7.4. SOLUTION OF CZECH REPUBLIC

Problem № 14: Einstein–de Haas experiment

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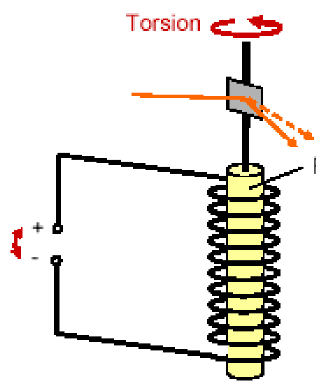
The problem

When you apply a vertical magnetic field to a metallic cylinder suspended by a string, it begins to rotate. Study this phenomenon.

Einstein–de Haas experiment was performed in 1915 with an iron cylinder and should clarify the cause of magnetism in ferromagnetic materials, results of their experiment were more than surprising, as the result was two times bigger than they'd expected. This 'inaccuracy' have been clarified several decades later.

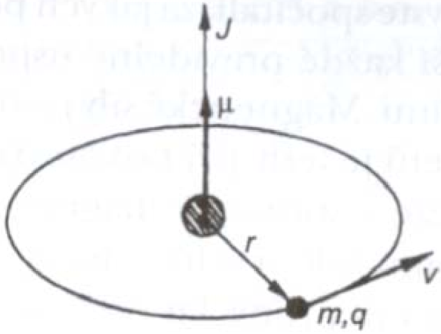
As the ferromagnetic material they used an iron cylinder hung on a thin thread. Around the cylinder is wound a coil. At the beginning, there are no moments of outer forces acting on the cylinder. Once a electric current goes through the coil, a magnetic field **B** is formed inside the coil. Due to this magnetic field, angular momentum of every single atom inside changes its direction so the total angular momentum is now not equal to zero. To preserve the total angular momentum (and to equal to the primal angular momentum, which was equal to zero), the cylinder starts rotating around its axis. If no thread was used, the cylinder would rotate as long as the magnetic field would be applied. Due to the torsion of the thread, a torsional momentum is formed (depending on the torsion modulus of the thread). The torsional momentum makes the cylinder stops rotating and makes it start rotating in a opposite direction (and thread gets straight). The thread will twist and straighten as the cylinder rotates with harmonic motion around its equilibrium like a torsional pendulum.



A mechanic analogy also exists: one stands on a rotatable pad and holds a horizontally revolving wheel (balance-wheel). When he turns the balance-wheel upside down, he starts revolving himself on the pad to preserve the total angular momentum.

First statement of classical mechanics says that if an electron moves along a circle (e.g. circulates around core due to centripetal force), its magnetic momentum and angular momentum are proportional.

Angular momentum **J** is defined as $\mathbf{J} = \mathbf{m}r \times \mathbf{v}$, where ***m*** is mass of electron; r is the distance from electron to the core and **v** is the velocity of



electron. Direction of angular momentum of the electron is perpendicular on the plane of trajectory. Radius r is perpendicular on the vector of velocity **v**, so the equation $\mathbf{J} = \mathbf{m}r \times \mathbf{v}$ can be written as:

$$J = mvr$$

Magnetic momentum μ of the electron circulating around the core is equal to $\boldsymbol{\mu} = I \cdot \mathbf{A}$, where **I** is the electric current and **A** the area inside the trajectory of the electron. Electric current is defined as charge which goes in time through any place at the trajectory, i.e. charge q times frequency of circular motion $I = q \cdot f$. Frequency is velocity divided by the length of trajectory $f = \frac{v}{2\pi \cdot r}$, so:

$$I = \frac{qv}{2\pi \cdot r}$$

The area is $A = \pi \cdot r^2$, magnetic momentum is then:

$$\mu = \frac{qvr}{2}$$

Magnetic momentum is perpendicular on the plane of trajectory, just like the angular momentum, their direction is the same, then:

$$\frac{\mu}{J} = \frac{\frac{qvr}{2}}{mvr} = \frac{q}{2m} \Rightarrow \boldsymbol{\mu} = \frac{q}{2m}\mathbf{J} \quad \text{(Orbital movement)}$$

Ratio of magnetic momentum and angular momentum is called the gyromagnetic ratio. This ration depends neither on velocity nor on the radius of trajectory. Magnetic momentum μ of every particle on the circular track is $(q/2m)$-multiple of angular momentum J. The electron's charge is negative (we'll call it $-q_e$):

$$\boldsymbol{\mu} = -\frac{q_e}{2m}\mathbf{J} \quad \text{(Orbital movement of the electron)}$$

This equation is valid also in the quantum mechanics. But we know that the orbital movement isn't the only causer of magnetism. Electron has also a spin (it's

like Earth and its rotation around its own axis) and like a consequence, there is also a spin angular momentum and related magnetic momentum.
From the quantum mechanics, we know that, the spin angular momentum of an electron is equal to:

$$J_{Spin} = \frac{n \cdot h}{4 \cdot \pi}$$

And spin angular momentum is:

$$\mu_{Spin} = \frac{n \cdot e \cdot h}{4 \cdot \pi \cdot m}$$

After substitution into the gyromagnetic ration:

$$\frac{\mu_{Spin}}{J_{Spin}} = \frac{4 \cdot \pi \cdot n \cdot e \cdot h}{4 \cdot \pi \cdot n \cdot m \cdot h} = \frac{e}{m}$$

Then:

$$\mu = -\frac{q_e}{m} J \quad \text{(electron spin)}$$

Generally, in every atom exist many of electrons and by compounding their spin and orbital movements the total angular momentum and the total magnetic momentum is formed. In spite of lack of classical mechanics explanation, in quantum mechanics is generally valid a statement that the direction of the magnetic momentum of an isolated atom is exactly opposite than the direction of angular momentum. Their ration don't have to be $-q_e / m$ or $-q_e / 2m$, though, but can be somewhere in between these values because the magnetic momentum is compound of orbital and spin portions. We can write their ration as:

$$\boldsymbol{\mu} = -g\left(\frac{q_e}{2m}\right)\mathbf{J}$$

Where g, so-called Landé Faktor, is a factor characterizing the state of atom. It's either equal to one for solely orbital movement, to two for solely spin movement and to any other number between one and two for a complicated system like an atom is. Landé factor is a non-dimensional constant. From the equation $\mu = -g\left(\frac{q_e}{2m}\right)J$ results that the magnetic momentum is parallel with the angular momentum, the size can be different though, depending on the Landé factor.
Magnetic momentum of an electron is in equation with the magnetization of the cylinder:

$$\mu = M \cdot V / N$$

Where M is magnetization, V is volume of the cylinder and N is the number of particles inside the cylinder.

Total angular momentum of the cylinder is:

$$J_{celk} = \sum_{i=1}^{n} m_i v_i r_i$$

$$J_{celk} = N \cdot J$$

After substitution μ and J:

$$M \cdot V = -g\left(\frac{q_e}{2m}\right) J_{celk}$$

In Einstein – de Haas experiment, magnetization angular momentum change within time because the system oscillates. Then, the time change of these quantities is observed:

$$\dot{M} \cdot V = -g\left(\frac{q_e}{2m}\right) D \wedge D = \dot{J}_{celk}$$

$$\Leftrightarrow g = -2\frac{m}{e} \cdot V \cdot \frac{\dot{M}(t)}{D(t)}$$

Experiment of Einstein and de Haas showed g-factor 2. So ferromagnetism is based on spin of the electron and not on the orbital angular momentum.
In Einstein – de Haas experiment, the cylinder behaves like a torsional pendulum, whose period of oscillation is:

$$T = \sqrt{\frac{2 \cdot \pi \cdot l_V \cdot m_T \cdot r_T^2}{r_V^4 \cdot \mu_V}}$$

Where l_V is the length of thread, m_T is the weight of cylinder, r_T is the radius of cylinder, r_V is the radius of thread and μ_V is torsion modulus of thread.
From the equation is obvious that period doesn't depend on current, voltage or on the used coil. If coil creates a bigger magnetic field, cylinder will revolve faster, on the other hand, it will deflect more, so the period won't change at all.