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# solution to the problem no. 14
# presented by the team of Ukraine

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### Water rise

Immerse the end of a textile strip in water. How fast does the water rise in the strip and what height does it reach? In which way do these results depend on the properties of the textile?

### Abstract

The phenomenon is studied as a hydrodynamic problem of flow in a capillary with account of viscosity and gravity forces. The structure of a textile is modeled as a set of capillaries with unique characteristic radius. There is the only fitting parameter in the proposed theoretical model. The differences in theoretical and experimental results are thoroughly analyzed. It is shown that they are connected with neglecting of some secondary effects, which do not influence the asymptotic for the height of water rise.

### Thanks

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### Overview

- Speed of water
- Continuity equation
- Height of water
- Experiment
- Conclusion

# 1 Speed of water

In this problem we deal with capillary properties of textile material. The space between threads in textile can be considered as a set of capillaries of very small diameters. That is why the water between the threads rises.
The speed of water rise on stripe (capillary) depends on the balance between viscosity, capillary, and gravity forces, which act on the column of rising water. The smaller the capillary radius, the more effective capillary force is. At the same time the viscosity force increases and hampers the water rise. Let's derive the equation of motion for the column of rising water:

$$\frac{\partial P}{\partial t} = F_{\text{cap}} - F_{\text{visc}} - F_{\text{gr}} \tag{1}$$

where:

$$\begin{aligned} \frac{\partial P}{\partial t} &= \frac{\partial m}{\partial t} v + \frac{\partial v}{\partial t} m && (2) \\ F_{\text{visc}} &= \alpha \pi \rho \nu R v && (3) \\ F_{\text{gr}} &= mg && (4) \end{aligned}$$

# 2 Continuity equation

Here we neglected the spatial dependence of the velocity of water flow along a capillary and approximated the gradient of corresponding component of the velocity by simple expression $\frac{\partial v}{\partial r} = \frac{v}{R}$ which is correct at least by an order of magnitude. From continuity equation we can conclude that the mass of water column is equal to

$$m = h \pi R^2 \rho \tag{5}$$

and, therefore,

$$\frac{\partial m}{\partial t} = 2 \rho \pi R^2 v \tag{6}$$

# 3 Height of water

The set of equations (1) and (6) can be solved numerically. As expected, the height of water column in textiles asymptotically approaches maximal height of water level, which reads as

$$H_0 = \frac{2\sigma}{R \rho g} \tag{7}$$

It is obtained from the equality of the gravity and the capillary forces.

# 4 Experiment

To verify our theoretical model we carried out the following experiment. We measured the time dependence of height of water rise for different textiles. To exclude evaporation from the surface of textile we put them in a hermetically sealed test-tube. The experimental data we obtained are shown on the graph by squares below. To compare experimental results with our theoretical model we plotted graphs for height of water column versus time. The capillary radius in thread we got from (7), and substituted it in our equation. The parameter $\alpha$ was fitted to get the better fitting. It is the only adjustable parameter of our model. From the physical reasoning we can conclude that its value should not exceed the value in well known expression for viscosity force of POISEUILLE flow in a tube.

# 5 Conclusion

Comparing theoretical and experimental dependence shows that in the beginning they are coincides with high accuracy, but soon after the theoretical curve increases more steeply and goes above the experimental data. This discrepancy can be easily explained as following. The proposed theoretical model does not take into account the outflow through the side branches of the capillaries. This effect is "switched off" at the very beginning of the rising because the main flow goes up swiftly. But when the vertical flow velocity decreases the definite amount of liquid penetrates in horizontal directions effectively damping the vertical velocity. But as time $t \to \infty$ this effect is again negligibly small compared with vertical flow. As a consequence, at any given moment of time the height of water column in our model is larger than in real experiment. In principle, this side outflow can be taken into consideration by applying equation of two-dimensional hydrodynamics. But in this case we would loose the simplicity of mathematics without acquiring any qualitatively new physical results.

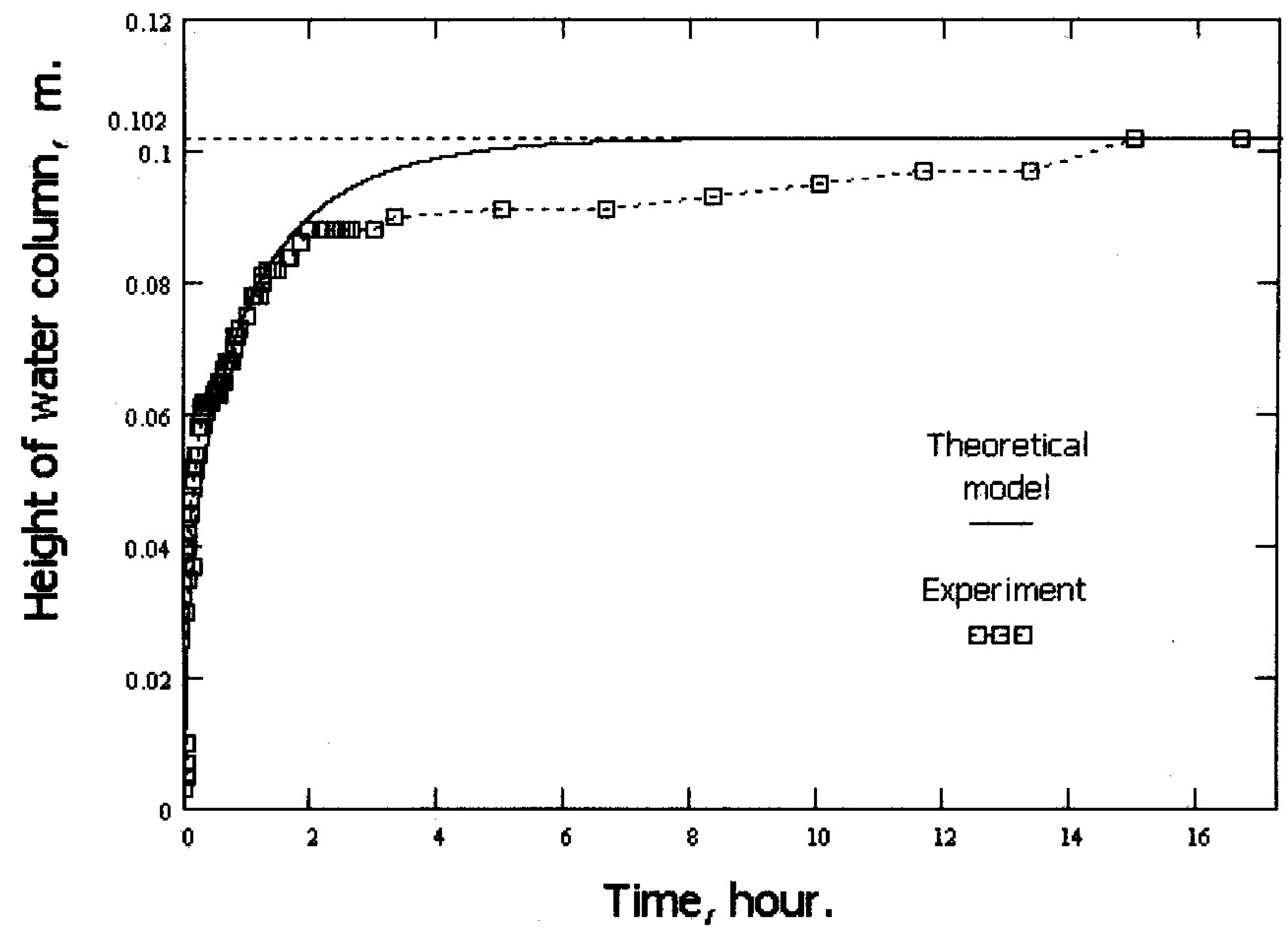


Figure 1: Silk

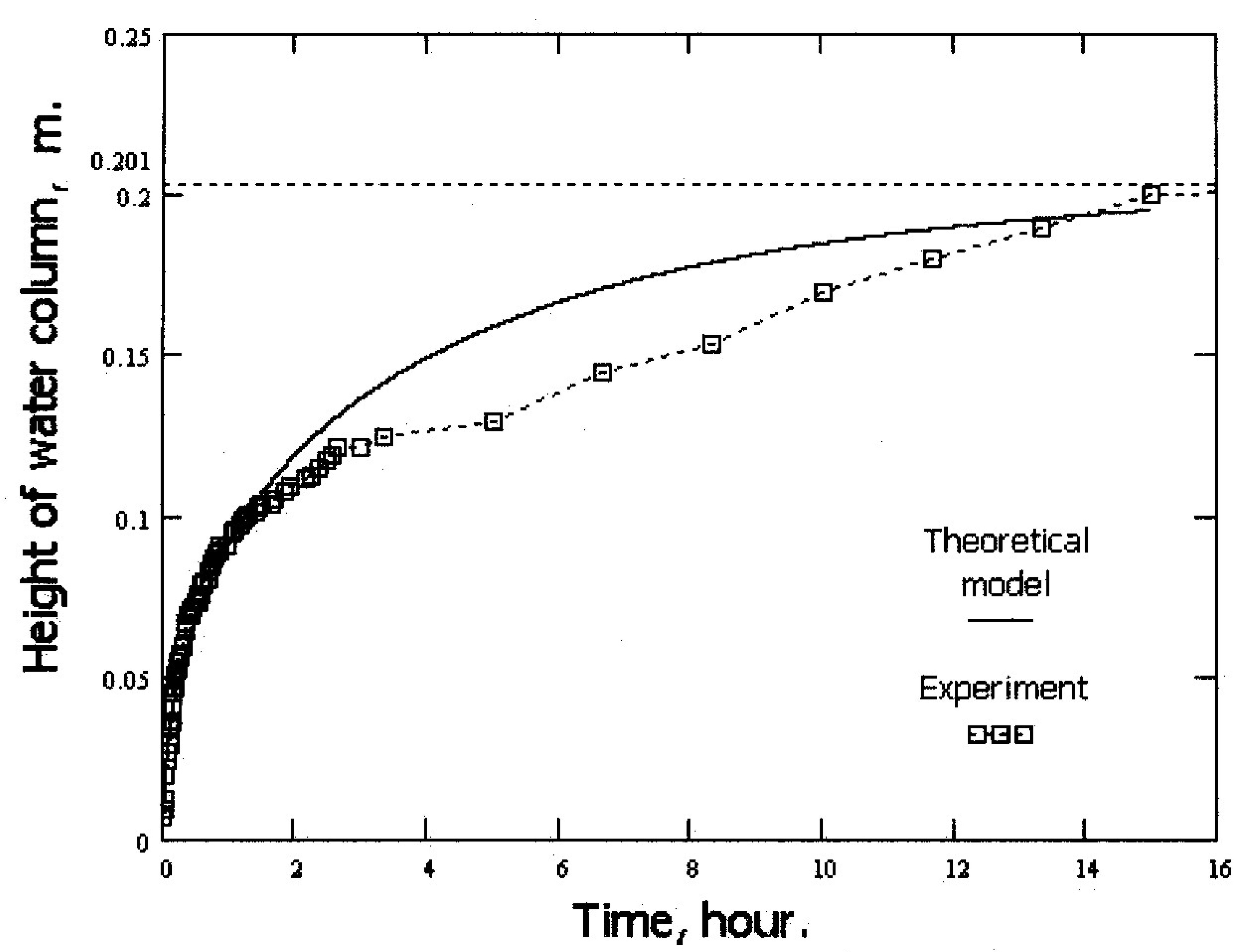


Figure 2: Cotton