7. PROBLEM № 14: EINSTEIN–DE HAAS EXPERIMENT

7.1. SOLUTION OF BRAZIL

Problem № 14: Einstein–de Haas Experiment

Emanuelle Roberta da Silva
Colégio Objetivo – São Paulo – SP

The problem

When you apply a vertical magnetic field to a metallic cylinder suspended by a string it begins to rotate. Study this phenomenon.

1. Objectives:

The original experiment, did in 1915, had the objective to determine the change in angular momentum which accompanies a known change in magnetic moment, or in other words, to determine the *gyromagnetic ratio.*Doing some experiments, we intend to prove our hypothesis that the cause of the cylinder movement is microscopic and interior, because there is no external torque acting on the cylinder. With experiments and theory, we will show that the movement is related with the angular momentum and magnetic moment of the electron and with the cylinder magnetization.

2. Theoretical Introduction:

Torque and angular momentum:

In order to make an object rotate, it is necessary that we apply a force. The torque is the vector product of the position vector and the force vector:

$$\vec{\tau} = \vec{r} \times \vec{F} \tag{1}$$

The angular momentum is defined as the vector product of the position vector and the linear momentum vector:

$$\vec{L} = \vec{r} \times \vec{p} \tag{2}$$

By the Newton's Second Law, we can relate the torque and the angular momentum:

$$\sum \overrightarrow{\tau_{ext}} = \frac{d\vec{L}}{dt} \tag{3}$$

When the resultant of the external torques is null, the angular momentum is constant, and we can enunciate the Conservation of the Angular Momentum Law:

If the resultant of the torques that are acting on a system is zero, the total angular momentum is conserved.

Magnetic Moment:

The magnetic moment is defined as the product of the electrical current and the area of the circuit:

$$\mu = iA \quad (4)$$

We can arrive at the classic relation between magnetic moment and angular momentum:

$$\mu = iA = q\left(\frac{v}{2\pi r}\right)(\pi r^2) = \frac{1}{2}qvr = \frac{1}{2}q\left(\frac{L}{m}\right)$$

$$\vec{\mu} = \frac{q}{2m}\vec{L} \quad (5)$$

The magnetic momentum of the electron, associated with its orbit is given by this equation:

$$\overrightarrow{\mu_1} = -g_l \mu_B \frac{\vec{L}}{\hbar} \quad (6)$$

where $\boldsymbol{\mu_B}$ is the Bohr magneto:

$$\mu_B = \frac{e\hbar}{2m_e} = 9,274.10^{-24}\, A.m^2 \quad (7)$$

Classically, the prediction for the *gyromagnetic factor* (*g*) is 1, but experimentally, the researches have found values near 2 , and it happens because the existence of the *spin* (the electron has an intrinsic angular momentum called spin).

So, we can calculate the magnetic moment of the electron due to the spin:

$$\overrightarrow{\mu_s} = -\frac{g_s \mu_B}{\hbar}\vec{S} \quad (8)$$

Where the *gyromagnetic fator* (*g*) is closer to 2.

Magnetization:

Magnetization is defined as the sum of the magnetic moments of the electrons that are in a solid.

$$\overrightarrow{M} = \sum_{i=1}^{N} \overrightarrow{\mu_i} \quad (9)$$

It is proportional to the magnetic field applied to the body.

$$\vec{M} = \chi_m \frac{\vec{B}}{\mu_0} \qquad (10)$$

where χ_m is the magnetic susceptibility of the material and μ_0 is the magnetic permeability. For the vacuum:

$$\mu_0 = 4\pi.10^{-7} \frac{T.m}{A}$$

Development:

Developing the equation that shows the relation between magnetic moment and angular momentum:

$$\overline{\mu_1} = -g \frac{e}{2m_e} \vec{L} \qquad (5)$$

$$\Delta\vec{\mu} = -\frac{ge}{2m_e} \Delta\vec{L}$$

$$\sum \Delta\vec{\mu} = -\frac{ge}{2m_e} \sum \Delta\vec{L} \qquad \qquad \sum \Delta\vec{\mu} = \frac{ge}{2m_e} \Delta\vec{L}_{mac}$$

$$\Delta\vec{M} = \frac{ge}{2m_e} \Delta\vec{L}_{mac}$$

$$\chi_m \frac{\Delta\vec{B}}{\mu_0} = \frac{ge}{2m_e} \Delta\vec{L}_{mac} \qquad (11)$$

Analyzing this last equation, we can conclude that a variation in the magnetic field will provoke a variation in the macroscopic angular momentum.

Magnetic field:

Inside a coil, the magnetic field is proportional to the electrical current:

$$B = ki \qquad (12)$$

3. Experiment:

Materials:

□2 Coils	□Iron cylinder	□Thread	□Oscilloscope
□Source of current	□Steel cylinder	□Banana conectors	
□Amplificator	□Functions generator		□Mirror
□Holder□	□Multimeter	□Laser	Rampart

EXPERIMENT 1:

We plugged the source of current in the coil with the banana connectors. We moored the thread in the holder and in the cylinder, and we put the cylinder inside the coil. We switched on the source of current and start to increase the current. We could see the movement of the cylinder, when we reach 3.12 ampères. *(pictures a and b)*

Coil:

- Number of turns: 760

Iron cylinder:

- Length: (3.50 ± 0.05)cm
- Diameter: (1.30 ± 0.05)cm
- Mass: (26.60 ± 0.05)g

EXPERIMENT 2:

We repeated the proceeding of the experiment 1, but with a source of current that gives us it in a sinoidal function. At this time we glued a small mirror on the cylinder. We used a laser that we put between the cylinder and a rampart. When the laser incise on the mirror, we could measure the amplitude, and looking to a oscilloscope, that we also connected in our source of current, we could measured the period. We changed the frequency in our functions generator and we obtained the following data: *(pictures c and d)*

Time (s) *	2.Amplitude(cm)
(18.0 ± 0.2)	(29 ± 1)
(16.1 ± 0.2)	(39 ± 1)
(13.8 ± 0.2)	(72 ± 3)
(11.1 ± 0.1)	(40 ± 2)
(9.9 ± 0.2)	(18 ± 1)

* time for 5 oscillations

Distance (cylinder – rampart) = (70.0 ± 0.5)cm

R = 9.5 Ω

U = 1.3 V

Coil

- Number of turns: 1000

Steel cylinder

- Length: (3.70 ± 0.05)cm

□Diameter: (0.15 ± 0.05)cm
□Mass: (1.14 ± 0.05)g

Calculating the gyromagnetic ratio:

Developing the equation (11) we can arrive in an equation to calculate the *gyromagnetic ratio*:

$$\frac{\chi_m}{\mu_0}\frac{dB}{dt} = \frac{ge}{2m_e}\frac{dL_{mac}}{dt} \qquad \frac{\chi_m}{\mu_0}k\frac{di}{dt} = \frac{ge}{2m_e}\tau$$

$$\frac{\chi_m}{\mu_0}ki_{\max}\cos(\Omega t) = \frac{ge}{2m_e}\tau$$

$$\tau = \frac{\chi_m 2m_e ki_{\max}}{\mu_0 ge}\cos(\Omega t) \tag{13}$$

$$\tau = A\cos(\Omega t) \qquad \sum\tau = I\theta''(t) \tag{14}$$

$$A\cos(\Omega t) - k\theta(t) - P\theta'(t) = I\theta''(t)$$

$$\theta''(t) + \frac{P}{I}\theta'(t) + \omega^2\theta(t) = \frac{A}{I}\cos(\Omega t)$$

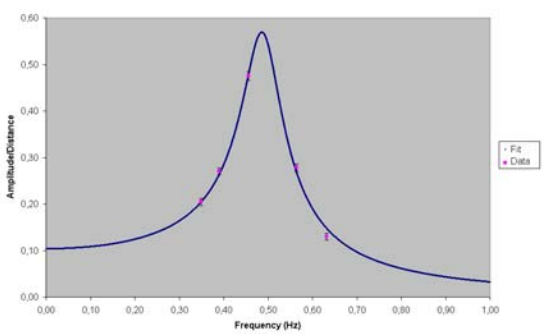
Solving this equation we have:

$$\theta(t) = \frac{A\cos(\Omega t + \Phi)}{I\sqrt{(\omega^2 - \Omega^2)^2 + P^2\Omega^2/I^2}} + \text{transient}$$

The frequency and the angle we could determine experimentally:

$$\Omega = \frac{2\pi}{T} \quad \text{and} \quad \tan\theta = \frac{ampl}{dist}$$

To determine the others parameters, we used a computer program that fit the parameters, so the distance of the curve to our points is the minimum possible. We obtained the following graphic:



After the software determined the parameters, we could arrive in a value for the *gyromagnetic factor*:

$$\frac{A}{I} = a$$

from the equation (13) we have:

$$Ia = \frac{\chi_m 2 m_e k i_{\max}}{\mu_0 g e} \qquad g = \frac{\chi_m 2 m_e k i_{\max}}{\mu_0 e I a} \tag{15}$$

Finally, we used our data:

- $a = 3.5$
- $I = 5.88\ 10^{-9}$ Kg.m²
- $e = 1.60\ 10^{-19}$ C
- $m_e = 9.11\ 10^{-31}$ Kg
- $i_{max} = 0.137$ A
- $\chi_m = 49$
- $k = 0.006$ T/A
- $\mu_o = 1.26\ 10^{-6}$ T.m/A

g = 17.7

Error Sources:

In our experiment, we can not affirm that the cylinder was in the middle of the coil, so the magnetic field was not constant.

Another possible error source are the perturbations on the system, because our system was not isolated of external perturbations, for example, air motion.

Finally, we have the influence of the Earth's magnetic field, that we could not minimize in our experiment.

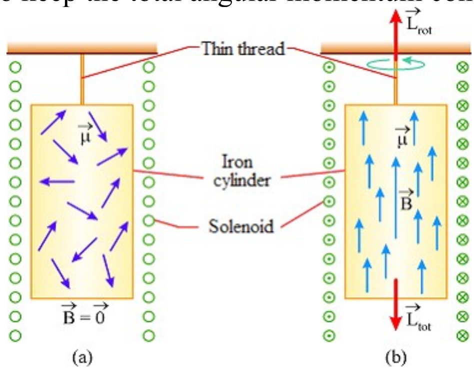
4. Analysis and Conclusions:

Firstly, we need to understand, why the cylinder begins to rotate when we apply a magnetic field on it.

Initially, the magnetic field is zero in the region where the cylinder is located, and the atomic magnetic moments are randomly oriented. The angular momentum have the same direction as the magnetic moment but pointed to the other side, due to this reason, there are also randomly oriented.

When we apply a vertical magnetic field on the cylinder, the atomic magnetic moments align on the direction of the magnetic field and consequently, the angular momentum align to the opposite side. So, the cylinder will have an angular momentum different of zero.

As there is no external torque acting on the cylinder, the angular momentum of it must be constant during the time. Because of the Conservation of the Angular Momentum, the cylinder begins to rotate to produce an angular momentum in order to keep the total angular momentum constant.

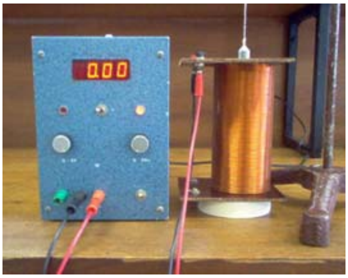


The rotation of the cylinder shows that the magnetic moment and the angular momentum are connected. Classically, the value predicted for the *gyromagnetic ratio* is 1. But doing the experiment, many researches found the numbers near to 2, and we also find a result different of 1. This results prove the theory about the existence of the *spin.*

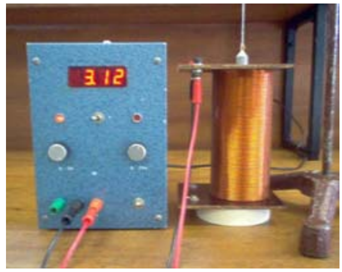
To conclude, we have a table showing the values for the *gyromagnetic ratio* calculated for different investigators, after extremely precise experiments:

Investigator	Year	g
BARNETT	1944	1.938 ± 0.006
MEYER	1951	1.936 ± 0.008
SCOTT	1951	1.927 ± 0.004
BARNET & KENNY	1952	1.929 ± 0.006
SCOTT	1955	1.919 ± 0.006
MEYER & BROWN	1957	1.932 ± 0.008
Scott (cylinder)	1960	1.917 ± 0.002
Scott (ellipsoid)	1960	1.919 ± 0.002

5. Pictures



Picture a = experiment 1



Picture b = experiment 1

Picture c= experiment 2

Picture d= experiment 2

6. Bibliography

- http://hyperphysics.phy-astr.gsu.edu/hbase/tables/magprop.html
- SCOTT,G.G.

Review of Gyromagnetic Ratio Experiments
- ATKINS, Peter e Julio de Paula.

Físico-química 7a edição, volume 2. LTC editora, 2004.
- FERRARO, Nicolau Gilberto; Caio S.V. Calçada; e Ronaldo Fogo.

Coleção Objetivo: 6- Eletricidade. Editora Sol, 2005.
- GALISON, P.

How experiments end. The University of Chicago Press, Chicago, USAlHALLIDAY, David; Robert Resnick; e Jearl Walker.Fundamentos de Física 6a edição, volumes 2 e 4. LTC editora, 2003.
- ROBORTELLA, José Luis de Campos; OLIVEIRA, Edson Ferreira de; ALVES FILHO, Avelino.

Física- Eletromagnetismo. Editora Ática, 1982
- TIPLER, Paul.

Física 4a edição, volumes 1, 2 e 3. LTC editora, 2000.