Problem № 17: Ocean Solaris

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Problem

A transparent vessel is half-filled with saturated salt water solution and then fresh water is added with caution. A distinct boundary between these liquids is formed. Investigate its behaviour if the lower liquid is heated.

As you see we have some conditions which we have to create in laboratory conditions to observe some specific and unexpected behaviour of the 'distinct boundary'. Experiments made by our team in our laboratory conditions gave such results:

While heating process, the boundary was rising and at the same time, the waves that occur in it disturbing the surface of the boundary. Let's explain the cause of this phenomenon.

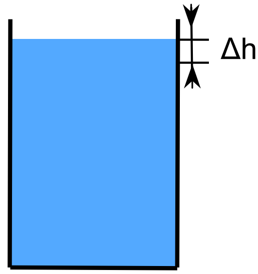
The causes the waves to occur are the convection flows create as the heating process begun. These convection flows exist because of temperature gradient, inside the salt solution. But there's one problem: these waves are impossible to be described using any method. This is because of the place of the convection flow pushes the boundary and other parameters are almost random, or factors that define these parameters are random, anyway they are impossible to be described in our model.

So let's look at another effect – rising of the boundary's height. Here everything is a bit easier. Here is the list of effects we consider in order to describe this raising:

1. Thermal expansion
2. Surface tension
3. Bubbles of gas
4. Convection flows

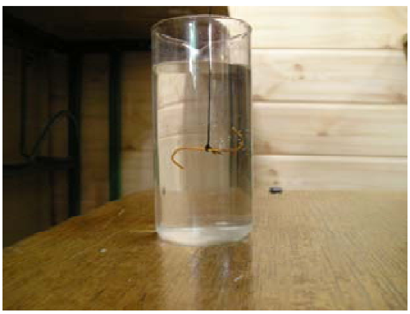
Thermal expansion

Thermal expansion is an effect of changing of the volume of the body (in our case it's salt solution) with increasing of its temperature. Using some approximations we got a result of changing height (Δh) of the boundary due to thermal expansion about 1mm. The same value was on experiment – about 2mm



Surface tension

Surface tension is not very important in our conditions, because of surface tension coefficient is very small. We have made both theoretical and experimental research on this question and had as a result surface tension coefficient is about $0.01N/m^2$.Comparing with water this coefficient is very small.



We used method of wire detachment for measuring surface tension coefficient (and this is the equipment we used fir this).

Bubbles of gas

Bubbles of gas have big influence on height of the boundary only when the liquid is near the boiling point. In this case big bubbles that can form in the case of narrow vessels like a piston that pushes water out. In wider vessels this effect is not worth considering (in our work we used quite wide one). So we consider case

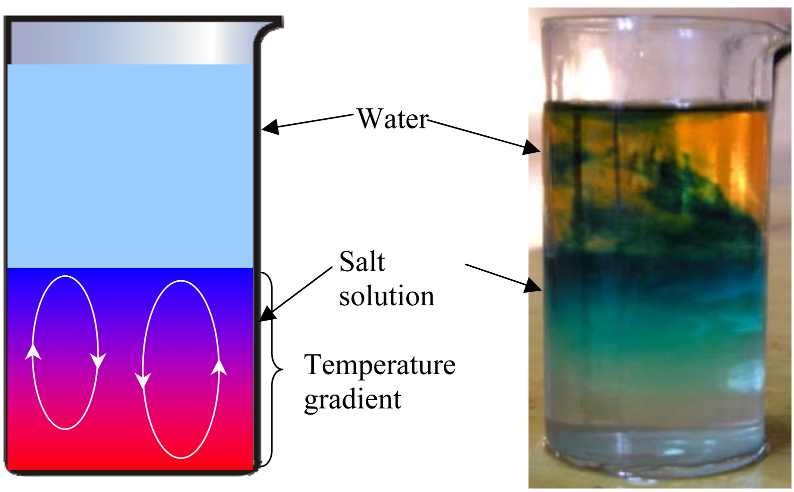
when bubbles of gas have no strong influence on behaviour of the boundary.

Convection flows

In our opinion convection flows have the biggest influence on the raising of the boundary. But this is right only when the conditions for the convection flow to occur are fulfilled, These conditions are: existence of the temperature gradient between different layers of liquid(in our case it is salt solution) and vessel shouldn't be very narrow(in this case bubbles and thermal expansion are important).Let's consider wide vessel(its cross section parameter is comparable with its height) and try to calculate some characteristics of the convection flow(this would be its speed).This will be done by dimension considerations:

Here's the list of values on which speed of the convection flow depends

- ✓ ΔT-temperature difference between upper and lower layers of the salt solution
- ✓ η-dynamic viscosity of the salt solution
- ✓ ρ-density if the salt solution
- ✓ β-thermal expansion coefficient



White arrows-scheme
of the convection flows

Here is the formula for the speed of the convection flow depending on the parameters, mentioned below:

$$V = \sqrt[3]{\frac{\eta \cdot g}{\rho}} \cdot f(\beta \cdot \Delta T),$$

where $f(\beta \cdot \Delta T)$ – dimensionless function, which we approximate with linear function, because $\beta \cdot \Delta T$ is quite small (for real values of β and ΔT, their product is also <<1), so in expansion into a Tailors series, we can neglect all terms, which power is more than 1. This means that $f(\beta \cdot \Delta T) \approx f(0) + A \cdot \beta \cdot \Delta T$, where A- is some coefficient and $f(0) = 0$ – because if the temperature difference is zero convection flows doesn't occur.

At last we get $f(\beta \cdot \Delta T) \approx A \cdot \beta \cdot \Delta T$.

Next step. Consider a small volume of salt solution, moving up from the bottom. Its density is smaller than surrounding liquid. Writing the Second Newton's law for this small volume, we can get formula for its acceleration when it gets the boundary:

$$a = \frac{\Delta \rho}{\rho} \cdot g ,$$

where $\Delta \rho$ is the density difference between water and salt solution, it is negative. Then knowing acceleration and initial speed we can calculate the height on which this small volume can "jump" above the boundary, and thus, thus the whole boundary (because there are a lot of convection flows at the same time). This looks this way:

$$h = \sqrt[3]{\frac{\eta^2 \cdot \rho}{g`}} \cdot \frac{(A \cdot \beta \cdot \Delta T)^2}{2\Delta \rho}`$$

This formula isn't very precise, because we neglect the changing of viscosity, while liquid flows from salt water to fresh, and the velocity of the flow is estimated by the dimensions considerations.

Except of these problems, we have to remind that during the time all parameters are changing.

Also we have to remember that in truth to say – relevant parameters (such as density, temperature, viscosity) are continuously changing from point to point, so we have to find distribution of these values. This problem is very difficult from mathematical point of view.

So we preferred the other way. We made an experiment, where the boundary height measured on time, and using the results selected the possible dependencies for relevant parameters on time. Than we made experiments with other initial density difference (different salt concentrations) and determined that the found dependencies work.

We got the next empirical formula:

$$h(t) = \sqrt[3]{\frac{\eta^2 \cdot \rho}{g}} \cdot \frac{(A \cdot \beta \cdot \psi \cdot t)^2}{2 \cdot (\Delta \rho_0 - \xi \cdot \sqrt{t})}`$$

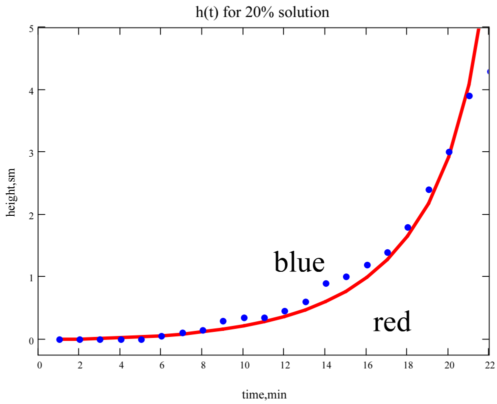
Coefficients ψ, ξ, A - are different for each kind of vessel and depend on the power of heating and quantity of fresh and salt water in the vessel. So we made experiments taking these requirements to regard, while stating previous formula.

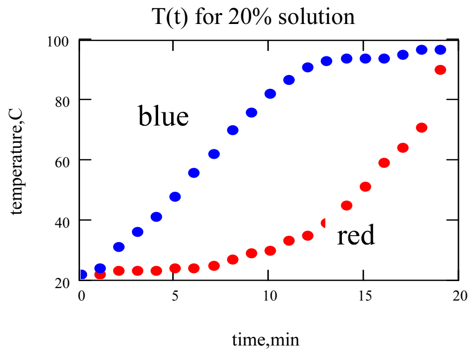
Here are the graphs we got using this formula and comparison between them and our experimental results.

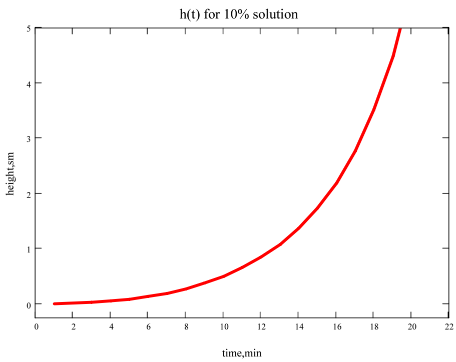
All graphs given for three different concentrations of salt in the solution-20%, 10%, 5%(the highest concentration that can be gained is 25% (for temperature $20\ ^{0}C$))

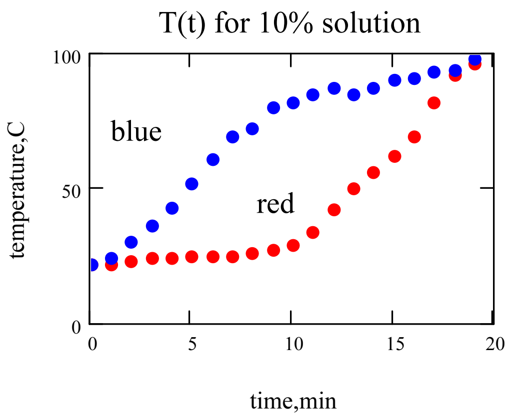
On T(t) graphs blue points-temperature of the water, red – of the salt solution

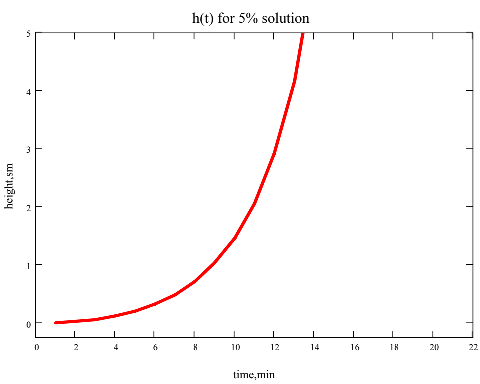
On h(t) for 20% solution blue points – our experimental results. The height is measured from initial position.

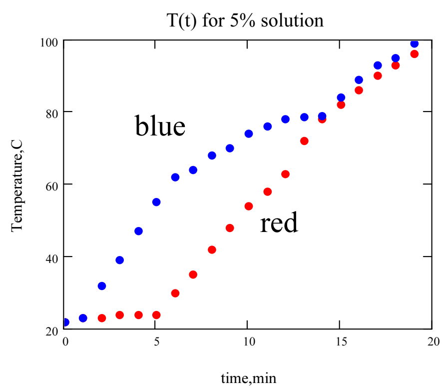




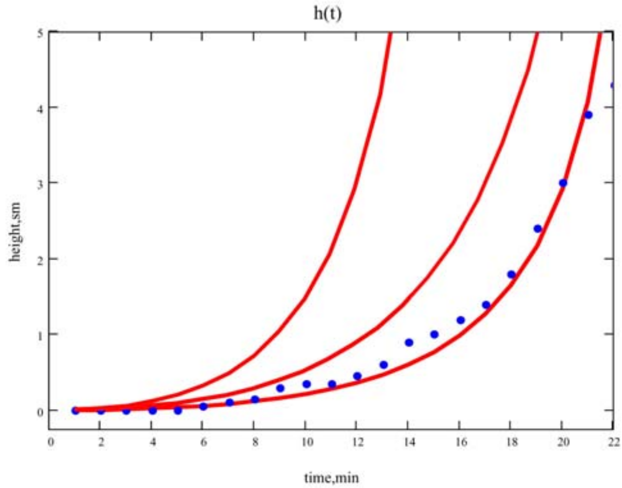








And here three graphs h(t) for three concentrations on one plot



As you see the less concentration, the bigger the acceleration of the boundary. Why you will ask. Because of density difference rely on acceleration of the boundary-the less concentration, the easier for convection to get to the water upwards, and further.

If you look at the graphs T(t) you see that at some moment temperature of the liquids becomes equal, this means that temperature at any point inside of the vessel is equal, convection stopped, but it is possible for the limit to exist at such conditions, if we heat all system very slowly. At this temperature boiling begins, so here bubbles begin to play more important part.

So we've made some theoretical research about the behaviour of the boundary between water and saturated salt solution when heating this system.

And we made experiments to compare them with our theoretical research. As you can see these results (I mean graphs h(t) and T(t)) in experiments and theory are very near and can be comparable.

Acknowledgement

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