PROBLEM № 14: EINSTEIN–DE HAAS EXPERIMENT

7.5. SOLUTION OF POLAND

Problem № 14: Einstein–de Haas Experiment

Team members: Tomasz Bobiński, Pawel Debski (Captain), Maciej Lisicki, Krzysztof Wojtowicz, Maciej Zielenkiewicz
*Team leaders**:** MSc. Stanislaw Lipinski, Karolina Kocko, Michal Oszmaniec*
XIV Stanislaw Staszic Secondary School, Warsaw,

The problem

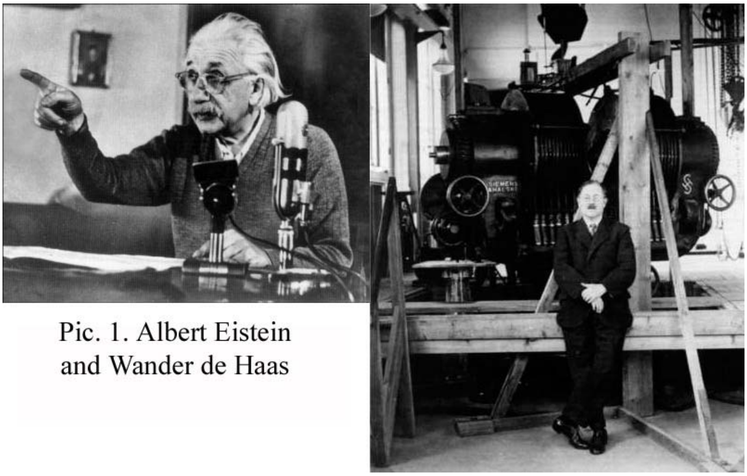
When you apply a vertical magnetic field to a metallic cylinder suspended by a string, it begins to rotate. Study this phenomenon.

The described phenomenon was firstly investigated in 1915, when Albert Einstein, searching for a proof of existence of Ampere molecular currents, conducted along with Wander de Haas, an experiment, which would confirm his theory.

Through the late years of the XIX cent. the scientific world was searching for the reason of magnetic properties of certain materials. In 1908 Richardson was the first one to mention that orbital motion of electrons is responsible for such behaviour of magnetic materials in presence of external magnetic field.

The described phenomenon was firstly investigated in 1915, when Albert Einstein (pic.1) , searching for a proof of existence of Ampere molecular currents, conducted along with Wander de Haas, an experiment, which would confirm this theory. The results of the experiment confirmed the theoretical predictions very well. Einstein published their work and did not put more attention on the analysis of the effect.

However, in later years many experimentators (including W.J. de Haas) tried to repeat this experiment, but their results were completely different. This uncertainty of the ultimate, correct result caused a discussion on Einstein's work and in further research,



Pic. 1. Albert Eistein and Wander de Haas

lead to discovery of the electron spin as partially responsible for magnetic behaviour of materials.

Theoretical explanation

At first, let's try to analyze a simple mechanical situation, in which we shall observe the basic effect which is in fact responsible also for Einstein – de Haas effect.

Let's consider a man sitting on a revolving chair, which is initially not rotating. A man holds a rotating bicycle wheel is such way, that its angular momentum vector is directed vertically (pic. 2). What happens, when we change the direction of the angular momentum vector?

Pic. 2

Obviously, as a consequence of the angular momentum conservation principle, the angular momentum vector of the whole system is constant, so the revolving chair starts to rotate if we change the angular momentum vector direction (pic. 3).

Pic. 3

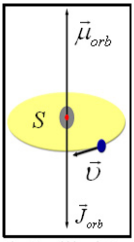
We shall now consider situation, in which we can observe the Einstein – de Haas effect. At first, let's analyse orbital motion of an electron (pic. 4)

Orbiting electron can be considered as a small current in a loop. Such a current has a magnetic moment:

$$\mu_{orb} = IS = \nu e \cdot \pi r^2 = \frac{e\upsilon\pi r^2}{2\pi r} = \frac{e\upsilon r}{2}$$

Magnetic moment is a vector, and its direction can be noted as:

$$\vec{\mu}_{orb} = \frac{e\left(\vec{\upsilon} \times \vec{r}\right)}{2}$$



Pic. 4 An orbiting electron (according to Bohr model)

An orbiting electron has also an angular momentum vector, which can be noted as:

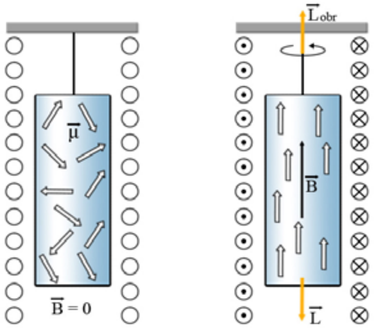
$$\vec{J}_{orb} = m\left(\vec{r} \times \vec{v}\right)$$

So the magnetic moment vector, which of course changes its direction in presence of external magnetic field, is strictly connected with the angular momentum vector of the atom. The relation is as follows:

$$\vec{\mu}_{orb} = -\frac{e}{2m}\vec{J}_{orb}$$

Basing on this relation, we can qualitatively explain the occurring effect. Let's describe it using a drawing (pic.5 and 6).

Initially, because of thermal motion and absence of external magnetic fields,



Pic. 5.6. Magnetic moment vectors in a probe.

the magnetic moment vectors of the atoms are randomly directed in the whole probe, so the resultant magnetic moment, as well as the resultant angular momentum, equals 0. When we apply a vertical magnetic field, the magnetic moment vectors tend to line up with the magnetic field lines, so they change their direction, so the resultant magnetic moment vector is directed vertically. We shall note, that this change is also connected with changes in angular momentum vectors, which then change their direction. The angular momentum conservation principle demands the resultant angular momentum vector to be 0, so the probe gains angular momentum directed oppositely to the resultant angular momentum vector of the atoms. This angular momentum causes the rotation of the probe.

We shall remember, that this is only a qualitative explanation. After an electron spin has been discovered, an important lacks in the experimental results could be explained. It occurred, that an electron has its own, internal angular momentum and magnetic moment, but the relation between those vectors is as follows:

$$\vec{\mu}_S = -\frac{e}{m}\vec{J}_S$$

We shall note that the proportionality factor between those vectors is 2 times bigger than when considering an orbiting electron.

This discovery revealed new facts connected with theoretical explanation of the behaviour of magnetic materials. It caused a discussion about the exact reason of the magnetic behaviour. Caused questions, like: what is more important in this phenomenon – orbital motion or electron spin? To solve this problem, precise quantitative experiments had to be conducted.

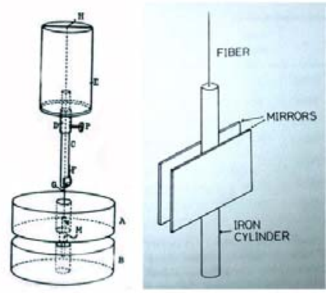
The idea was to measure the proportionality constant, called the g-factor or Lande factor:

$$\vec{\mu}_{metal} = -g\left(\frac{e}{2m}\right)\vec{J}_{metal}$$

If the g-factor value was closer to 1, it would confirm the hypothesis, that crucial for this effect are the orbiting electrons, if it was closer to 2, it would prove, that elector spin has the key role for this phenomenon.

Experimental Approach

We wanted to use the same method, which Einstein and de Haas described in their work in 1915. Their idea was to apply an external field and measure the deflection of an iron cylinder. Their setup consisted of coils, generating the magnetic field and a cylinder suspended on a glass fibre, with a mirror attached to it (pic. 6 and 7).



Pic. 7.8. Einstein's original scheme of experimental setup

The cylinder can be treated as a torsional pendulum – using a resonance phenomenon, we can amplify the oscillations. Therefore, the frequency of current in the coils, generating the magnetic field equals the natural frequency of oscillations of the cylinder, the oscillations will be amplified and easier to observe.

To measure the deflection we have used a laser and a screen.

To calculate the value of the g-factor, we have tried to use Einstein's method, but to make the measurements easier, we have modified it, as did the Berlin Technical University students. The formula for calculating g, given with no derivation, is as follows:

$$g = \frac{m_e}{q} \cdot \frac{U_{ind} \cdot V}{N_2 \cdot \mu_0 \cdot A \cdot X \cdot \omega \cdot \beta \cdot \alpha_{\max}}$$

m_e – electron mass
q – electron charge
U_{ind} – inducted voltage
μ_0 – magnetic premeability of vacuum
ω – resonant angular frequency
X – moment of inertia
β – damping constant

Pic. 11 First experimental setup.

So we had to measure various parameters of the whole system.

We have constructed our first experimental setup (pic. 11), hoping to measure necessary parameters. As it occurred, it was very imprecise and allowed us only to estimate g as about 0.002.

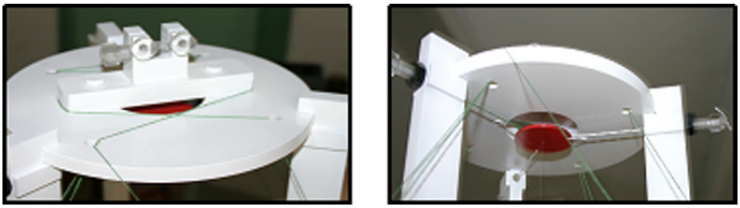
These wooden constructions, which can be seen in this photography are Helmholtz coils, used to compensate the nonvertical components of earth magnetic field.

Pic. 12 Second experimental setup.

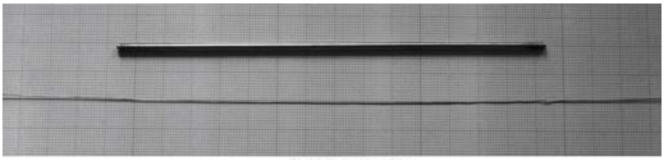
We have decided to build much more precise experimental setup, which would allow us to conduct measurements (pic.12).

This time the setup was very stable, symmetric, with special devices used to center the cylinder inside the coil and regulate the tension of the suspension (pic.13).

Our cylinder was long (150 mm) and very thin (2.4 mm), to avoid any nonuniformities of the magnetic field inside the coil. It was made of ARMCO (contained 99% of iron) We have suspended it on a CuNi wire (with diameter of 0.2 mm) (pic. 14).

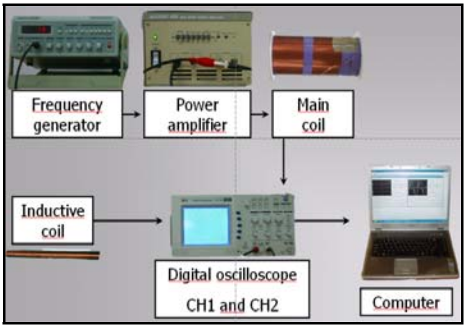


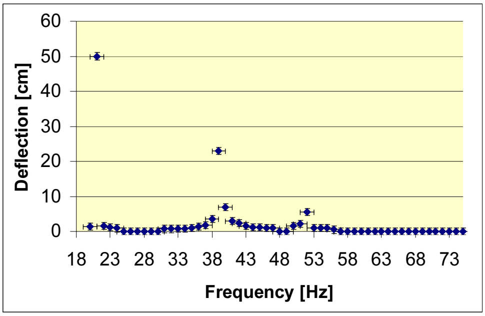
Pic. 13 String tension controller and centering device



Pic. 14. CuNi wire and iron cylinder

To generate the external magnetic field, we have used a frequency generator, connected to a power amplifier and a digital oscilloscope. The signals generated by presence of an iron cylinder inside the coil were gathered by an inductive coil, and this signal was also analyzed using a computer.

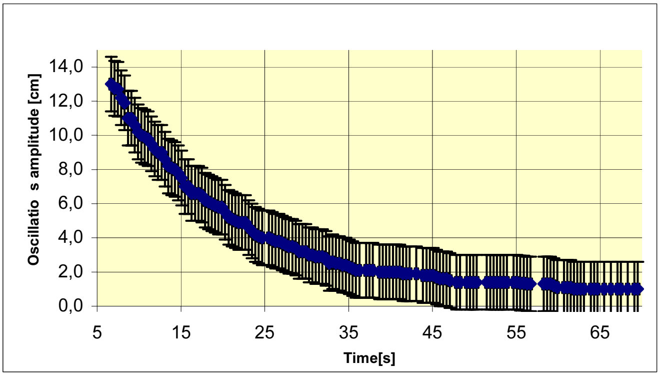


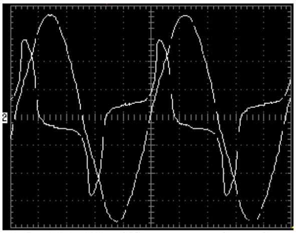


First of our measurements was the estimation of the resonant frequency of the cylinder. Obtained results are presented in the above graph:

We have measured the deflection on the screen while changing the external magnetic field change frequency (our generator). Three harmonics can be clearly visible. We therefore assume the lowest and strongest one, at about 19 Hz, to be the resonant frequency of the cylinder.

Another important parameter was the damping constant. We could measure it by turning off the generator at the resonant frequency and measuring the deflection decrease in time. Then, using GNUPlot, we have fitted a curve, showing the expected dependence, as it is shown below:





Pic. 15 Oscilloscope signals

The expected dependence was:

$$f(t) = Ae^{-\beta t}$$

After estimating curve-fitting parameters, we could estimate the damping constant as:

$$\beta = 0.060 \pm 0.005$$

The inducted voltage was measured using the digital oscilloscope, as it can be seen in the illustration (pic. 15).

The two signals in this screen are signals from main coil (sinusoidal) and from the inductive coil (in this signals, peaks mark the demagnetisation of the cylinder, as the current in main coil changes its direction).

$$\begin{aligned}
&m = 9.1 \cdot 10^{-31}\ \mathrm{kg}\\
&q = 1.6 \cdot 10^{-19}\ \mathrm{C}\\
&\mu_0 = 1.257 \cdot 10^{-6}\ \mathrm{T^2m^3J^{-1}}\\
&V = 6.79 \cdot 10^{-7}\ \mathrm{m^3}\\
&U_{ind} = 2.2\ \mathrm{V}\\
&N_2 = 600\\
&A = 4.52 \cdot 10^{-6}\ \mathrm{m^2}\\
&X = 3.744 \cdot 10^{-9}\ \mathrm{kg{\cdot}m^2}\\
&\omega = 131.95\ \mathrm{Hz}\\
&\beta = 0.06\\
&\alpha_{\max} = 5.22 \cdot 10^{-2}\ \mathrm{rad}
\end{aligned}$$

$$\begin{aligned}
&\frac{\Delta V}{V} \approx 16.5\%\\
&\frac{\Delta X}{X} \approx 16\%\\
&\frac{\Delta\omega}{\omega} = \frac{\Delta f}{f} \approx 5\%\\
&\frac{\Delta\alpha_{\max}}{\alpha_{\max}} \approx 14\%\\
&\frac{\Delta\beta}{\beta} \approx 8.4\%\\
&\frac{\Delta U}{U} \approx 4.5\%
\end{aligned}$$

After conducting necessary measurements, we were ready to estimate the value of g and analyse possible sources of error in our measurements.

The total error of our measurements was therefore:

$$\sqrt{\left(\frac{\Delta V}{V}\right)^2 + \left(\frac{\Delta X}{X}\right)^2 + \left(\frac{\Delta\omega}{\omega}\right)^2 + \left(\frac{\Delta\alpha_{\max}}{\alpha_{\max}}\right)^2 + \left(\frac{\Delta\beta}{\beta}\right)^2 + \left(\frac{\Delta U}{U}\right)^2} = 0.296$$

The calculated value of g:

$$\boxed{g = \frac{m_e V U_{ind}}{q N \mu_0 A X \alpha_{\max} \omega \beta}} \qquad \boxed{g = 1.61 \pm 0.48}$$

Therefore we can draw some conclusions on this effect:

1. Einstein – de Haas effect is connected with atoms and electron having angular momentum and magnetic moment. In fact, professional laboratory measurements estimate g as about 1.8, so it proves that electron spin and magnetic moments have bigger influence on the behaviour of magnetic materials in presence of external field.
2. Change in direction of magnetic moments vectors causes change in angular momentum of the probe; rotation is the effect of the angular momentum conservation principle.
3. This effect has a wide historical background and had many implications on the concept of magnetism of matter (from molecular currents to spin of the electron).
4. Although quantitative analysis is very hard, but it is possible to do it in school conditions. In our case, we've obtained value, which is completely sufficient, even surprisingly close to professionally measured value.

Acknowledgement:

We'd PhD. Piotr Kossacki from Warsaw University for valuable discussion discussion and theoretical help. We'd like to thank Mr Krzysztof Bobiński from Warsaw Univeristy of Technology, for help in experimental setup construction and PhD. Jan Grabski fromWarsaw Univeristy of Technology, for advice, theoretical support and providing us with electronic equipment. We'd also like to thank MSc. Anna Mazurkiewicz from Jozef Poniatowski Secondary School for help in search for theoretical materials.