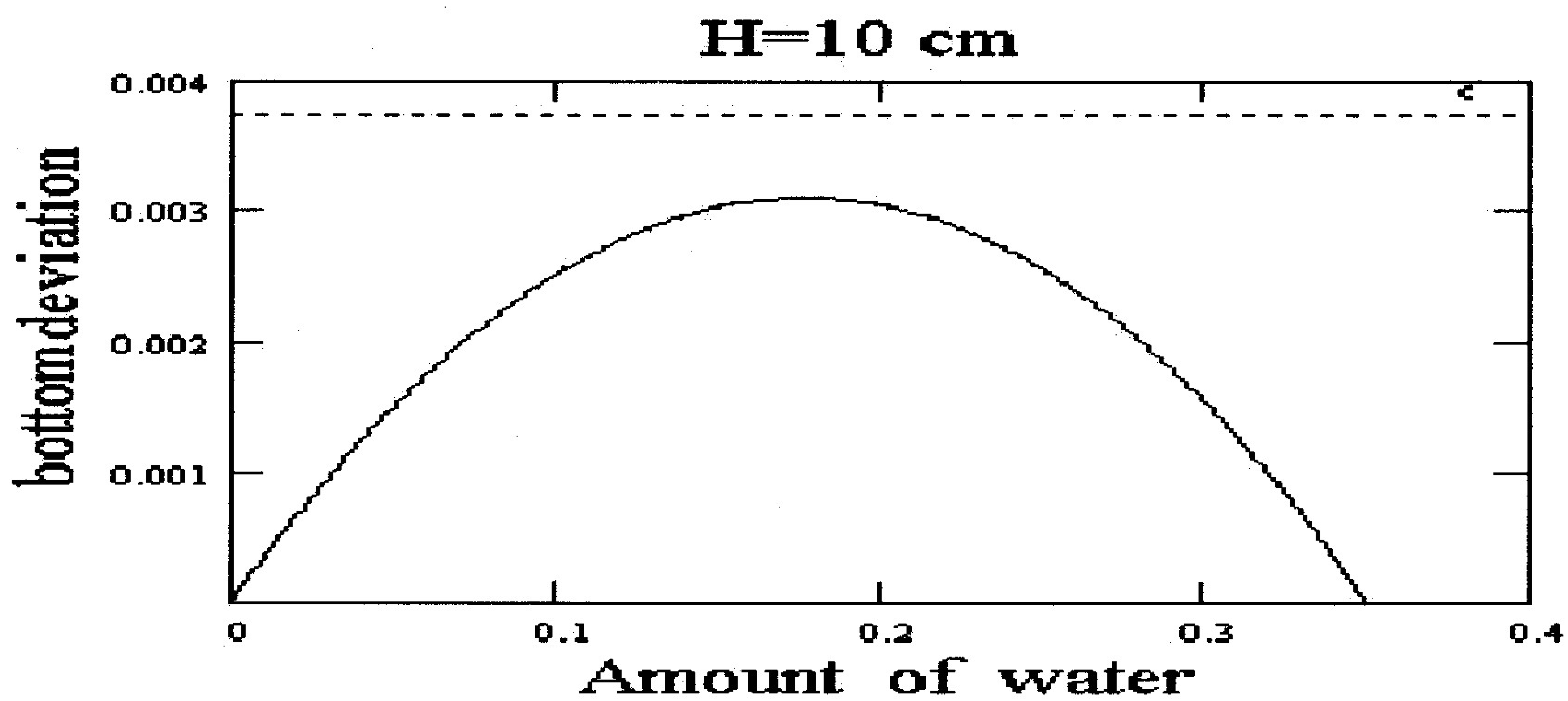
We presented them by the graphs in figure 3. The value Δ_0 is shown by dashed line, the amount of water means the height of the water column. Figure 3 (top) corresponds to the situation where one manages to perform the trick with any amount of water as has been discussed above. Figure 3 (middle) describes the possibility of performing the trick in given intervals of height of water. Here is the maximal height h_1. h_1 and the minimal one h_2 are the rots of the equation $\Delta = \Delta_0$. In particular, the case $H = H^*$ presented by figure 3 (bottom), the roots h_1, h_2 merge. At this height there exists the only root of the equation $\Delta = \Delta_0$. The value H^* is near 38.2 cm. So, the answer for this problem is the height

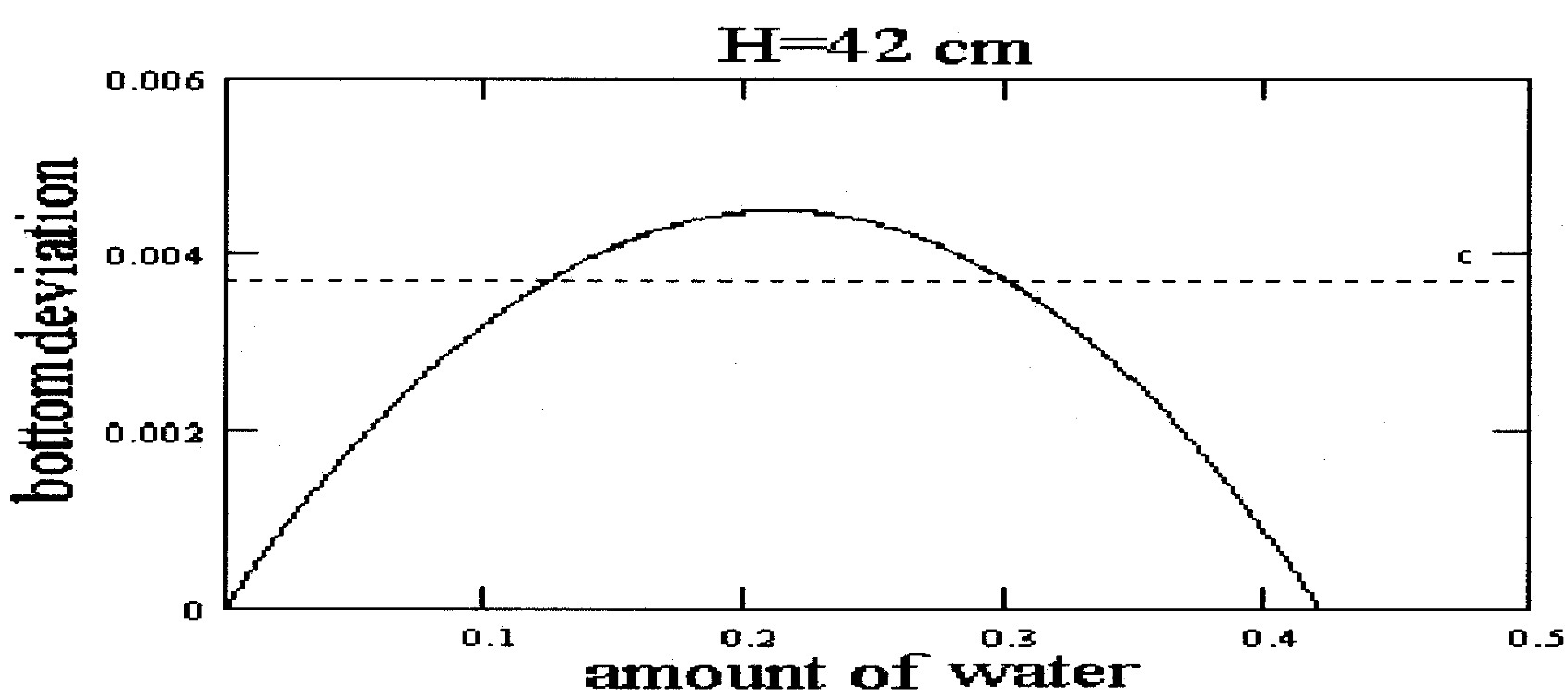
$$h = H \frac{1 + \delta_0 + \sqrt{(1 + \delta_0)^2 - \pi_0 \delta_0}}{2}$$

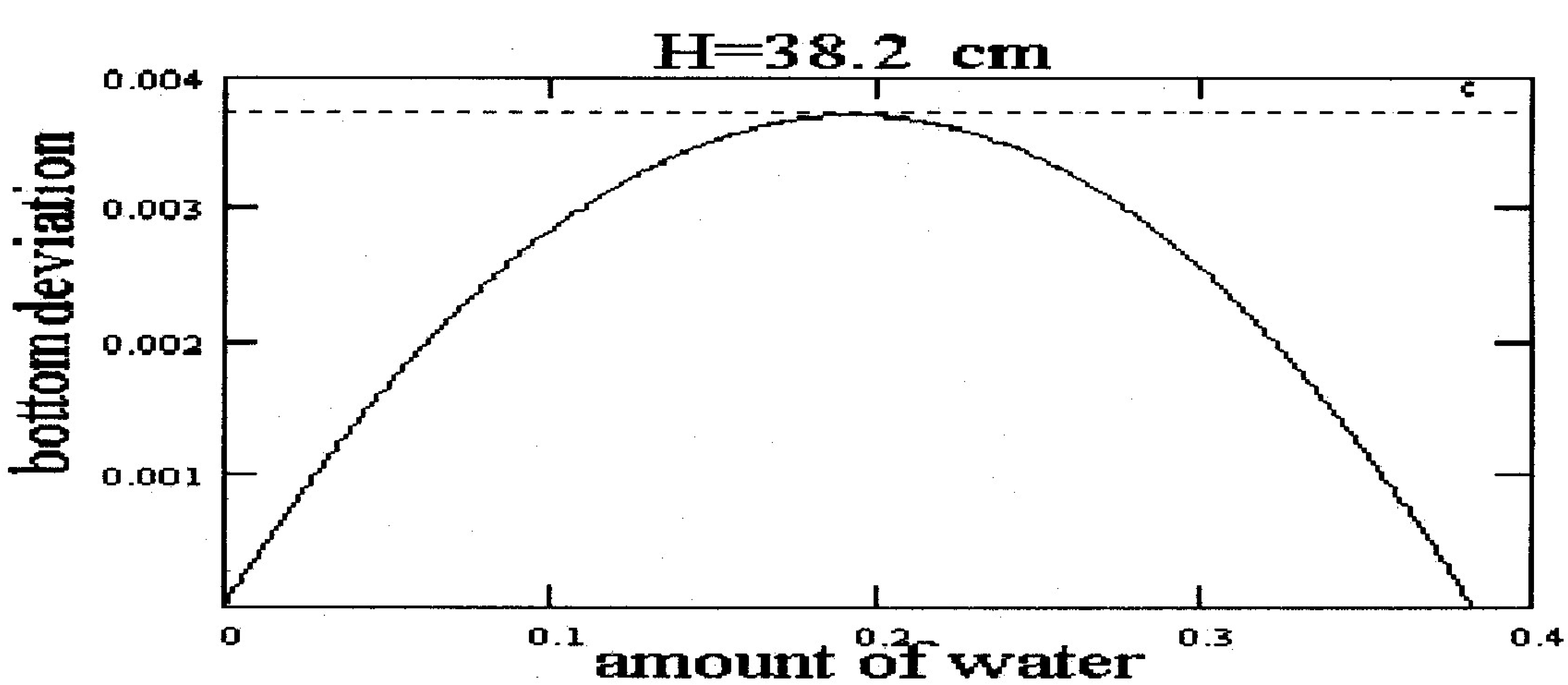
$$\delta_0 = \frac{\Delta_0}{H}$$

$$\pi_0 = \frac{P_0}{\rho g H}$$

– the minimal root ($H \geq H^*$). In opposite case the trick can be performed at any amount of water, which is sufficient for forming of water film. The existence of characteristic height H^* was confirmed experimentally. It corresponds to the border of stability region. Such a situation resembles the bifurcation phenomenon when smooth change of some parameters leads to drastic change of state of the system.


H=10 cm
0.004
0.003
0.002
0.001
bottom deviation
c
0
0.1
0.2
0.3
0.4
Amount of water


H=42 cm
0.006
0.004
0.002
0
bottom deviation
c
0.1
0.2
0.3
0.4
0.5
amount of water


H=38.2 cm
0.004
0.003
0.002
0.001
bottom deviation
c
0
0.1
0.2
0.3
0.4
amount of water