

11<sup>th</sup> IYPT '98  
solution to the problem no. 13  
presented by the team of Hungary

**Rope**

How is it possible that a very long and strong rope can be produced from short fibers?  
Prepare a rope from fibers and investigate its tensile strength.

**Abstract**

A theoretical model of a rope is given. The stresses arising in the tearing process are calculated based on the model. The rope is found to be a 'self-blocking' system. Extensive measurements are presented and compared with the model. They are found to be in excellent agreement. Interesting observations are discussed and explained, such as the small 'jumps' on the stress-strain curve, or why the outer fibres break first ?

**Thanks**

The members of the hungarian team thank Professors Péter Gnädig and Géza Tichy (Eötvös Loránd University, Budapest Departments of Atomic and Solid State Physics, respectively) for their advice and helpful discussions on the following two problems.

**Overview**

- Friction force
- Natural fibres
- Tearing of the external fibres
- Tearing of one single fibre
- Results of our measurements

## 1 Friction force

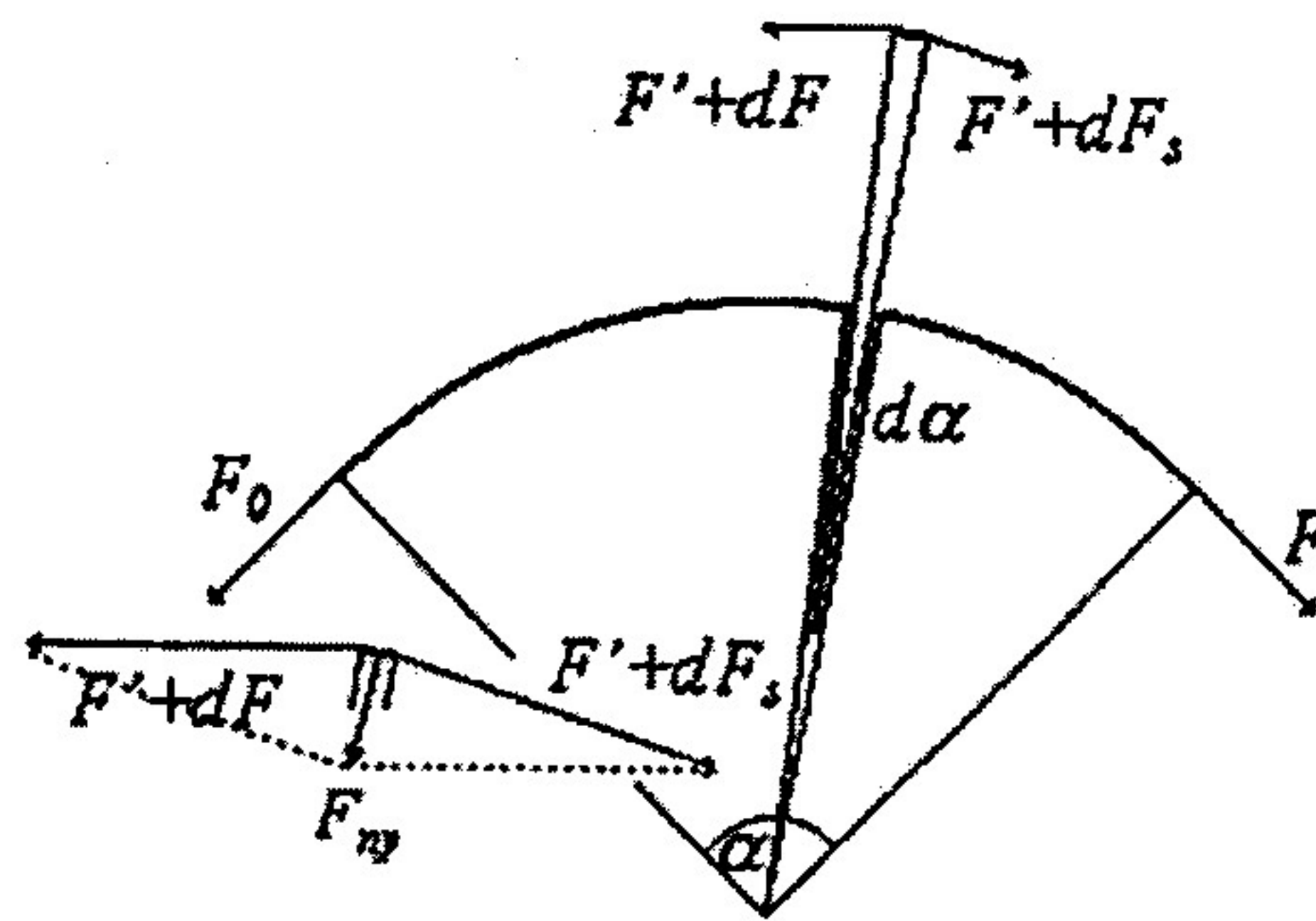


Figure 1: Forces

In this position let's examine the arc belonging to a very small central angle. Besides the tensile force, friction also acts on the rope element:  $dF = dF_s$ . The resultant of the forces stretches the rope on to the arc. This force can be figured on this way:

$$\frac{F_e}{2} = \sin \frac{d\alpha}{2} F'$$

Since  $d\alpha (\ll 1)$  is far less than one

$$\sin \frac{d\alpha}{2} \approx \frac{d\alpha}{2}$$

so the previous equation will change:

$$F_e = d\alpha \cdot F'$$

Since  $F_e$  strains the rope onto the arc the friction force can be figured on this way:

$$\begin{aligned} -dF &= dF_{\max} \\ &= \mu_0 \cdot F' d\alpha \\ dF' &= \mu_0 F' \cdot d\alpha \end{aligned}$$

divided the equation  $dyF'$  we get this formula:

$$\frac{dF'}{F'} = \mu_0 \cdot d\alpha$$

If we add the forces from  $F_0$  to  $F$  and the angles from 0 to  $\alpha$  we get the next equation:

$$\begin{aligned} \int_{F_0}^F \frac{dF}{F'} &= -\mu_0 \cdot \int_0^\alpha d\alpha \\ \ln \frac{F}{F_0} &= -\mu_0 \cdot \alpha \end{aligned}$$

it follows:

$$\frac{F}{F_0} = e^{-\mu_0 \alpha}$$

where  $e = 2,71....$

We get how big a force  $F$  we need to balance the rope pulled by force  $F_0$  if it meets on  $\alpha$  central angle with cylinder jacket of  $\mu_0$  friction index.



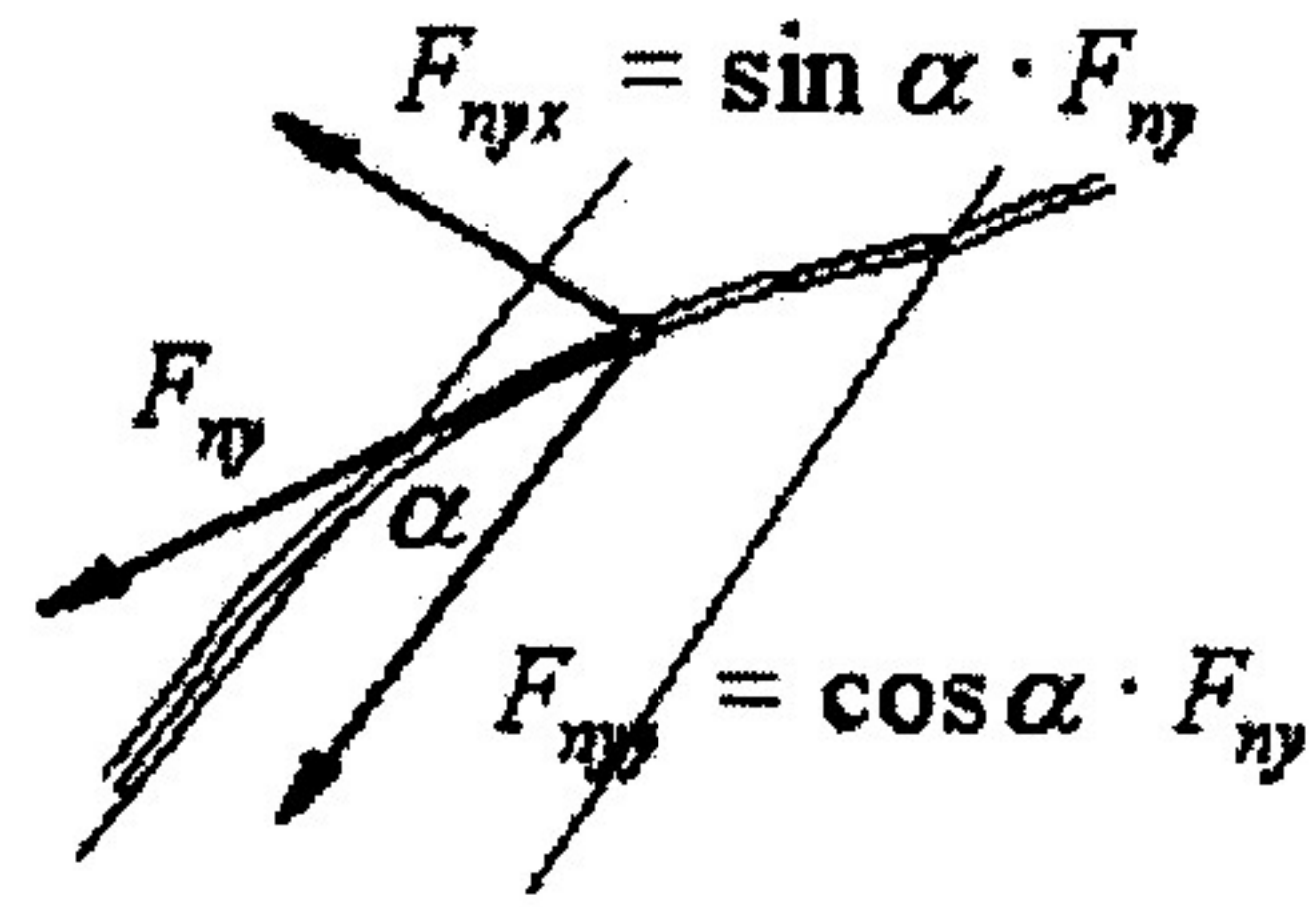


Figure 2: Natural fibres

## 2 Natural fibres

Naturally the single fibres twist on one another inside the twisted ropes so the force  $F$  that pulls them is not perpendicular to the radius of single ropes. If we reduce the tensile to its components and put its projections into the previous relation we get the friction force appears between the single fibres.

$$F = \sin \alpha' \cdot F_0 \cdot e^{-\mu_0 \cdot \alpha}$$

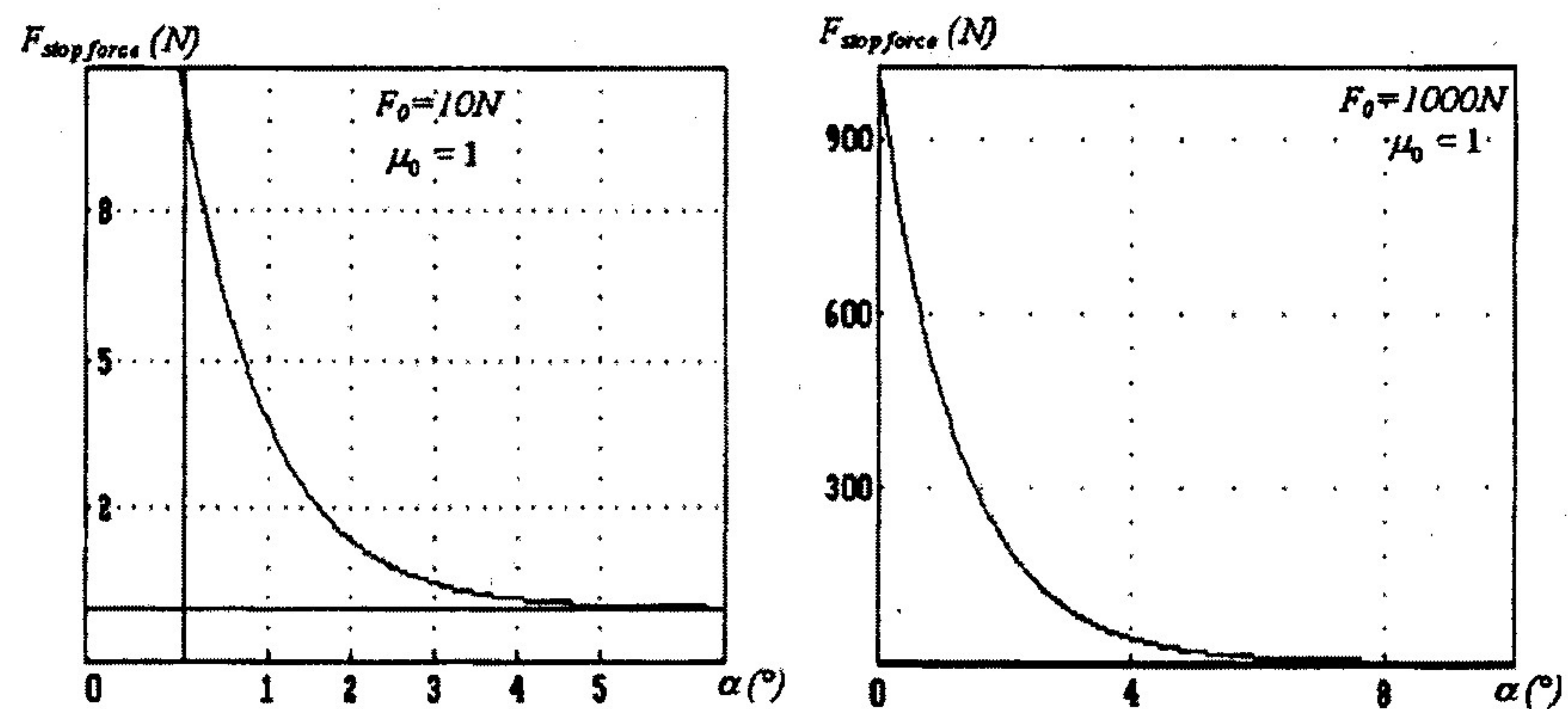


Figure 3: You can see the connection between the central angle and the force what stop the slipping in the case of  $F_0 = 10 \text{ N}$  and  $F_0 = 1000 \text{ N}$ .

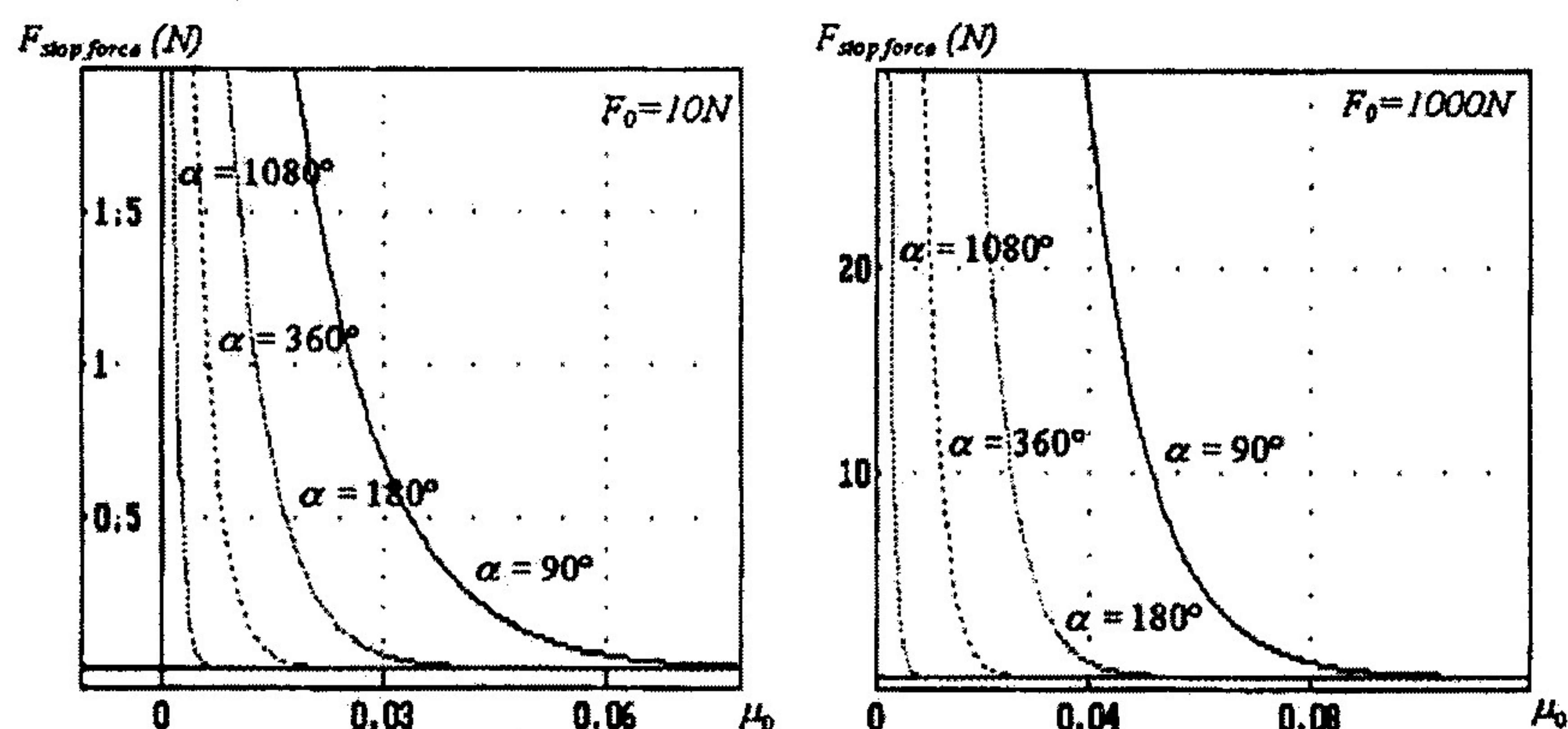


Figure 4: You can see the connection between the friction factor and the force what stop the slipping in the case of  $F_0 = 10 \text{ N}$  and  $F_0 = 1000 \text{ N}$ .



### 3 Tearing of the external fibres

If we examine the lengthening of the external fibres in the case of a given lengthening of the middle fibres we can observe this:

$$x = \frac{2 \cdot r \cdot \pi}{\cos \alpha} \rightarrow x' = \frac{2 \cdot r \cdot \pi}{\cos \alpha}$$

where  $\alpha > \beta$ , so  $\Delta x > \Delta d$ .

We can see that the lengthening of the external fibres is bigger than that of the middle fibres,

$$\Delta l = \frac{1}{E} \cdot \frac{F \cdot l_0}{A}; \left( \frac{\Delta}{l} = \frac{1}{E} \cdot \frac{F}{A} \rightarrow \text{relative lengthening} \right)$$

They are strained more, so they tear first.

### 4 Tearing of one single fibre

Next we examine the tearing of one single fibre. In the fibres, the influence of the  $F$  tensile induced by tension of the size:

$$\sigma = \frac{F}{A}$$

This can determine whether the single fibre will tear or not.

During the lengthening the cross section of the rope will decrease as well:

$$\frac{-\Delta d}{d} = \mu \cdot \frac{\Delta l}{l} \rightarrow \mu = \frac{\frac{-\Delta d}{d}}{\frac{\Delta l}{l}} < \frac{1}{2}$$

In the fibres of the rope however, the tension arises not only from lengthening, the single fibres want to slip on one another but shearing tensions as well.

$$\tau = \frac{F}{A}$$

Although to a little extent, but the shearing tension contributes to the tearing of the fibre.

### 5 Results of our measurements

We examine the lengthening of the ropes in the case of different forces with the help of a tearing machine.

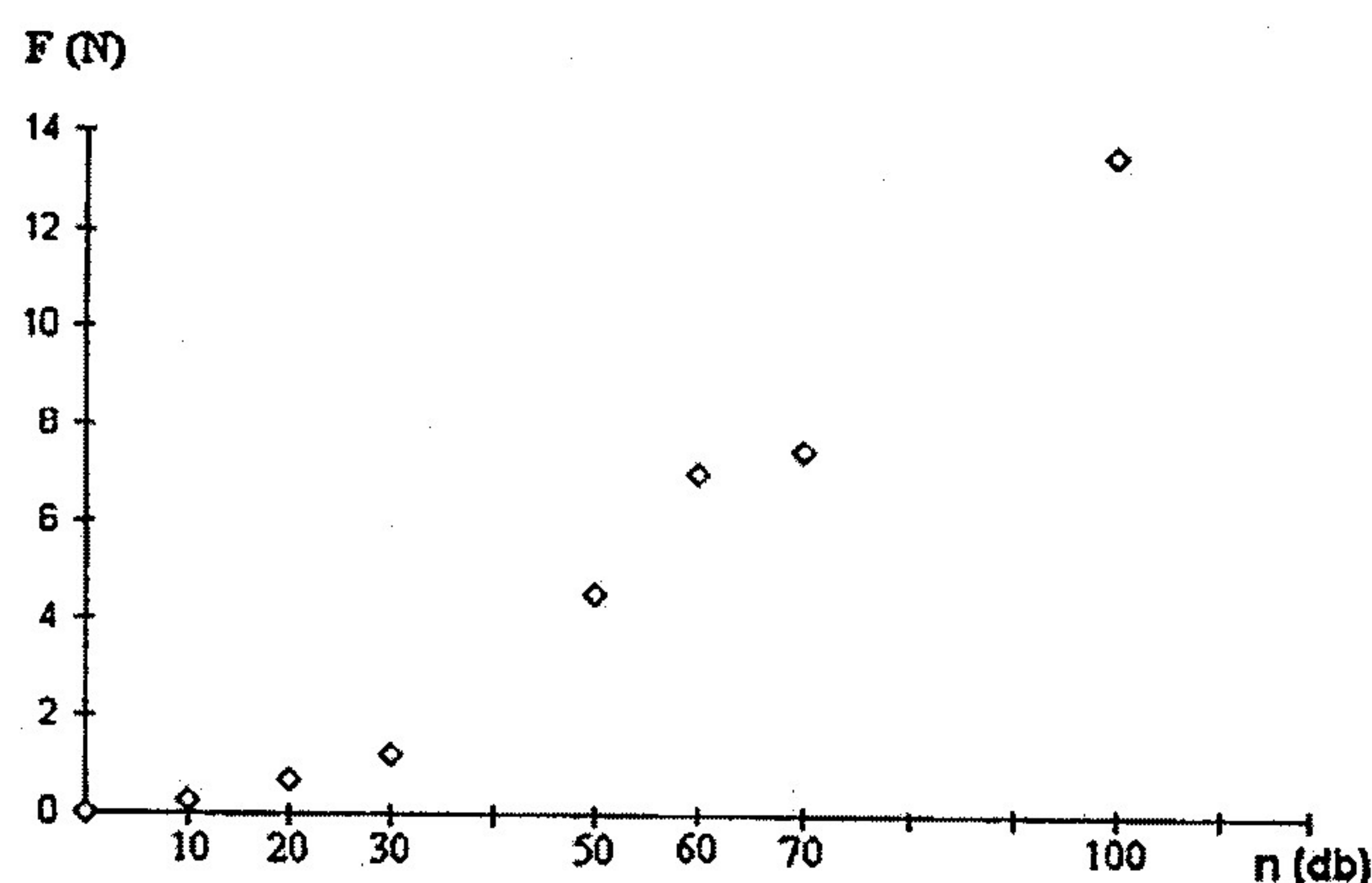


Figure 5: This graph plots the changing of the tearing force against the number of the fibres creating the rope in the case of “damaged” rope.

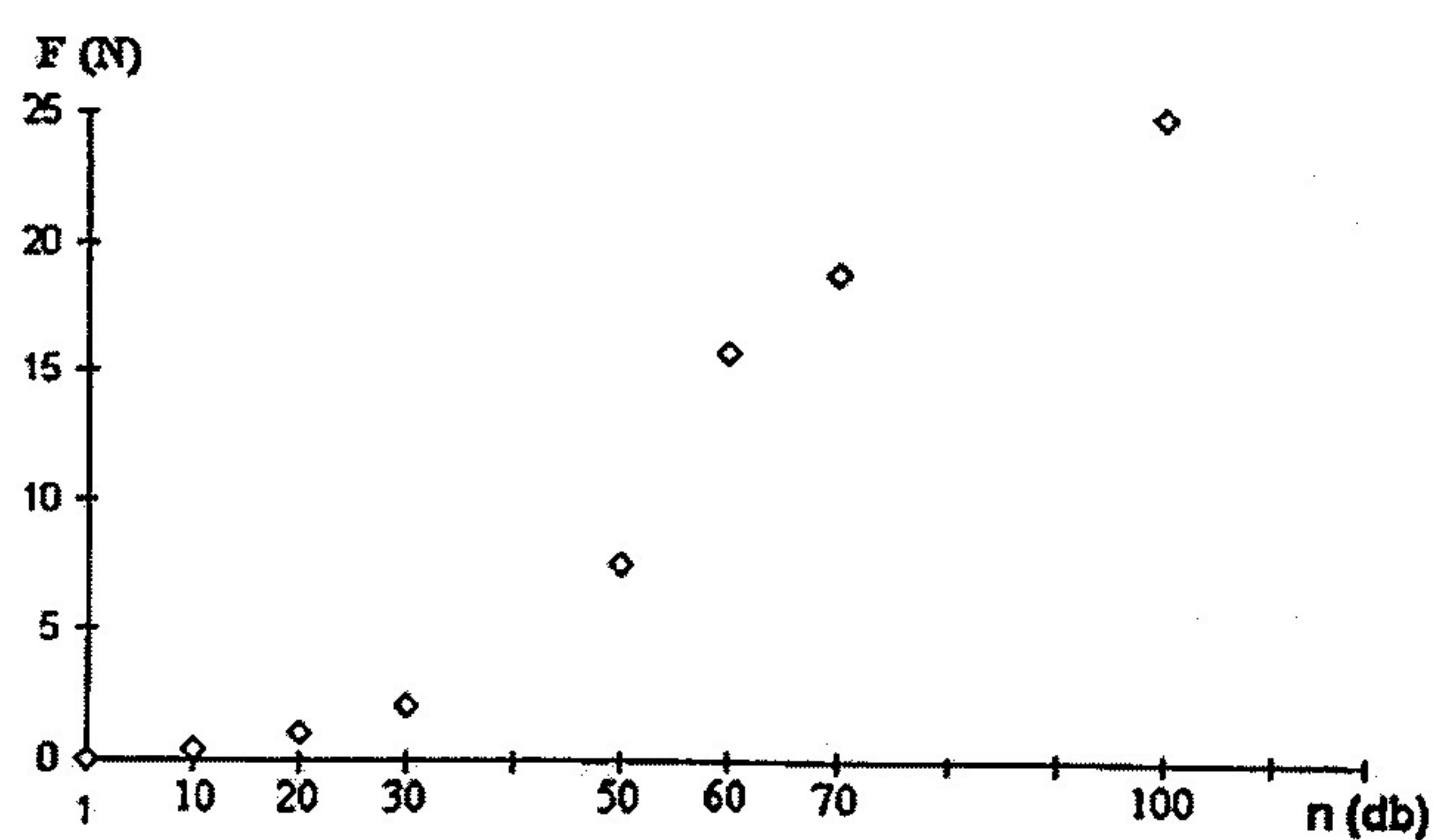


Figure 6: This graph plots the changing of the tearing force against the number of the fibres creating the rope in the case of knotted rope.

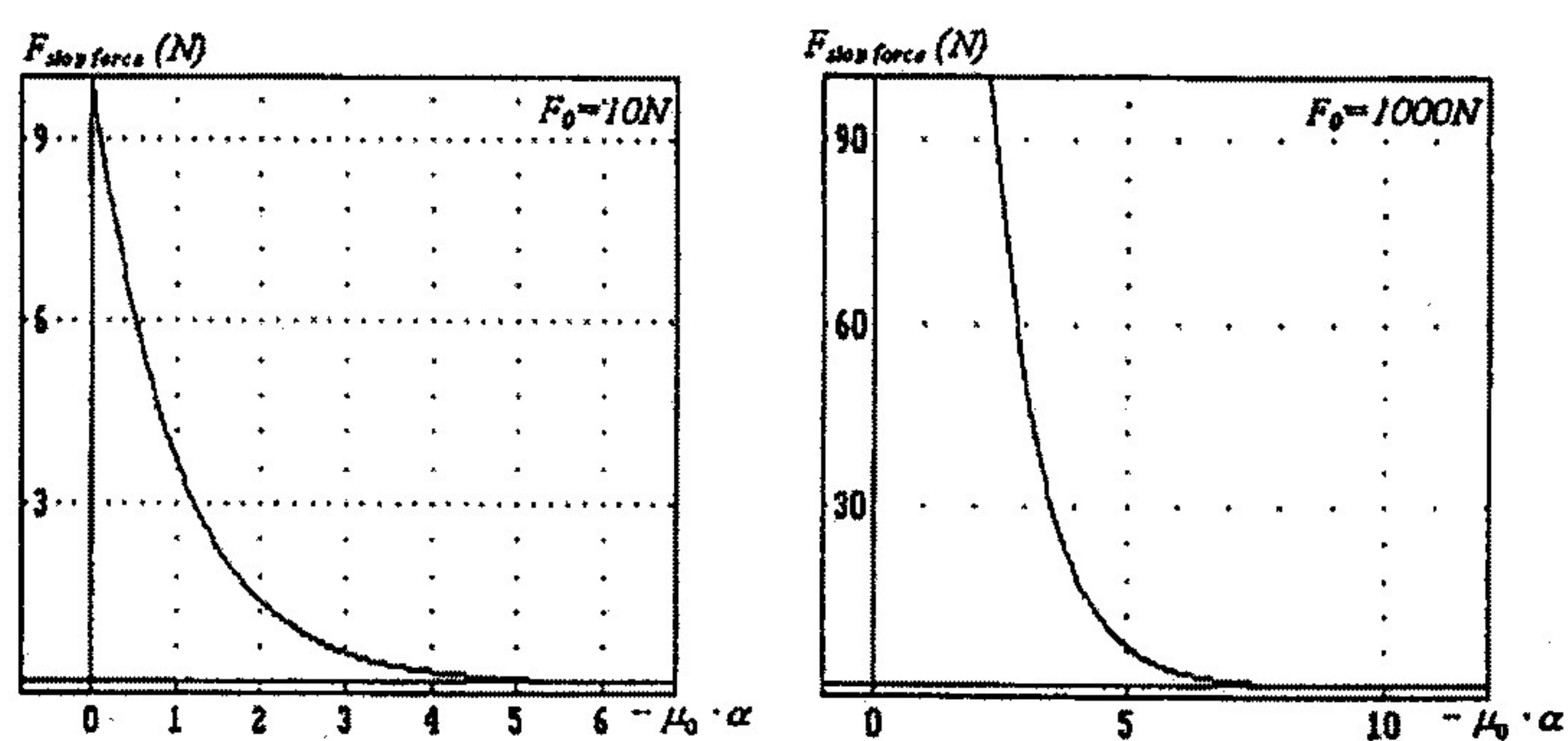


Figure 7: You can see the connection between the central angle multiplied by the friction index and the force stops the slipping in case of  $F_0 = 10\text{ N}$  and  $F_0 = 1000\text{ N}$ .

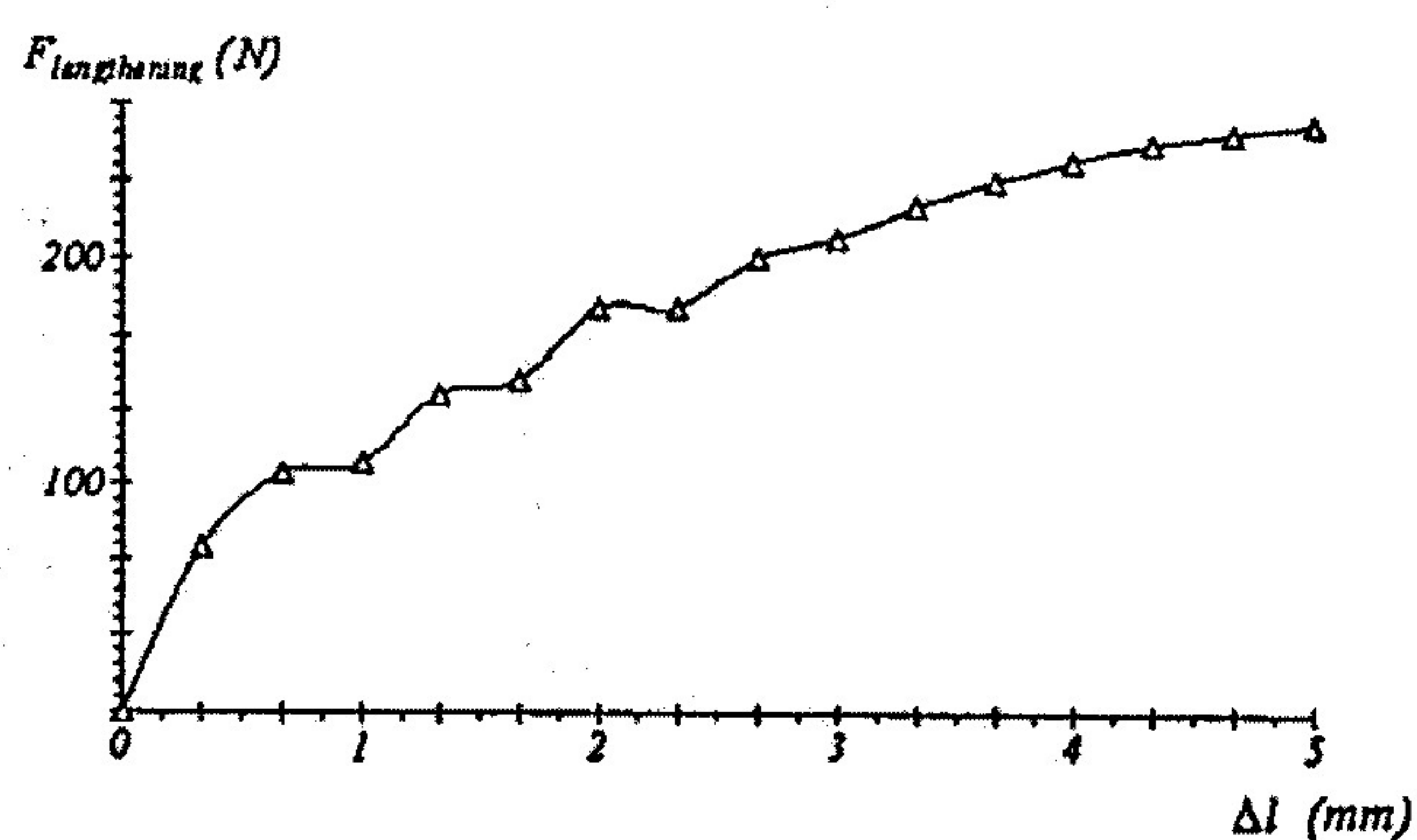


Figure 8: This graph shows the result of our measurement in case of rope number 1 ( $d = 4\text{ mm}$ ).



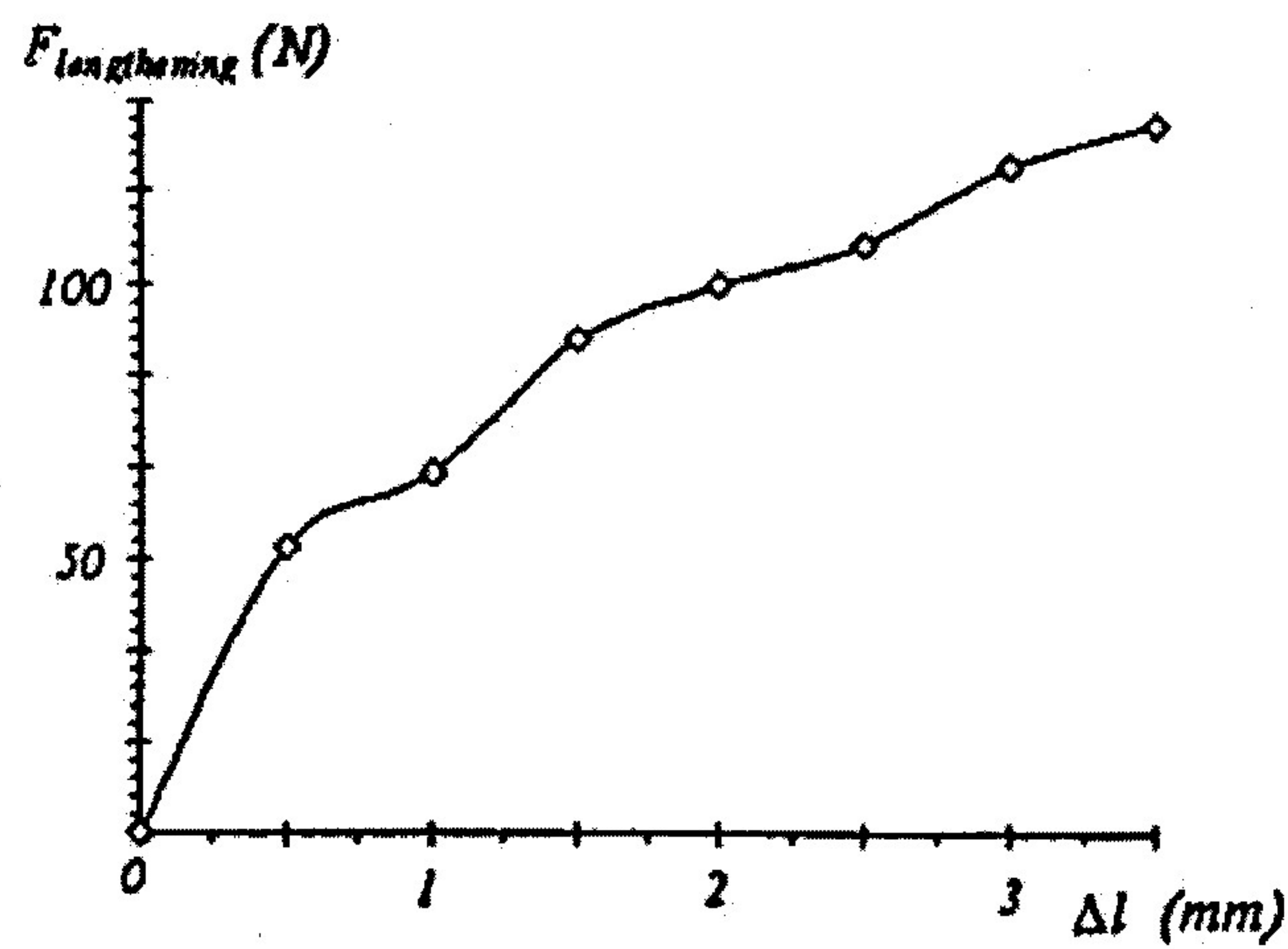


Figure 9: This graph shows the result of our measurement in case of rope number 2 ( $d = 2.8$  mm).

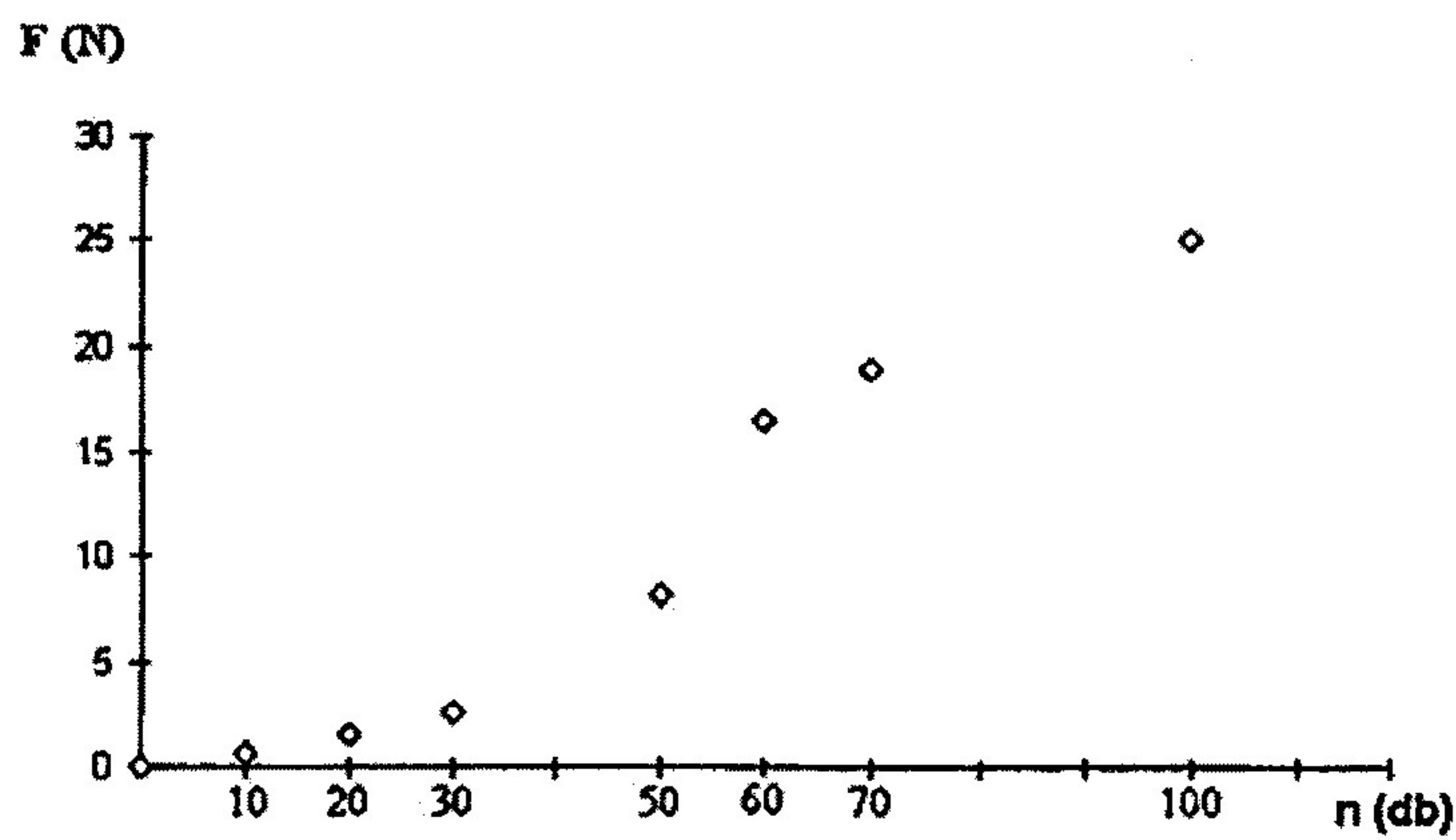


Figure 10: This graph plots the changing of the tearing force against the number of fibres in the rope.

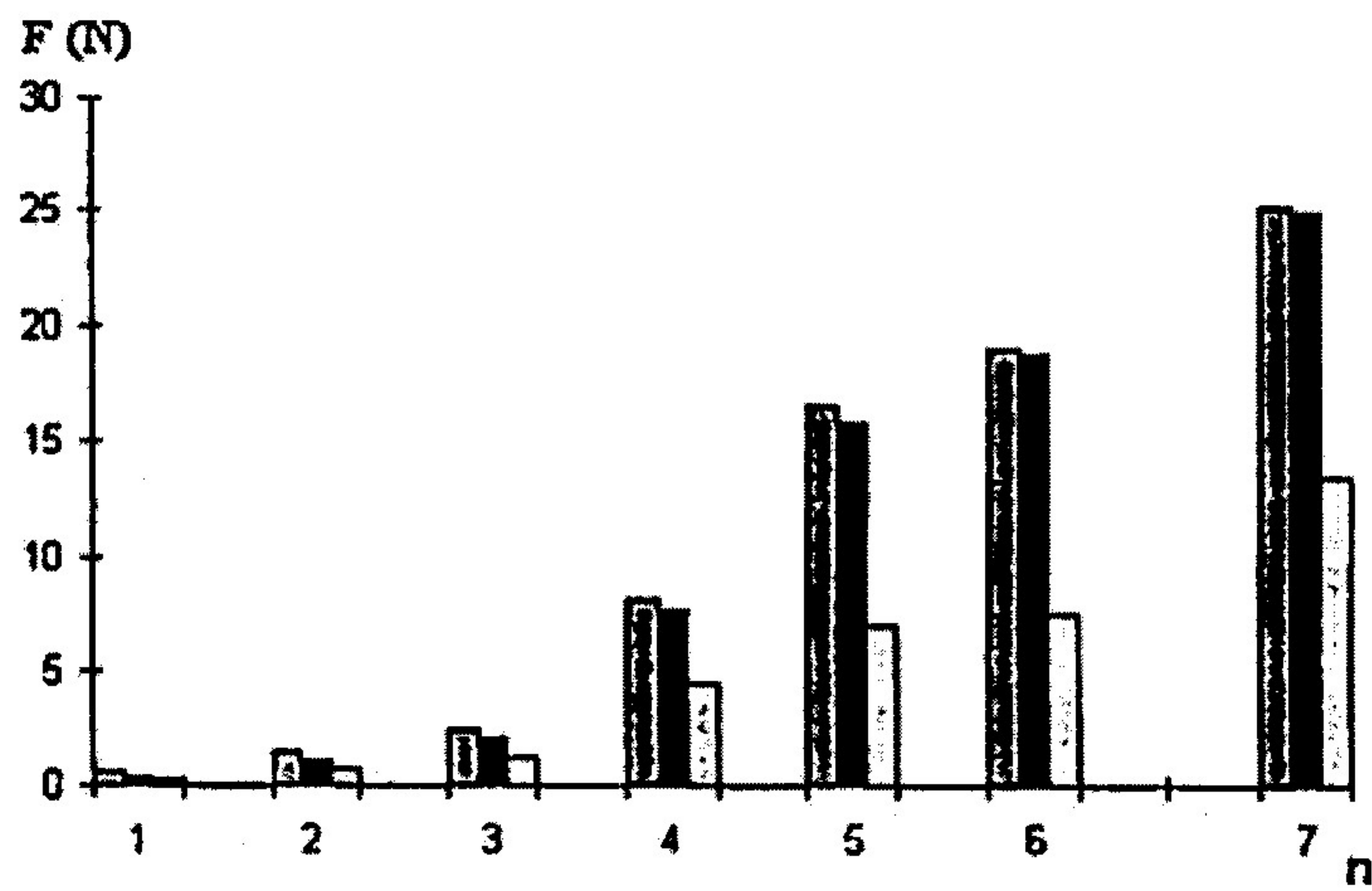


Figure 11: You can see the tearing force in case of damaged (light grey), knotted (black) and normal (dark grey) rope in case of different diameter.