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solution to the problem no. 11
presented by the team of Belarus

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Current system

In a Petri dish (shallow bowl), small metal balls, e.g. 2 mm in diameter, are immersed in a layer of castor oil. The inner rim of the dish contains an earthed metal ring. Above the centre of the dish there is a metal needle which does not touch the oil surface. Investigate what happens when the voltage between needle and earth is about 20 kV.
Warning: The high voltage should be obtained by means of a safe generator, e.g. an electrostatic generator!

Overview

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- Conclusion

1 Experiments

For our experiments we used metal balls (approximately 4 mm diameter). When the 25 kV voltage applied to the system balls form lines in radial direction. While such cautionary is touching the ring by one end and another end is sufficiently close to the needle; the discharge occurs and farther alignment into lines breaks off.

2 Explanation

2.1 Induction phenomenon

This process is explained on the basis of electrostatic induction phenomenon. When the conductive body is placed into electrostatic field the charge separation occurs. As a result body (in our case metal balls) polarises and gains dipole moment, magnitude of which is proportional to the value of electrostatic intensity in the centre of body before its location into the field (1).

$$p = 4\pi\varepsilon_0 R^3 E \tag{1}$$

2.2 Mirror-image method

In order to describe electrostatic field in the system let us apply mirror-image method. Let's consider conductive ball placed into a field of point-charge on the basis of this method. In such a case all points of sphere must have the same potential value and total charge of sphere must equal zero. In order to satisfy these conditions field outside of ball can be represented as field of three point charges (initial and two image charges). Magnitude and position of charge $\dot{q}$ is chosen from the condition of zero surface potential ((2) and (3)):

$$\dot{q} = -q\frac{R}{L} \tag{2}$$

$$a = \frac{R^2}{L} \tag{3}$$

Where
q - magnitude of initial charge,
R - sphere radius,
L - distance between the centre of sphere and initial charge,
a - distance between the centre of sphere and image charge.

$\ddot{q}$ charge doesn't change surface potential but provide realisation of term that total charge of sphere equals zero (4).

$$\ddot{q} = -\dot{q} \tag{4}$$

Field of theses charges coincides with the field of initial charge and charges induced on sphere only outside the ball. It's obvious that inside a metal body electric field is absent.

2.3 Field in any place of space

Now we can easily calculate magnitude of summarised field in any place of space and build up pictures of forces lines. The results can be seen in the figure 1. The image is obtained from computer simulation.

It is well-known that dipole situated in non-uniform field is pulled into the region of more intense field. In dipole approach force of pulling can be calculated by the (5). It's clearly seen that is proportional to

$$F = p\frac{\partial E}{\partial r} = 4f_1\varepsilon_0 R^3 E\frac{\partial E}{\partial r} \tag{5}$$

the value of product of field intensity and derivative of E with respect to r. It follows from this that force is inversely proportional to the distance from point charge to centre of sphere into the fifth power.

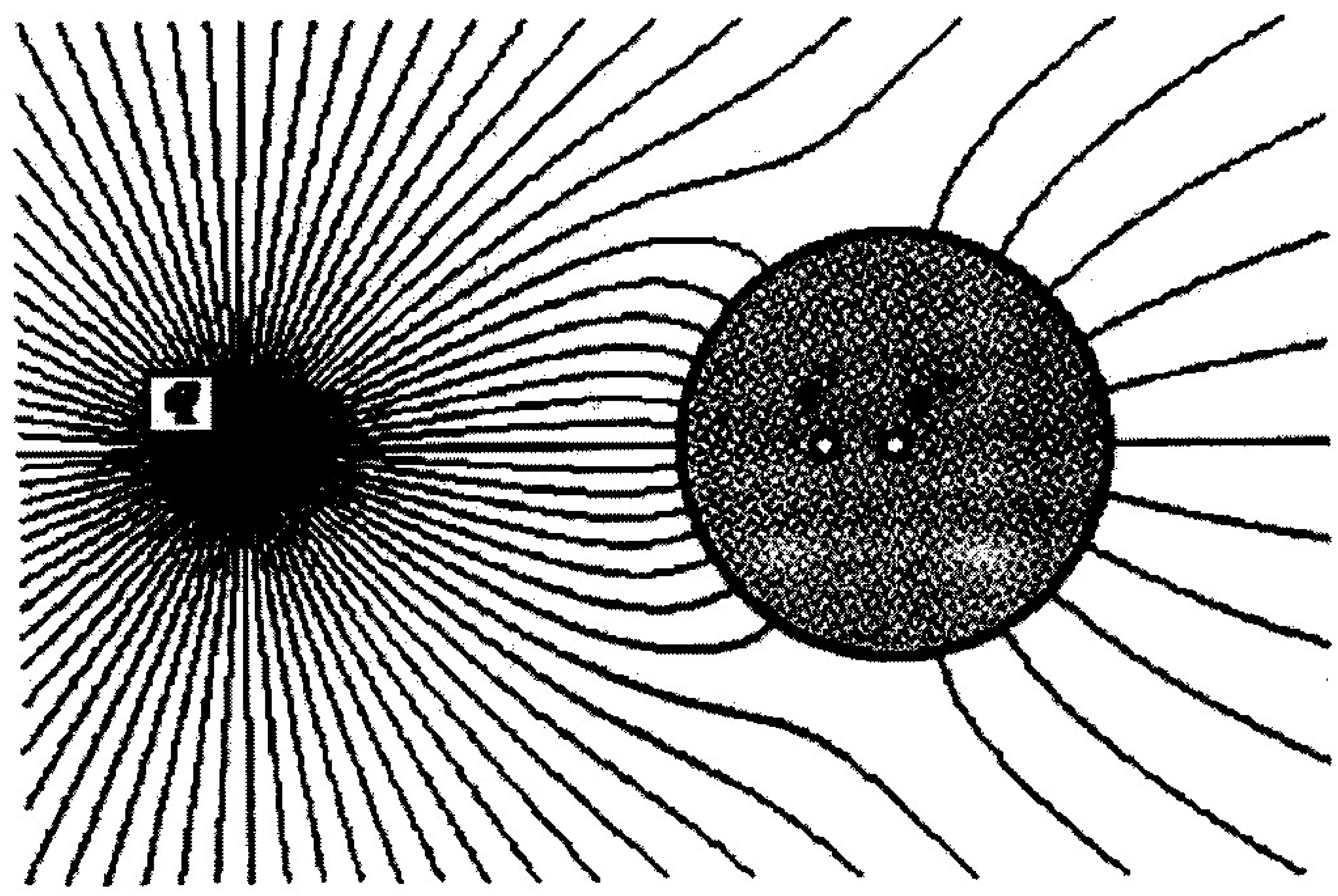

Figure 1: Solitary sphere in the field of a point charge

F (N)	r (cm)
0.08	3
0.28	2
2.22	1
17.79	0.5

Dependence of pulling force on the distance from the point-charge is represented on the figure 2. This force is steeply increasing while the distance between initial charge and sphere decreases. Therefore it's more convenient to plot this graph in logarithmic scale. It is clearly seen when

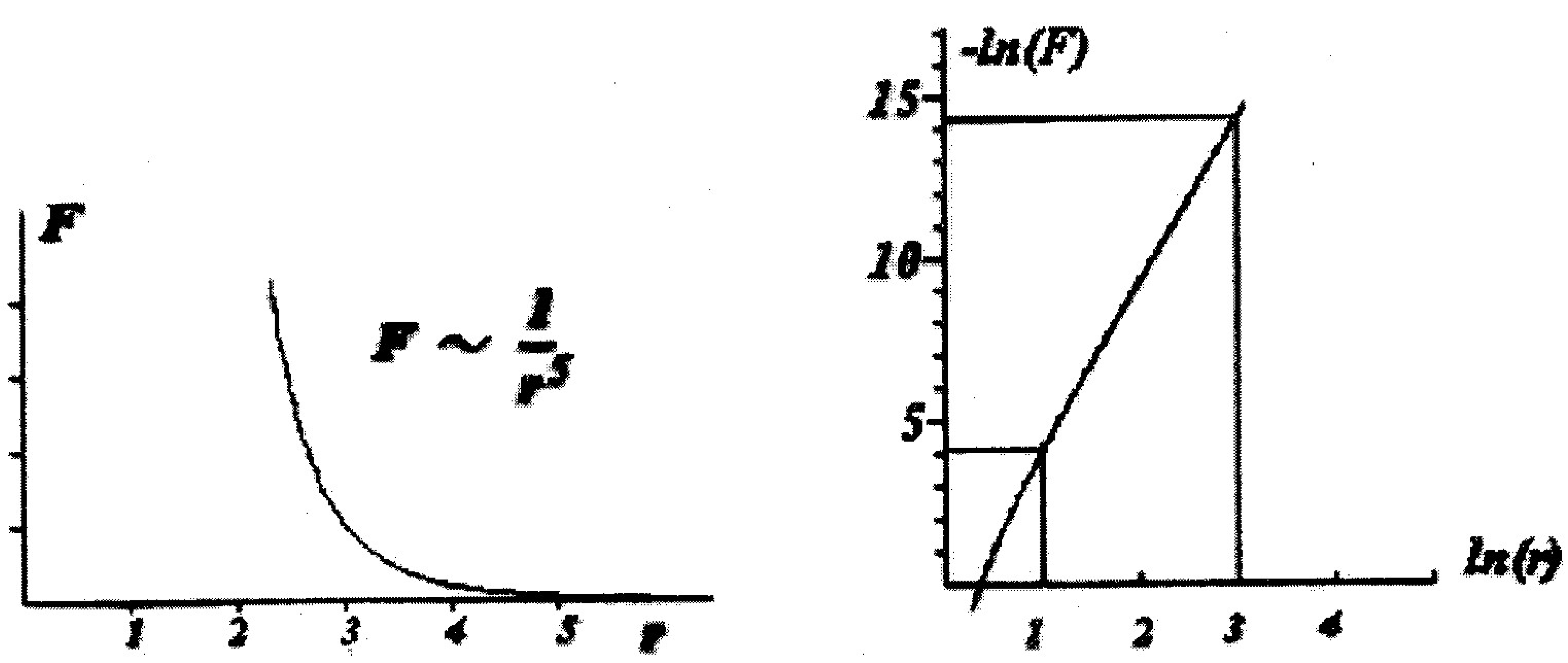

Figure 2: Force of pulling

distance tend to infinity $F \sim \frac{1}{r^5}$. It proves the possibility of application of dipole approach.

Let's put into the field another sphere. It's obvious that the charges $\dot{q}$ and $\ddot{q}$ are induced on its surface, besides the interaction between first and second sphere comes into existence. In order to take into account this fact we resort to mirror-image method again. Let's consider charge $\dot{q}$ on the first sphere. This charge induces in the second sphere charge the magnitude and position of which is calculated on the basis of (2) and (3). At the same time induced charge induces on the first sphere another charge. We can repeat this procedure infinite number of times. But in computer simulation fifty images for each charge is enough. In such a case the error of the potential is less than 10^{-10}, and deflection of total charge of spheres from zero is less than 10^{-13}. The force line picture for this complicated system is obtained from computer simulation and shown on the figures 3–5.

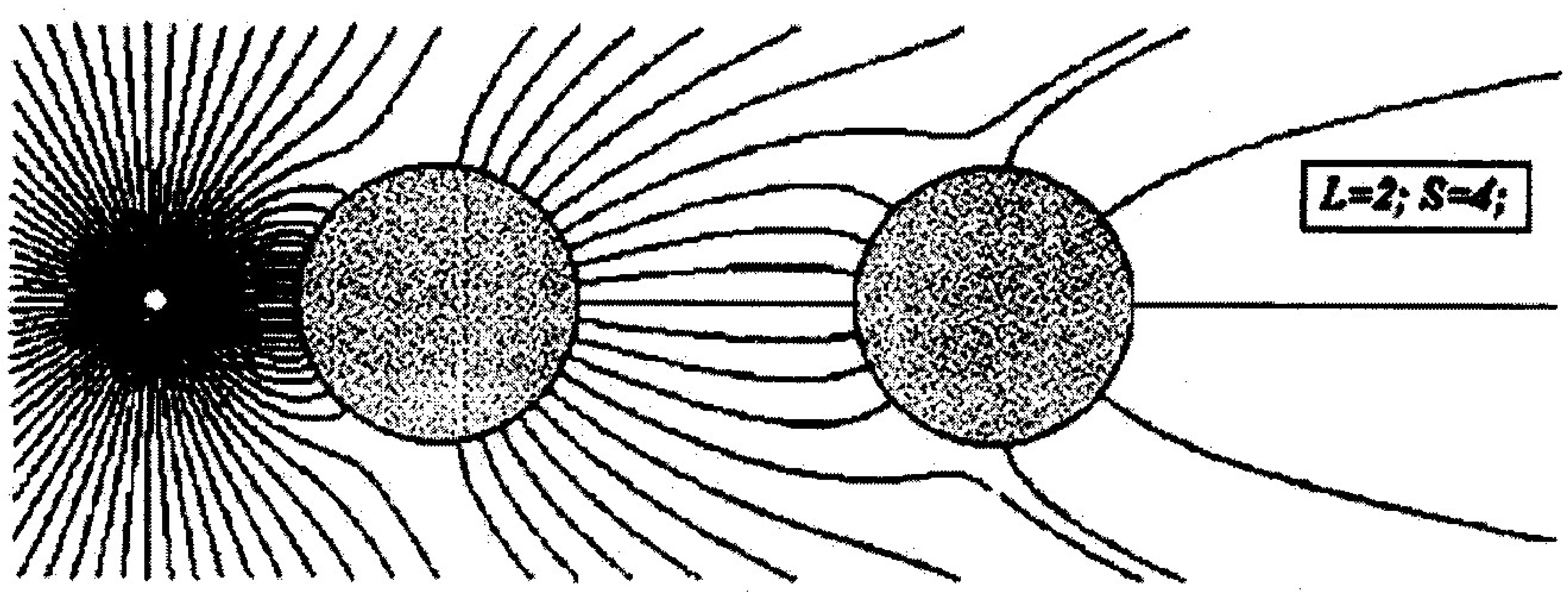

Figure 3: Two solitary balls in the field of a point charge

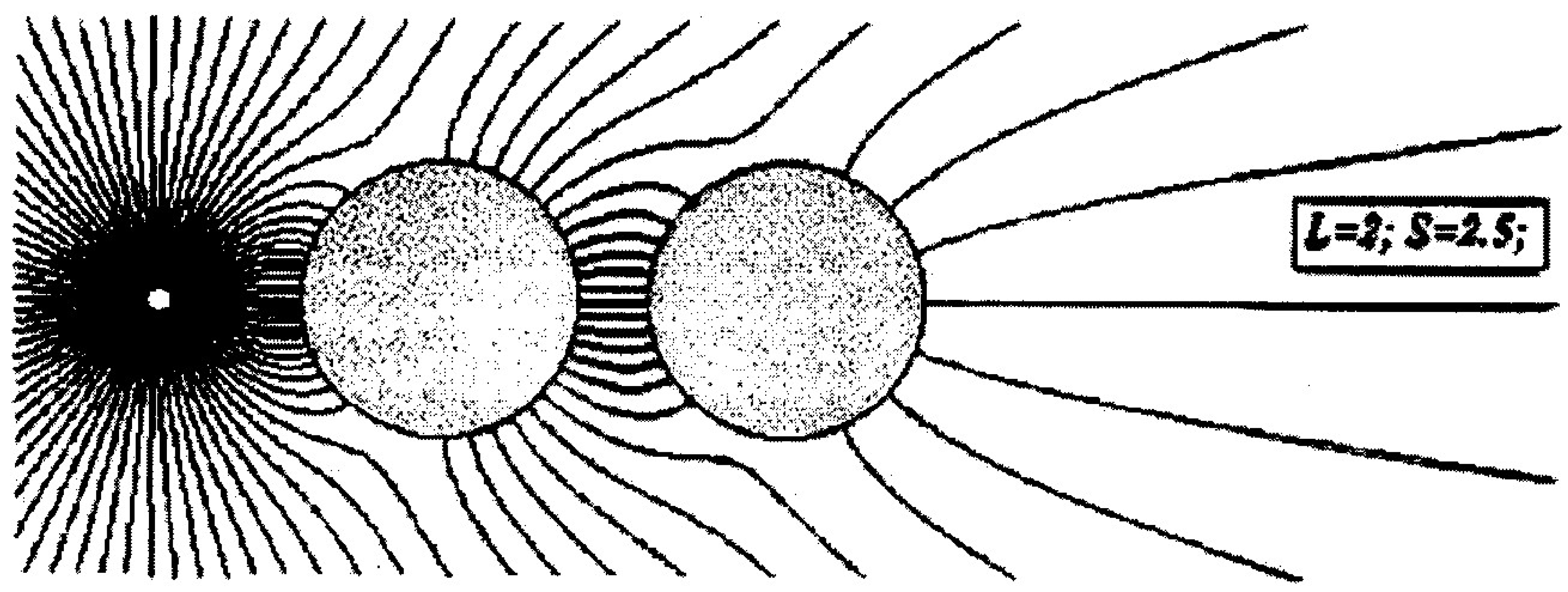

Figure 4: Two solitary balls in the field of a point charge

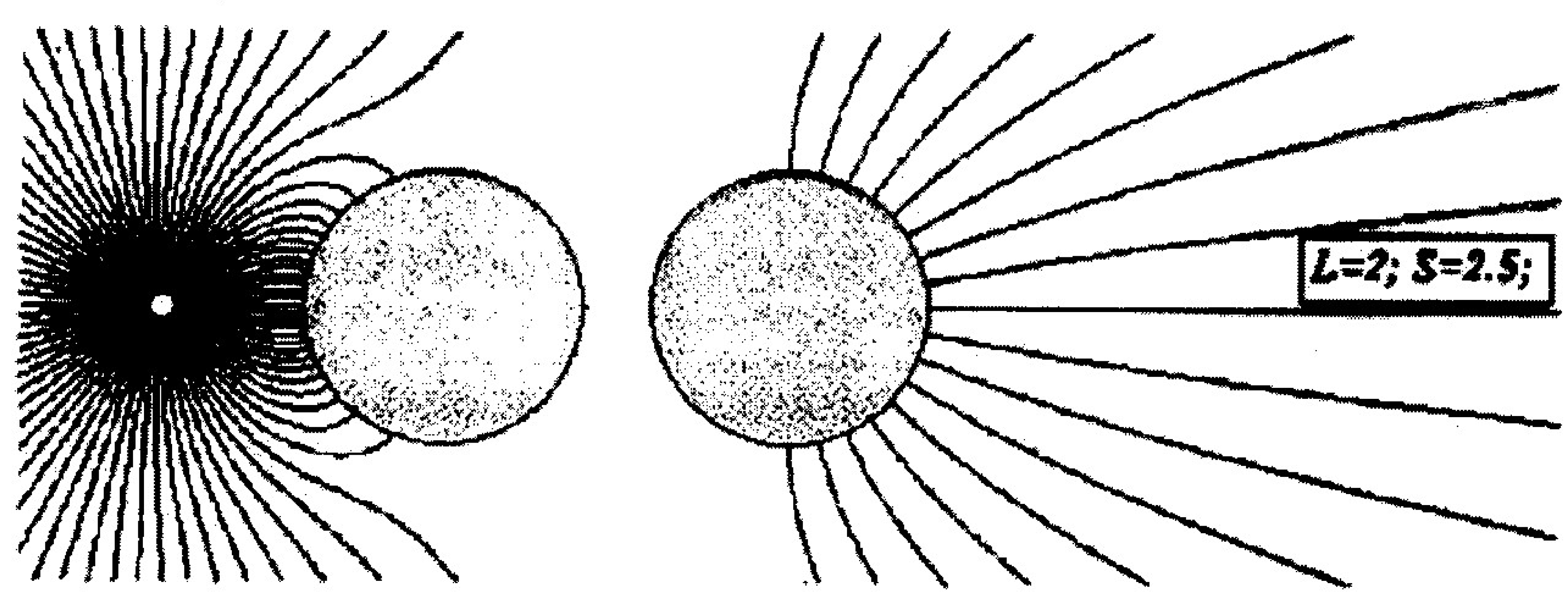

Figure 5: After breakdown between the balls

2.4 Interaction force

Because of superposition principle and mirror-image method the dependence of interaction force on the distance between spheres was obtained. It is shown on the graph in figure 6.

It can be seen that when the bodies approach to each other, electrostatic field and force between them increases up to occurrence of breakdown. In the result of breakdown, equalisation of potentials and redistribution of charges between two spheres take place. And dipole with greater dipole moment appears. The magnitude of field in such a system greatly increases. It appears when we compare figures 3 to 5 (number of lines is proportional to the field intensity). We obtained dependence of field intensity E and product $E\frac{dE}{dr}$ on the separation r from the needle. It can be seen on the figure 7. The first graph shows the plotted functions without any objects placed into the field and second graph show it when the metal spheres are put into the field. It is easy to notice that the field near the second sphere increases 4 times and the product $E\frac{dE}{dr}$ 10 times. We've already said that this magnitude is very great because the force of attraction which acts on the third sphere is proportional to the value of that product.

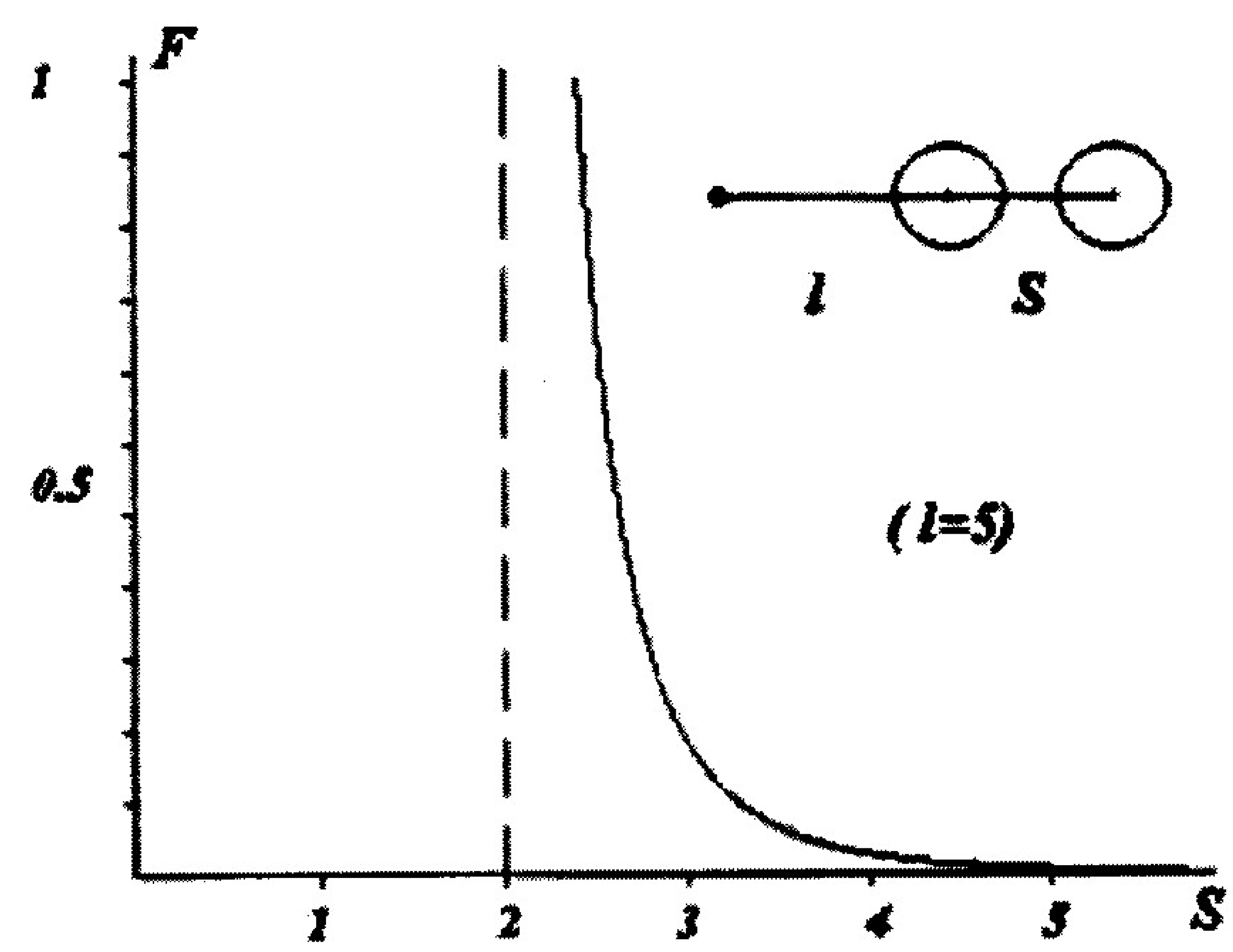

Figure 6: Interaction force

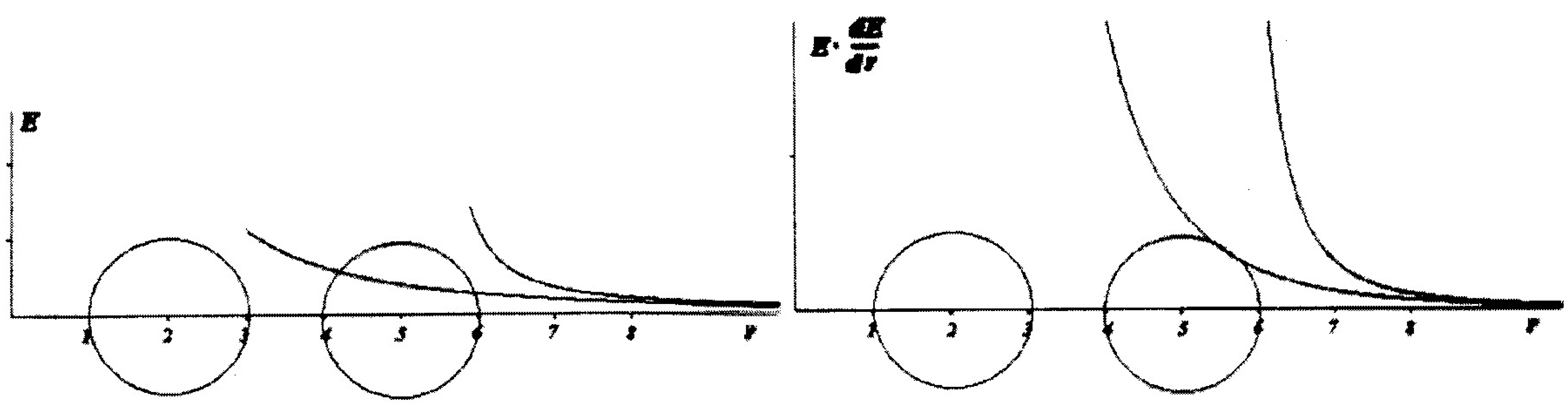

Figure 7: Electrostatic intensity

2.5 Connections of the balls

It's obvious that after connection of the third ball the field will also grow up. And on this process we'll continue with increasing of field around the chain. By the time when the circuit is closed the field outside the chain will be practically equal to zero. It is an explanation of the that other lines of balls stop their formation after breakdown.
Before this moment we've considered the field in the system as a field of point-charge. It is approximation which we've done for description the behaviour of balls into system. But we calculated a real field. We considered the Laplace equation for a cylindrical system with axis symmetry. Then we availed oneself on net method of computation. As a result we get the structure of electrostatic field into our system, which is represented in figure 8. On the basis of this calculation we get

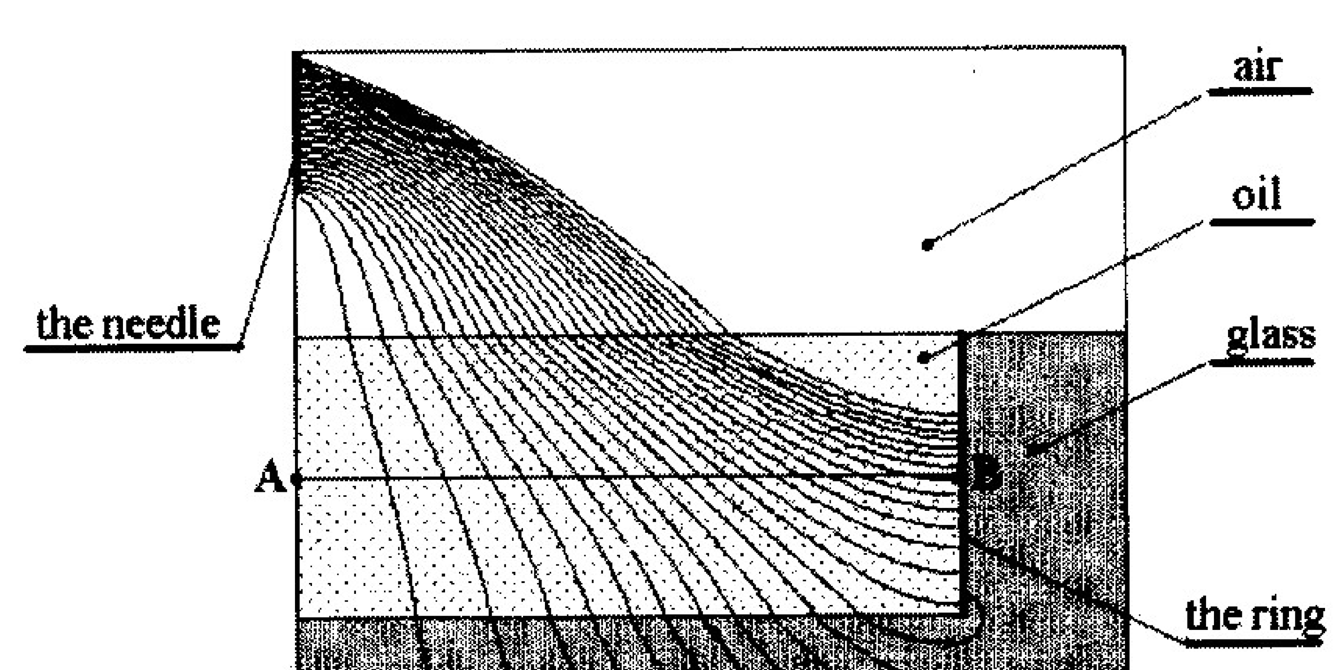

Figure 8: Electrical field in the experimental set-up

dependence of potential on the distance from needle (figure 9). Analogous operation was done for horizontal projection of field intensity and product E and $\frac{dE}{dr}$ (figure 10).
All of these computations were received for line AB in the figure 8. Despite graphs (figure 10) are differed from graphs in figure 7, we can affirm that all of appropriateness, which were received for them, remain the same.

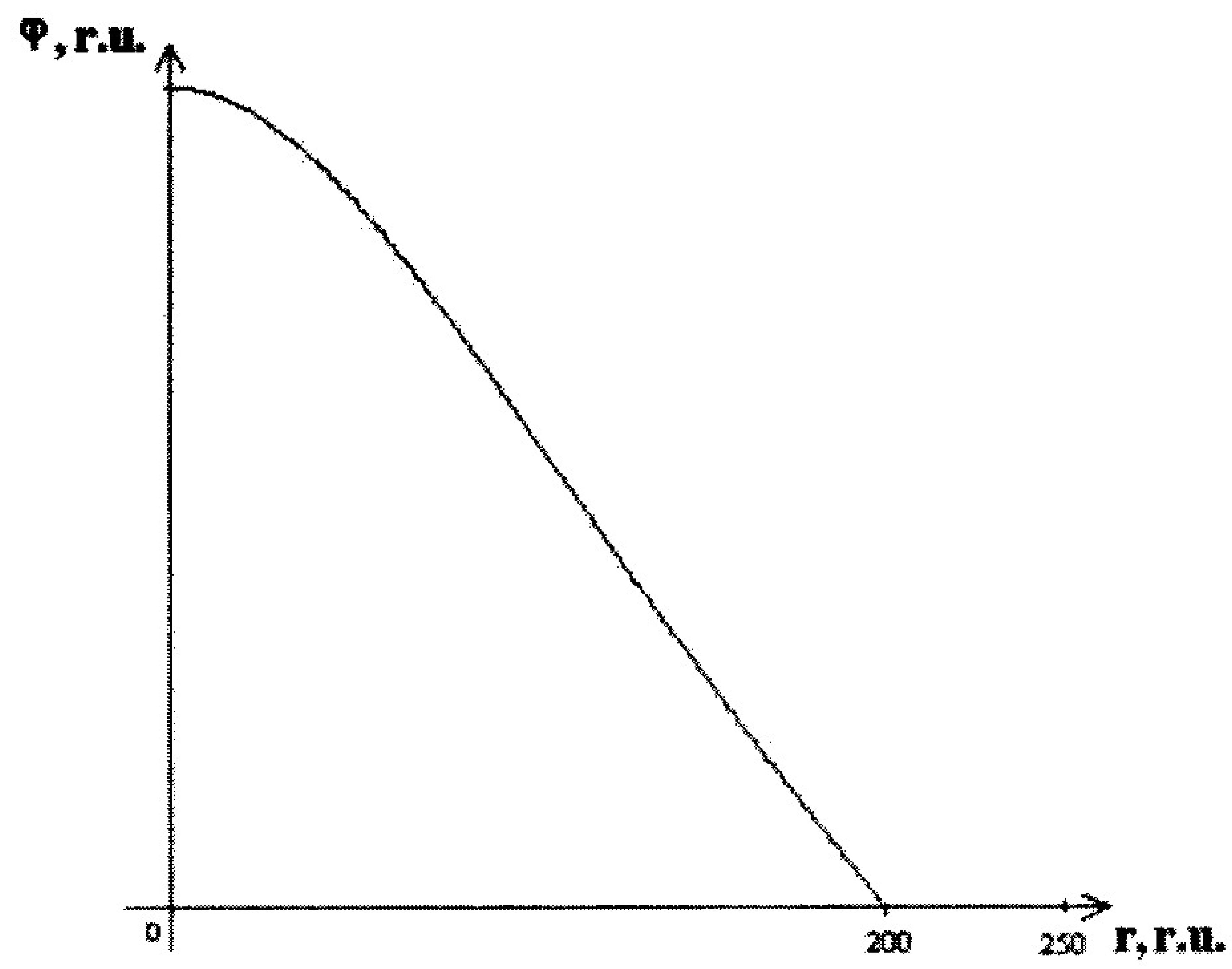

Figure 9: Potential depending on the distance of the needle

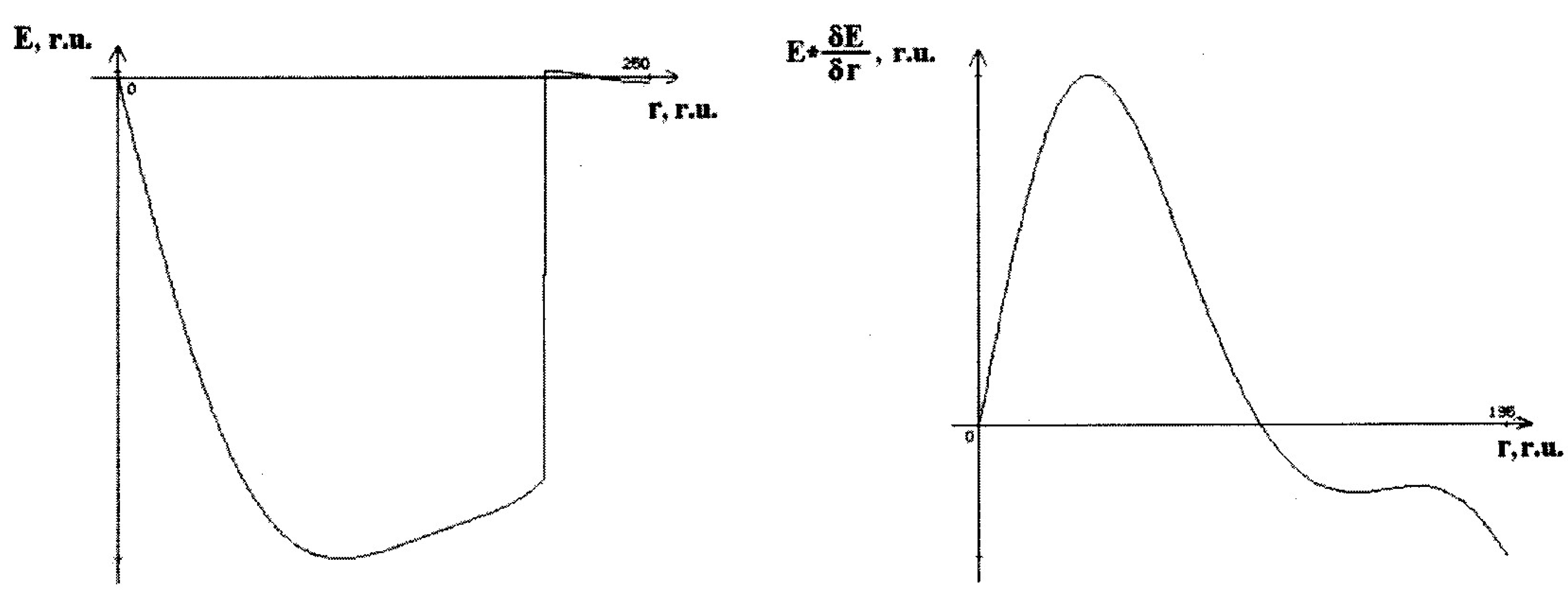

Figure 10: Horizontal projection

2.6 Alignment of metal balls

Alignment of metal balls into the lines is not the only process that takes place in the current system. Many other phenomena are observed such as formation of creator on the surface of castor oil under the needle, or discharge between the needle and surface of oil take place. Each of these phenomena needs its own consideration. We decided to explain alignment of balls because it is the very outstanding and beautiful phenomenon in the current system.

3 Conclusion

Thus, the current system is a real fount of different nice phenomena which illustrate variety of nature of electricity. Among all phenomena we chose alignment of balls in lines for consideration. We only carried its qualitative investigations out, because it is very difficult to do any quantitative measuring. But this problem interests us and, in some time, who knows, may be you will know about our new investigations this system.