4. Problem №4: Whispering Gallery

Solution of Bulgaria

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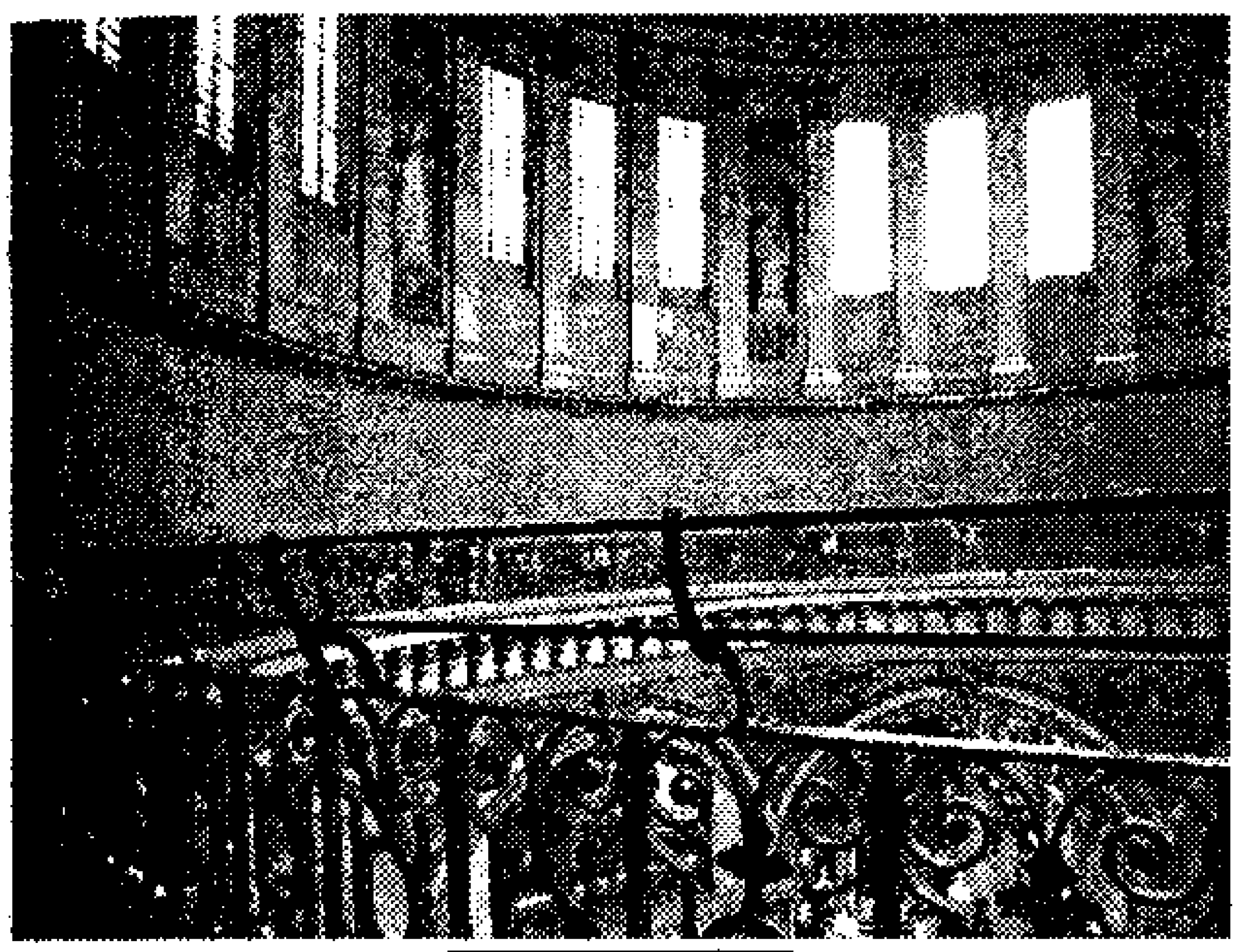
The Problem:

The Whispering Gallery at St Paul's Cathedral in London, for example, is famous for the fact that the construction of the circular gallery makes a whisper against its walls on the side of the gallery audible on the opposite side of the gallery. Investigate this phenomenon.

The Whispering gallery at the "St. Paul" cathedral in London and Relay's explanation

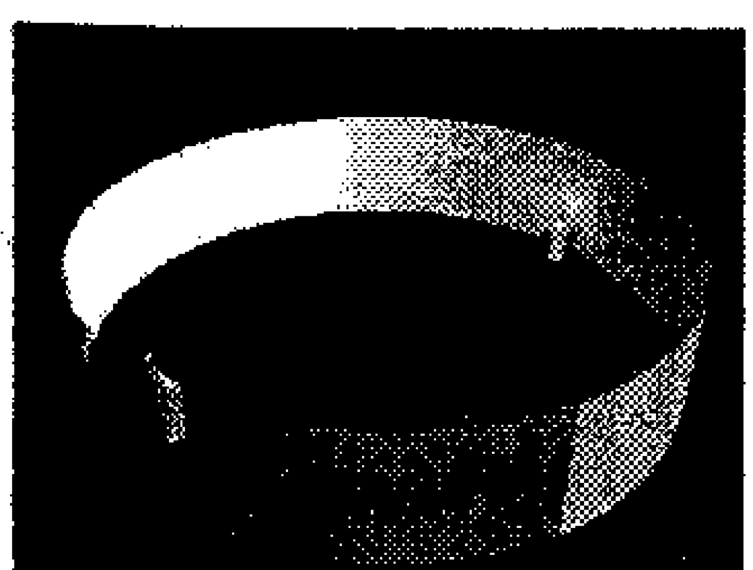
The whispering gallery is a smooth cylindrical wall in the "St. Paul" cathedral in London. A whisper produced near the wall of the gallery is well audible on the diametrically opposite side of the wall, despite the fact that the diameter of the hall is above forty meters. Relay created a large enough model of the whispering gallery and conducted a series of experiments in order to explain the observed phenomenon. For that purpose he creates sound with a whistle, placed near the wall of the cylindrical surface of the model and registers the occurrence of a sound wave with the help of a candle's flame.



••• .1.

With three ingenious experiments (fig. 2a, 2b, 2c) Relay proves that the sound wave travels with the smallest losses near the wall of the gallery.

When he places a barrier in the middle of the gallery's model, the flame flickers, which is a sign that the sound wave occurs, but placing barriers near the walls of the model and conducting the experiment once more, he notices the flame does not flicker.



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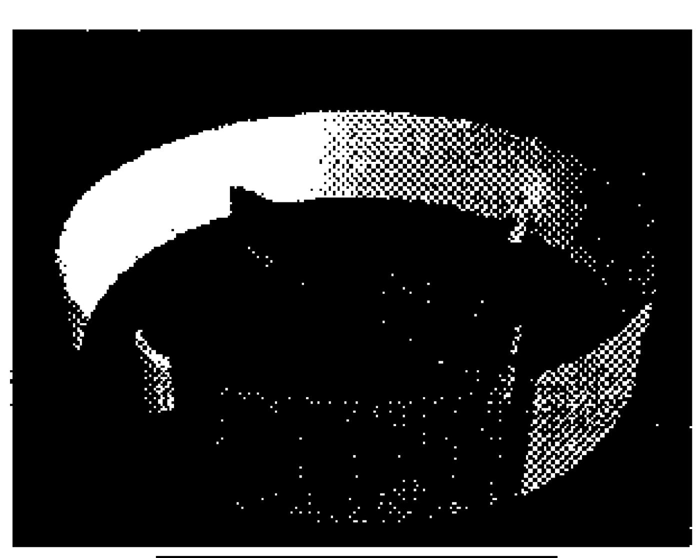

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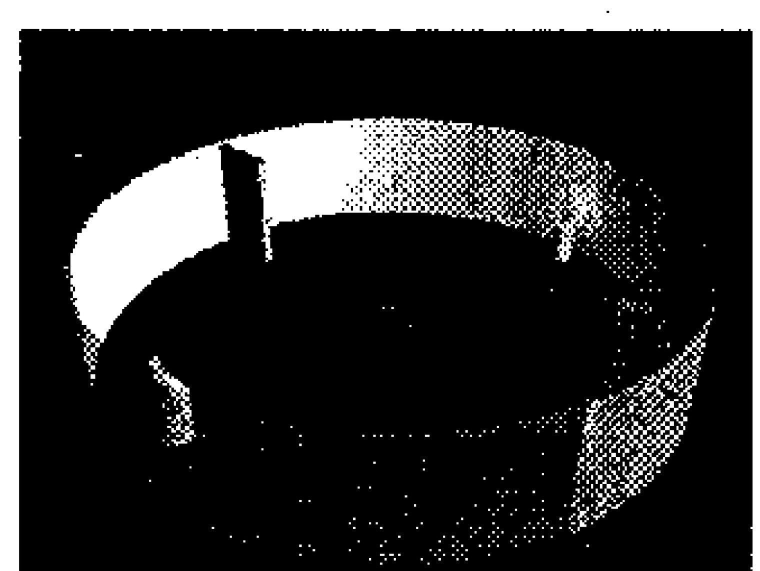


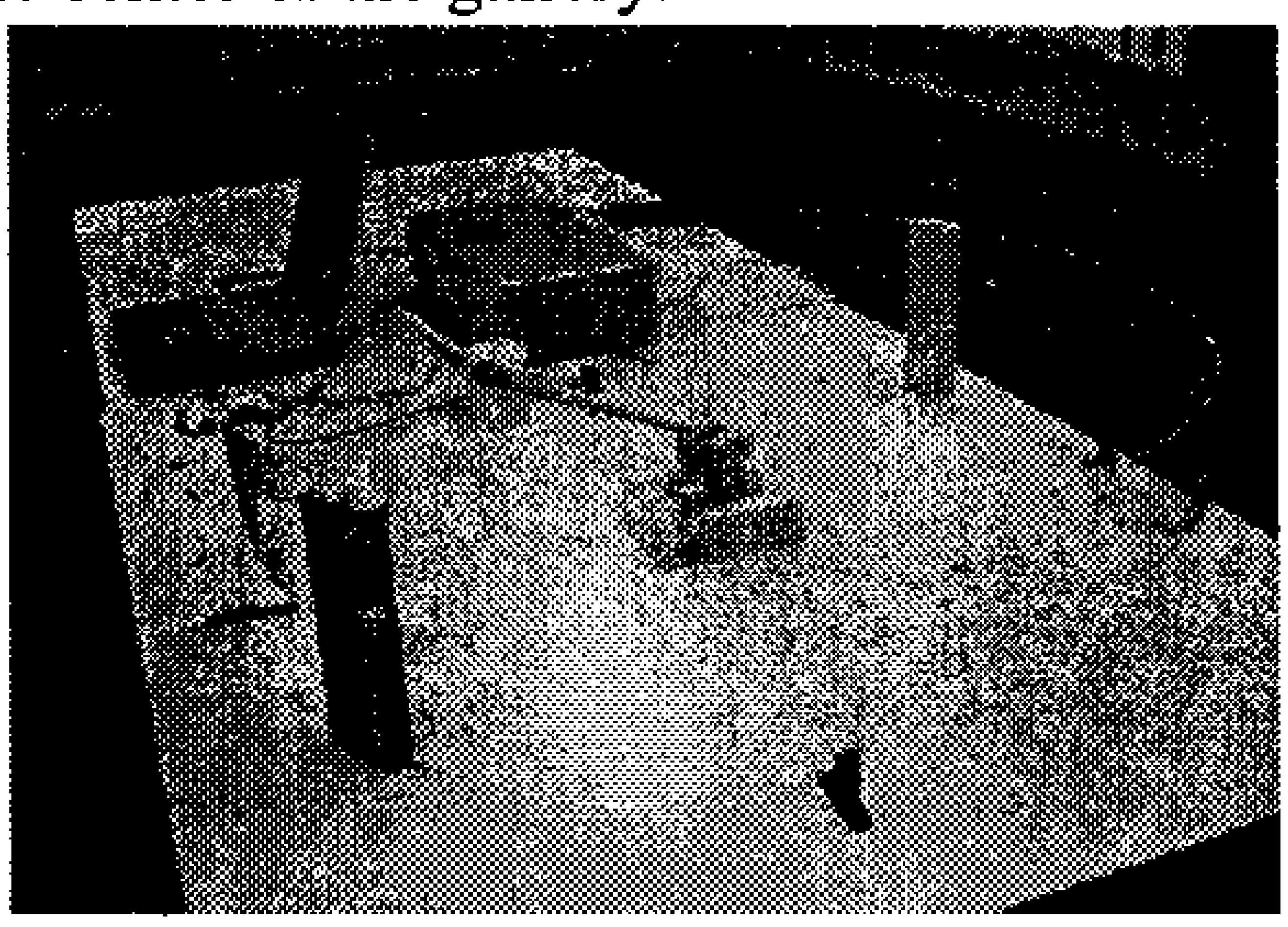
•••. 2•

We decided to repeat Relay's experiments and, using a microphone and a loudspeaker, make a sound intensity map of the gallery. Our hypothesis was that the intensity should be the largest near the walls of the gallery and the lowest behind the barriers if such are placed as shown on fig. 2c.

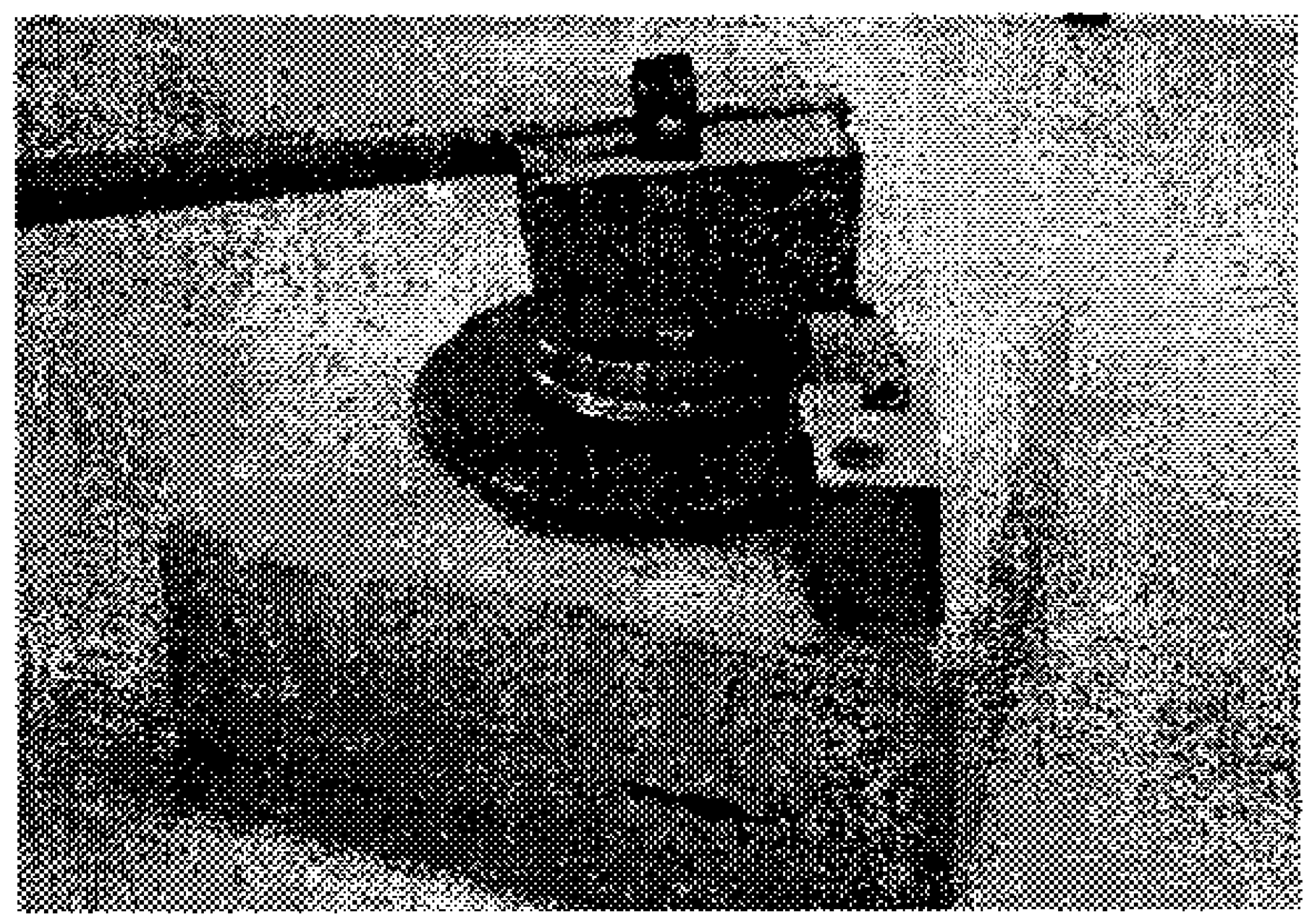
Experimental stage

The experimental stage is presented on figure 3a. The microphone is placed on an arm, which could be rotated around an axis, coinciding with that of the gallery – a Plexiglas cylinder with a diameter of 79 cm. The structure of the rotation mechanism that creates sound markers (clicks) every 36 degrees rotation is shown on fig. 3b.

The scanning of the area inside the cylinder is made in concentric circumferences with a decreasing radius. Scanning is impossible only in a circular area with a diameter of 25 cm in the center of the gallery.



•••. 3•

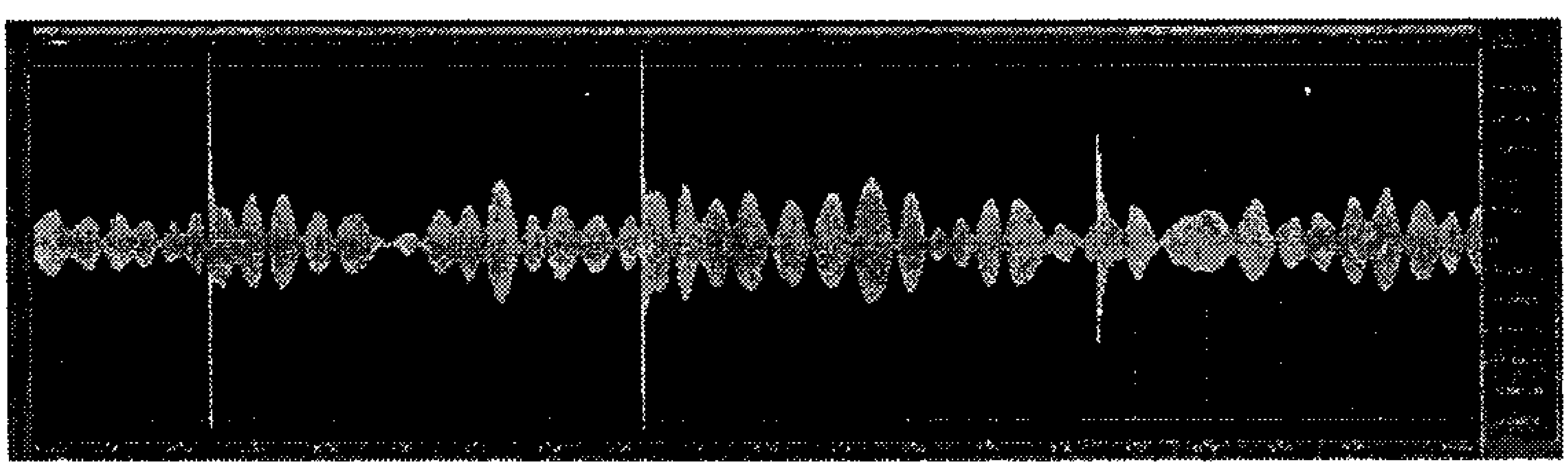


•••. 3•

The experiment

The first experiment was conducted with sound, which had a frequency of 10 000 Hz. This presupposed the occurrence of a standing wave inside the gallery, similar to the figures of Hladni. A part of the sound lane, made while scanning the most outer circumference, laying 3 cm away from the gallery's wall.

One could clearly see the sound markers – the sudden peeks, and the complicated standing wave pattern, which has 13 maxima between each two markers /fig.4/. When we approximated the results, using the maxima, taken from the whole area of the gallery, we got the sound map shown on fig. 5.

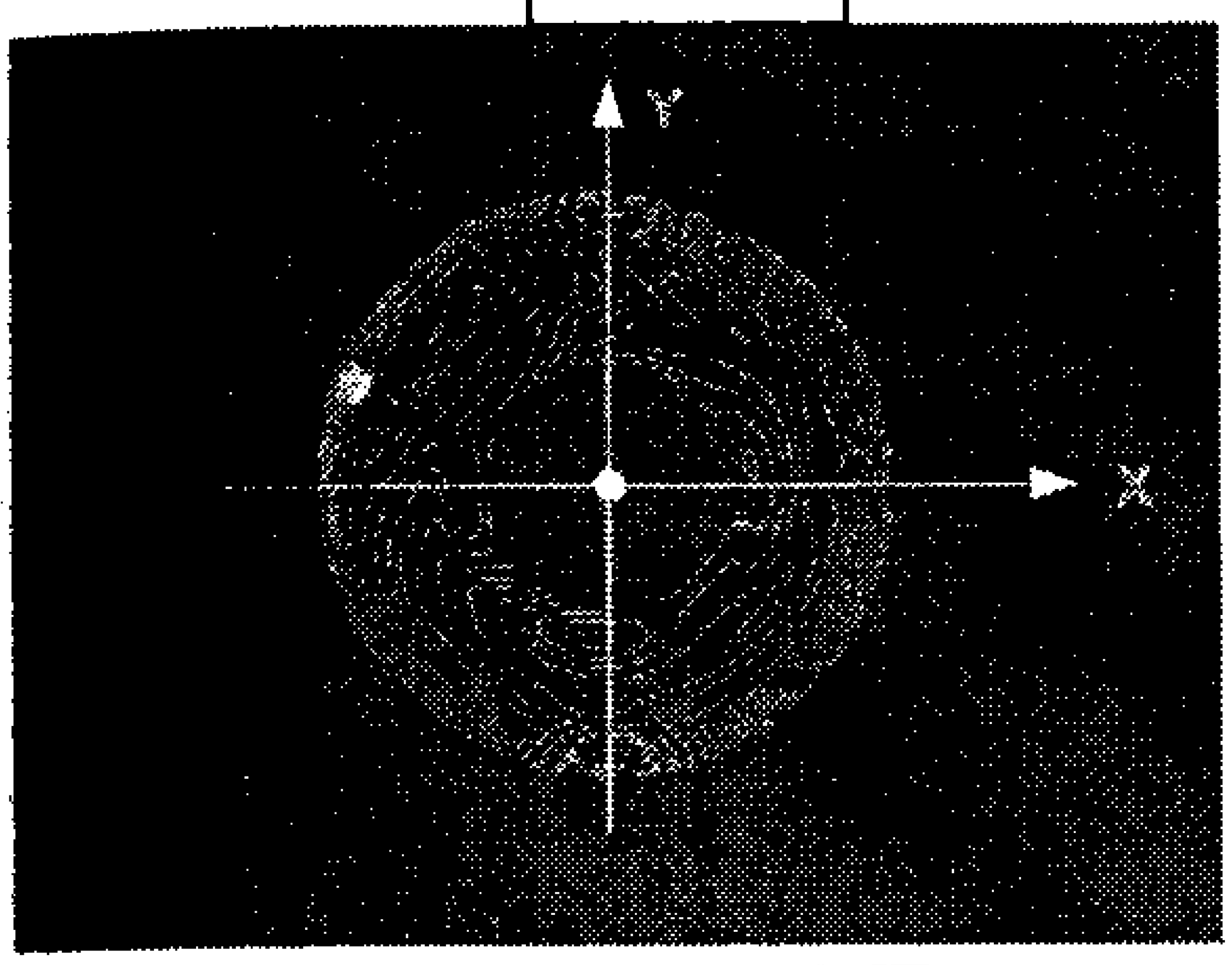


•••.4

•••. 5

The complicated distribution of the maxima does not allow us to see Relay's wave, but there is an obvious division into two areas – a central one and a peripheral one – of the map.

In order to avoid the complicated interferential structure we used white noise in the course of our second experiment. On fig. 6a and fig. 6б you can see a sound map, composed of isolines of the intensity, as well as a 3d graphica.



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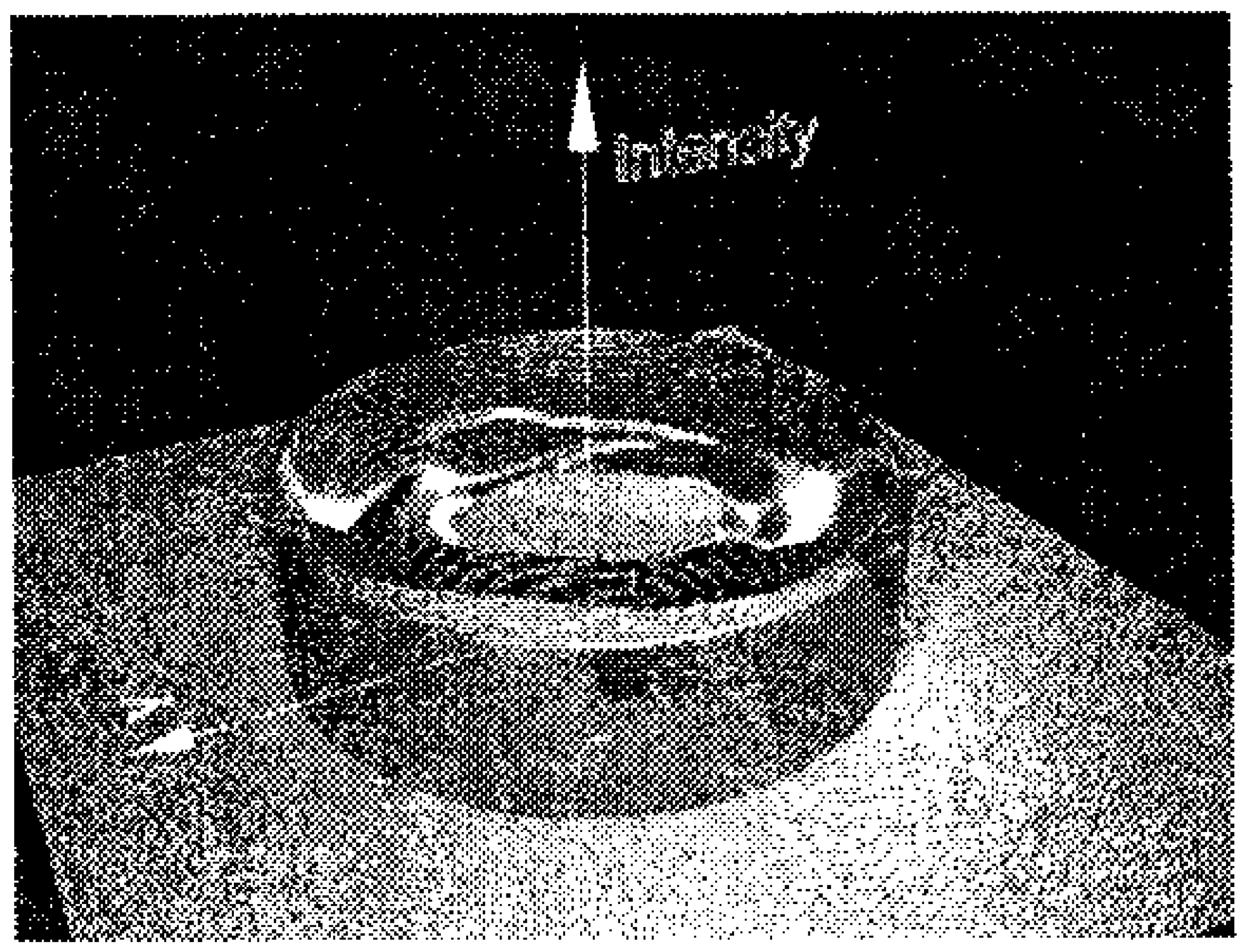
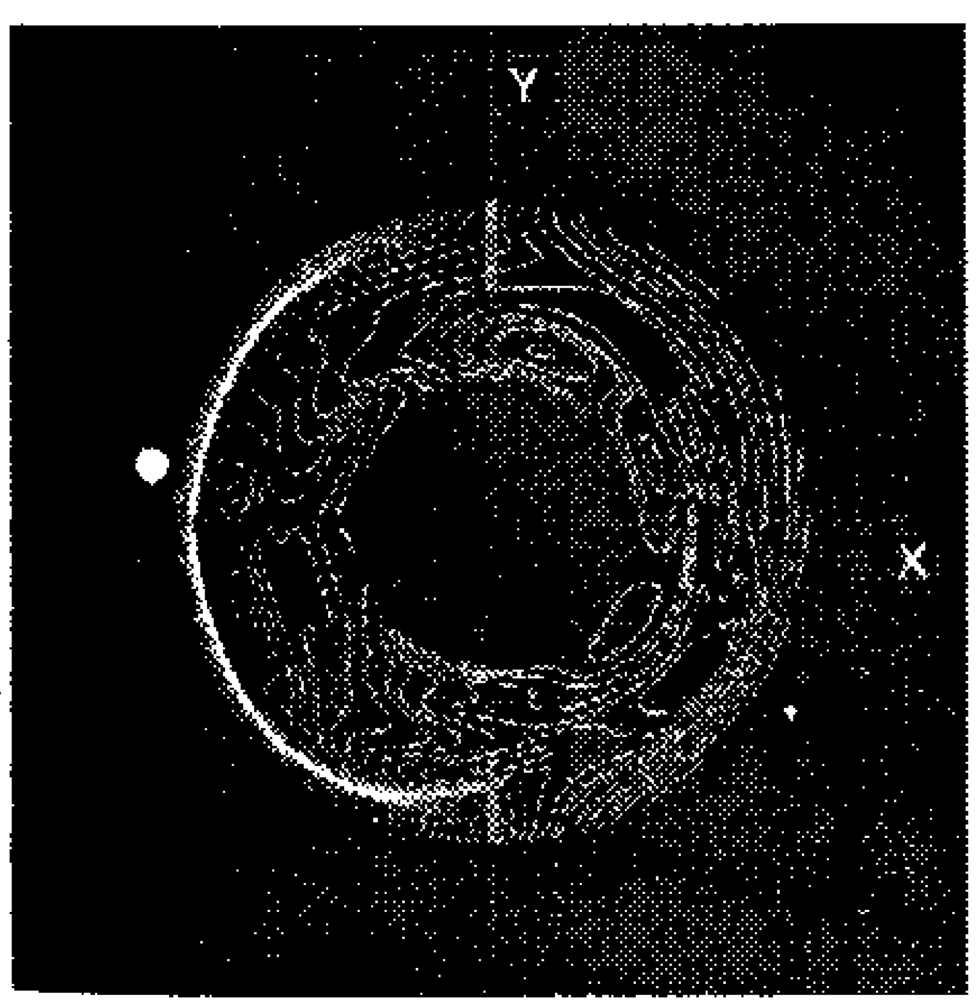


•••. 6•

Relay's wave is in the area where the intensity is greatest. In our model it has an average width of 6 cm. The position of the loudspeaker is marked with a cylinder, near the wall of the gallery.



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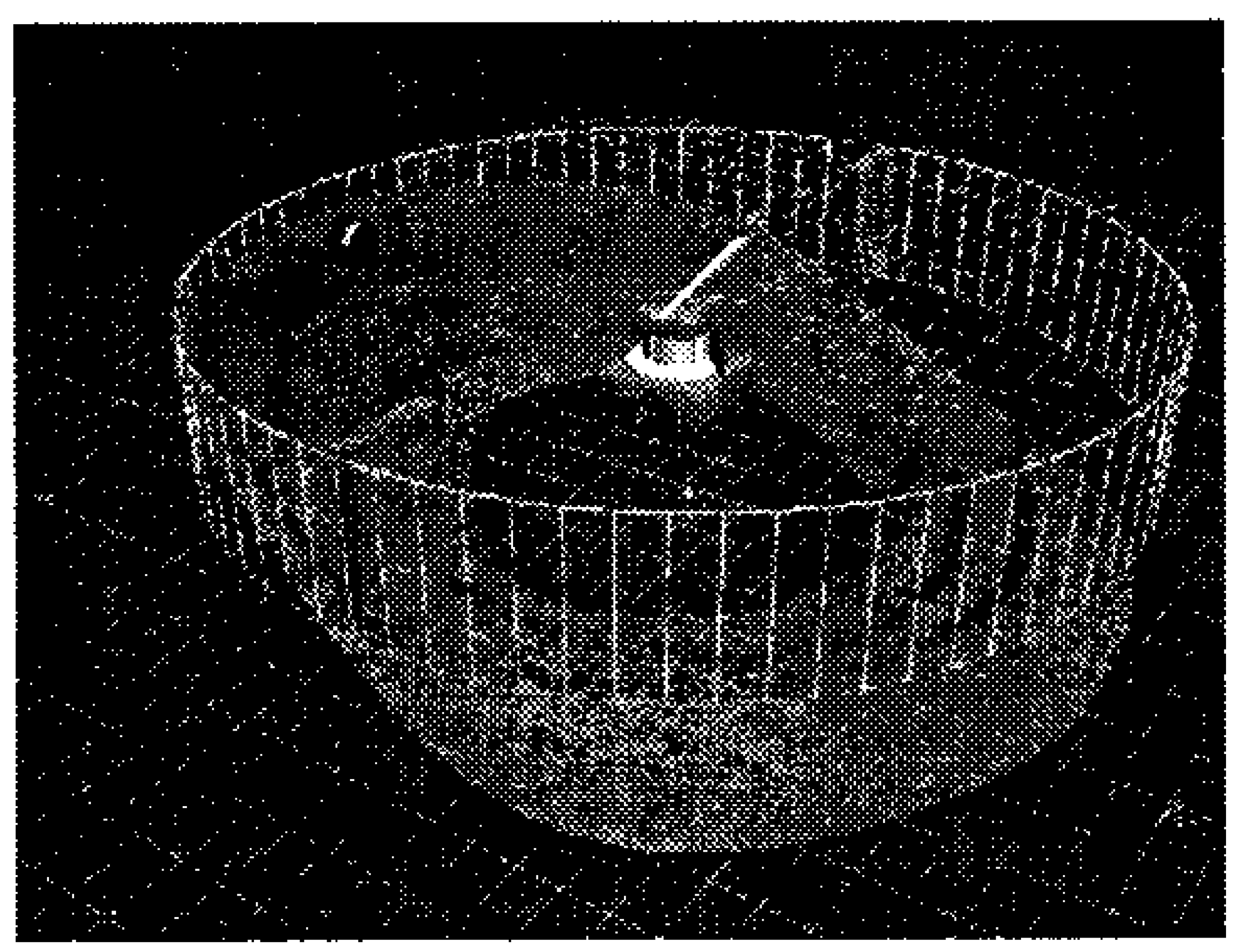

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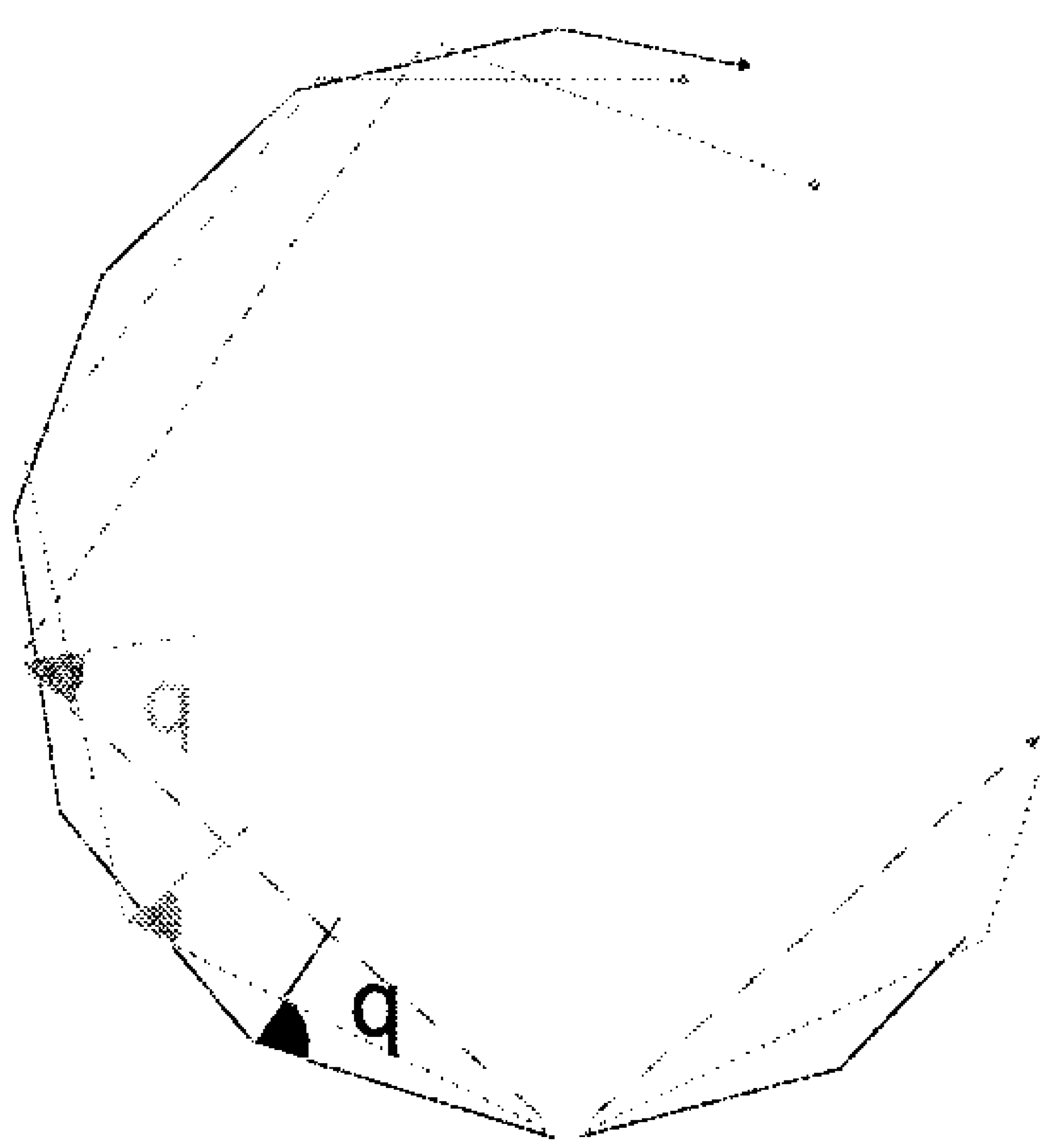
During our third experiment we put two barriers, as shown on fig. 2c. The map we composed of the gathered results shows unequivocally that the Relay wave exists only before the barriers. On fig. 7a and fig. 7б one could easily spot that wave.

The effect could be explained in the following way. The sound emitter is considered a point and the travel of the waves is marked with a ray, coming from that point.

The reflection coefficient will be different for different rays and the largest for those rays, which fall at the greatest angle.



•••.8

$$V = \frac{m.\cos\theta - \sqrt{n^2 - \sin^2\theta}}{m.\cos\theta + \sqrt{n^2 - \sin^2\theta}}$$

$$\sin^2\theta \le n^2$$

$$n = \frac{C_{pleksiglas}}{C_{air}} \quad ; \quad m = \frac{\rho_{pleksiglas}}{\rho_{air}}$$

$$\lim_{\theta \to \pi/2} V(m,n,\theta) = 1$$

In our case the sound of air in plexiglass is $C_{pleksiglase}$ = 4200 m/s, and C_{air} *=330 m/s in air.*

The densities are: □$_{pleksiglase}$ = *1260 kg/m³* , and for air □$_{air}$ *=1,3 kg/m³*.

The ray, drawn with the darkest line on fog.8 will travel with the least losses. That explains why the intensity of the sound wave near the wall is the greatest. This part of the wave, traveling with the least losses near the wall of the gallery is the Relay wave. Similar effects exist in the propagation of electromagnetic waves and these phenomena carry the group name “Whispering gallery effects”.

References:

1. Крауфрод, Ф., Волны, т.III, 1976г.
2. Персел.Е., Берклиевский курс физики, , т.II, Наука, 1972г.
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