1.2. PROBLEM № 3: ELECTRIC PENDULUM – IYPT 2004

SOLUTION OF AUSTRIA

Problem № 3: Electric Pendulum

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(Power Point Presentation)

The problem

Use a thread to suspend a ball between the plates of a capacitor. When the plates are charged the ball will start to oscillate. What does the period of the oscillations depend on?

Structure

- Basic consideration
- Experimental Setup
- Observation (1)
- Assumption
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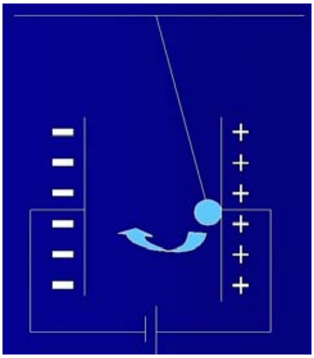
Basic consideration

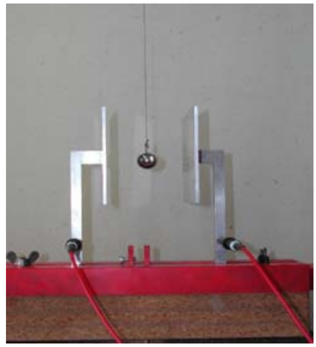
The cause of the oscillation will be the fact that whenever the ball touches a capacitor plate it will be charged as the plate and therefore be repelled afterwards. Simultaneously it will be attracted by the oppositely charged plate and the process will carry on.

The issues of the investigation are:

⇒ Why does the oscillation start?
⇒ What are the parameters which influence the oscillation?

Experimental Setup

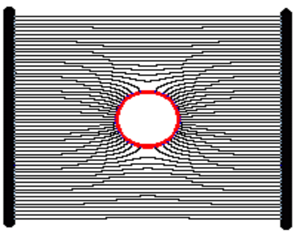




- two capacitor plates in variable distance (in the order of 10-1m)
- thread of 2,5m
- balls of iron (d = 1,91cm, m=28,7g), wood (d1= 2,02cm, m1= 3,8g; d2= 3,32cm, m2 = 15,1g), aluminium (hollow; d = 2,52cm, m = 12,3g) and table tennis (d = 3,75cm, m = 2,8g)

Observation (1)

The ball symmetrically situated between the capacitor plates would not move:



(fixed homogeneous field assumed)
Little asymmetry starts the process:
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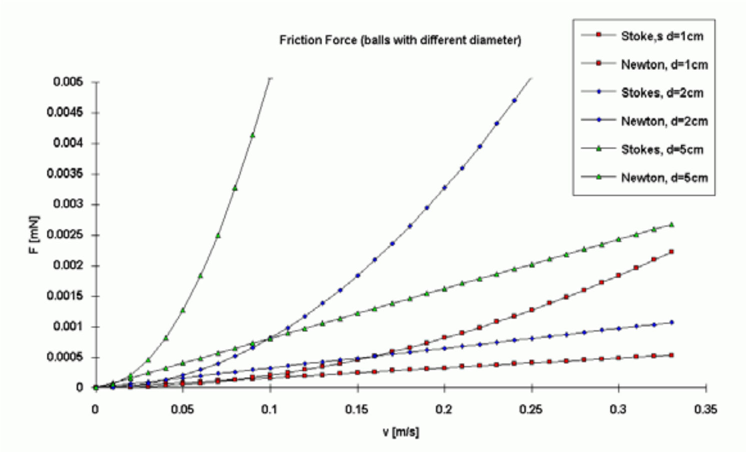
Assumption

- infinite length of the suspending thread
- Newton's friction law $F_{Newton} = c_D \rho A v2/2$ (1)
- accelerated motion of the charged ball until the gained energy in the electric field equals the energy loss due to friction
- total elastic reflection at the plates

Quantitative Estimation (1)

Newton’s friction:



Newton‘s friction more likely than Stokes‘ friction since for air viscosity Newton‘s friction outweighs Stokes‘ friction for velocities > 0.05m/s.

Quantitative Estimation (2)

Equation of motion: $m\frac{dv}{dt} = F_{el} - F_{Newton}$ with $F_{el} = QE = Q\frac{U}{D}$

$$Q = \alpha dU$$

D … distance of the plates
D … diameter of the ball

(2) $$m\frac{dv}{dt} = \alpha\frac{dU^2}{D} - \beta d^2 v^2$$

β … constant variables in Newton‘s friction law

α = 2πε0

Steady state: $\frac{dv}{dt} = 0 \qquad v = U\sqrt{\frac{\alpha}{\beta Dd}}$ (3)

Quantitative Estimation (3)

Period of the oscillation: $$T = 2\frac{(D-d)}{v} \equiv \frac{2(D-d)}{U}\sqrt{\left(\frac{\beta Dd}{\alpha}\right)} \qquad (4)$$

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With a thread of finite length: $T_{max} = 2\pi\sqrt{\frac{l}{g}}$

l … length of the thread
g …gravitational acceleration

Thus

⇒ velocity proportional to the voltage

⇒ the period of the oscillation is inversely proportional to the voltage
⇒ the upper limit for the period is given by the length of the thread

Quantitative Estimation (4)

Total reflection of the balls: 1 iron ball
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2 table tennis ball
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Energy loss:

$$\mathrm{K} \quad \Delta E = \frac{mv^2}{2}$$

proportional to v^2 as Newton's friction → no change of the dependence of v from U

⇒ Dependence on the mass (total elastic) :
→ no analytical solution of the equation of motion
→ with evidence shown in the experiments

Quantitative Estimation (5)

Fully inelastic case:

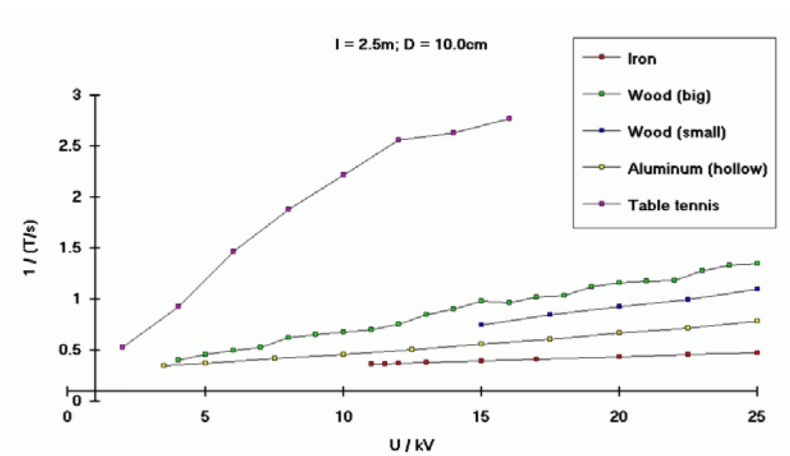
- predominant energy loss at the reflection
- uniformly accelerated motion
- full stop at hitting the electrodes

Then follows

$$m\frac{dv}{dt} = QE \text{ with the solution } D = \frac{\alpha d U^2}{2mD} T_U^2$$

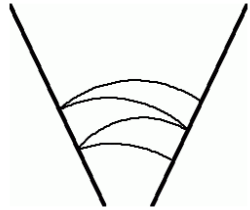
and the period
$$T_U = \frac{D}{U}\sqrt{\frac{2m}{\alpha d}} \tag{5}$$

Results



- inverse period of the oscillation 1/(T/s) as a function of U→ all balls show a linear dependence on U] less inelastic reflection → more evidence of the dependence table tennis ball).

Special Arrangement



In this case the ball is deflected at every bounce and moves outward until the outward motion which is caused at every reflection is compensated by the inward motion due to the torque build up.
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Conclusions

- Energy loss ~ v^2 for all balls
- 1/U dependence for the period of oscillation for all balls
- 1/U dependence increases elasticity
- for low voltages the period of oscillation is limited by the free oscillation period of the ball

HANDOUT "ELECTRIC PENDULUM"

$$F_{Newton} = c_D A\rho / 2v^2 \tag{1}$$

$$m\frac{dv}{dt} = \alpha\frac{dU^2}{D} - \beta d^2 v^2 \tag{2}$$

$$\frac{dv}{dt} = 0 \quad v = U\sqrt{\frac{\alpha}{\beta D d}} \tag{3}$$

$$T_U = 2\frac{(D-d)}{v} \equiv \frac{2(D-d)}{U}\sqrt{\left(\frac{\beta D d}{\alpha}\right)} \tag{4}$$

$$m\frac{dv}{dt} = QE \quad D = \frac{\alpha d U^2}{2mD}T_U^2 \quad T_U = \frac{D}{U}\sqrt{\frac{2m}{\alpha d}} \tag{5}$$