PROBLEM № 9: SOUND IN THE GLASS

4.2. .SOLUTION OF UKRAINE

Problem № 9: Sound in the Glass

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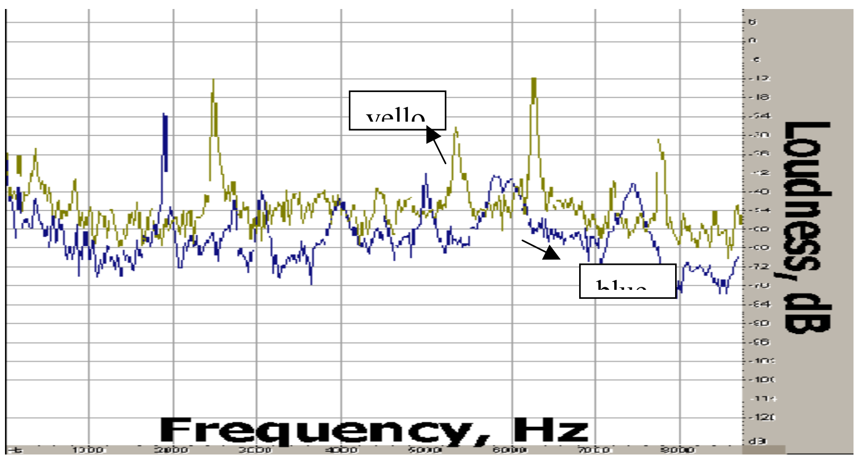
The problem:

Fill the glass with water. Put a tea-spoon of salt into the water and stir it. Explain the change of the sound produced by the clicking of the glass with the tea-spoon during the dissolving process.

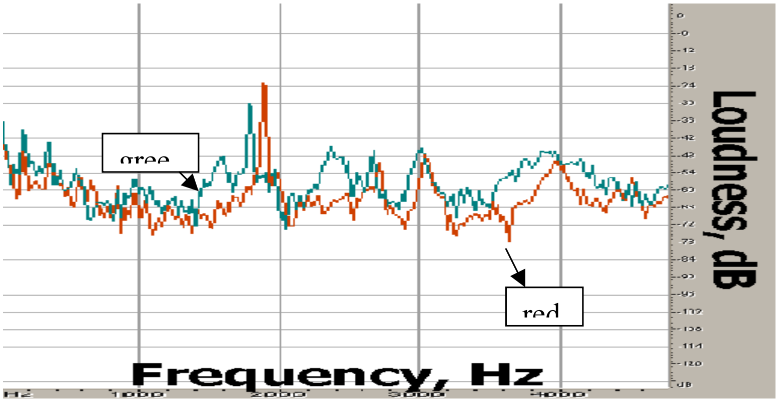
To solve the problem the aim of the work is always needed. We will look for the frequency change of clicking, which appears after stirring in the glass, experimentally and theoretically.

We wanted to observe the phenomenon so took a glass of water and recorded sounds of clicking for the empty glass, for the glass filled with water and for the glass with water and dissolved coffee in it. Using computer programms frequency analysis was done:



Here you can see **blue line** that represents frequency analysis for the glass with water and yellow line for the empty glass. It is obvious that water makes the sound duller, its frequency lower. We can use a mechanical similarity: the glass will be interpreted as a spring with a weight and it oscillates with some frequency; adding of the water corresponds to increasing of the weight.

On the next graph **red line** represents frequency analysis for the glass with water and **green line** for water with coffee.



What changes with adding coffee? When we put a tea-spoon of coffee or other soluble in water material into the glass and stir it, the dissolving starts. It causes the changing of the gas dissolubility (as you know some gas always exists in water). So the gas evolves in the form of bubbles. Because of this compressibility and density of the liquid changes consequently the speed of the sound also changes. This works for small bubbles, which don't influence on the sound path, which we observe in the glass. Digressing into the mechanical similarity we can say that dissolving of coffee means changing rigidity for the spring and mass for the weight.

As mechanism of the phenomenon is understood, we can start mathematical investigation of the problem. In our model such assumptions were made:

1. Wave that travels in the water is longitudinal;
2. Bubbles that form in the water are small R_b b A_{oscill};
3. While sound travels in the glass compression of water and gas occurs adiabatically.

To find the frequency change we need new compression modulus and density of the water with bubbles.

So long as compression of water and gas occurs adiabatically:

$$pV_{gas}^{\gamma} = const \quad \Rightarrow \quad dpV_{gas} + \gamma p dV_{gas} = 0 \qquad \textbf{(1)}$$

$$\Delta p = -k\Delta V_{water} = -\frac{\gamma p}{V_{gas}}\Delta V_{gas} \qquad \textbf{(2)}$$

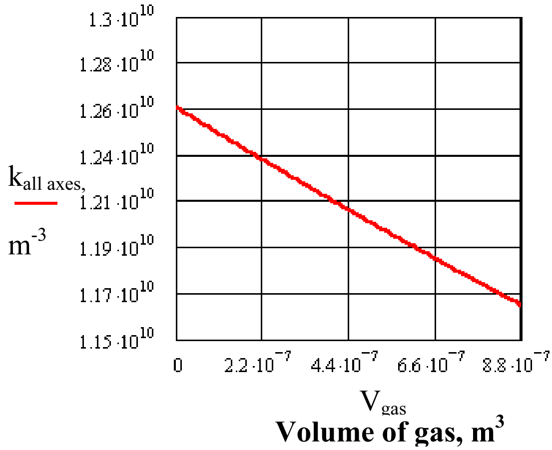
$$k = \frac{1}{V_{water}K_{water}} \qquad \textbf{(3)}$$

$$\left(\frac{\partial p}{\partial V_{all}}\right)_S \approx \left(\frac{\Delta p}{\Delta V_{gas} + \Delta V_{water}}\right) = \left(\frac{-\frac{\gamma p}{V_{gas}}}{1 + \frac{\gamma p}{kV_{gas}}}\right) = -\frac{k}{1 + \frac{kV_{gas}}{\gamma p}} \qquad \textbf{(4)}$$

$$k_{\textbf{all axes}} = \frac{k}{1 + \frac{kV_{gas}}{\gamma p}} \qquad \textbf{(5)}$$

Compression modulus

d is the characteristical geometrical size of the glass with water.



$$\rho_{new} = \frac{\rho_{water} V_{water}}{V_{gas} + V_{water}} \Rightarrow \frac{\rho_{old}}{\rho_{new}} = 1 + \frac{V_{gas}}{V_{water}} \quad \textbf{(6)}$$

$$C_{sound} = \sqrt{\frac{E}{\rho}} \Rightarrow \frac{C_{new}}{C_{old}} = \sqrt{\frac{\rho_{old}}{\rho_{new}\left(1 + \frac{kV_{gas}}{\gamma P}\right)}} \quad \textbf{(7)}$$

$$\nu_0 = \frac{C_{old}}{d} \quad \textbf{(8)} \qquad \nu_1 = \frac{C_{new}}{d} \quad \textbf{(9)}$$

$$\Delta\nu = \nu_1 - \nu_0 = \frac{C_{old}}{d}\left(\frac{C_{new}}{C_{old}} - 1\right) = \nu_0 \left(\sqrt{\frac{1 + \frac{V_{gas}}{V_{water}}}{1 + \frac{kV_{gas}}{\gamma p}}} - 1\right)$$

In our experiments we saw:

Frequency before dissolving, Hz	**Frequency after dissolving, Hz**	$\Delta\nu$, Hz	$\frac{\Delta\nu}{\nu}$
1880	1800	80	0.04
3036	2910	126	0.04

The only problem for theoretical solution is to find the volume of the gas that evolved. At the same time definite volume of gas corresponds to the definite frequency change. So the experiment gives us information about the frequency

$$\nu_0 = \frac{C_{old}}{d} \quad \textbf{(8)} \qquad \nu_1 = \frac{C_{new}}{d} \quad \textbf{(9)}$$

$$\alpha = \frac{1}{V_{water}} \qquad \beta = \frac{k}{\gamma p}$$

and

we can find the volume of gas theoretically. Then we'll try to estimate this value in the other way. But our lust formula is too complicated and can be simplified.

Substitution of these magnitudes gives:

$$\mathbf{K} = 4.5 \cdot 10^{-10}\,\mathbf{Pa^{-1}} \qquad \mathbf{V_{water}} = 0.177\ \mathbf{m^3}$$

$$\mathbf{p} = 10^5\,\mathbf{Pa} \qquad \gamma = \frac{\mathbf{C_p}}{\mathbf{C_V}} = 1.4$$

$$\frac{C_{new}}{C_{old}} = \sqrt{\frac{1 + \frac{V_{gas}}{V_{water}}}{1 + \frac{kV_{water}}{\gamma p}}}$$

$$\boxed{\Delta \mathbf{V_{gas}} = 8.8 \cdot 10^{-7}\,\mathbf{m^3}} \quad \longrightarrow \quad d\nu = \nu_0 \left[-\frac{1}{2}(\beta - \alpha) \right] dV_{gas}$$

$$\boxed{\Delta V_{gas} = \frac{-2}{(\beta - \alpha)} \frac{\Delta \nu}{\nu_0} = -\frac{2\Delta \nu}{\nu_0} \cdot \frac{K_{water}\gamma p}{1 - K_{water}\gamma p} V_{water} \quad \textbf{(10)}}$$

We heard that after some time sound of the clicking becomes the same as before appearance of any bubbles. It means all of them have risen to the surface. By these considerations we can estimate the average size of the bubble and consequently their volume.

We equalize buoyant and resistant forces, thinking that velocity becomes constant very quickly.

$$F_{res} + mg = F_b \qquad mg \ll F_{res} \ , \ mg \ll F_b$$

$$\left.\begin{aligned} &F_{res} = 6\pi\eta RV, \quad V = \frac{L}{t} \\ &F_b = \frac{4}{3}\pi R^3 \rho g \end{aligned}\right\} \longrightarrow F_{res} = F_b$$

$$\eta = 10^{-3} \ \boxed{\text{Pa.}} \qquad L \simeq 10^{-1} m \qquad \rho = 10^3 \frac{kg}{m^3}$$

$$t = 15s \qquad n \simeq 10^3$$

$$R = 3 \cdot \sqrt{\frac{\eta L}{2\rho g t}} \longrightarrow \Delta V_{gas} = \frac{4}{3}\pi R^3 n$$

$$\boxed{\Delta V_{gas} \simeq 6.7 \cdot 10^{-7} m^3}$$

As you see both results for the volume of the evolved gas are comparable with each other so frequencies got in the experiment and predicted by our theory are in quite good agreement.

In conclusion I would like to add that such bubbles lead to one more effect which wasn't considered here: high frequencies are dampened in the water with such bubbles. But nevertheless considerations of the effect of the frequency shift because compressibility and density of the liquid change gives good results for explanation and prediction of the phenomenon.

Special thanks to:

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