SOLUTION OF HUNGARY

Problem № 16: Obstacle in a funnel

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The problem

Granular material is flowing out of a vessel through a funnel. Investigate if it is possible to increase the outflow rate by putting an 'obstacle' above the outlet pipe.

Introduction

Granular materials are more common than one would guess; let us just think about agriculture, building-trade or plastic industry. For handling so many times with these in some aspects extraordinary materials, people have thoroughly studied their properties in order to find a more efficient way of storage and usage.

Background

A granular material consists of many macroscopical (i.e. above 10 µm) particles. In this range of dimension there are three main forces acting inside the system: the gravity force, the repulsive force between touching particles and the friction force between particles at contact points.

Probably the most interesting property of granular systems is that if applying stress onto the aggregation, above a certain threshold of stress the particles may jam up. The cause of this jamming is that particles form so called force chains in compressional dimensions. These chains can be modeled as linear strings of rigid particles in point contact. Chains only support mass along their own axis so they are strictly collinear. They end on the walls of the container (if there is any, in other cases they end on the bottom of the aggregation), thus there is a significant pressure on the wall and/or on the bottom. The force applied to the granules is mediated to the walls by the force chains. If the force in a chain is too large or its direction changes, then the force chain is broken. This can happen when the granular material is stirred or moved, and afterwards a network of new force chains is formed.

We investigated the phenomenon referred to experimentally. For our measurements glass funnels of three different sizes were used (diameters 0.6 cm, 0.8 cm and 1.2 cm). As granular materials we chose sand, semolina, lead-balls (diameters 0.25 cm and 0.5 cm) and plastic 'balls' of an irregular shape. And finally, we used a wide range of obstacles. The funnels were attached to an

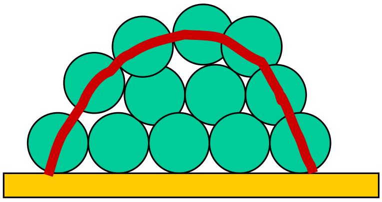

Fig. 0 Simplified conception of a force chain building up in an aggregation

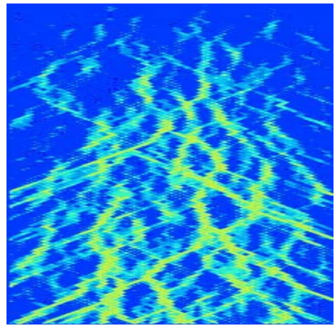

Fig. 2 Experimental observation of force chains building up in a granular system (Ref. [1])

eprouvette-stand. Material of given volume (125 cm^3) was poured into the funnel (while the outlet pipe was kept closed)

Fig. 3 Granules, funnels and obstacles used

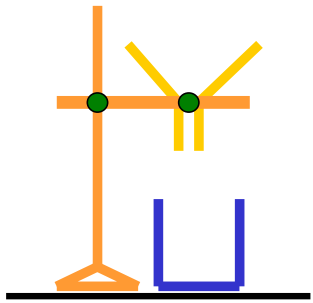

Fig. 4 Schematic figure of the way of attachment

As our first experiment, we measured the flow time of each material in the funnels without an obstacle (t_o). Afterwards as further experiments we measured the flow rate by placing an obstacle inside the funnel (t). We varied some parameters: shape of the obstacle (blunt (dull), peaked and spherulitic), diameter of the obstacle (0.3 cm, 0.8 cm, 1.2 cm) and the distance of the obstacle from the mouth of the funnel (h).

We assumed namely, that following parameters may be relevant:

- the size of the particles (d)
- the shape of the particles
- the material of the funnel – may influence friction
- the material of the obstacle – may also influence friction
- the diameter of the obstacle (d_{obs})
- the diameter of the funnel mouth (Φ)
- the distance between the obstacle and the mouth of the funnel (h)

Our observations Three different cases were analyzed. In the first case the obstacle acted really as an obstacle: the flow was hindered. In the second case the obstacle surprisingly increased the flow rate – this was the main point of our study. We also had cases in which the obstacle had no influence on the flow rate at all.

During experiments we noticed the fact that the outflow time reducing effect only appears if $d_{obs} > 3d$, so we had to take this into account in our investigations. This means that we did not go through all measurements in cases like pouring big lead-balls (d = 0.5 cm) into a medium-sized (Φ = 0.8 cm) funnel.

1. Firstly, *concerning the obstacle*s see Table 1.

Obstacle	Average flow time (s)
None	3.8
Blunt (all sizes)	3.8
Peaked (0.8 cm)	3.4
Peaked (1.2 cm)	3.6
Spherulitic	*3.26*

Table 1 Comparison for one material (small lead-ball, d = 0.25cm) in a given funnel (Φ = 1.2 cm) at given mouth-obstacle distance (h = 1 cm)

As it is clearly noticeable, the obstacle with a spherulitic end had the largest influence on the flow rate.

2. It is also important to observe, that there is a *certain region* ($\Delta h = h_{max} - h_{min}$) in which the time of outflow is reduced. This value depends strongly on the granules investigated.

	h_{min} (cm)	h_{max} (cm)	Δh (cm)
small lead-balls	1	2.3	1.3
big lead-balls	1	2	1
Semolina	0	1.6	1.6

Table 2 The region (Δh) in which reduced time of outflow can be found in case of spherulitic obstacle in a funnel (Φ = 1.2 cm)

3. Let us now investigate what happens if we change the *mouth-obstacle distance*!

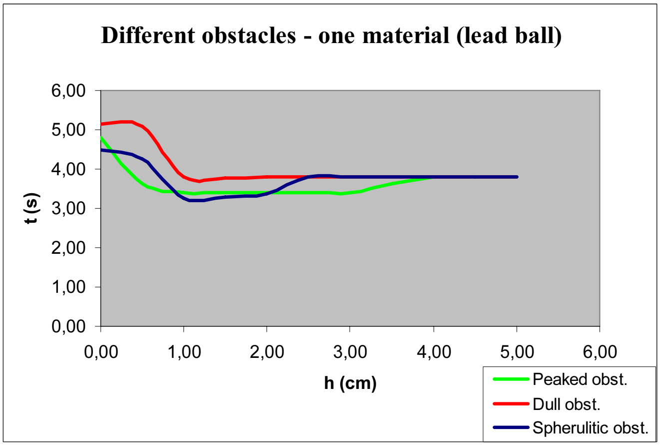

Fig. 5 Comparison of the obstacles by changing the mouth-obstacle distance (material was given)

As one can see there is a well-defined height at which the reduced time of outflow reaches its maximal value. It is also very remarkable that the previously recognized schema (see figure 5) remained: the spherulitic obstacle had the largest outflow time reducing influence on the granules, followed by the peaked obstacle and finally the blunt one. However the peaked obstacle is special in some way: the reducing effect appears even at relatively small mouth-obstacle distances. This may be explained with its conical shape

Explanation

As already told, force chains building up in a granular aggregation end on the walls and bottom of the vessel (in our case there is no 'bottom' of the vessel, for we investigate funnels).This is not the case if there is some kind of obstacle inside the system: the obstacle hinders the formation of the 'basic' chains, a different force chain network forms where some chains will end on the obstacle itself. This means of course a smaller compression of the bottom particles, which allows them an easier motion or flow in optimal cases (see figure 8). However, what do we mean by optimal case? By optimal case we mean that if placing the obstacle too near the mouth, it will really act like a physical obstacle, and the rate of outflow is reduced. The other extreme case is when we place our obstacle too far from the mouth of the funnel. In this case force chains build up even in regions below the obstacle, so the compression on the bottom particles becomes significant again and jamming appears.

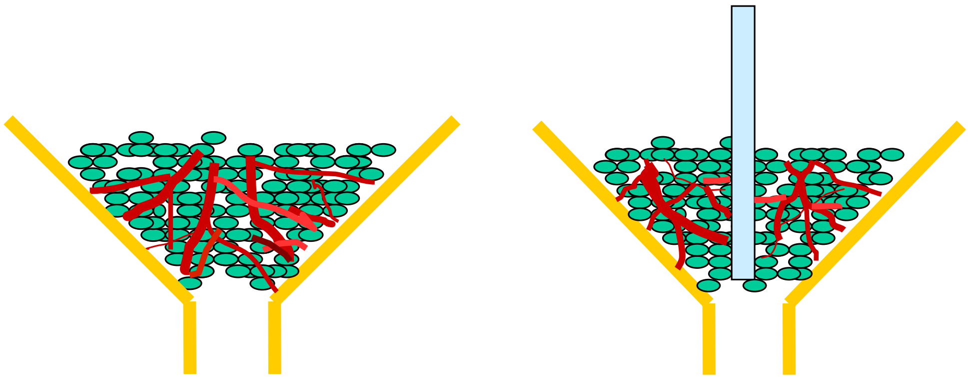

Fig. 6 The schematic diagram of a granular system in a funnel without an obstacle and with an obstacle. The red lines indicate force chains

The model

Probably the most difficult part of our investigation was to set up an appropriate model. Granular systems are namely far more complicated than non-granular ones, and thus the physical description is very complex.
But before introducing our model, let us make a detour to another, in some aspects surprisingly similar phenomenon, called pedestrian escape panic. That means, if a room is crowded with people and somehow they are forced to leave the place (for example if flames come up for some reason), just like granular particles, people may jam up at the exits. However it is a known fact that columns placed near the gates help people to get out. This observation is very similar to our original topic. That means a crowd can be modeled as a self-driven many-particle system.

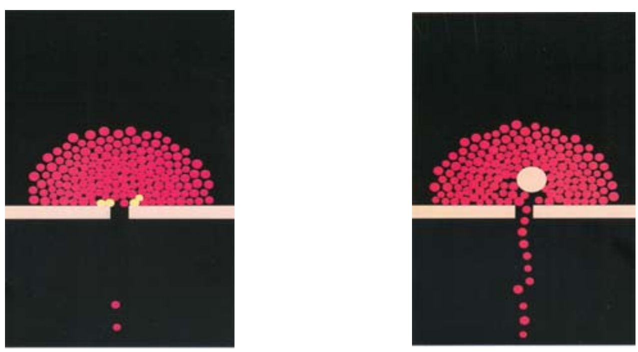

Fig. 7 Influence of a column placed in front of an exit (Ref. [4])

Escape panic can be correctly modeled. In Ref. [4] the following equation is suggested for the description:

$$m_i \cdot \vec{a}_i = m_i \cdot \frac{d\,\vec{v}_i}{dt} = m_i \frac{v_0 \cdot \vec{e}_i - \vec{v}_i}{\tau} = \sum_j \vec{F}_{ij}^{rad} + \sum_j \vec{F}_{ij}^{tg}$$

where **a** stands for the acceleration of the particles, $\mathbf{v_o}$ for the desired speed, $\mathbf{e_i}$ the desired direction, $\mathbf{v_i}$ the adapted actual velocity divided by a certain characteristic time τ. The interaction forces shown in the equation mean the radial (F^{rad}) and the tangential (F^{tg}) components of the force rising between two particles (see figure 10).

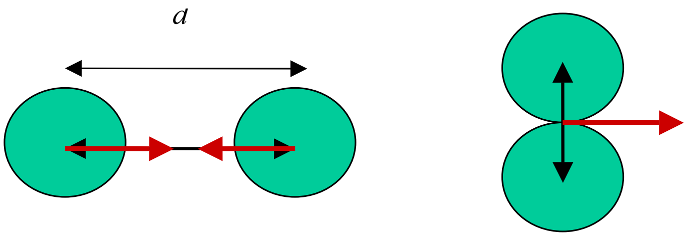

Fig. 8 Radial and tangential interaction force between two pedestrians

We applied this model to our problem, as well.

Although the main idea was the same, there are some remarkable differences between the two models. Let us now compare granular flows and panicking crowds.

The main difference is that granular flows are always accelerated by gravity instead of varying factors like in crowds of people. It is also extremely important to emphasize that while the tangential force (F^{tg}, friction force) is proportional to the force compressing the surfaces in both systems, the radial force cannot be given by such an idea. Namely the repulsive force rising for keeping off of each other is a long-range interaction ($d > r_1+r_2$) which does not exist in granular systems. Additionally, the repulsive force rising when colliding is a short-range interaction ($d = r_1+r_2$) which does not appear in a panicking crowd.

Despite these remarks the difference between the granular system model and the escape panic model (Ref. [4]) is relatively small, so most of the calculations and simulations made for an escaping crowd are also valid for granular systems investigated here.

Finally, we got the following equation as our mathematical model for the problem:

$$m_i \cdot \vec{a}_i = m_i \cdot \frac{d\,\vec{v}_i}{dt} = m_i \frac{v_0 \cdot \vec{e}_i - \vec{v}_i}{\tau} = \sum_j \vec{F}_{ij}^{rad} + \sum_j \vec{F}_{ij}^{tg} \underbrace{\left(+m_i \cdot \vec{g}\right)}_{gravity}$$

Conclusion

As you could see the movement of granular materials differs from the motion of liquids as well as the moving of single particles, and this was what made our job hard for it is very difficult to describe this extraordinary movement quantitatively. However with the help of another model we could draw our own one, as well.
Finally, to answer the basic question, yes, we clearly found that it is possible to increase the outflow rate by using a kind of obstacle in the funnel. The cause of this phenomenon is that the obstacle influences the building up of force chains in our granules the stability of which thus decreases drastically.

References:

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