2. Problem •2: Shades

Solution of Korea

Problem •2: Shades

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The Problem:
If small non-transparent objects are illuminated with light, patterns in the shadows are observed. What information can be obtained about these objects using these patterns?

Abstract
This problem told us to answer "What information can be obtained with illuminating small non transparent objects with light". There are many theories and equations related to this phenomenon. Representative ones are Fresnel and Fraunhofer equations. These equations explain a relationship between obstacles, light source which illuminates obstacles, and screen where diffraction patterns appear. Also, using Babinet's principle which says that diffraction patterns made by two complementary gratings are same, I applied those equations to get information of small non transparent objects. In the experiment, at first, I simply set a device and checked if experimental results satisfy the equations. Secondly, I checked if there is any difference between diffraction patterns made by scattered and non scattered obstacles. I found there are differences when I put obstacles randomly or regularly. I found also found that shapes of small objects also affects the results. Finally, I applied these result to real small non transparent objects. With seeing diffraction patterns and measuring distances between center of diffraction pattern and minima rings of it, applying results to Fresnel and Fraunhofer equations, I could get several information of small non transparent objects.

1. Theoretical Backgrounds

1.1. Fresnel diffraction and Fraunhofer diffraction

Fresnel diffraction supposes distances between light source, obstacles and screen are infinite. Also it treats light waves as spherical waves. Fraunhofer diffraction supposes distances between those are finite. It treats light waves as plane waves. Under these supposed conditions, we can make equations related to diffraction. Following is deriving process of Fresnel diffraction equation.

1.2. Derivation of Fraunhofer diffraction equation

Amplitude of diffracted waves in case of Fresnel diffraction and Fraunfoher is defined as below

$$U \propto \int \frac{e^{ik(r+r')}}{rr'} d\Omega \qquad\qquad I \propto |U|^2$$

(U : amplitude, I : intensity, r : distance between detection point and aperture,
r′: distance between source and aperture)

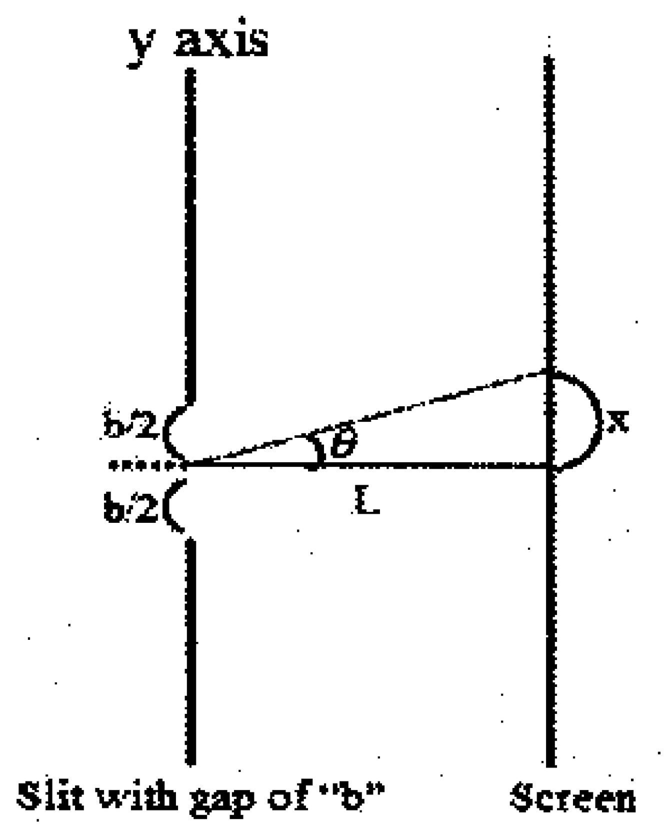
As we ignore the effect of spherical wave, there only left factors related with plane waves as following

$$U \propto \int e^{ikr} dA$$

From now on, I will use this relation to derive diffraction equation of each obstacle.

1.2.1. In case of diffraction patterns made by slits

The situation is as following image.



In case of slits, amplitude of diffracted waves is as below.

$$U \propto \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{iky\sin\theta} dy = \frac{e^{ik\frac{b}{2}\sin\theta} - e^{-ik\frac{b}{2}\sin\theta}}{ik\sin\theta}$$

$$\therefore I = I_0 \left[b\frac{\sin\beta}{\beta} \right]^2 \qquad \left(\beta = \frac{1}{2}kb\sin\theta\right)$$

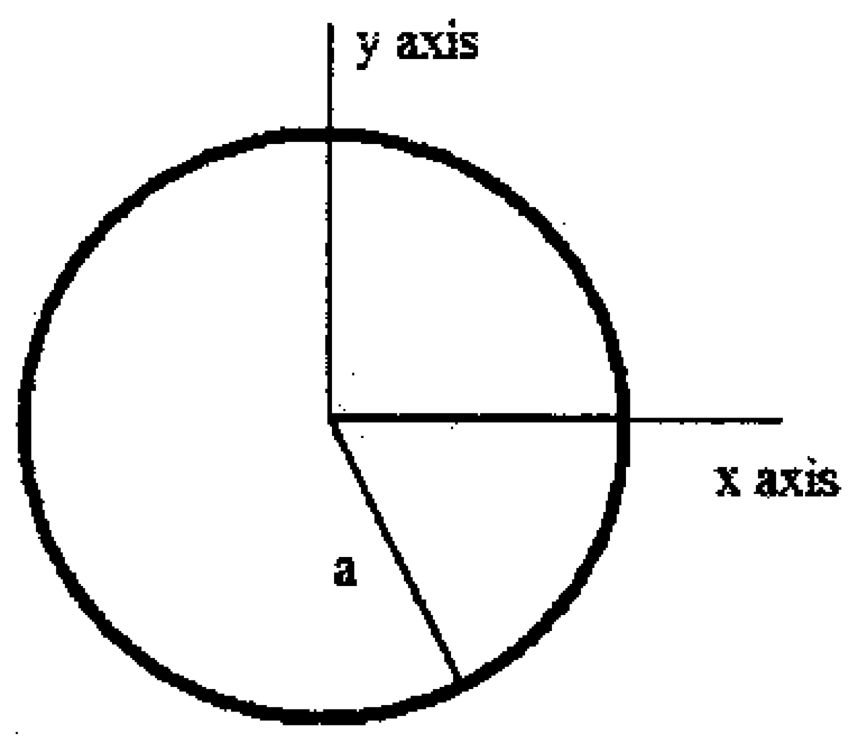
Therefore, the zeros of the diffraction pattern appear when

$$n\pi = \beta \; (\because I = 0) = \frac{kb\sin\theta}{2} = \frac{\pi b\sin\theta}{\lambda} \qquad \left(\because k = \frac{2\pi}{\lambda}\right)$$

$$\therefore b\sin\theta = n\lambda$$

$$\therefore n\frac{\lambda}{b} = \frac{x}{L} (n = 1,2,3.....)$$

1.2.2. In case of diffraction pattern made by circular aperture
The situation is as following image, a circular obstacle with radius of "a".



Intensity of diffraction pattern is,

$$I \propto \left[\int_{-a}^{a} \int_{\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} e^{iky\sin\theta} dxdy \right]^2 \qquad I = I_0 (2\frac{J_1(\beta)}{\beta})^2$$

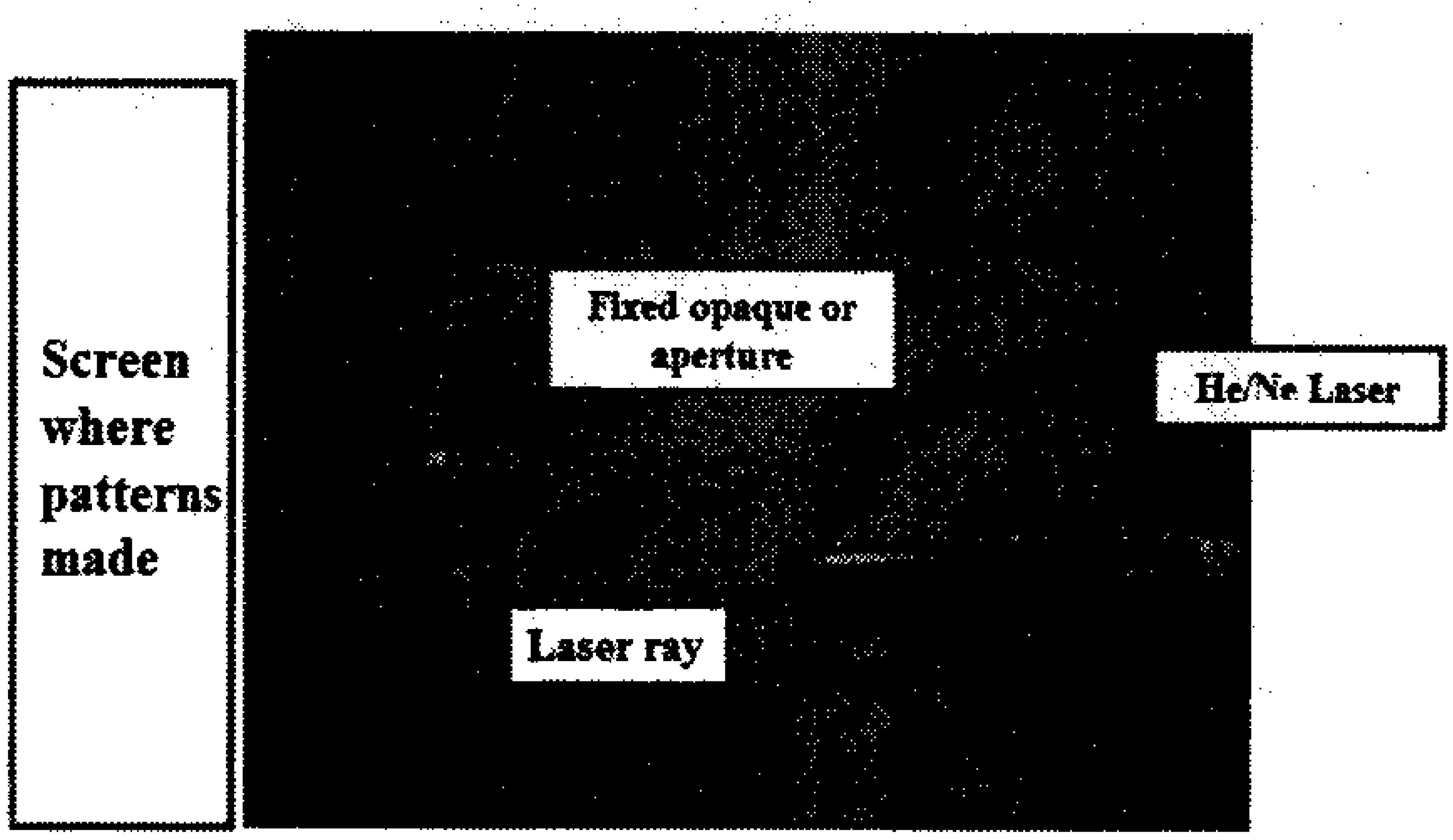
$$\therefore m\frac{\lambda}{2a} = \frac{r}{L} \qquad (m = 1.220,\ 2.233,\ 3.238...)$$

2. Experiment

2.1. Experiment no.1

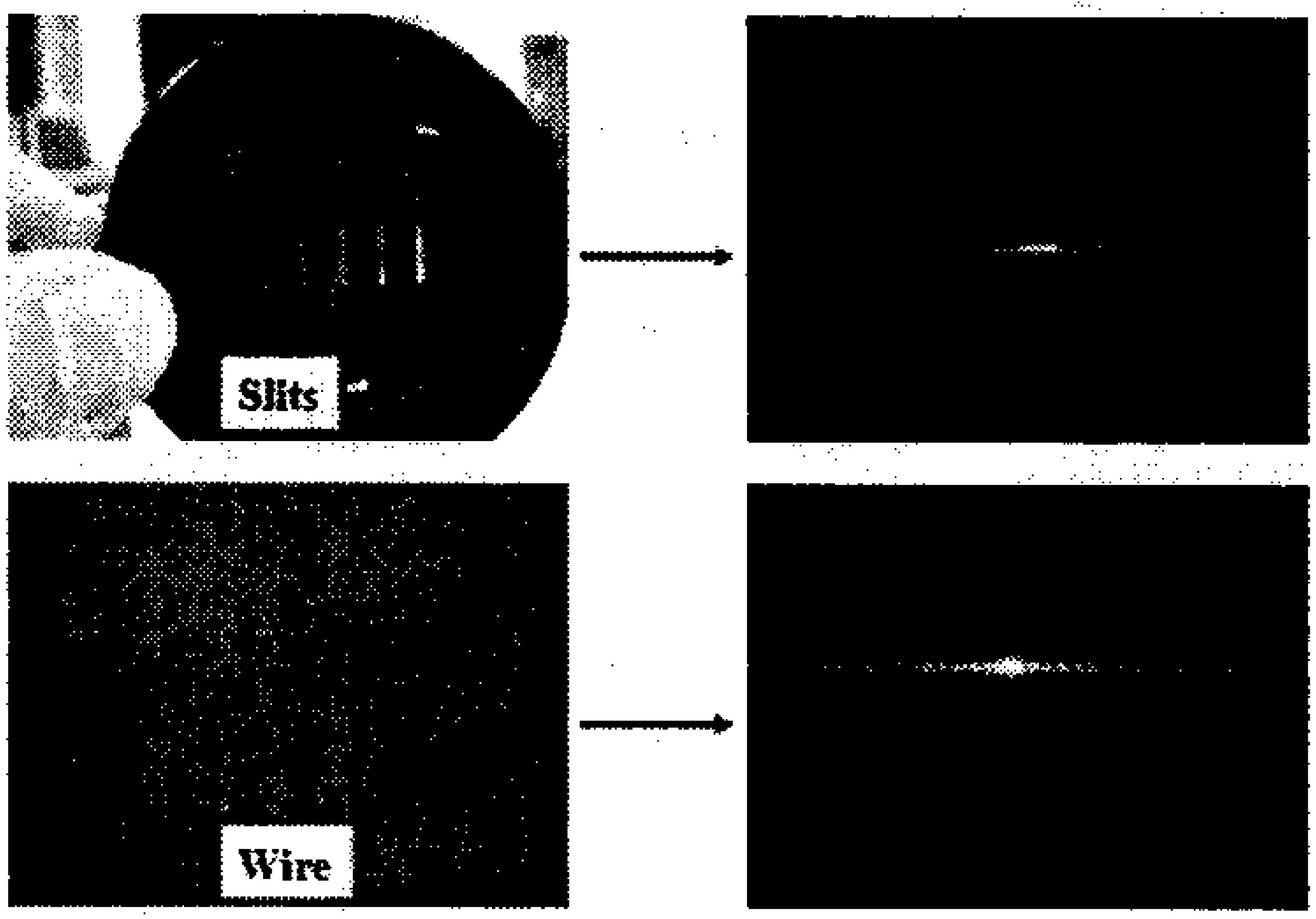
In this experiment, I made simple diffraction patterns with using apertures and opaque (slits, circle, triangle and square). Also I compared diffraction patterns from various opaque.

2.1.1. Experimental set up

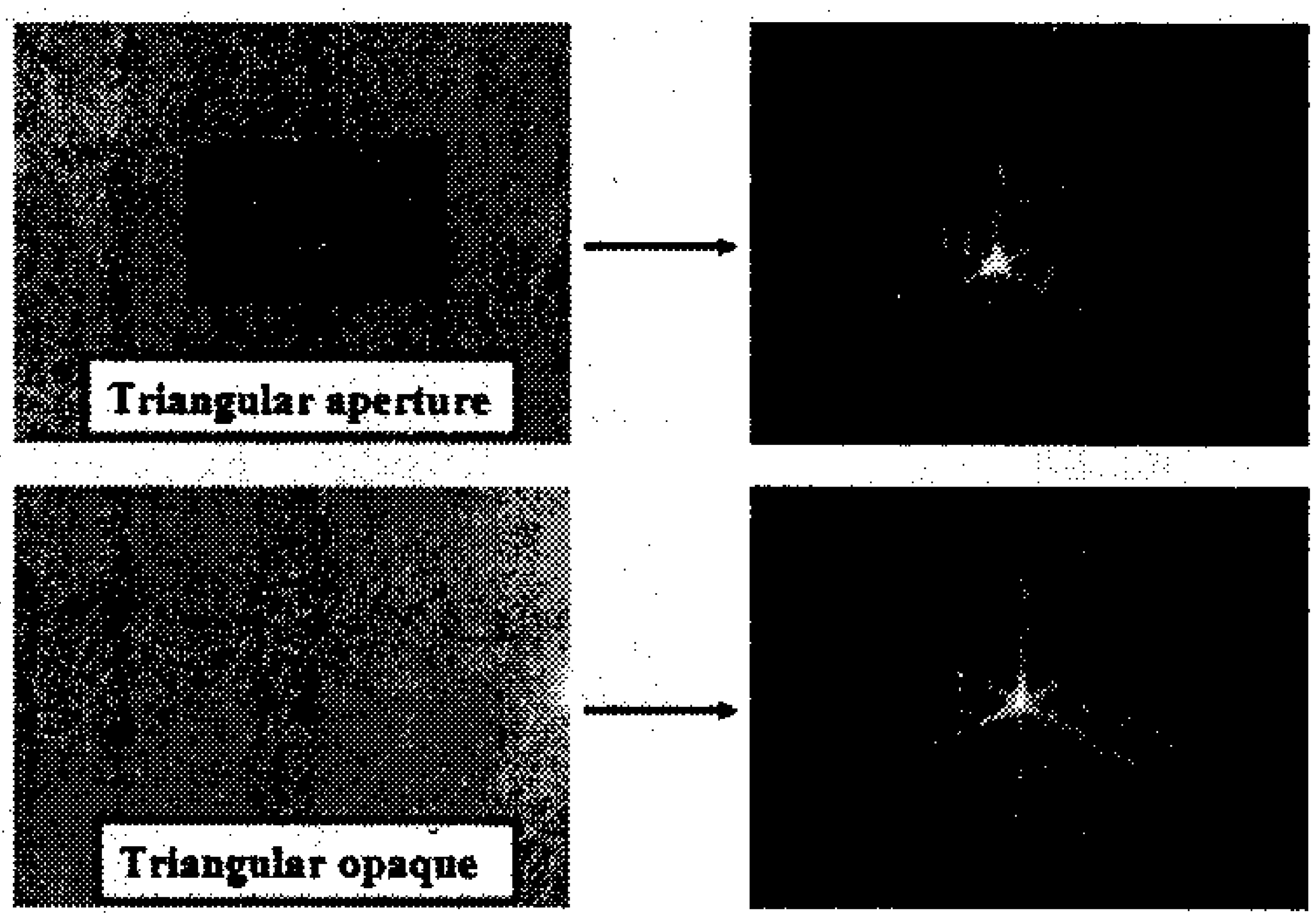


2.1.2. Experimental results

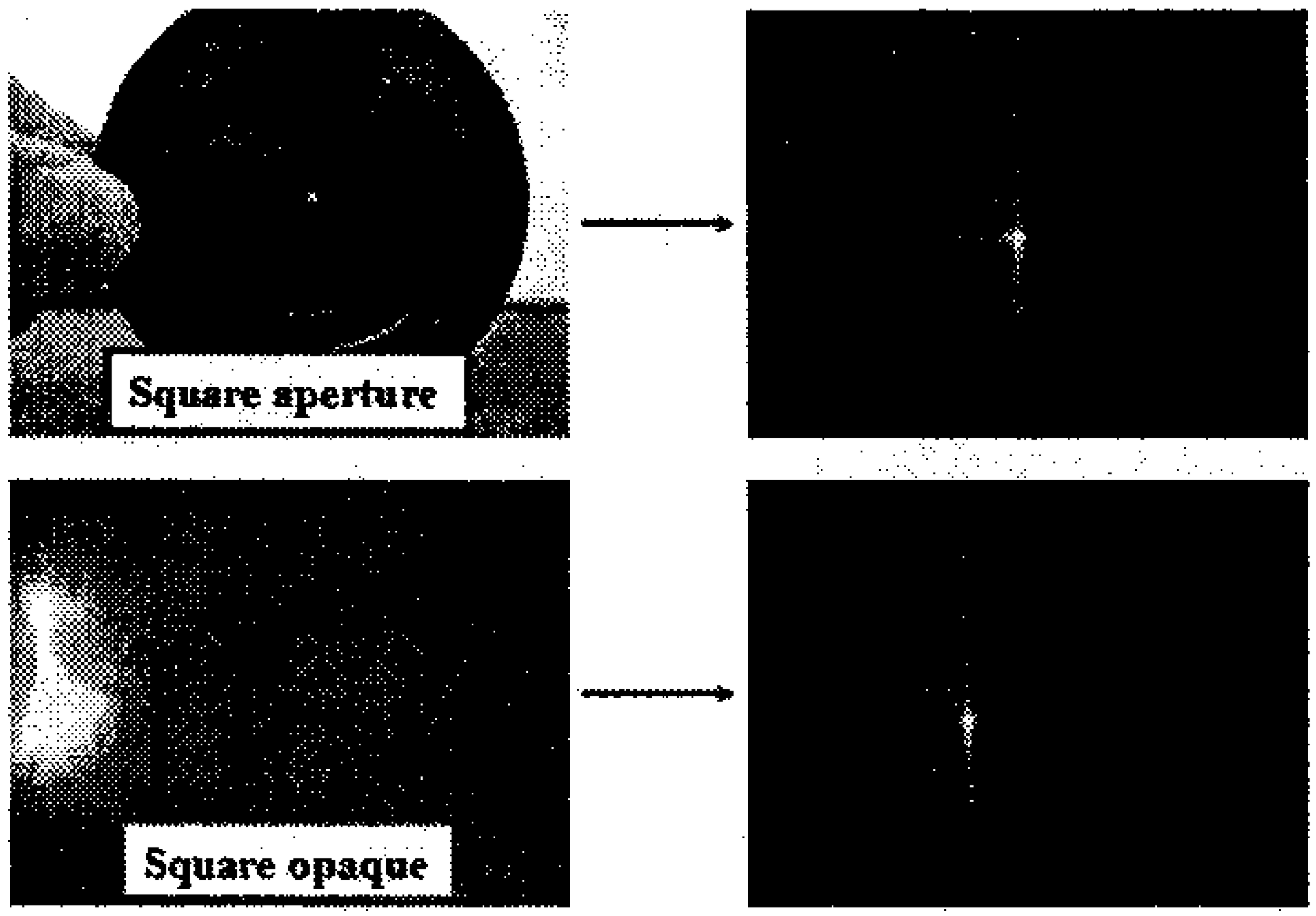
2.1.2.1. Results with slits and wires



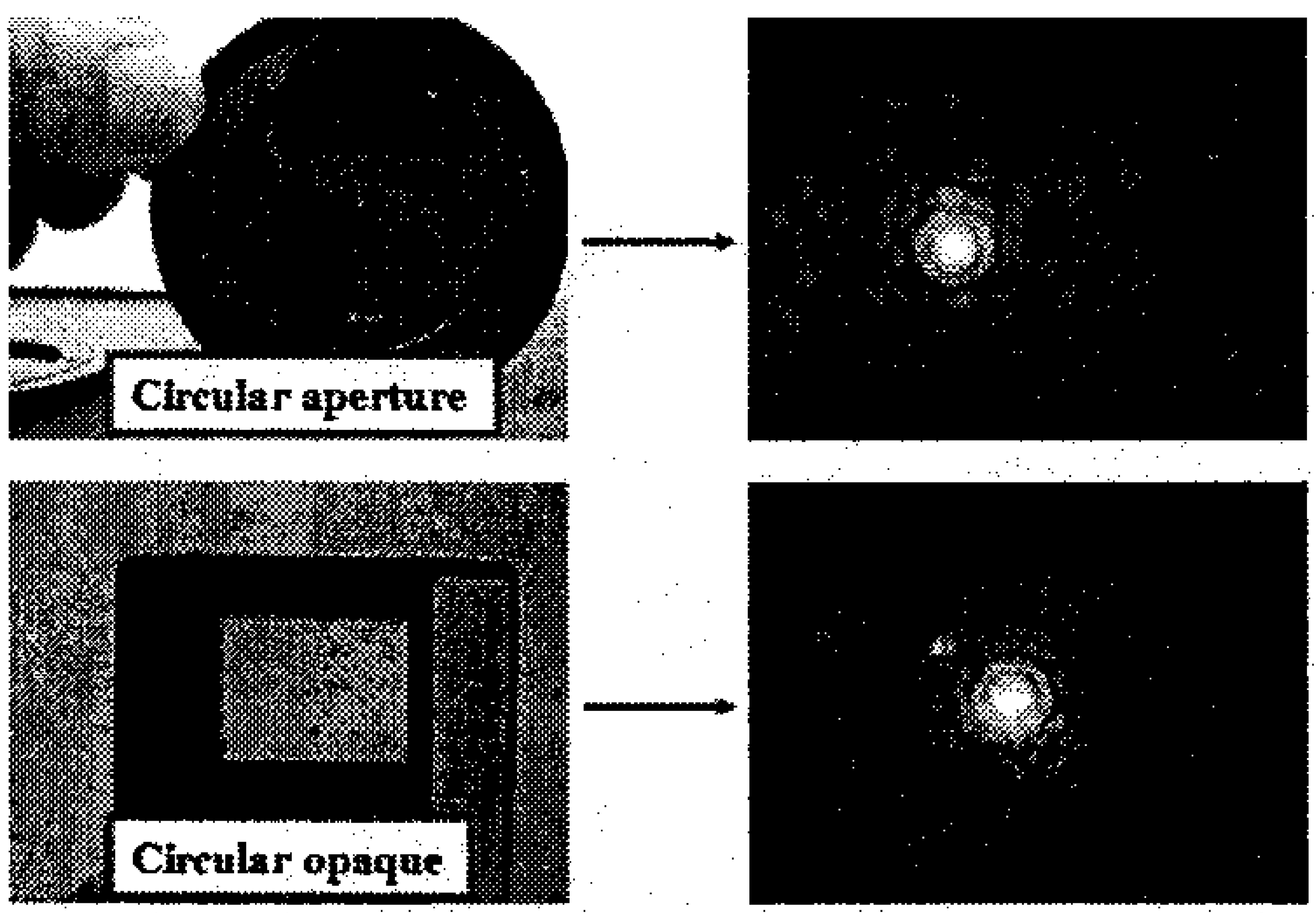
2.1.2.2. Results with triangular aperture and opaque



2.1.2.3. Results with square aperture and opaque

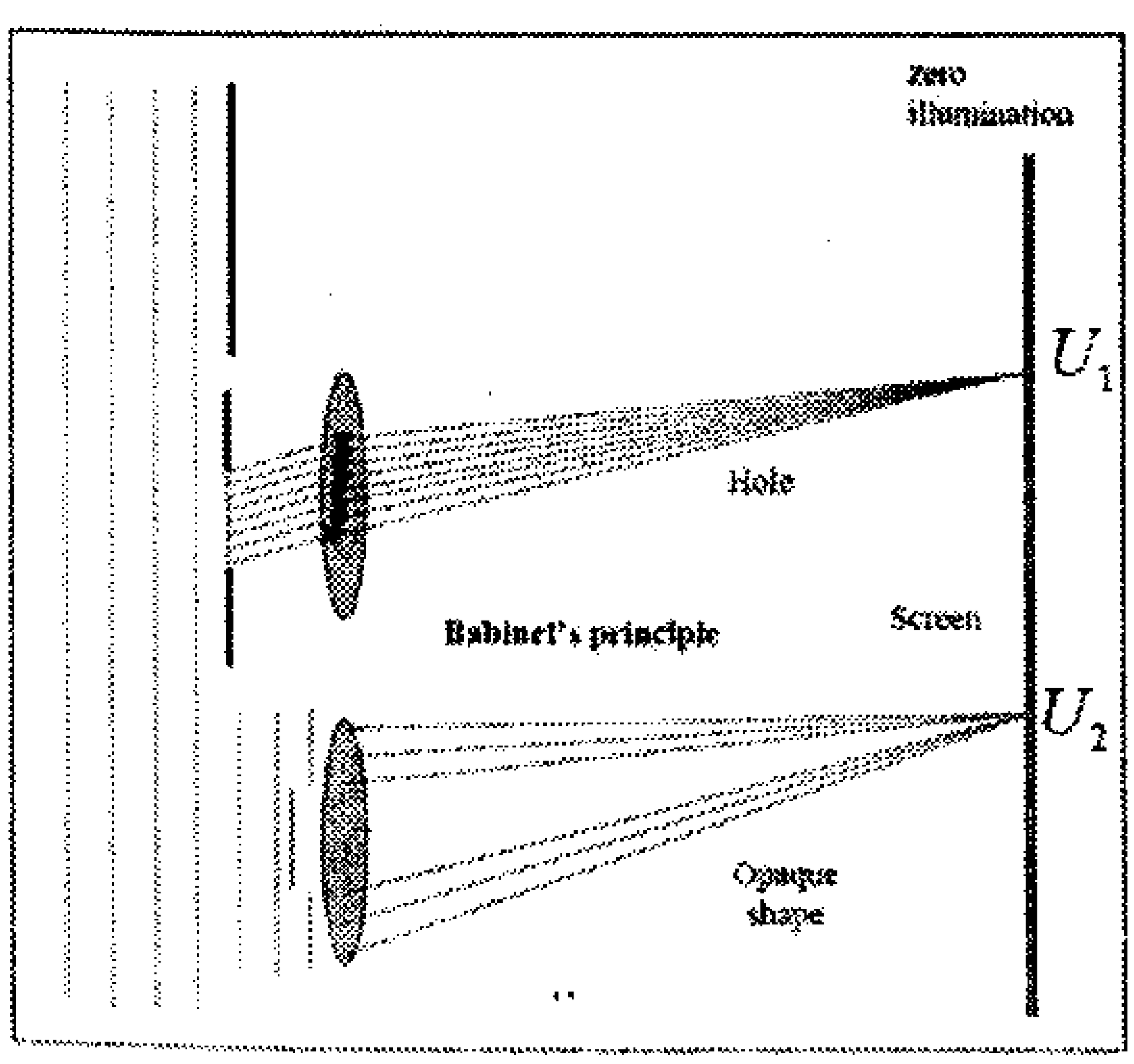


2.1.2.4. Results with circular aperture and opaque



2.1.3. Babinet's principle

Babinet's principle says that diffraction patterns which made by two complementary gratings are same. Upper results show the principle in visual.



Amplitude

$$U_1 + U_2 = 0$$

$$\downarrow$$

$$U_1 = -U_2$$

$$\downarrow$$

$$|U_1|^2 = |-U_2|^2$$

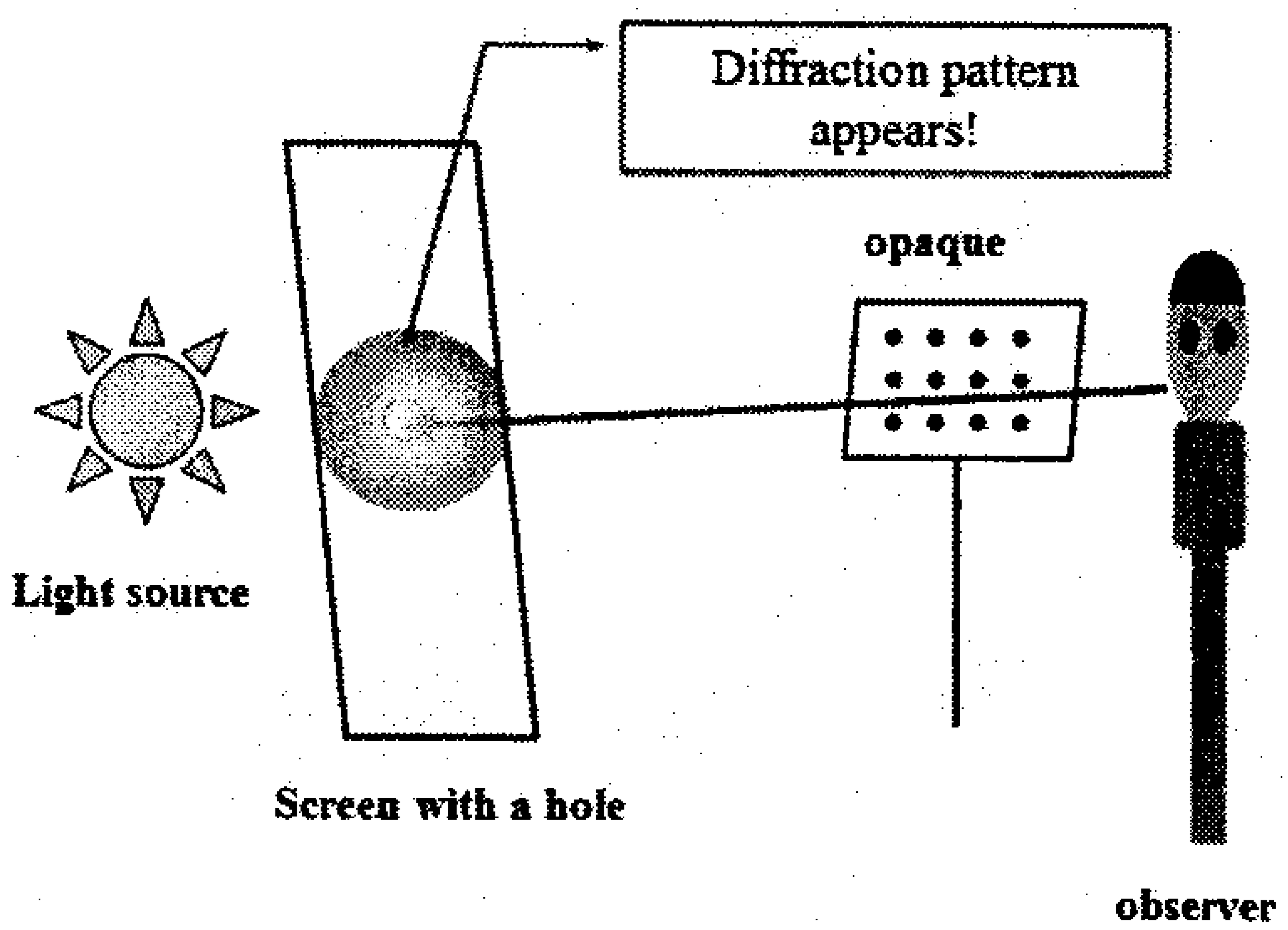
$$\downarrow$$

$$I_1 = I_2$$

2.2. Experiment no.2

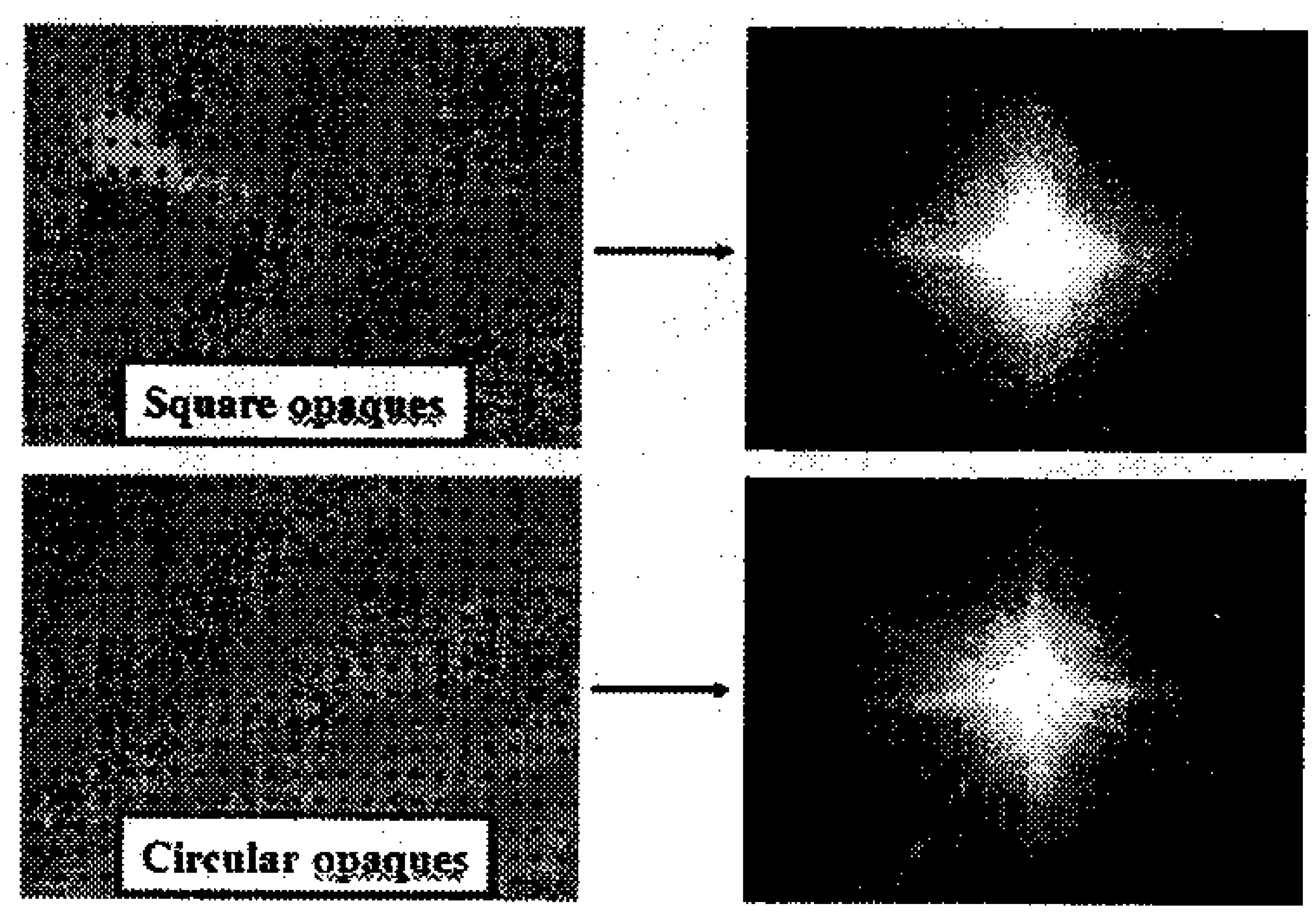
In this experiment, I arranged small opaques randomly and regularly. Then, I illuminated light, observed diffraction pattens, and compared results

2.2.1. Experimental set up

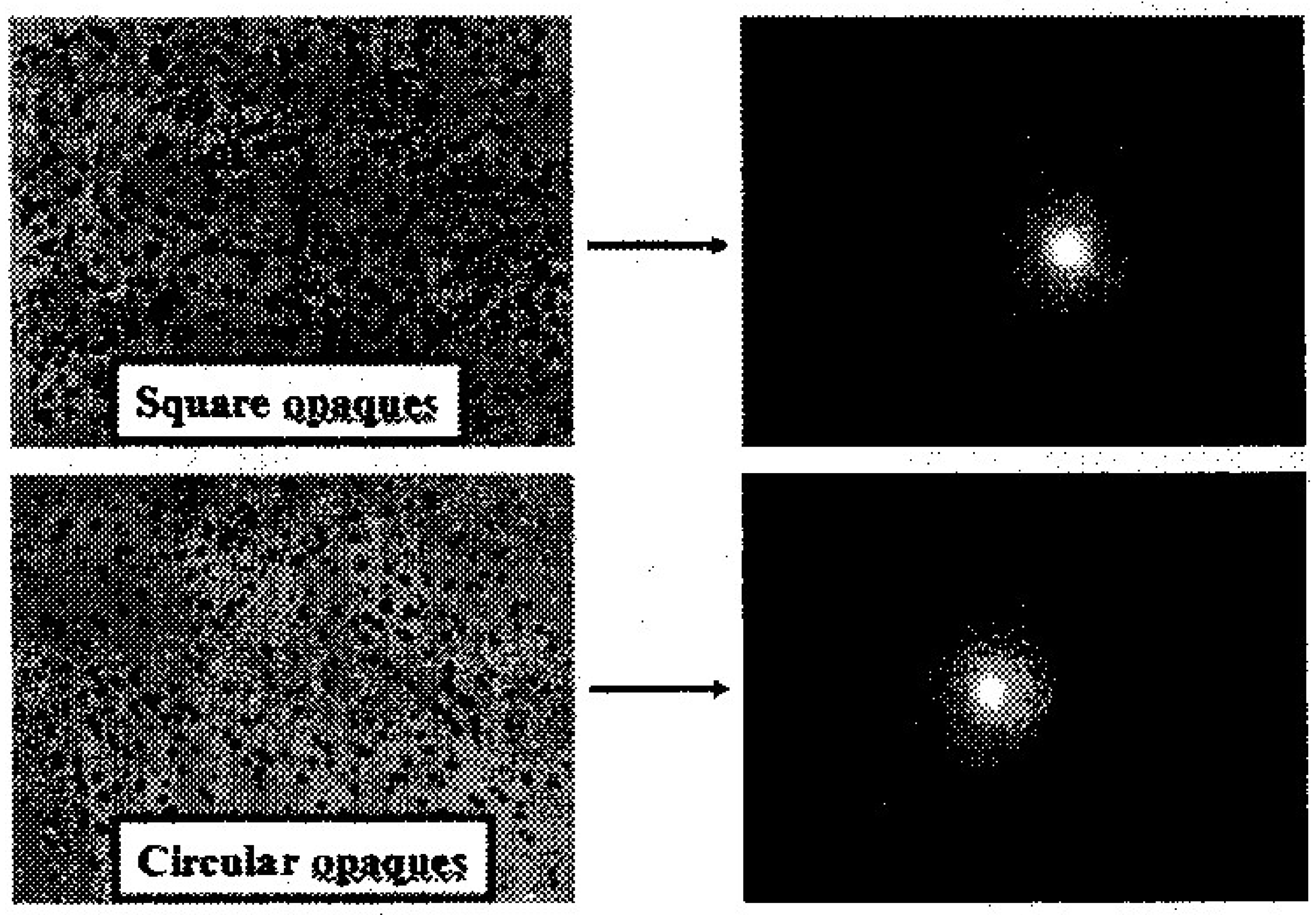


2.2.2. Experimental results

2.2.2.1. Results with regulary arranged square and circular opaques

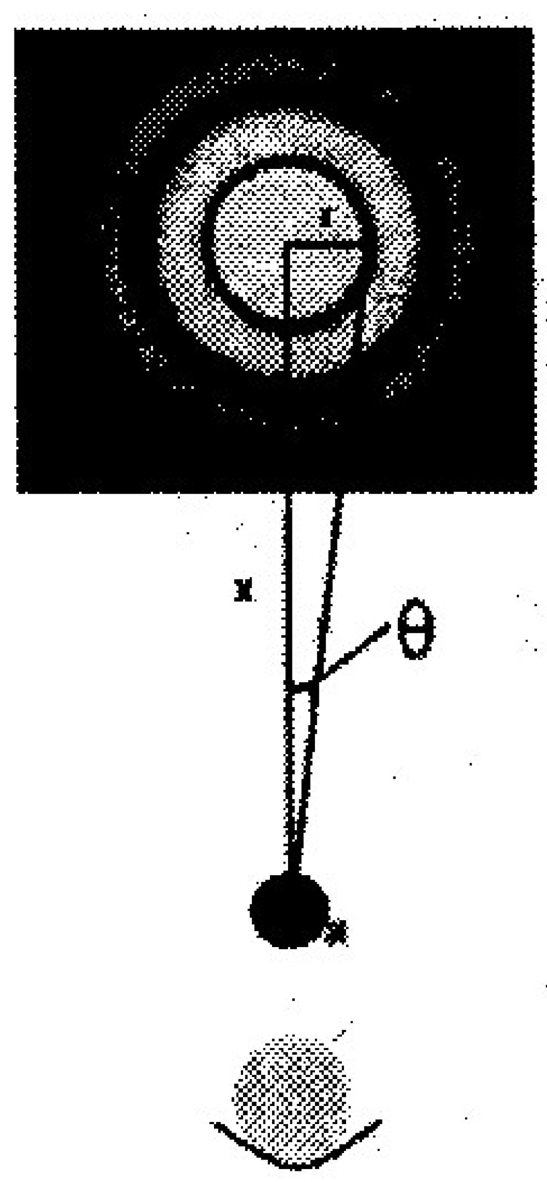


2.2.2.2. Results with randomly arranged square and circular opaques



2.3 Experiment no.3

In this experiment, I used blood corpuscles, pine flowerpots and fox's wool. With these objects, I illuminated light and tried to find information related to them. Also I used diffraction equation to measure sizes of small objects as follows.



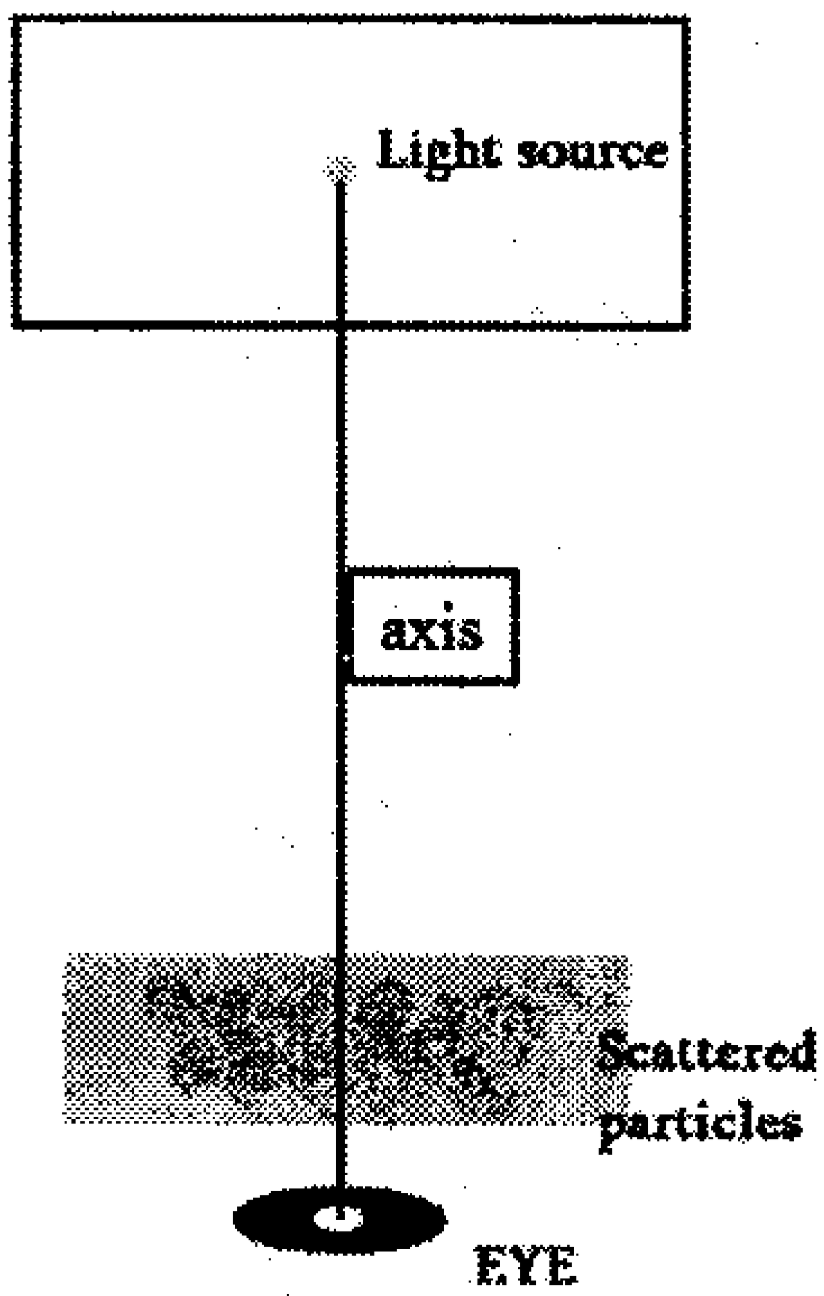
• Minima on Diffraction pattern

$$\sin\theta = m\frac{\lambda}{a} (m = 1.220, 2.233, 3.238...)$$

(m is defined with Bessel function)

$$\therefore m\frac{\lambda}{a} = \frac{r}{x} (when, \theta << 1)$$

I measured values of x, r and theta and applied these values into upper equation. Then I could get values of a. I put 1.22 as a value of m when r was distance between center and 1st minimum and 2.233 when r was distance between center and 2nd minimum.



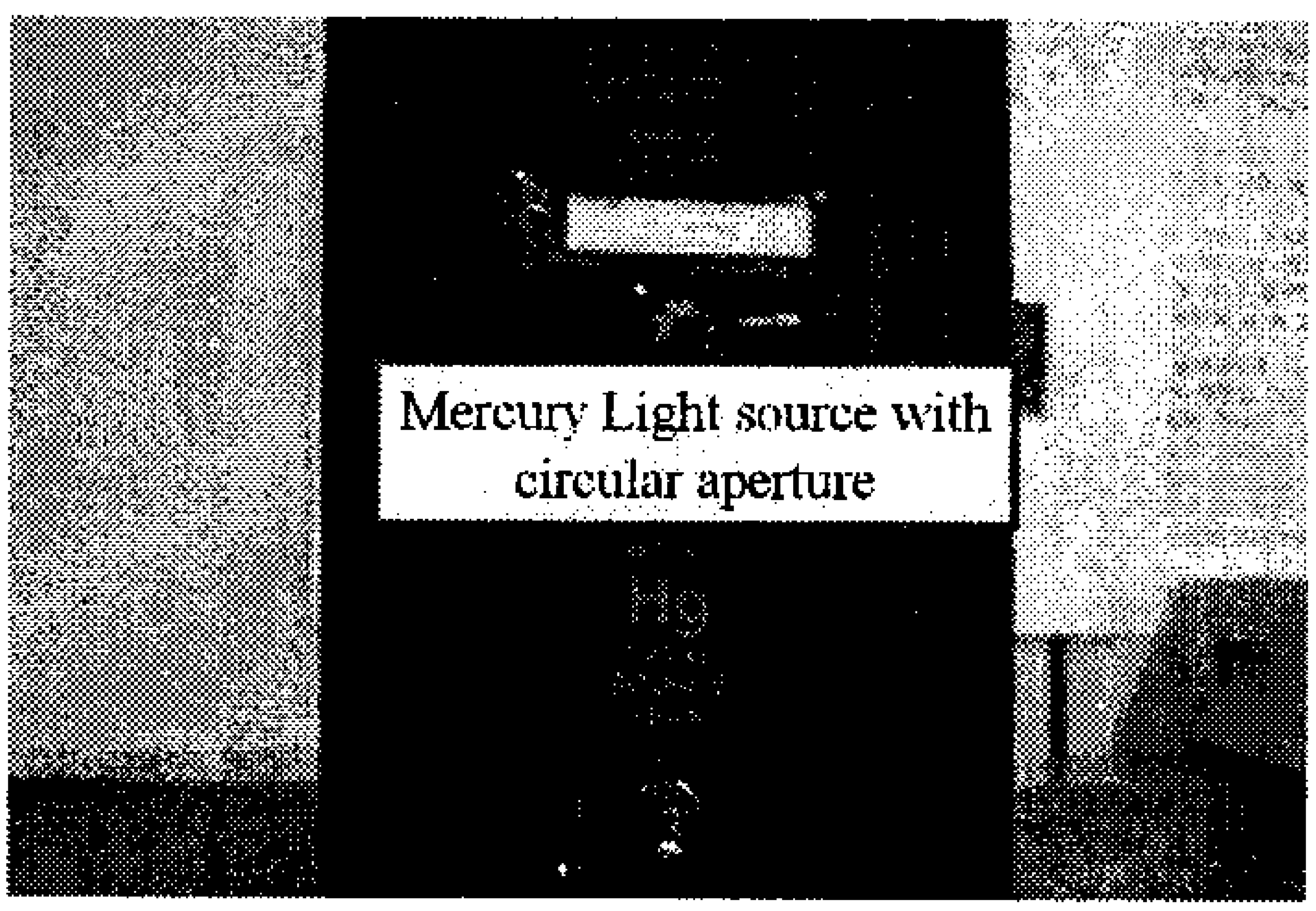
• The light source and eye are on the same axis

• Then, diffraction patterns by each particle are constructed
→Their path lengths are same

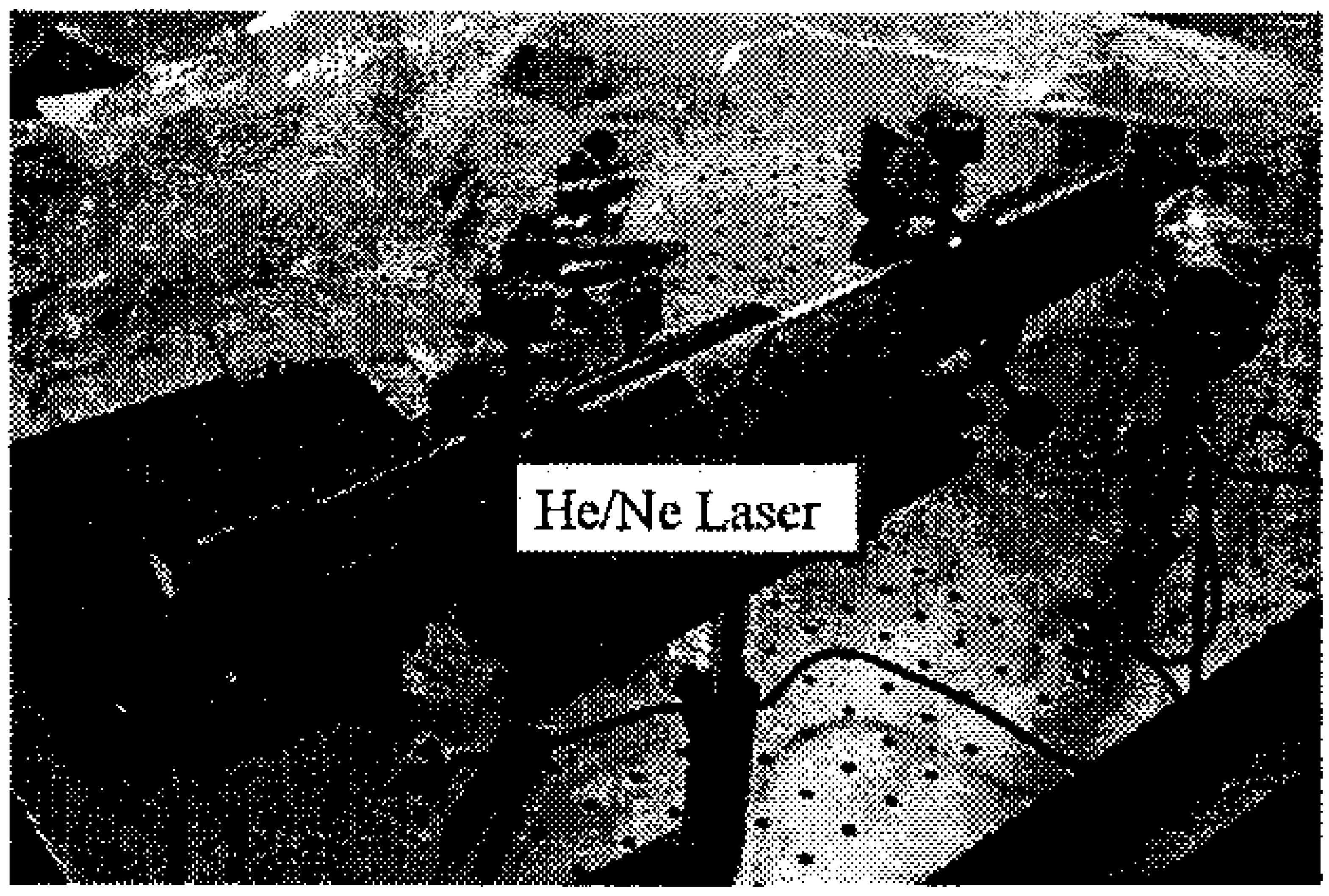
• We can say,
Pattern by a particle
= Pattern by many particles

Upper image explains why diffraction patterns made by scattered particles and non scattered particles are same. It works only when light source and eyes are on same axis. Then path lengths of light beam toward eyes are same and it makes the patterns same. So I could say that the pattern made by a particle is same as the pattern made by several scattered particles. It enabled me to use diffraction equation to get sizes of small objects.

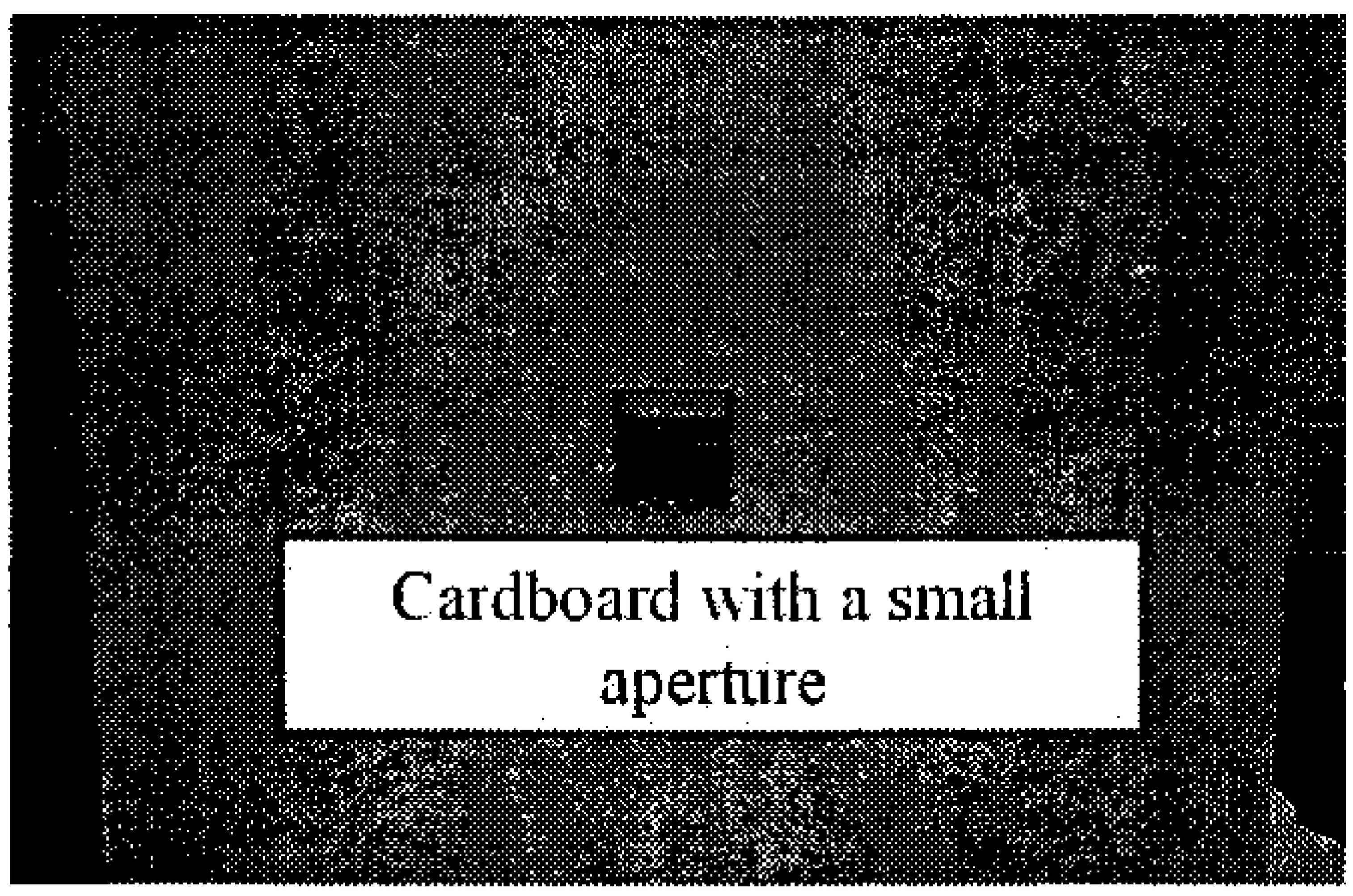
2.3.1. Experimental devices and set up



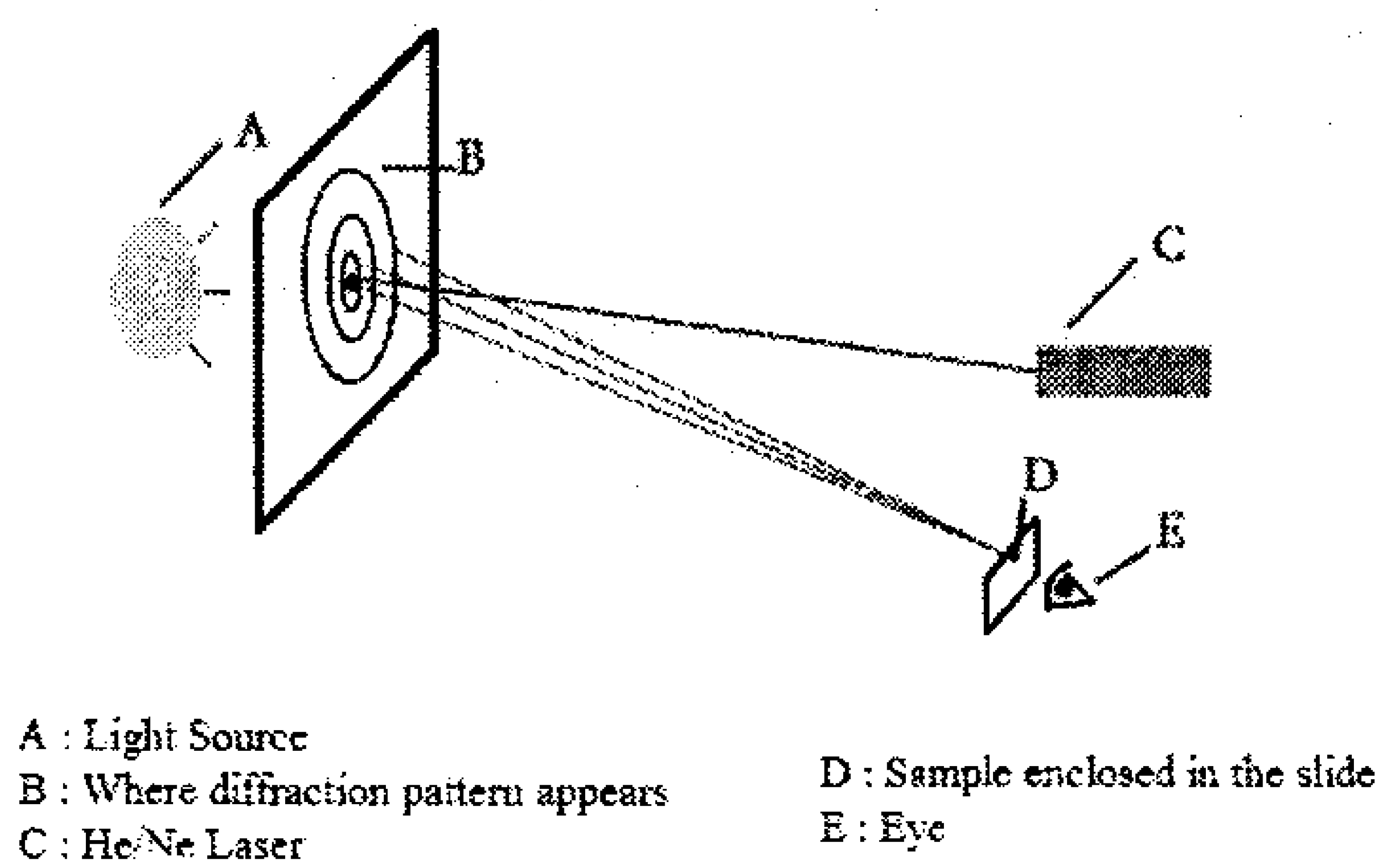
< Mercury light source with circular aperture >



< He/Ne Laser (red) >

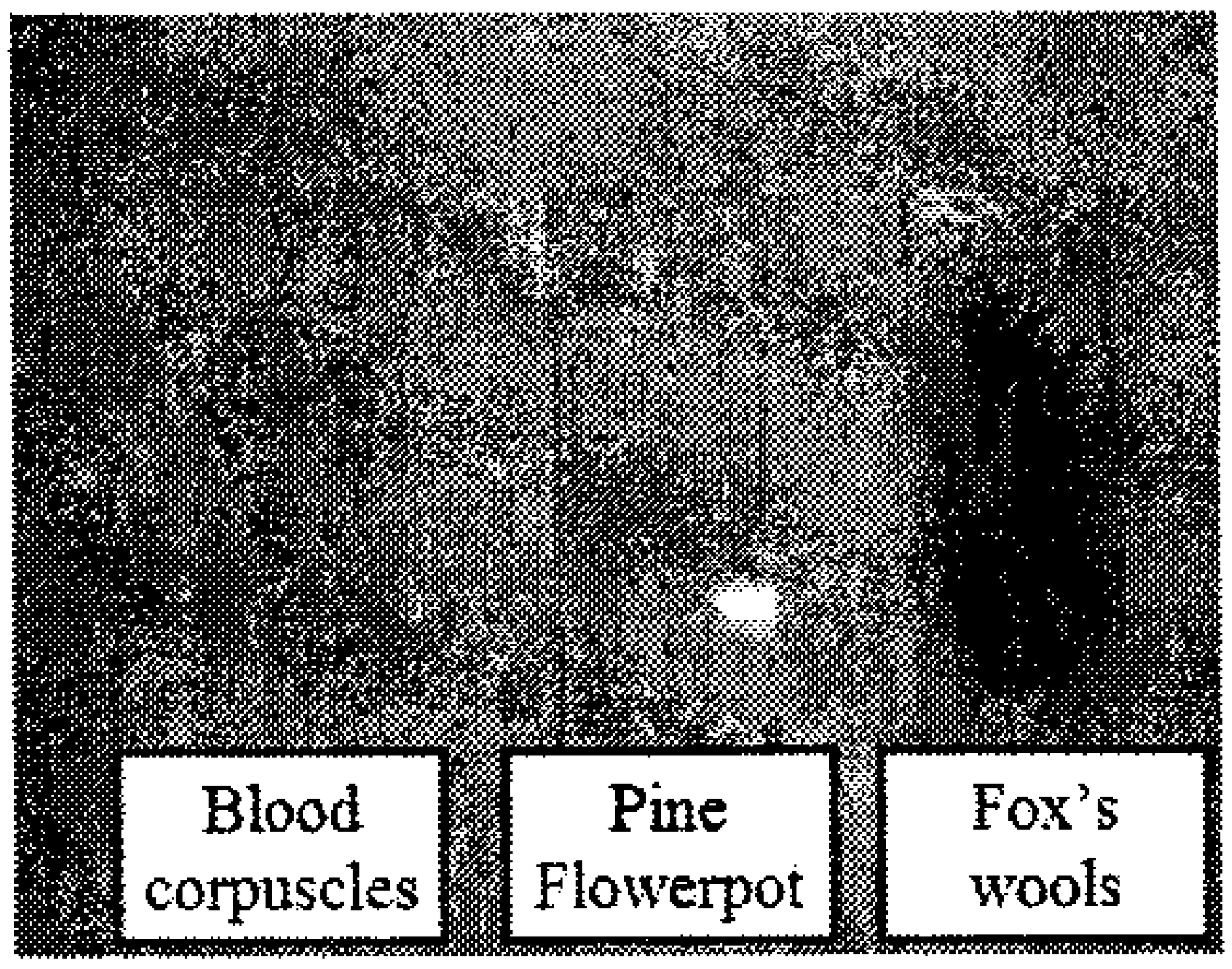


< Cardboard with a small aperture >



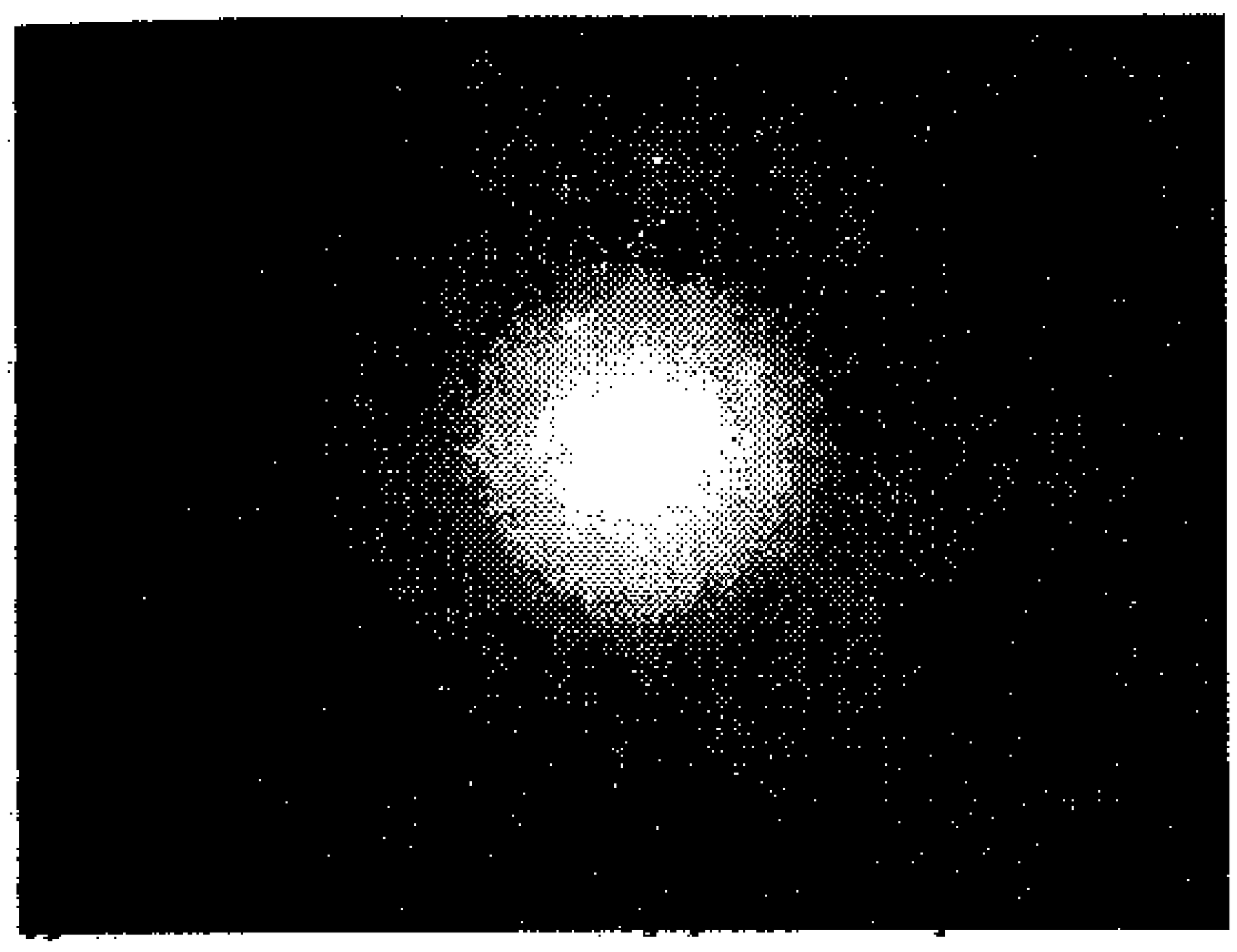
< Experimental Set Up >

2.3.2. Experimental samples



< Experimental samples : blood corpuscles, pine flowerpot, fox's wools>

2.3.3. Image of diffraction pattern made by mercury light source



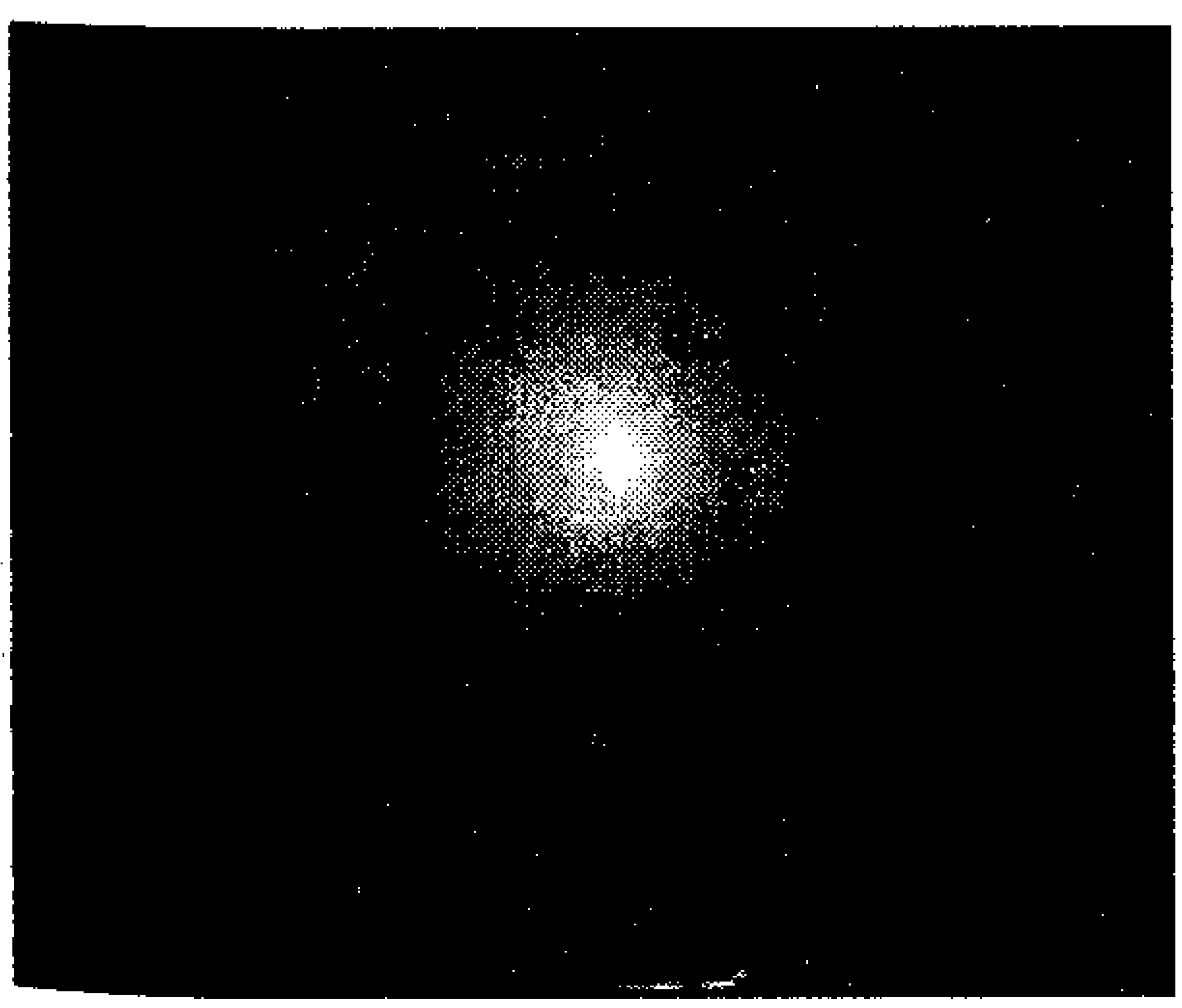
Diffraction pattern made by Hg light source >

2.3.4. Experimental set up

2.3.4.1. Experiment no.3-1

In this exeperiment, I used green filtered mercury light source whose wavelength is 546nm. I observed patterns which appeared on screen. Then, I measured distances between center of diffraction pattern and each minima with He/Ne laser (I marked points where minima exist with laser, marked with pen, and measured distances by using a ruler).

2.3.4.1.1. Experimental results – image



Diffraction pattern made by green filtered Hg light source >

2.3.4.1.2. Experimental results

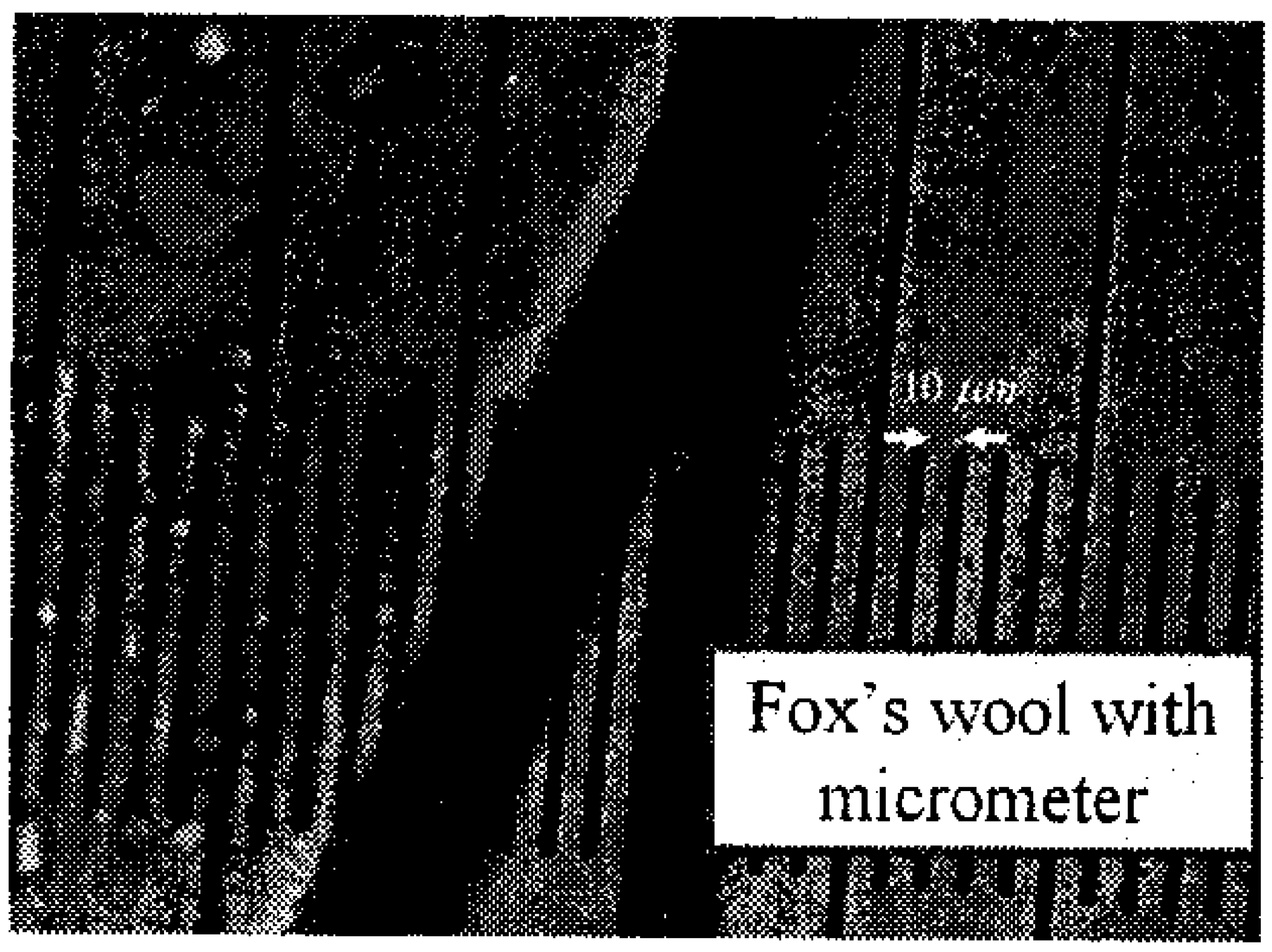
Distance		1m	2m	3m	Average
Pine flowerpot	R1 R2	1.6cm 3cm	3.2cm -	4.8cm -	41.1 microm
Blood corpuscles	R1 R2	9.2cm -	18cm -	28.5cm -	7.1microm
Fox's wool	R1 R2	3.2cm -	7cm -	10.5cm -	19.1microm

< R1 : distance between center of diffraction patterns and 1st minimum,
R2 : distance between center of diffraction patterns and 2nd minimum >

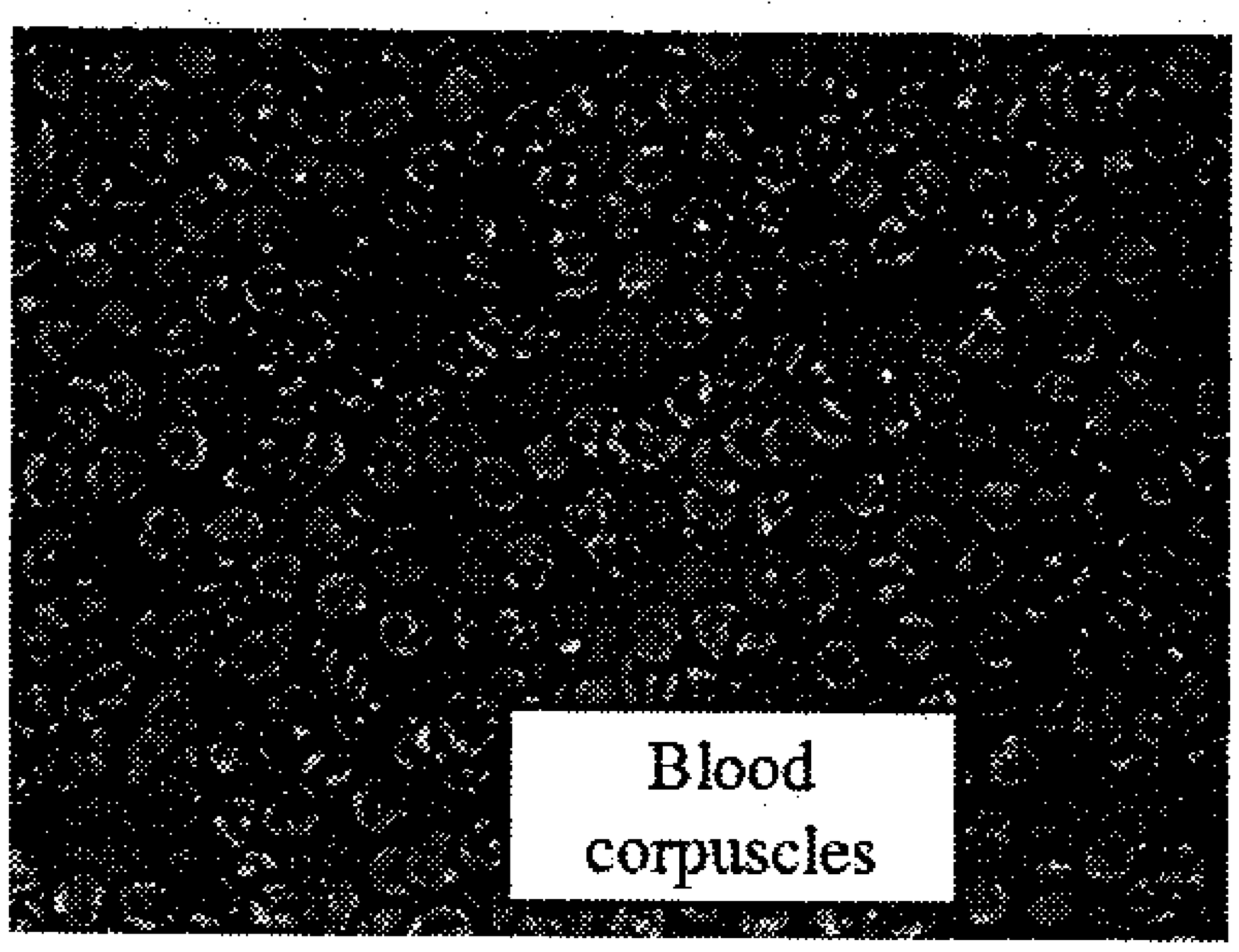
2.3.4.2. Experiment no.3-2

In this experiment, I used microscope to measure real sizes of small objects. I set microscope on slide glasses for measurement. I tried 7 times each, calculated average values and compared with results of experiment no.3-1.

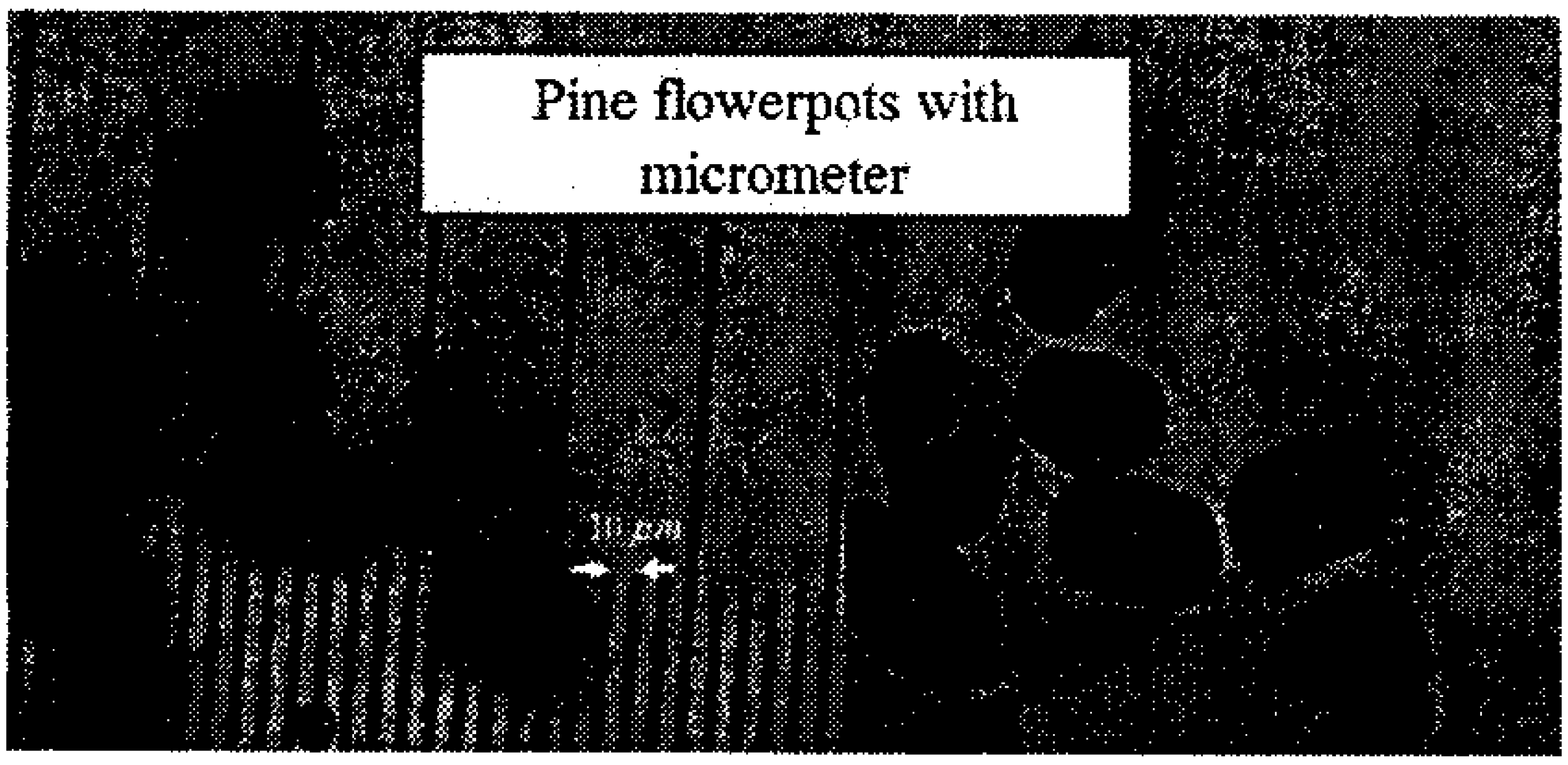
2.3.4.2.1. Microscopic photos of samples



< Microscopic photo of fox's wools with micrometer >



< Microscopic photo of blood corpuscles >



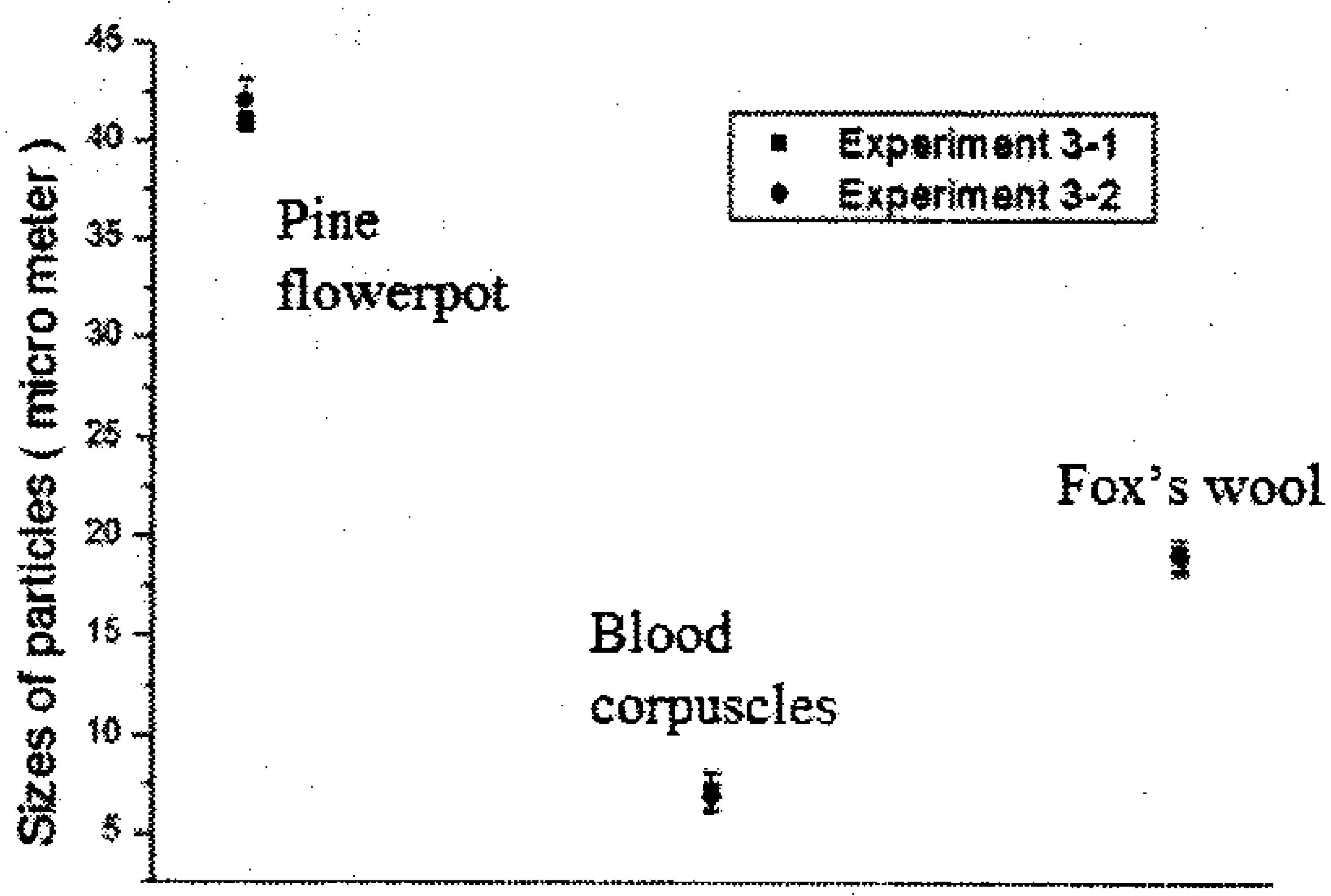
< Microscopic photos of pine flowerpots with micrometer >

2.3.4.2.2. Experimental results

Trials	*1*	*2*	*3*	*4*	*5*	*6*	*7*	Avg
Pine	38	40	46	50	41	42	44	*43*
Blood	6.4	7.2	8.0	6.8	7.2	6.4	6.6	*6.9*
Fox's wool	19	18	21	20	19	18	18	*19.0*

Dimension : μm

2.4. Comparison between results



As the graph shown above, results between experiment 3-1 and 3-2 are almost same within error range. So we can say that the accuracy of measurement using light is pretty high enough to be trusted. Also it guaranteed the theory which says that diffraction patterns made by scattered pariticles and a same particle are same when light source and eyes are on same axis.

3. Conclusion

The problem "Shade" wanted us to find out information of small objects by using patterns in the shadow when we illuminate light through small objects. In my experiment, I interpreted the patterns as diffraction patterns by illuminating light through particles. The equations I derived in the theoretical part helped me to find out real sizes of small objects with diffraction pattern. Critical factors of this measurement were distance between particles (eyes) and screen and center of diffraction pattern and minima. Actually, in real experiment, I could only see 1^{st} and 2^{nd} minima because upper 2^{nd} minima were too dim to observe. Also I was sure that the accuracy of this measurement is enough to be believed by measuring real values by using microscopes. Values gained with experiment 3-1 and 3-2 were almost same within error range.

In experiment no.1 and no.2, I tried to find simple diffraction patterns and tested the accuracy of equations. Also I found that diffraction patterns made by regularly arranged particles depend on their arrangement (In my experiment, I arranged them in lines so the patterns appeared rectangular). In case of randomly arranged particles, their diffraction patterns appeared to be circular shapes regardless of their shapes.

In conclusion, information I could get from patterns were shapes of objects, sizes of objects and state of arrangement of objects.

4. References

[1] Introduction to modern optics, by Fowles.
[2] Optics, by Hecht
[3] Fundamental of physics, by Halliday