

IV. Solutions of the problems for the 19th IYPT

1. Problem №1: Froth

Solution of New Zealand

Problem №1: Froth

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The Problem:

Investigate the nature of the decay in height of the 'froth' or 'foam' on a liquid. Under what conditions does the froth remain for the longest time?

Abstract

The statement of the problem pertained to the investigation of the decay of froth with respect to time. In order that such an investigation could be fully explored it was felt that a definition of froth itself would not be fully satisfactory. As such, a brief explanation of the structure of froth was deemed necessary to explain how and why froth decays in height over time.

It was found that froth both changes in structure and decays in height over time. The latter formed the basis of the solution since it provided readily measurable quantities. The decay in froth height is well documented and most sources suggest that froth decays exponentially. It was found that this was true of the surface of the froth with respect to some reference point, but the same could not be said of the entire froth body, on the surface of a liquid. Froth contains residual fluid and over time this fluid drains from the froth structure. As a result the dimensions of the body of froth have two variables: the surface height of the liquid and the height of the froth. It was felt that the "nature of the decay in height of froth or foam on a liquid" could only be fully investigated by examining both of these variables. The model suggested combines both the mechanisms referred to and thus suggests that the decay in froth height is not simply exponential. Unfortunately, due to the complexity of bubble formation and development with time, the model suggested relies on constants found practically. Practical work was completed with relative ease in comparison to other problems and it was found that a narrow vessel and an optimum temperature of approximately 7°C produced foam which remained for the longest time.

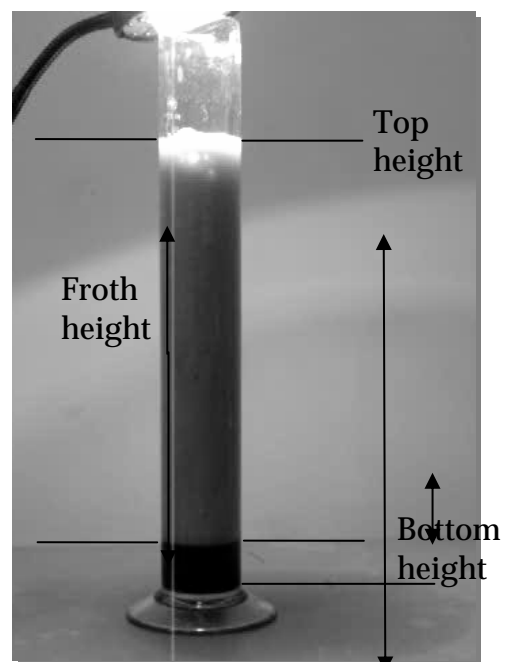
Definitions

Height - difference between the *top height* and the *bottom height* - froth height.

Froth or foam - A *colloidal*, liquid gas system. High gas volume, composed of a high number of gas bubbles in a liquid matrix.

Colloid - Liquid continuous medium, gaseous dispersed medium.

Conditions - parameters that affect the decay of froth.



Interpretation

Establish the conditions upon which the phenomenon depends. Set these conditions such that the decay of froth or foam is the slowest.

Initial Observations

At the top of the froth:

- Lower liquid content.
- Liquid drains out.
- Foam quickly coarsens.
- Bubbles size increases before rupturing.

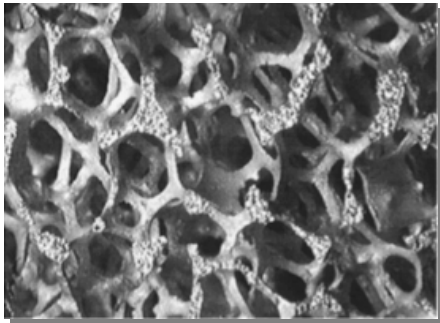
At the bottom of the froth:

- High liquid content.
- Liquid drains out.

How is froth generated?

- Gases are less soluble at higher temperatures.
- Henry's law - concentration of a dissolved gas is proportional to the partial pressure of that gas in equilibrium with that liquid.
- When a carbonated beverage is opened - sudden drop in pressure and an increase in temperature - CO₂ is supersaturated.

The structure of froth



Aqueous foams are connected by Plateau borders.

Liquid-filled channels between three adjacent bubble

Single Plateau border

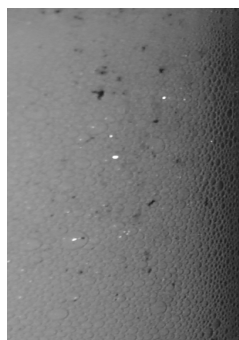
Froth drainage

Liquid drains from the froth under gravity.

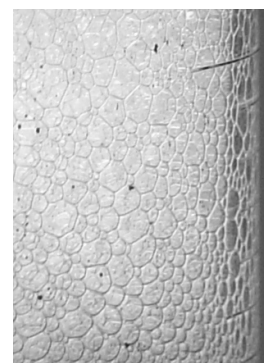
Foam structure changes from a round to hexagonal shape. (This is also as a result of diffusion).

Due to liquid draining under gravity and diffusion ...

emulsion of circular facets bubbles meeting surrounded by liquid. plateau borders.



hexagonal lattice with with an angle of 120° at



Diffusion

- Diffusion occurs as a result of the Second Law of Thermodynamics.
- Pressure of 193 kPa needed for 2.6 volumes of CO₂.
- Migrate to atmosphere at 0.22 kPa until thermodynamic equilibrium reached.
- CO₂ - bi-polar gas freely diffuses through bubble membranes. N₂ does not.
- Bubble size increases - their shape becomes hexagonal - membranes eventually rupture.
- Process of froth coarsening.

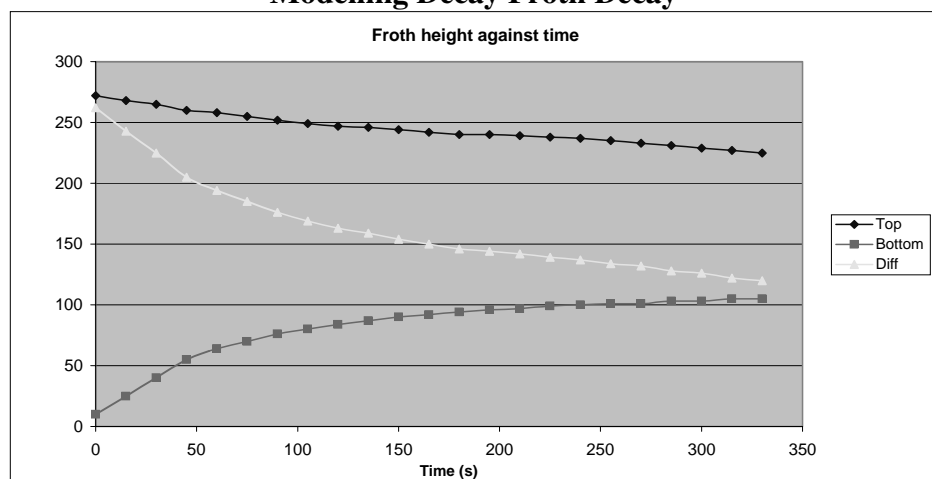
Froth coarsening

- Individual cells continually evolve so as to reduce the total interfacial area in the system - *coarsening* or growth of the characteristic bubble size.
- Occurs by two processes:
 - coalescence*
 - disproportionation*.
- *Coalescence* - merging of two bubbles which occurs when the film between them ruptures
- *Disproportionation* - described by the Young-Laplace equation for the differential pressures between the inside and outside of a bubble due to surface tension.
- Pressure is inversely proportional to the bubble radius.
- CO₂ diffuses from smaller bubbles where pressure is higher into interconnected larger bubbles.

Froth decay

- Foam structure coarsens and larger bubbles continue to expand.
- Membranes thin until they reach a critical thickness.
- Film ruptures are spontaneous.
- Collapse occurs at the crown surface.
- Rupture or diffusion of dipolar CO₂ directly to the atmosphere through the CO₂ permeable bubble film.

Modelling Decay Froth Decay



Froth height - non exponential relationship

Top height decay

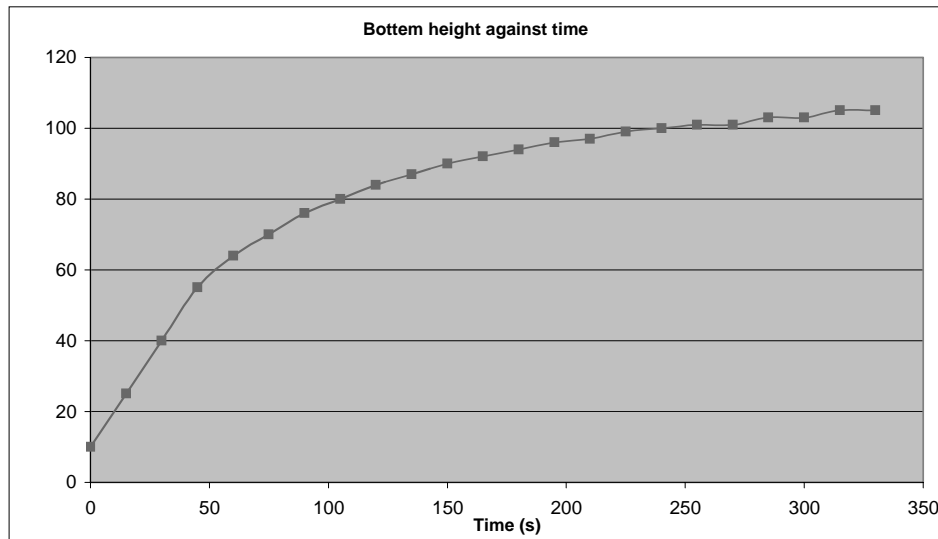
- Recall that the top decays largely due to diffusion.
- Diffusion is a spontaneous natural process.
- Thus the rate of decay is proportional to the foam height at time t:

$$\frac{dh}{dt} = -\lambda h$$

- Height therefore follows the exponential law:

$$H(t) = H_0 e^{-\lambda t}$$

Bottom height



- Bottom height modelled by:

$$h(t) = -e^{\text{int}} e^{-\lambda t} + h(\text{max})$$

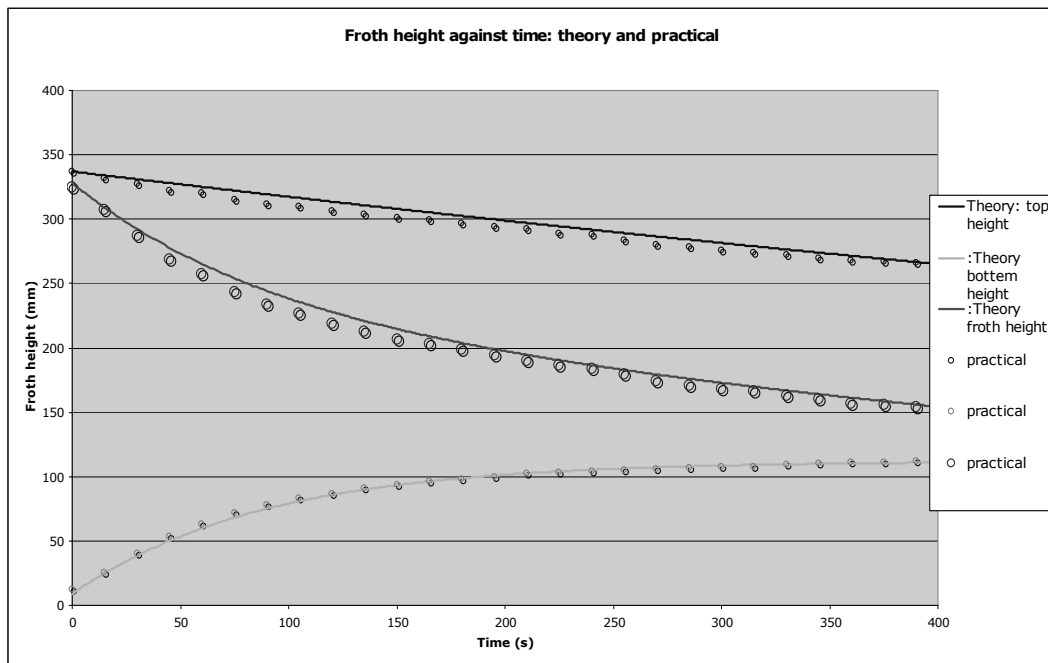
Froth height decay

- Froth height modelled by:
- λ_1 , λ_2 and int must be found experimentally.

$$F(t) = H_0 e^{-\lambda_1 t} + e^{\text{int}} e^{-\lambda_2 t} - h(\text{max})$$

-
- int is the intercept from a graph of the natural logarithm of height (derived from the bottom height) against time.

Initial comparison between practical results and theoretical functions.



Parameters

1. Temperature
2. Vessel diameter
3. Flow rate
4. Vessel angle
5. Gas composition
6. Surface tension

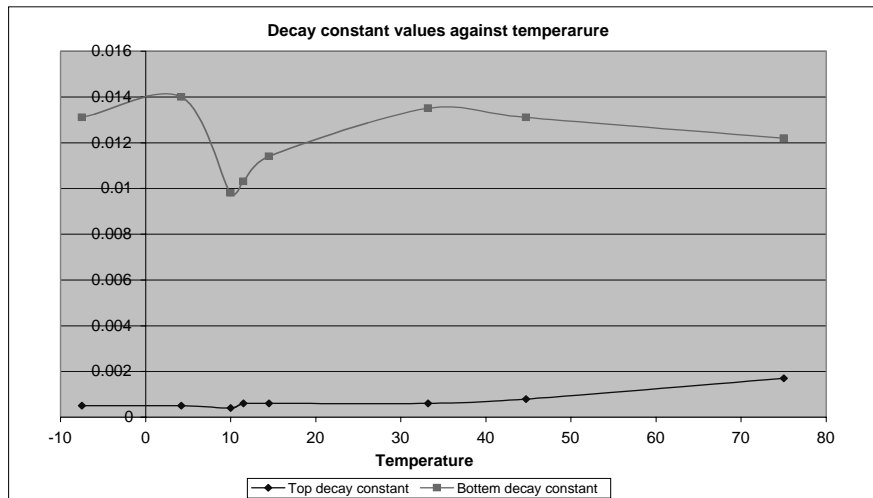
Other considerations

- Improperly cleaned vessel may alter the surface tension, especially if a detergent is involved.
- Particulates, imperfections in the vessel and other surfaces (e.g. ice) act as nucleation sites for the production of CO_2 .
-

PRACTICAL

- Froth was generated using Coke and two varieties of beer: Steinlager and Rhineck.
- Measuring cylinder 5.5 cm in diameter and 45.0 cm tall was used.
- Masking tape was attached along it's length.
- Top height and bottom height were marked off every 15 seconds for beer, every 10 seconds for Coke.

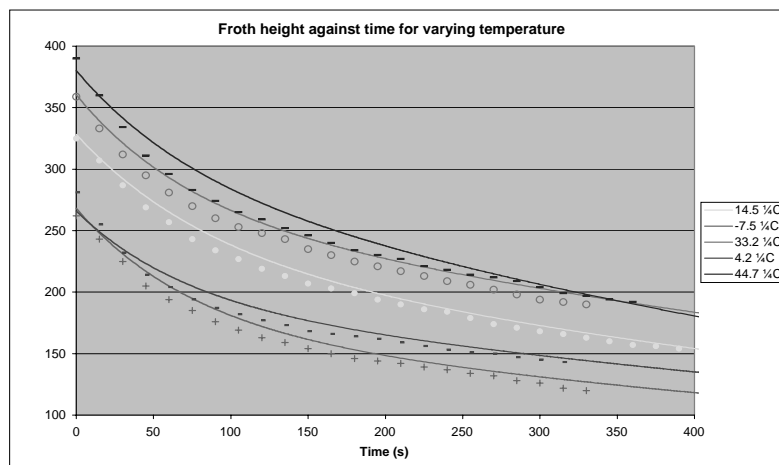
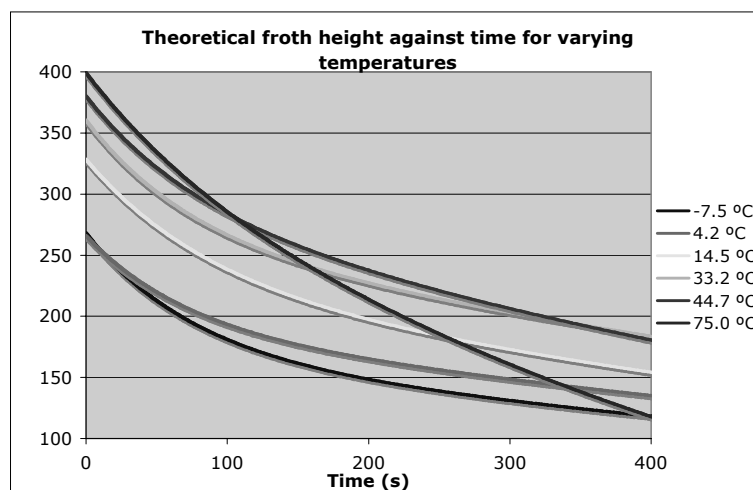
Parameter 1: Temperature

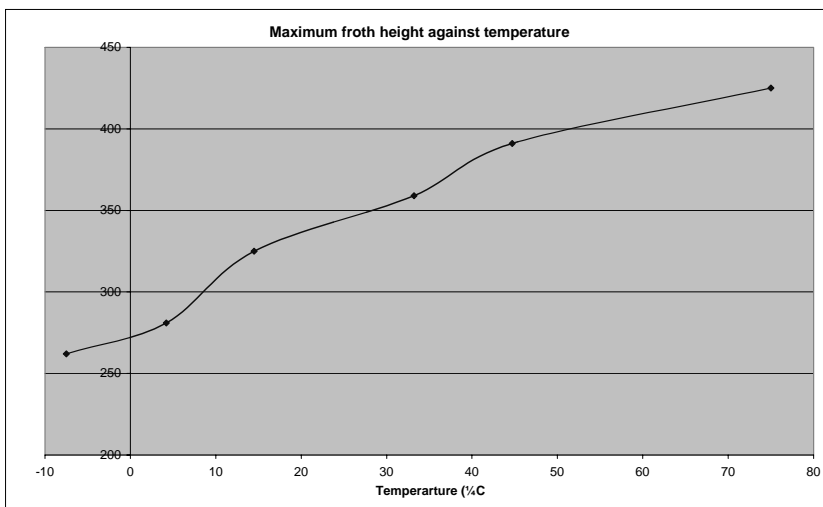
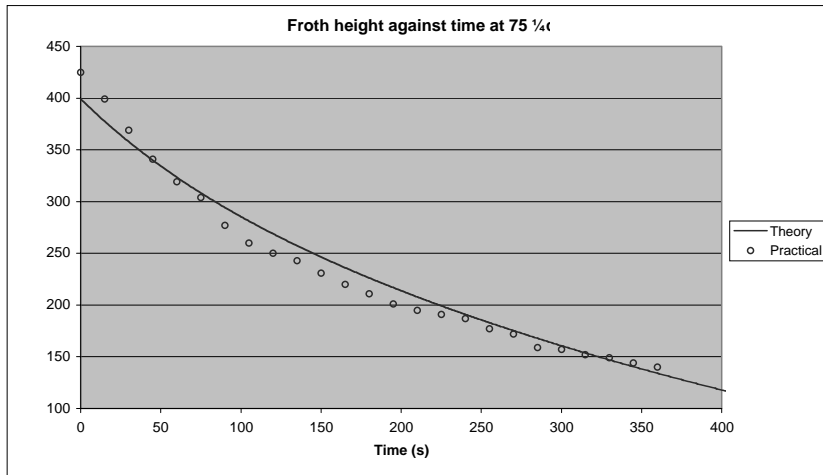


Initial practical conducted to determine the variation of decay constants with temperature.

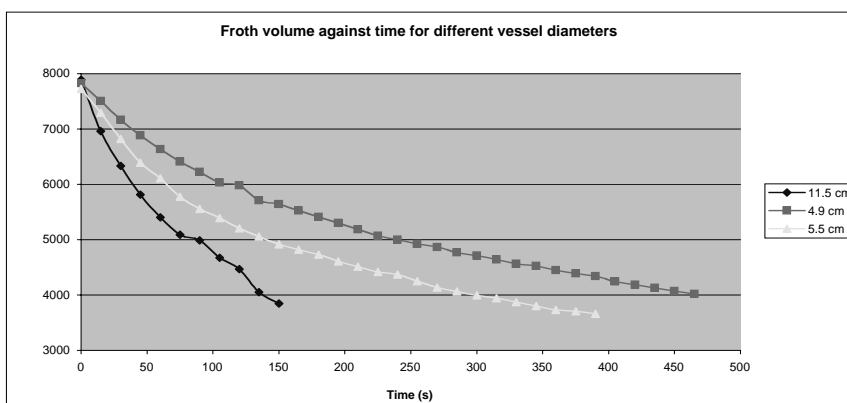
Top - diffusion.

Bottom - viscosity and Plateau border size.



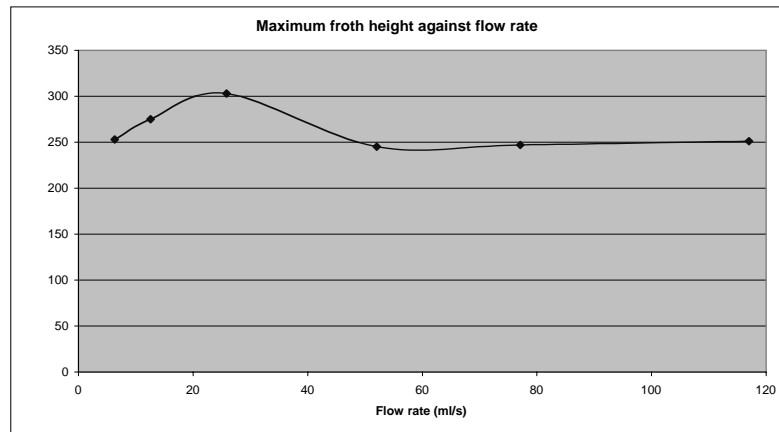


Parameter 2: Vessel Diameter



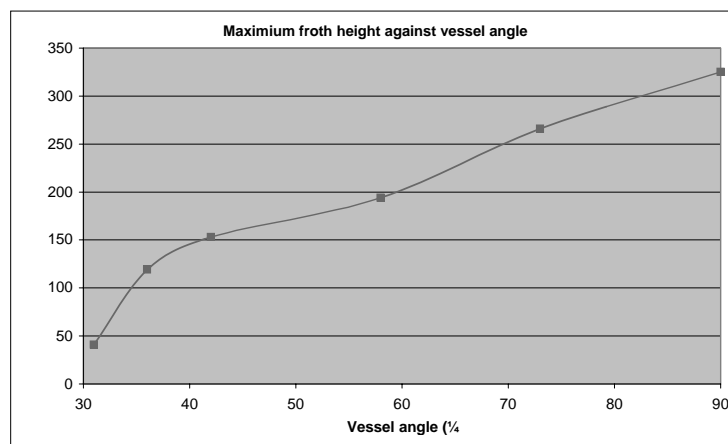
Froth volumes are compared. Decay constant increases with increasing vessel diameter – surface tension and smaller surface area to volume ratio.

Parameter 3: Flow Rate



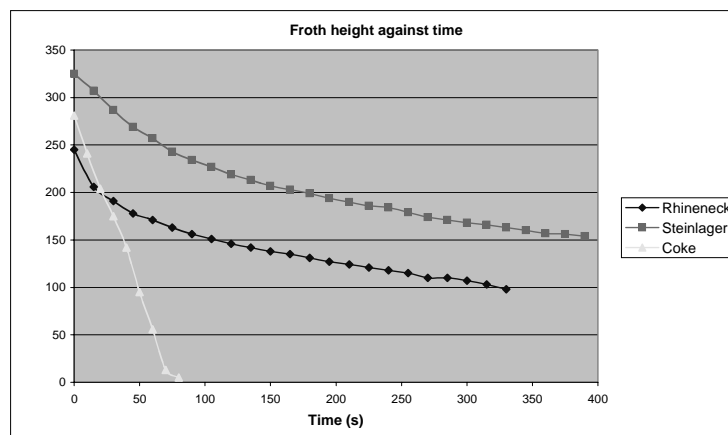
A flow rate of about 25 mL/s gives the highest froth height. Decay constants are unaffected.

Parameter 4: Vessel Angle

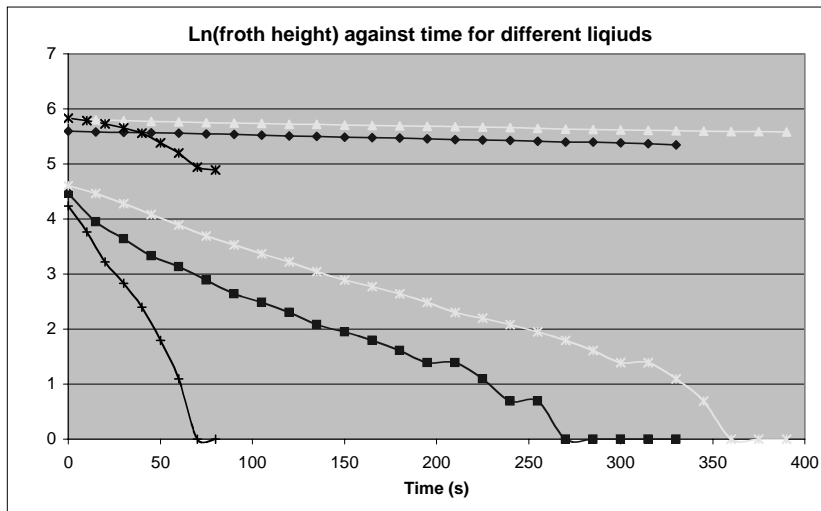


Decay constants are unaffected

Parameter 5: Gas Composition

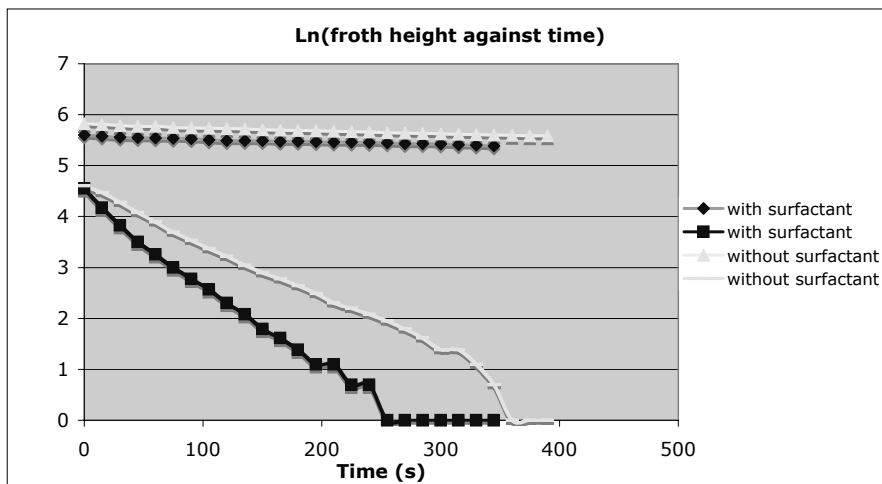


Coke - higher content of CO₂ - larger decay constant. Bi-polar and easily diffuses through bubble membranes.



Log-linear graphs show the difference in decay constants more clearly.

Parameter 6: Surface Tension



Surface tension was altered by the addition of a surfactant - detergent.

Surface tension is reduced. λ top height decreases by 16.7%

λ bottom height decreases by 18.0%

CONCLUSION

“Under what condition does the froth remain for the longest time?”

⇒ Small λ , large initial froth height.

- Narrow vessel - small surface area to volume ratio.
- Compromise temperature $\approx 10^0$ c
- High = large initial froth height
- Too high = decay constant starts increasing
- Without surfactant - 17% reduction in surface tension
- Viscous liquid.
- Low CO₂ content.
- Flow rate of 25 ml/s into a vertical vessel.

Appendix

$$\frac{dh}{dt} = -\lambda h$$

$$\Rightarrow \int \frac{dh}{h} = -\lambda \int dt$$

$$\Rightarrow \ln(h) = -\lambda t + c$$

$$\Rightarrow h = e^{-\lambda t + c}$$

$$\Rightarrow h = Ae^{-\lambda t}$$

$$\Rightarrow H(t) = H_0 e^{-\lambda t}, \quad \text{initial condition } H(0) = H_0$$

Derivation of top height:

Derivation of bottom height:

Let $h(t)$ represent the bottom height.

Consider the rate at which liquid drains from the froth given by:

$$|h(t) - h(\text{max})|$$

$$\Rightarrow \ln|h(t) - h(\text{max})| = -\lambda t + \text{int}$$

$$\Rightarrow h(t) - h(\text{max}) = e^{-\lambda t + \text{int}}$$

$$\Rightarrow h(t) = -e^{\text{int}} e^{-\lambda t} + h(\text{max})$$

$$\text{Note: } h(\text{max}) = \frac{V_{\text{total}}}{\pi r^2}$$

Derivation of froth height:

Froth height = Top height - bottom height \pm

$$\Rightarrow F(t) = H_0 e^{-\lambda t} - [-e^{\text{int}} e^{-\lambda t} + h(\text{max})]$$

$$\Rightarrow F(t) = H_0 e^{-\lambda t} + e^{\text{int}} e^{-\lambda t} - h(\text{max})$$

Error Analysis

- Top height error increased with time. Initially ± 2 mm, increasing to ± 10 mm.
- Bottom height could be continually measured to ± 1 mm
- Difference in height: initially ± 3 mm, increasing to ± 11 mm

Laplace's formula

$$\Delta p = \sigma_{lv} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\text{contact angle } \cos \theta = \frac{\sigma_{wv} - \sigma_{wl}}{\sigma_{lv}}$$

How is froth generated?

- 1.8 grams of CO₂ will dissolve in 1 litre of water at 20⁰C.
- Gases are less soluble at higher temperatures.
- Henry's law - concentration of a dissolved gas is proportional to the partial pressure of that gas in equilibrium with that liquid.
- When a carbonated beverage is opened - sudden drop in pressure and an increase in temperature - CO₂ is supersaturated.
- CO₂ (and trapped air) condense.
- Bubbles are dispersed through the liquid- froth.