SOLUTION OF AUSTRIA

Problem № 2: Stubborn Ice

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(Power Point Presentation)

The problem

Put a piece of ice (e.g. an ice cube) into a container filled with vegetable oil. Observe its motion and make a quantitative description of its dynamics.

Structure

- Supposition
- Experimental Setup
- Observation
- Interpretation
- Quantitative Estimation
- Conclusions
- Literature

Supposition

1. Dynamics dependent on the following parameters:
⇒ difference between the densities of ice, oil & water → buoyancy
⇒ temperature of the oil
⇒ friction
⇒ surface energy
⇒ height of the container
2. Not considered:
⇒ different temperatures/densities in the ice
⇒ layers of different densities in the oil
⇒ boundary effects at the walls of the vessel, etc.

Experimental Setup

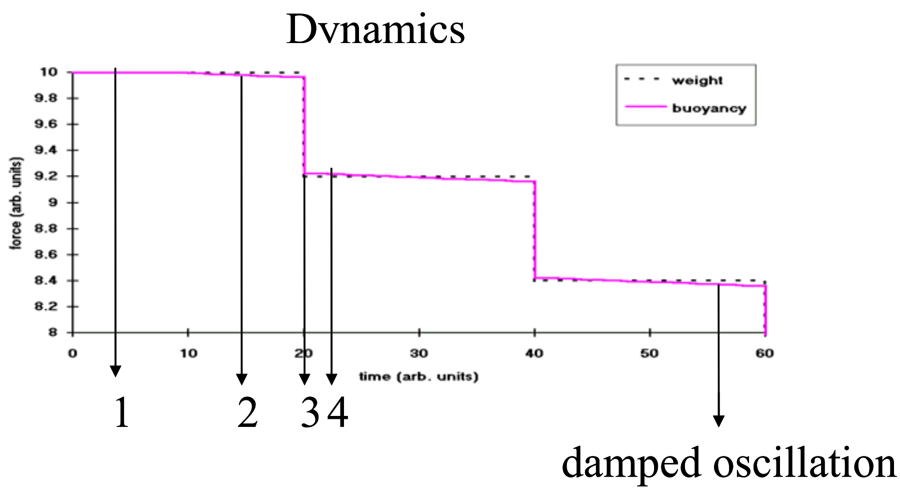
- large container (height: 13cm, diameter: 7cm)
- ice (average density): ρ_{ice} = 917kg/m3
- water: ρ_{water} = 1000kg/m³
- vegetable oils (room temperature): ρ_1 = 925kg/m³
  ρ_2 = 923,3kg/m³ (olive oil)
  (ρ of vegetable oils varies usually between 910kg/m³ and 928kg/m³)
- ice cubes of different shape and volume

Observation

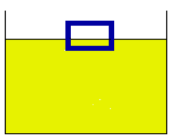
ρoil < ρice: ice descends to the bottom accelerating until the accelerating force due the weight of the cube equals the retarding force due to the friction (Stokes friction)

ρice < ρoil < ρwater : the cubes show vertical oscillation – several up and down motions may occur

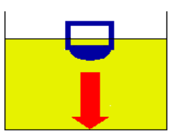
No dependence on the shape and no qualitative dependence on the volume of the cubes can be observed.



Interpretation

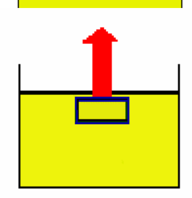


1 cube swims due to buoyancy – ice starts melting and forms water drops until weight is bigger than buoyancy



2 molten water sticks to the cube (surface energy) average density of the combined bodies increases and becomes larger than the density of the oil: the assembly moves down

3 surface tension does not hold the drop attached: the drop seperates and moves downwards



4 the buoyancy increases (less water) – the cube slows down – the cube might rise again until damping stops the process

Quantitative Estimation (1)

Newtons Law: $\mathrm{M}\, a = F - \beta\eta v$

$F = \mathrm{M}\, g - FB$

M … total mass
a … acceleration
M g…weight of the ice cube plus weight of the water
F_B …buoyancy force due to the dispelled oil

The buoyancy force results to: $F_B = g\, \rho oil\, (\mathrm{V}ice + \mathrm{V}water)$ (1)

Quantitative Estimation (2)

Friction force βηv (linearly depending on velocity) :
To good approximation: mass of the ice cube mice linearly decreasing with time due to melting

$\mathrm{M}_{ice} = \mathrm{M} - \alpha\, t$

$\dot{\mathrm{Q}} = k \mathrm{A} \Delta \mathrm{T} \qquad \Delta \mathrm{T} = \mathrm{T}_{oil} - \mathrm{T}_{icewater}$

$\mathrm{M}_{water} = \alpha\, t$

A … surface area of the cube

$$F = g[-\mathrm{M}(\frac{\rho_{oil}}{\rho_{ice}} - 1) + \rho_{oil}(\frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}})\alpha t] \qquad (2)$$

The first term is the buoyancy reduced weight of the ice cube (negative) and second term is positive since $\rho water$ is bigger than that of oil and is linearly increasing with time.

Quantitative Estimation (3)

The acceleration for the ice/water combination is

$$a \equiv \frac{dv}{dt} = g\left[-\left(\frac{\rho_{oil}}{\rho_{ice}} - 1\right) + \rho_{oil}\left(\frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}}\right)\alpha t\right] - \frac{\beta\eta}{M} v \qquad (3)$$

and gives the velocity

$$v(t) = \frac{\mathrm{M}}{\beta\eta} g\left[\left(\frac{\rho_{oil}}{\rho_{ice}} - 1\right) + \frac{\alpha}{\beta\eta}\rho_{oil}\left(\frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}}\right)\right]\left(e^{\frac{-\beta\eta}{\mathrm{M}} t} - 1\right) + g\frac{\alpha}{\beta\eta}\rho_{oil}\left(\frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}}\right) t \qquad (4)$$

Conclusions

- The drops may oscillate because of the interaction between the increasing density of the combined water/ice –system and the buoyancy force
- The phenomenon will stop when the ice cube is so small that its buoyancy force cannot withstand the weight of the new water drop formed.
- Such movement results from the theory and can be observed as shown in the movie.

Literature:

- Density of cooking oil http://hypertextbook.com/facts/2000/IngaDorfman.shtml
- Density of ice http://hypertextbook.com/facts/2000/AlexDallas.shtml
- Density of water http://www.ucdsb.on.ca/tiss/stretton/chem2/data19.htm

HANDOUT "STUBBORN ICE"

$$F_B = g\,\rho_{oil}\,(V_{ice} + V_{water}) \tag{1}$$

$$F = g[-M(\frac{\rho_{oil}}{\rho_{ice}} - 1) + \rho_{oil}(\frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}})\alpha t] \tag{2}$$

$$a \equiv \frac{dv}{dt} = g\left[-\left(\frac{\rho_{oil}}{\rho_{ice}} - 1\right) + \rho_{oil}\left(\frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}}\right)\alpha t\right] - \frac{\beta\eta}{M}v \tag{3}$$

$$v(t) = \frac{M}{\beta\eta} g\left[\left(\frac{\rho_{oil}}{\rho_{ice}} - 1\right) + \frac{\alpha}{\beta\eta}\rho_{oil}\left(\frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}}\right)\right]\left(e^{\frac{-\beta\eta}{M}t} - 1\right) +$$

$$+ g\frac{\alpha}{\beta\eta}\rho_{oil}\left(\frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}}\right)t \tag{4}$$