PROBLEM № 14: EINSTEIN–DE HAAS EXPERIMENT

7.2. SOLUTION OF BULGARIA (AMERICAN COLLEGE OF SOFIA)

Problem № 14: Einstein–de Haas Experiment
(Power Point Presentation of the National Team of Bulgaria)
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The problem:
When you apply a vertical magnetic field to a metallic cylinder suspended by a string it begins to rotate. Study this phenomenon.

Historical Background


The quoted works of Einstein and de Haas, all published or reported in Germany reveal the idea and the results of an experiment to prove the existence of the so called molecular currents of Ampere, which interpret the magnetic properties of materials.

Nature of magnetism of the elements

- Atoms of the elements contain electrons
- Electrons move in orbits and “spin” around their axises
- They are the currents in the atoms (molecular Ampere’s currents) => reason for magnetic properties of atoms
Electrons possess:
- Orbital magnetic moment and spin magnetic moment
- Orbital angular momentum and electron spin mechanical momentum

At the time Einstein performed his experiment only the orbital motion and the relevant magnetic moment were known. The modern concept of the atom involves orbital, spin magnetic moments and their interactions.

**Moments and Momenta**

**Orbital moments**
- orbital angular momentum \( L = l \hbar \)
- orbital magnetic moment \( \mu_l = -(e/2m).L = \gamma_l \cdot L \)

\[ \mu_l = \gamma_l \cdot L, \text{ where } \gamma_l = g_l \cdot \frac{e}{2m} \]

**Spin moments**
Spin mechanical moment
\( s = \frac{1}{2} \hbar \)
Spin magnetic moment
\[ \mu_s = -(e/2m) \cdot s \]
where \( \gamma_s = g_s \cdot \frac{e}{2m} = \frac{\mu_s}{s} = -(e/m) = 1.76 \times 10^{11} \text{C/kg} \)

**Mixed magnetism**
\( j = (l + m_s) \cdot \hbar \)
\[ \mu_j = \gamma_j \cdot j, \text{ where } \gamma_j = g_j \cdot \frac{e}{2m} \]
\[ g_j = 1 + j(j+1) + m_s (m_s +1) - l(l+1) - \frac{m_s (m_s +1)}{2j(j+1)} \]

The theory of atomic magnetism defines the mechanical momenta of the electron (orbital and spin) and its magnetic orbital and spin momenta. The ratio between the magnetic to the mechanical moment is known as the gyromagnetic ratio, \( g \). In the case of spin-orbital interaction this ratio is known as the Landé factor \( g_j \).

**Domains**
- Magnetic domains are regions in a crystal with different directions of the magnetizations
- Ferromagnets
Magneto-mechanical phenomena

- The sum of the magnetic moment vectors of the domains can be said to be the vector of the magnetic moment of the substance.
- Let $\mathbf{I}$ be the vector of magnetization, $V$ is the volume of the body

$$\mathbf{I} \cdot V = S\mu_d$$

- Let $Q$ be the total mechanical momentum of the domains

$$Q = \sum L_d = \chi I V = \chi \cdot \sum \mu_d$$

Magneto-mechanical phenomena

- When the body is not magnetized $\mathbf{I} = \mathbf{0} \Rightarrow Q = \mathbf{0}$
- When the body is magnetized $\mathbf{I}$ is no more $\mathbf{0}$
- Then according to the formula $Q = \chi I V$, $Q$ also changes
- According to the law of conservation of mechanical momentum

$$Q_{\text{tot}} = Q_D + Q_B$$

- In the beginning the sum of the mechanical momenta of the domains is $\mathbf{0}$ and the body is not moving $\Rightarrow$ the total momentum is $\mathbf{0}$
- $Q_D \neq \mathbf{0} \Rightarrow \mu B \neq \mathbf{0}$
- So we must observe the spinning of the body

In fact in Einstein-de Haas experiment the orientation and reorientation of domains, which are effects on a larger than molecular scale have been detected. The change of the magnetic moment of the domains causes a macro magneto-mechanical effect, which results in the rotation of the body.
### Experiment Comparison

<table>
<thead>
<tr>
<th>Einstein's experiment</th>
<th>Our experiment</th>
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<tbody>
<tr>
<td><img src="image1" alt="Einstein's experiment" /></td>
<td><img src="image2" alt="Our experiment" /></td>
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</tbody>
</table>

*Picture from: Albert Einstein-selected scientific works, "Nauka", Moscow 1966*

### Experiment Setting

![Experiment Setting Diagram](image3)

### Laser

**Semiconductor laser**

1.5 euro from the market
• Wire
  • Wire made of Tungsten with
  • thickness of 15 mm
  • Connected to a reel
Frequency – measured
• Torsion balance

Sample
• The cylinder
• Nail (Fe)
• Two parts of brass (Cu and Zn) up and down
• Weight 7,052 g
• Length 7,44 cm
• Diameter 4mm
• Mirror – aluminum foil

The brass parts above and below the iron nail possess non-ferromagnetic properties. In our experiment they serve to damp the parasitic libration motion, which occurs at magnetization of the iron part.

Solenoids
• Two solenoids connected in series
• 12000 turns each
• Magnetic field of about 20 G is created
• Alternative current of 6 mA

**Generator**
Generator of sinusoidal vibrations with changing frequency
Creates resonance to strengthen the effect
Beating effect, when slightly different frequency is set

**Experiment**

\[
\gamma = \frac{\omega J}{I.V}
\]

**Video**
To run the videos that show the experiment performance, the observed oscillations and the beating effect, see [www.acs.bg](http://www.acs.bg). Look in the link “student life”.

\[
\gamma = \frac{I.V.T_e}{\varphi J_s \Lambda}
\]

The major factor calculated is the gyromagnetic ratio of the suspended body in our experiment.
Parameters

I – vector of magnetization, its table value is

I = 1,59155.106 A/m

V – volume of the sample (only ferromagnetic part)

\[ V = \pi r^2 h = 0,9349.10^{-6} m^3 \]

\[ \gamma = \frac{I.V.T_e}{\phi.J_s.\Lambda} \]

- \( r \) – radius of the cylinder \( r = 0.002 \) m
- \( h \) – length of the cylinder \( h = 0.0744 \) m

\( \phi \) – angle of declination, experimentally found:

\[ \phi \approx 180^\circ = 3,14 rad \]

- \( T_e \) – experimental time for measuring the angle of declination

\[ T_e = \frac{1}{f} = 1.43 s \]

\[ \gamma = \frac{I.V.T_e}{\phi.J_s.\Lambda} \]

- \( f \) – frequency of the torsion balance \( f = 0.07 \)Hz

\( J_s \) – moment of inertia of the sample

- for cylinder:

\[ J_s = \frac{1}{2} m_i \cdot r_i^2 \]

for our sample:

\[ J_s = \frac{1}{2} m_b \cdot r_b^2 + \frac{1}{2} m_f \cdot r_f^2 = 1,7883.10^{-6} kg.m^2 \]

\[ \gamma = \frac{I.V.T_e}{\phi.J_s.\Lambda} \]

\( m_b \) and \( r_b \) – mass and radius of the brass parts

\( m_b = 0,003359 \) kg and \( r_b = 0,0015 \) m

\( m_f \) and \( r_f \) – mass and radius of the ferromagnetic part

\( m_f = 0,007052 \) kg and \( r_f = 0,002 \) m

\( \Lambda \) – decrement of decrease

\[ \Lambda = \ln \frac{\phi_b}{\phi_{n+1}} = \ln \frac{180}{179.5} = 0.0028 \]
\[ \gamma = \frac{IVT}{\phi J_s \Lambda} \]

\( \phi_n \) and \( \phi_{n+1} \) are two consecutive angles of declination. The difference is about 0.5 degrees.

Final Calculation

\[ \gamma = \frac{1.59155 \times 10^6 \cdot 0.9349 \cdot 10^{-6} \cdot 1.43}{3.14 \cdot 1.7883 \cdot 10^{-3} \cdot 0.0028} = 0.1353298 \times 10^{11} \]

This will be true if:

\[ g \approx 0.15 \]

Modern experiments performed with precision much higher than that of Einstein and de Haas give a value of about 2 for the g factor. Our result differs substantially from it.

Sources of error

- The torsion balance is not centered
- Brass part – Barnett’s effect
- Earth’s magnetic field
- Noises and vibrations
- Domains that do not get remagnetized
- Eddie currents

Conclusions

- Einstein – de Haas experiment was successfully demonstrated
- The observed magneto-mechanical effect was predicted and explained using quantum mechanics concepts

References

- Albert Einstein – Selected Scientific Works, "Nauka", Moscow 1966
- Experimenteller Nachweis der Amperschen Molekularstromen, Naturwissenschaft, 1915,3,237-238
- Electricity, S.G.Kalashnikov, "Nauka", Moscow, 1977