8 Problem № 11: Singing tube

8.1. Solution of Australia

Problem №11: Singing tube

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The Problem:
It is possible to stabilize an inverted pendulum It is even possible to stabilize an inverted multiple pendulum /one pendulum the top of the other/. Demonstrate the stabilization and determine on which parameters this depends.

1. Abstract
The Rijke or Singing Tube is a vertical tube with metal gauze inserted in the lower half. After heating the gauze, a loud sound is produced. By investigating many aspects of the Singing Tube phenomenon, it was discovered that during a compression, cool air is drawn in and heated, this causes its pressure to change, augmenting the pressure maximum. This action creates and sustains the acoustic wave, however when determining the optimum gauze position, the two factors of consideration disagree. The Rayleigh Criterion and Rayleigh Index provide a more precise model of the relationship between gauze position and sound intensity. Expressions for heat transferred from the gauze, and where this heat was lost to provide insight into the mechanisms which determine how long the singing can be sustained, and thus what variables to experimentally investigate, these included tube length, diameter, material, shape and the gauze's heating time.

2. Interpretation
We were asked to investigate the 'singing' produced by an open tube over a flame. We defined 'singing' as a loud sound with definite frequencies, and minimal variation in sound intensity. We tried producing a sound with just a tube over a flame; however, we did not count this quiet and raspy sound as 'singing'. Research suggested that adding a piece of gauze inside the pipe would enhance the noise to a louder and measurable 'singing' tone. Therefore, we concentrated on this interpretation of a singing tube (also called the Rijke[8] tube).

3. Basic Theory
There were 2 phases of heating, in the initial heating phase, the Bunsen heats the gauze, yet no sound is produced. After removing the heat source, the tube 'sang' for a period of time before stopping. To understand how this creates sound, some basic theory is needed.

3.1. Waves
Sound or Acoustic waves are longitudinal, meaning that the particles vibrate in the direction of the wave's propagation. A sound wave is made of compressions (areas of high pressure) and rarefactions (areas of low pressure). Particle displacement and pressure variations can be modelled by the sinusoidal functions [1]:

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[8] P.L. Rijke was a professor of physics at the University of Leyden in the Netherlands when, in 1859, he discovered this phenomenon

$$s(x,t) = s_m \cos(kx - \omega t)$$

$$\Delta p(x,t) = \Delta p_m \sin(kx - \omega t)$$

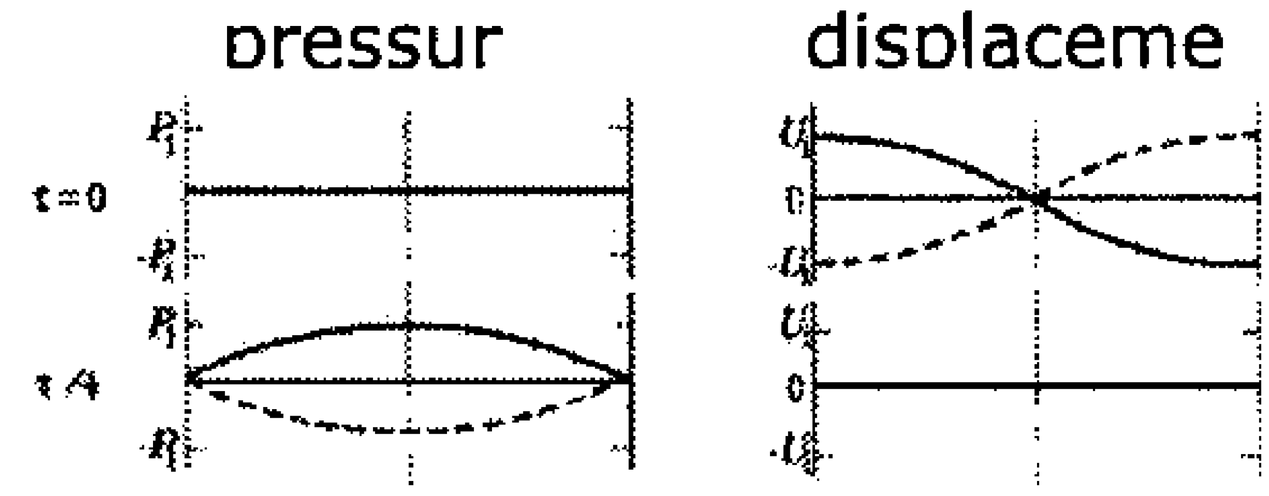


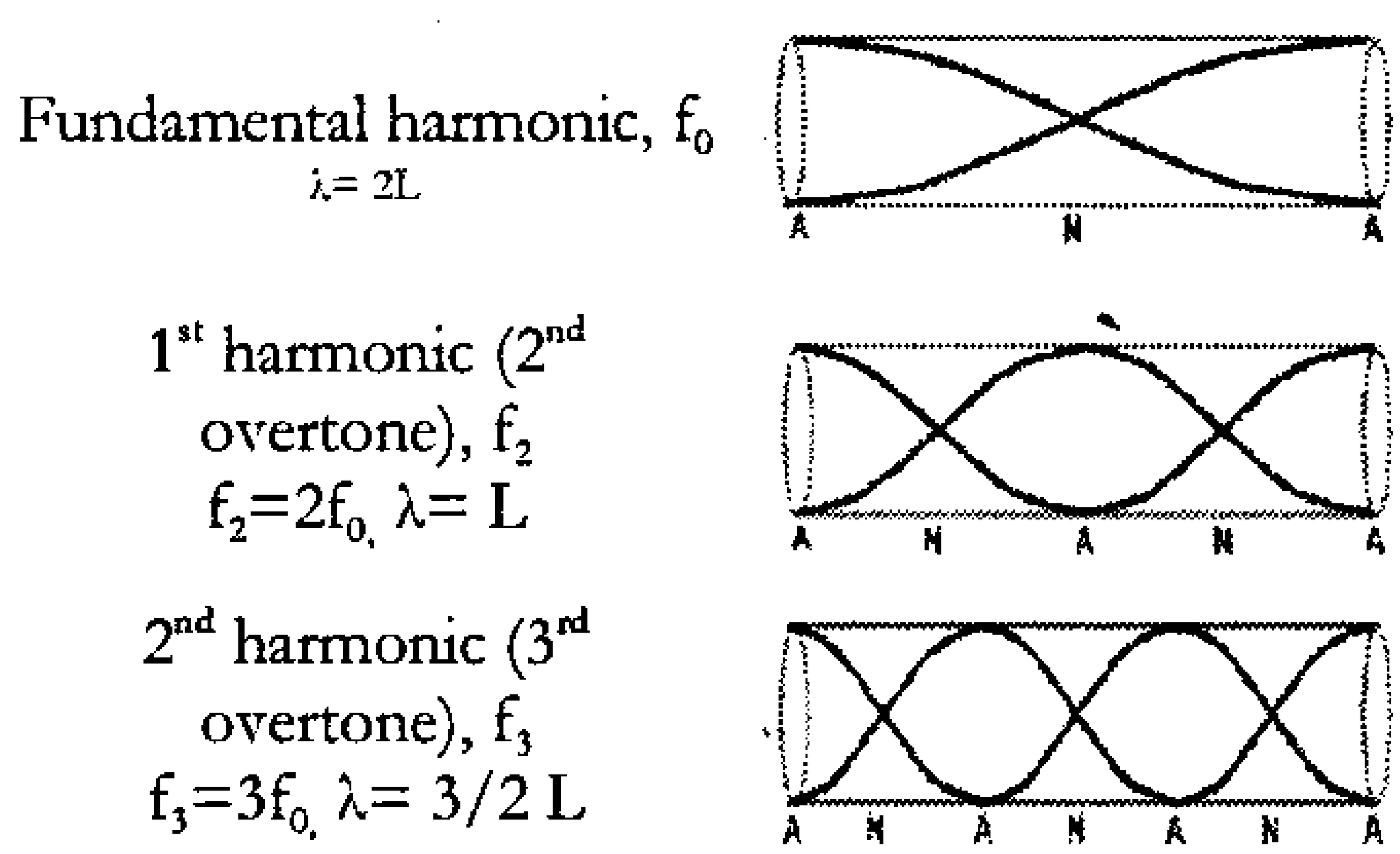
Figure 11 Pressure and Displacement functions are out of phase

Where ω is the angular velocity, p is the pressure, k is the wave number, s_m is the displacement amplitude and p_m is the pressure amplitude. The pressure and displacement functions are out of phase.

Hot air is less dense than cool air, so will rise. An initial compression is caused by the rising hot air, as the static air above it will 'bunch up' causing a compression. This hot moving air will also cause a pressure difference between the inside of the tube and the ambient outside pressure, so a pressure barrier at the end is formed. Some of the initial compression will then reflect of this barrier back down the tube. The reflected compression will constructively or destructively interfere with the rising hot air forming a standing wave.

3.2. Standing Waves and Resonance

The standing wave is cause by the interference of the reflected wave on itself. A standing wave consists of nodes (areas of least particle and velocity displacement) and antinodes (areas of maximum particle and velocity displacement). The standing wave causes us to hear a continual tone. Resonance is caused when the standing wave is continually reinforced, the amplitude is increased creating a very loud sound, the air column in the pipe is said to resonate.



3.3. Harmonic frequencies

For the tube to resonate, the tube length must be a multiple of half wavelength of the standing wave, we can express this as $n\lambda = 2L$, where λ is the wavelength, and L is the tube length. The first harmonic is called the fundamental harmonic (f_0) and only has one nodal point in the tube. Higher harmonics are multiples of the fundamental (see figure 2).

From the wave equation, $v = f\lambda$, where v is the velocity of sound, and the relationship above, we can derive a formula for the frequency [2].

$$v = f\lambda = f\frac{2L}{n} \qquad n\lambda = 2L \qquad f = \frac{nv}{2L}$$

However, the air particles also vibrate with a slight sideways motion, and beyond the end of the tube. This results in a necessary 'end correction' [3], where d is the diameter of the tube.

$$f = \frac{nv}{2L + 0.8d}$$

For the fundamental frequency (n=1), and at a temperature of 34°C (where v, the speed of sound is $351 ms^{-1}$), the frequency becomes:

$$f = 175.5\left(\frac{1}{L + 0.4d}\right)$$

4. Sustaining the acoustic wave

Some of the wave and thus energy escapes and is emitted at the end of the tube. Therefore heat energy must be added to sustain the oscillations. There must also be a continuous air flow; we demonstrated this by turning the tube to horizontal, which removed the thermal convection. As predicted, the tube did not sing.

4.1. Velocity and Pressure Fluctuations

The flow past the gauze is a combination of two motions. There is a uniform upward velocity, caused by the rising hot air, and a varying velocity, u' caused by the sound wave (oscillatory particle vibration velocity).

There is an ambient or mean pressure, as well as the varying pressure, p' caused by the compressions and rarefactions of the acoustic wave.

4.2. Amplifying the wave

For half the cycle, the varying and uniform velocities are in the same direction, which is when the particles around the gauze are vibrating upwards, in the same direction as the thermal convections. Therefore, air will be drawn up into the tube until pressure reaches a maximum. Most of this air will already be warm (having been expelled from the hot tube during the last cycle). However just before the pressure reaches a maximum, some cool air is drawn in, this is because the uniform thermal convection pushed some of the warm air out of the top of the tube last cycle. This cool air is quickly heated by the hot gauze, so there is a large heat transfer. This causes the (previously cool) air's pressure to increase, adding to the pressure maximum. Therefore, although energy is being lost at the top of the tube, every cycle a small part of the gauze's heat energy is used to increase the pressure maximum, thus amplifying the wave.

4.3. Optimum gauze position

Since the singing is caused by cool air increase in pressure, there two things which must be considered when determining the optimum gauze position. First, the placement of the gauze should be such that the amount of cool air heated is optimised. As it is the combination of varying and uniform particle velocity that determines how much cool air gets heated, this would suggest that the gauze should be placed where the varying velocity is at a maximum, at the anti-node, for the fundamental frequency, the anti-node is at the end of the tube.

The second aspect to consider is where the cool air's pressure increase will have most impact. The pressure increase reinforces the varying acoustic pressure, but at an anti-node, the varying pressure is zero, so placing the cool air's pressure increase would have no effect. The varying pressure is at a maximum at a node, so according to this, the gauze should be placed at a node, which is at the middle of the tube for the fundamental frequencies.

These aspects clearly disagree, so all this tells us is that the optimum position is a compromise, somewhere between the end and middle of the tube. The Rayleigh Criterion is a mathematical description of the interaction between pressure and velocity fluctuations and allows for more precise optimisation of gauze placement.

5. The Rayleigh Criterion

As energy escapes at the top end of the tube, to sustain the acoustic wave, heat energy must be added at certain points, according to the Rayleigh Criterion [5]; "If heat be given to the air at the moment of greatest condensation, the vibration is encouraged". This means that most heat must be added in a compression. Please see section V of the Onera short lecture course of Combustion instabilities in liquid rocket engines [6] for a mathematical proof and derivation of the Rayleigh Criterion.

5.1. Velocity and Heat Transfer

When there is a large heat transfer, many particles get heated and thus, many rise, this creates a large varying velocity. So the heat transfer, Q' creates the varying velocity, u'.

As mentioned previously, when cool air is drawn in, heat is transferred easily so the heat transfer, Q' is large. When the varying velocity, u' is down and against the upward flow, there is little airflow, less cool air is drawn in, and the gauze is surrounded by warm air. Therefore there is less heat transfer, Q' is small.

So Q' varies with u', and there may be a time lag (as it is a cause and effect, rather than simultaneously). This can be expressed by: $Q' \alpha\, u'$. Where τ is the time lag. In summary, the heat transfer is creates the varying velocity, and it is sustained by the varying velocity, so once the cycle is initiated, the waves are sustained.

5.2. Rayleigh Criterion Integral

The Rayleigh Criterion can be expressed as "If p' > 0 and Q' > 0 or p' < 0 and Q' < 0, the wave is sustained" where p' is the acoustic pressure, and Q' is the heat transfer. A convenient way to express this statement is in the following integral, where T is the period of the wave, and R is the Rayleigh index [7]:

$$R = \frac{1}{T}\int p'Q'\,dt$$

The Rayleigh Criterion can now simply be expressed as if R > 0 the wave is amplified. Sound intensity will be greatest when I is maximised.

5.3. Optimum gauze position

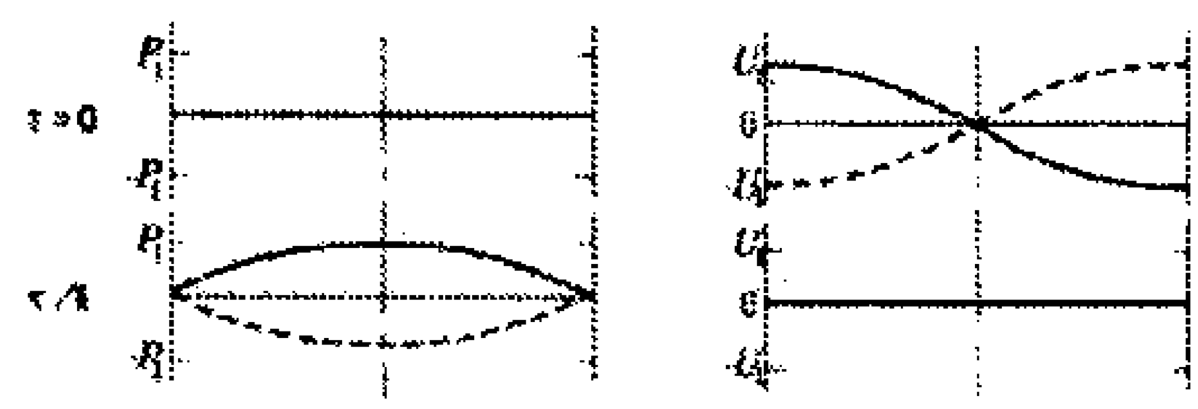
The optimum gauze position (for sound intensity) will occur where the Rayleigh Index, R, is maximised. It was seen in section 5.1, that the heat transfer is proportional to the varying velocity, $Q' \alpha\, u'$ so R is proportional to the product of acoustic pressure and velocity.

$$R = \frac{1}{T}\int p'Q'\,dt \;\Leftrightarrow\; R \propto \frac{1}{T}\int p'u'\,dt$$

$$\therefore\; R \propto p'u'$$

Now we need to maximise p' u'. It was explained in section 3.1, that the acoustic pressure and velocity are sinusoidal:

$$u'(x,t)=u'_m \cos(kx-\omega t)$$
$$p'(x,t)=p_m \sin(kx-\omega t)$$



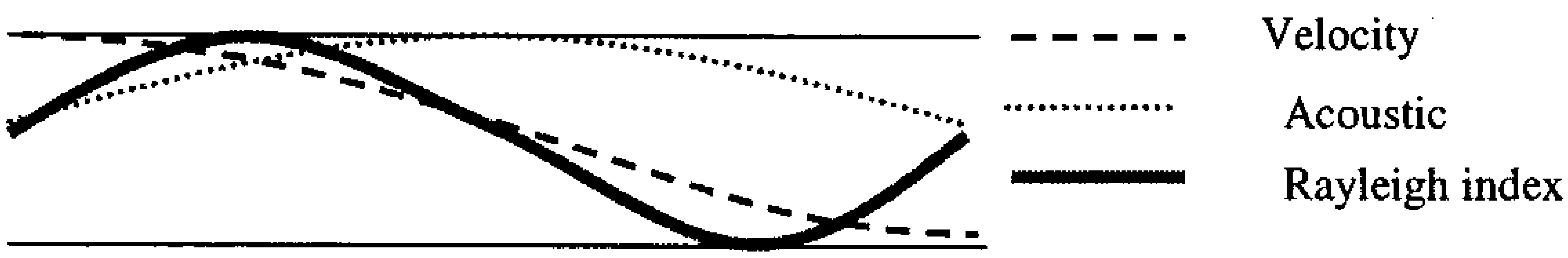
Since we are only concerned with proportionality, and u_m and p_m are constants (maximum amplitudes), they can be neglected:

$$R \propto \cos(kx-\omega t)\sin(kx-\omega t)$$

Here we can substitute in the trigometric identity, 2 sinx cosx = sin2x:

$$R \propto \frac{1}{2}\sin 2(kx-\omega t)$$
$$R \propto \sin 2(kx-\omega t)$$

This function has a similar shape to the function for acoustic pressure, but it has half the period, therefore, R will be at a maximum halfway between the end of the tube and the node in the centre of the tube, where the pressure has a maximum:



N.B. waveforms are for comparative purposes only; the amplitude of the Rayleigh index would be different due to constants of proportionality

Figure 4 Diagram showing that the Rayliegh index is proportional to the product of velocity and pressure, and is at a maximum 25% up the tube, thus this is the optimum position for the gauze

So the optimum gauze position is where R is at a maximum, which is 25% up the tube, and the sound intensity should follow a similar sinusoidal curve to the Rayleigh index as the gauze is moved along the tube. This supports the qualitative explanation presented in section 4.4.

5.4. Gauze in upper half of tube

However, by this logic, a sound should be created when the gauze is 75% up, as this is also halfway between the fundamental node and anti-node, however, the cool air is drawn towards the centre just before the pressure maximum, and is pushed up past the centre after the pressure maximum. When the gauze is in the top half, the

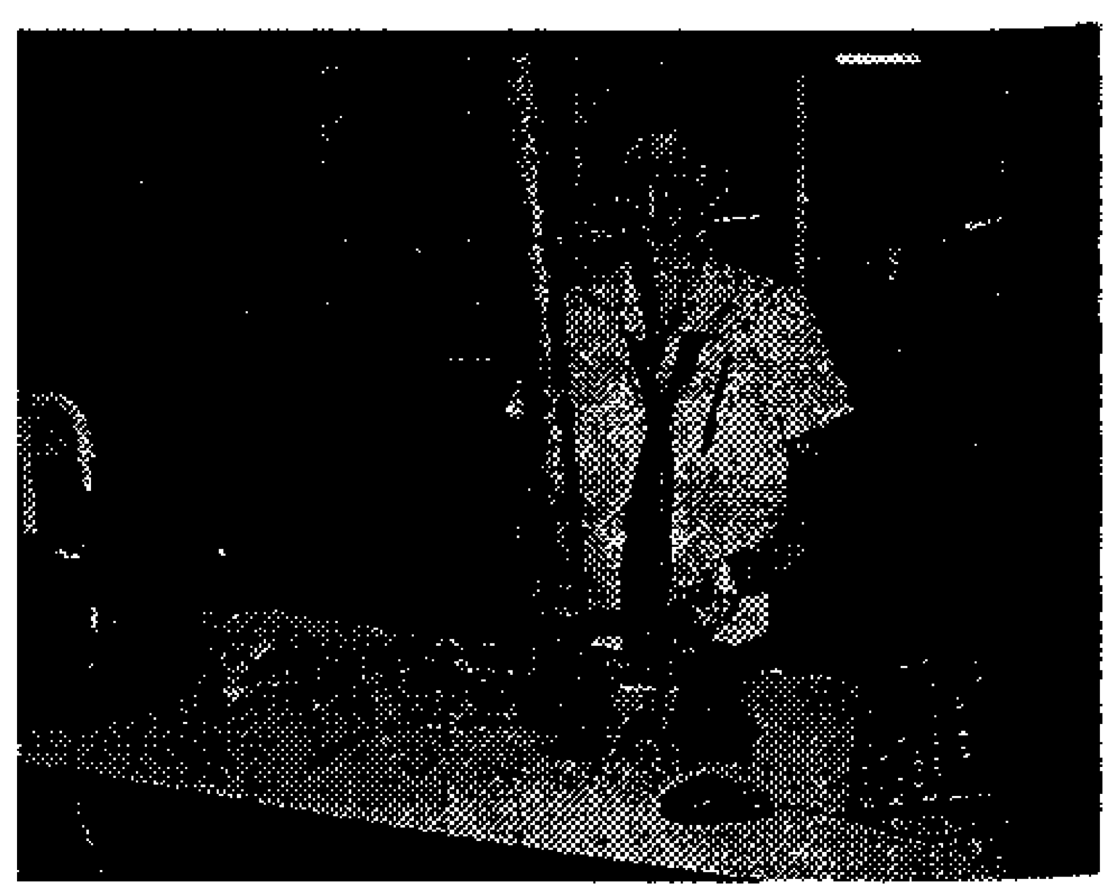


Figure 12 Using a vacuum cleaner to reverse the air flow, changing the optium gauze position

cool air is heated just before the pressure minimum, so the increase in the cools air's pressure cancels the acoustic decrease in pressure, thus diminishing the sound.

We proved this by reversing the direction of the thermal convections with a vacuum cleaner. This pushed cool air in from the top rather than the bottom. When using an artificial air flow, a sound was created with gauze in the top half.

6. Heat Transfer

It has been demonstrated that the gauze will transfer little heat in a rarefaction, and lots in a compression, but how much heat is transferred, and what effect will this have on the singing time? Expressions for the amount of heat transfer will be derived in the following sections.

6.1. Heat Transfer Expressions

Let us first examine where the heat from the gauze is lost to heating the air to provide the uniform thermal convection, sustaining the acoustic wave (heat transferred to cool air, discussed in section 7), and losses, which will be predominantly through conduction to the tube walls. The heat absorbed by the gauze through heating must transformed to the energy forms listed above, as energy is conserved, these must be equal.

This can be expressed in a differential equation as;

$$\frac{dQ_{gauze}}{dt} = \frac{dQ_{convection}}{dt} + \frac{dQ_{emitted\ wave}}{dt} + \frac{dQ_{losses}}{dt}$$

Where dQ/dt is the rate of heat transferred (measured in Joules per second). The standard equation for heat transfer to the gauze during heating is [2]:

$$\frac{dQ_{absorbed\ by\ gauze}}{dt} = c_g m_g \frac{dT_{gauze}}{dt}$$ Where c_g is the specific heat capacity of the gauze, m_g is the mass of the gauze, and dT/dt is the change in temperature of the gauze. The heat lost through convection to the air can also be modelled by the standard heat transfer equation [2]:

$$\frac{dQ_{\text{lost through convection}}}{dt} = c_{air} m_{air} \frac{dT_{air}}{dt}$$ Where c_{air} is the specific heat capacity of air, m_{air} is the mass of air, and dT/dt is the change in temperature of the air, after passing the gauze. The average rate at which kinetic energy, E_k of a sound wave escapes can be expressed as: [1]

$$\frac{dQ_{emitted\ wave}}{dt} = \frac{dE_k}{dt} = \frac{1}{16}\rho A v \omega^2 s_m^2$$ Where ρ is the density of air, A is the cross-sectional area of the tube, v is the velocity of the wave, ω is the angular velocity, and s_m is the displacement amplitude. Most of the losses will be thermal conduction to the tube walls, this can be modelled as follows:

$$\frac{dQ_{tube}}{dt} = kA(T_{hot} - T_{cold}) \quad \Leftrightarrow \quad \frac{dQ_{tube}}{dt} = kA\frac{dT_{tube}}{dt}$$

Where k is the thermal conductivity co-efficient. These components can now be combined, so the original equation:

$$\frac{dQ_{gauze}}{dt} = \frac{dQ_{convection}}{dt} + \frac{dQ_{emitted\,wave}}{dt} + \frac{dQ_{losses}}{dt}$$ Becomes:

$$m_g c_g \frac{dT_{gauze}}{dt} = m_{air} c_{air} \frac{dT_{air}}{dt} + \frac{1}{16}\rho A v \omega^2 s_m^2 + kA\frac{dT_{tube}}{dt}$$

6.2. What can this expression tell us?

This expression reveals the limiting factor which affects how much heat must be given to the gauze for singing to occur, and how long the tube will sing for. The tube will start singing when:

$$\frac{dT_{gauze}}{dt} \geq \frac{1}{m_g c_g}\left(m_{air} c_{air} \frac{dT_{air}}{dt} + \frac{1}{16}\rho A v \omega^2 s_m^2 + kA\frac{dT_{tube}}{dt} \right)$$

To keep singing, the gauze must have enough heat or a large enough temperature gradient to provide constant heat to the air convections and losses to the tube. So the tube will stop singing when:

$$\frac{dT_{gauze}}{dt} \leq \frac{1}{m_g c_g}\left(m_{air} c_{air} \frac{dT_{air}}{dt} + \frac{1}{16}\rho A v \omega^2 s_m^2 + kA\frac{dT_{tube}}{dt} \right)$$

Therefore, this expression reveals many factors and variables which affect the onset and discontinue of the singing. The type of gauze will have an effect since the expression starts, $\frac{1}{m_g c_g}$, and both the mass and specific heat capacity are properties of the gauze, since it is the inverse of mass, the tube should sing for earlier and for longer when a heavier gauze is used, since a heavier gauze could absorb more heat. The atmospheric conditions will also affect the singing, since at different temperatures and humidities, the specific heat capacity of air changes. Also, if the air starts of warmer, then the change in temperature as it passes the gauze (dT_{air}/dt) would be less. This is why all the trails for an experiment were conducted on the same day, or when the temperature is similar (within 3 degrees). The cross sectional area, A, which is directly proportional to the diameter, appears in all three terms on the left had side of the heat transfer expression ($m_{air} = Av\rho t$) and so that is why the cross sectional area was a controlled variable in experiments, and tube diameter was chosen to be investigated as an independent variable. s_m

Figure 13 vacuum cleaner used to replace the thermal convection, the tube sings in a horizontal in 3rd term is the displacement amplitude, it describes how far the particles vibrate and thus determines sound intensity. This is why either sound intensity or singing time were measured, because they effect each other.

In the very last term, $kA\frac{dT_{tube}}{dt}$, k is the thermal conductivity co-efficient, which is a property of the material, this highlighted the importance that the tube material has, which is why the material was investigated as an independent variable, and controlled in all other experiments.

7. Stopping the singing

There are three methods of stopping the tube from singing; hold the tube horizontally, place heat source under tube, and let the gauze cool.

When the tube is held horizontally, there is no thermal convection, so no cool air brought in contact with the gauze, so the wave cannot be amplified (we demonstrated that it was the lack of thermal convection preventing amplification by using a vacuum clean to produce artificial convections). When the Bunsen burner or another heat source is under the tube, the air passing the gauze has already been heated, so with no heat transfer possible, there is no pressure increase to sustain the acoustic wave. As the gauze losses its heat energy to the passing air, it will gradually cool, until it is no longer able to supply enough energy to facilitate a pressure increase, and sustain the wave.

8. Cold Gauze

The change in temperature between the gauze and the air causes the phenomenon; therefore, a similar sound can be produced with a gauze which is cooler than the surrounding air. A weak sound was produced when cold gauze (from freezer is sufficient in the Brisbane summer) was inserted 25% up the tube. The cold gauze causes a downwards convection current, so warmer air is drawn in at top just before the pressure maximum, and reaches the gauze in the bottom half just before the pressure minimum, the warm air is rapidly cooled by the gauze, its pressure decreases, enforcing the pressure minimum.

9. Our Investigations

9.0. General Procedure

The 1mm steel gauze was inserted ¼ of the way up each tube. The Bunsen burner was slid under the tube and 10 seconds later, the Bunsen burner was removed and another person recorded the singing with the microphone. This was repeated for all the tubes, five times then averaged.

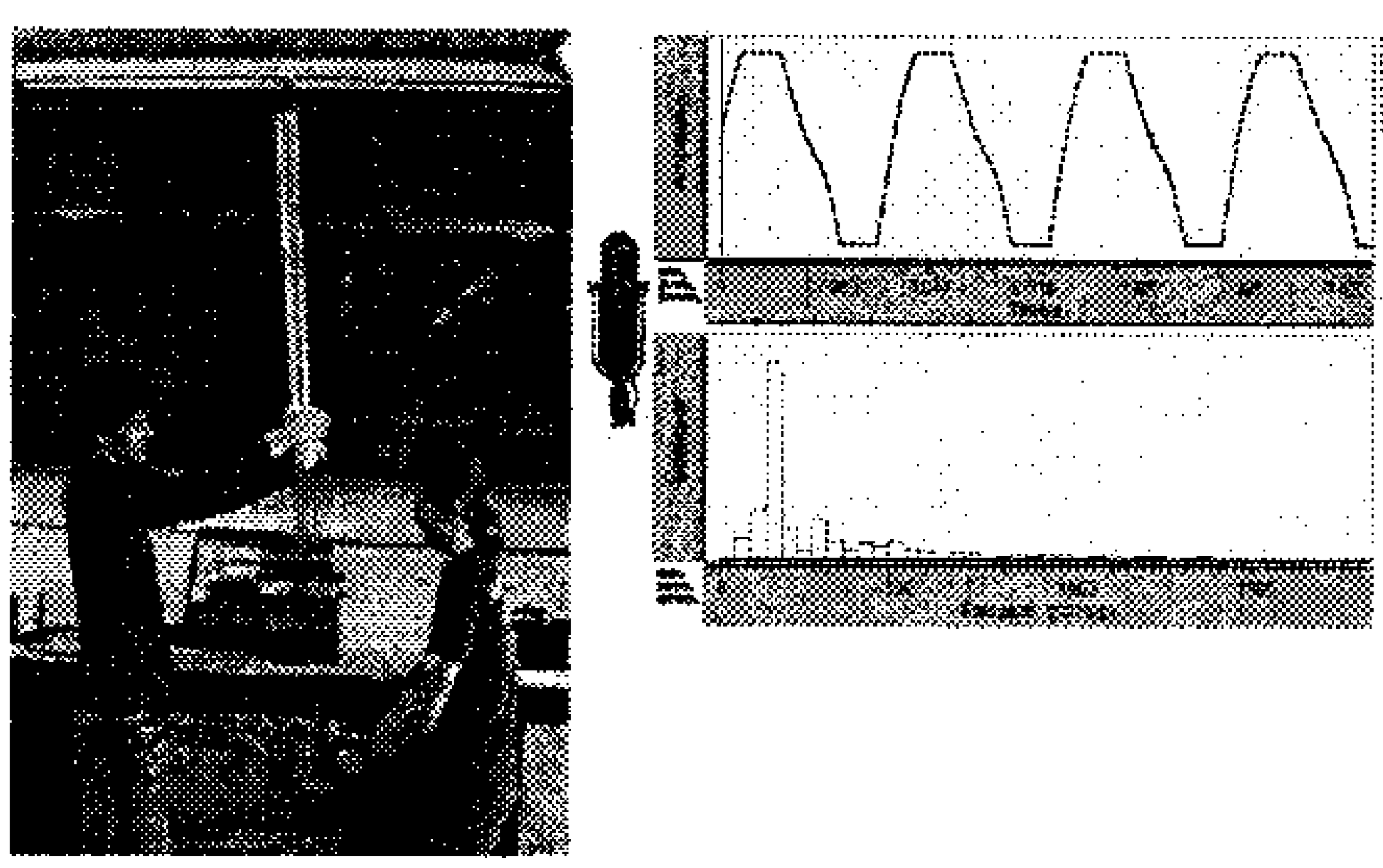


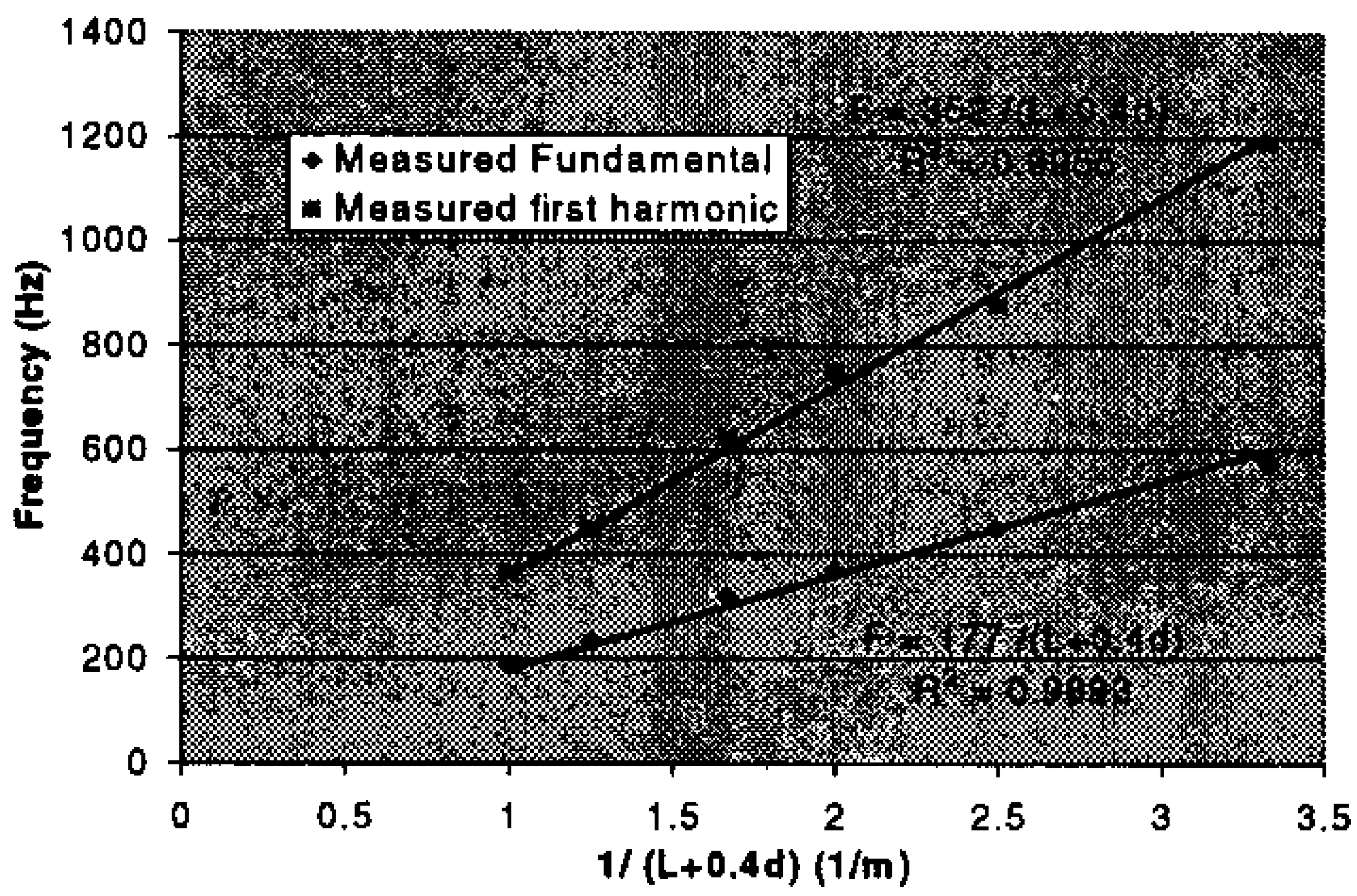
Figure 14 Sample datum collected and demonstration

9.1. Tube Length

9.1.1. Hypothesis

As the tube length increases, the wavelength of the sound will increase, and therefore its frequency will decrease. The frequency and tube length should have the following relationship: $f = 175.5\left(\frac{1}{L+0.4d}\right)$. As the tube diameter was constant, the graph of frequency vs. $\frac{1}{L+0.4d}$ should be linear.

9.1.2. Results



There was an audible difference in pitch between the 'singing' from the larger and smaller tubes, this is evidence that the frequency was changing. The longer tubes produced a weaker tone.

Figure 4 Graph of results from Tube Length experiment, linear relationship

9.1.3. Discussions

We derived in section 4.3 that the fundamental frequency (at 34°C) was given by: $f = 175.5\left(\frac{1}{L+0.4d}\right)$ So in the above graph, a linear relationship predicted, with a gradient of 175.5. In the experimental results, the linear relationship is reasonable since the datum points are scattered on both sides of the line, the R^2 value is 0.9893, and the uncertainties are small. The measured gradient was 177 ± 4. Therefore, the Harmonic equations derived in section 4 are accurate (within the uncertainty) and our hypothesis was supported. We have also consolidated the knowledge on which these equations were derived.

9.1.4. Conclusions and Reflections

In section 9.1.3.1. it was observed that the sound volume and therefore intensity decreased as the tube length increased, this was unexpected, and upon further qualitative experiments, when heated for a longer period of time, the longer tubes 'sung' with more intensity than the shorter ones. One possible explanation for this discrepancy is that in longer tubes, the gauze is further from the Bunsen burner flame, so more of the flames heat may have been used to heat the longer air column and tube wall in the longer tubes than in the short tubes. Hence, an improvement to the procedure would be to measure the temperature of the gauze, and stop heating once a certain temperature had been attained, rather than heating for a specific duration of time.

9.2. Tube Diameter

9.2.1. Hypothesis

As the tube diameter increases, the frequency will decrease. The relationship should be $f = 175.5\left(\frac{1}{L+0.4d}\right)$. In this experiment, the tube length, L was constant, so the graph of frequency vs. $\frac{1}{L+0.4d}$ should be linear, with a gradient of 175.5.

9.2.2. Results

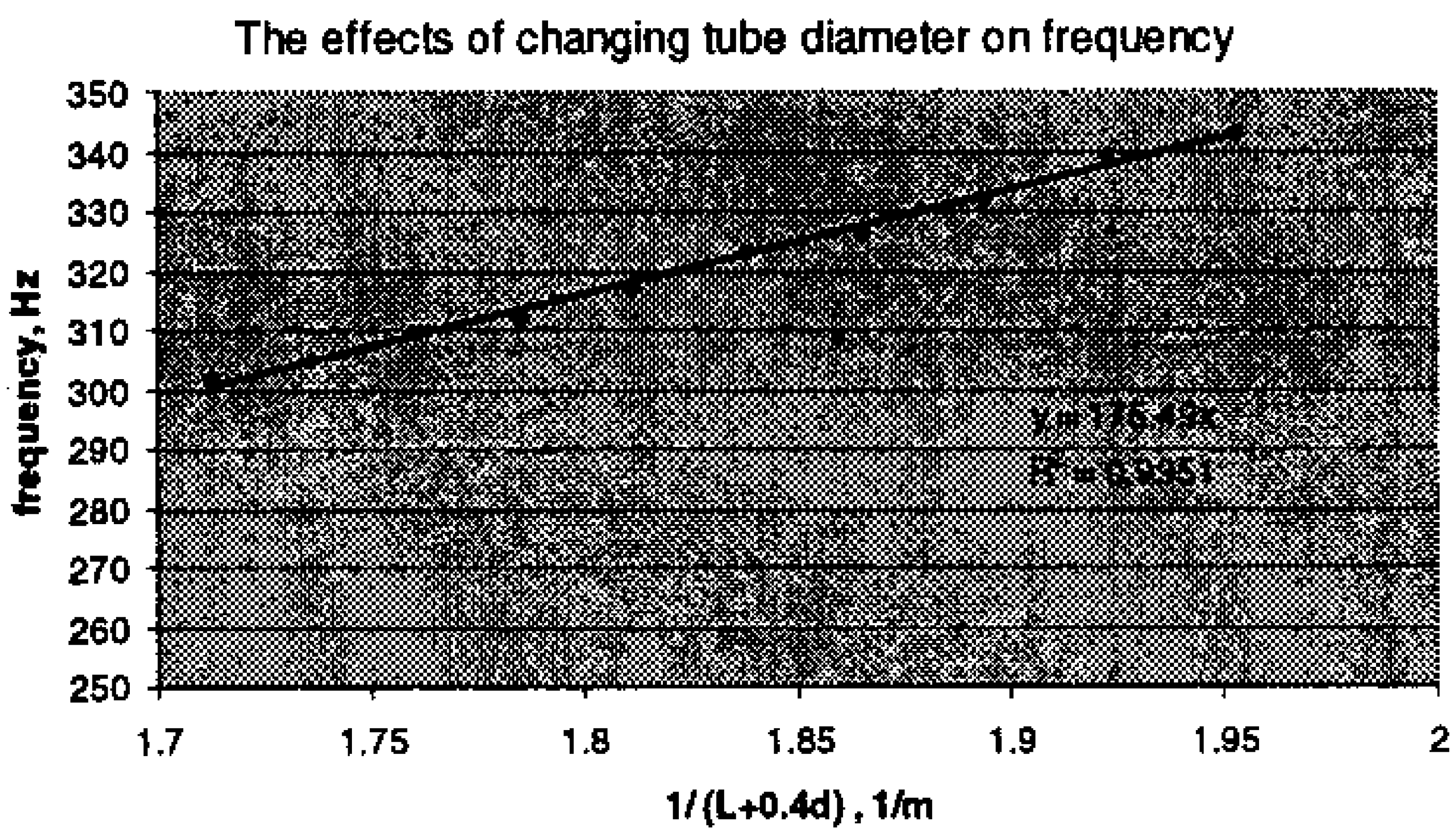


Figure 6 Graphed results from Tube Diameter experiment, linear relationship

For tubes bigger than 20cm in diameter, it became increasingly difficult to produce a sound. The larger diameter tubes had a larger sound intensity until they stopped producing a sound. There was very little audible variation in pitch, since the frequency only varied ~10%.

9.2.3. Discussions

We derived in section 4.3 that the fundamental frequency (at 34°C) was given by: $f = 175.5\left(\frac{1}{L+0.4d}\right)$Experimentally, there was a linear relationship between the frequency and 1/(L+0.4d). Since the points at scattered both sides of the line, with an R^2 value of 0.9952, this relationship is reasonable. In the above graph, the predicted gradient was 175.5, and the measured gradient was 174 ± 5. Therefore, (within the uncertainties), it has again been show that these Harmonic equations are accurate. We have also supported our hypothesis, that as the tube diameter increases, the sideways vibration of the particles and therefore the tube's end correction increases, thus frequency will slightly decrease. However, it was observed in section 5.2.3.1 that this effect was less noticeable than it was when changing the tube length in experiment 5.1. This is because in our derived equations, tube diameter is multiplied by 0.4, so is decreased, and the diameter of all the tubes was less then their length, thus changing the diameter of the singing tube had less of an effect than changing the length. The increase in sound intensity with increasing tube diameter is due to a larger air flow since there is more hot gauze available to heat the air and cause the thermal convections.

9.2.4. Conclusions and Reflections

In section 5.2.3.1, it was observed that it became difficult to produce a sound when the tube diameter was greater than 0.20m, and the sound intensity decreased. A possible explanation for this is that will a wide tube, the barrier of pressure difference necessary for the travelling wave to reflect and cause a standing wave becomes unstable and weak, due to a less defined 'end' of the tube. This would cause only some or none of the wave to be reflected, producing a weaker standing wave. One possible method of testing this would be to use a very sensitive pressure probe, so detect the strength and location of the barrier, such a sensitive instrument was not available for our use.

9.3. Tube Shape:

9.3.1. Hypothesis

Since it is the column of air that resonates, not the tube its self, changing the horizontal shape of the tube should have no effect, as long as the tube still contains a region of air.

9.3.2. Results

Shape Approximation	\ /	\ /	/ \	/ \	> <	((
% change in diameter	200	400	50	25	0 (centre =200%)	0
observation	Weak sound	No sound	Normal sound	No sound	Gauze wouldn't stay in, No sound	Slightly weaker sound
Frequency	221	-	226	-	-	213

Table 4 Frequency and observation results from Tube Shape experiment

9.3.3. Discussion

There did not appear to be any clear trend in changing frequency (all values were within the 23Hz margin of error). In addition, our hypothesis was supported in the fact that the frequency is independent on the shape (this is because the tubes purpose is to channel the rising hot air, and provide boundaries for the air column that resonates, neither of these properties are affected by slight changes in tube shape.)

9.3.4. Conclusions and Reflections

Tube shape has little effect on frequency.

It was observed in section 5.3.3.1 that for some shapes, the sound was weak or non-existent, which contradicts the hypothesis that shape has no affect. A possible explanation is that it is not the shape of the tube that is inhibiting sound production, but rather the diameter at either end. As experienced in section 9.2.3.1, if the diameter at the top or bottom of the tube is too great, no definite barrier of pressure exists, so the standing wave cannot by created. A possible further experiment would involve using tubes of the same proportion, or same percent increase in diameter, but are smaller, as this would determine if a breakdown in the pressure barrier was responsible for the weaker sound. The curved tube may have produced a weaker sound because the Bunsen's heat was not directly under the gauze (it was under the end of the tube), so the gauze may not have been heated as effectively.

9.4. Tube Shape:

9.4.1. Hypothesis

Since it is the column of air that resonates, not the tube its self, changing the vertical cross section of the tube should have no effect on the frequency and sound intensity, as long as the tube still contains a region of air.

9.4.2. Results

	Circular	Square	Triangle
Frequency (Hz)	235	235	237
Observation	Loudest sound	Weak sound	Weak sound

Table 5 Frequency and observational results from the Tube Shape (horizontal) experiment

9.4.3. Discussion

Since sound is a longitudinal wave, and it is the air not the tube that resonates, there was no noticeable effect on frequency (within the 10Hz uncertainty), supporting the hypothesis. The observation of the sound being strongest with the circular tube was unexpected, and difficult to explain, our best inference is that there is more air flow in circular tube because the triangular and square tubes had corners, and a larger internal surface area. This may cause a larger boundary layer, so the air flows slower in the corners and the gauze in the corner areas doesn't heat as well, so the useable area in the square and triangle is effectively reduced to that of a smaller circle (see figure 4). This would account for the decrease in sound intensity.

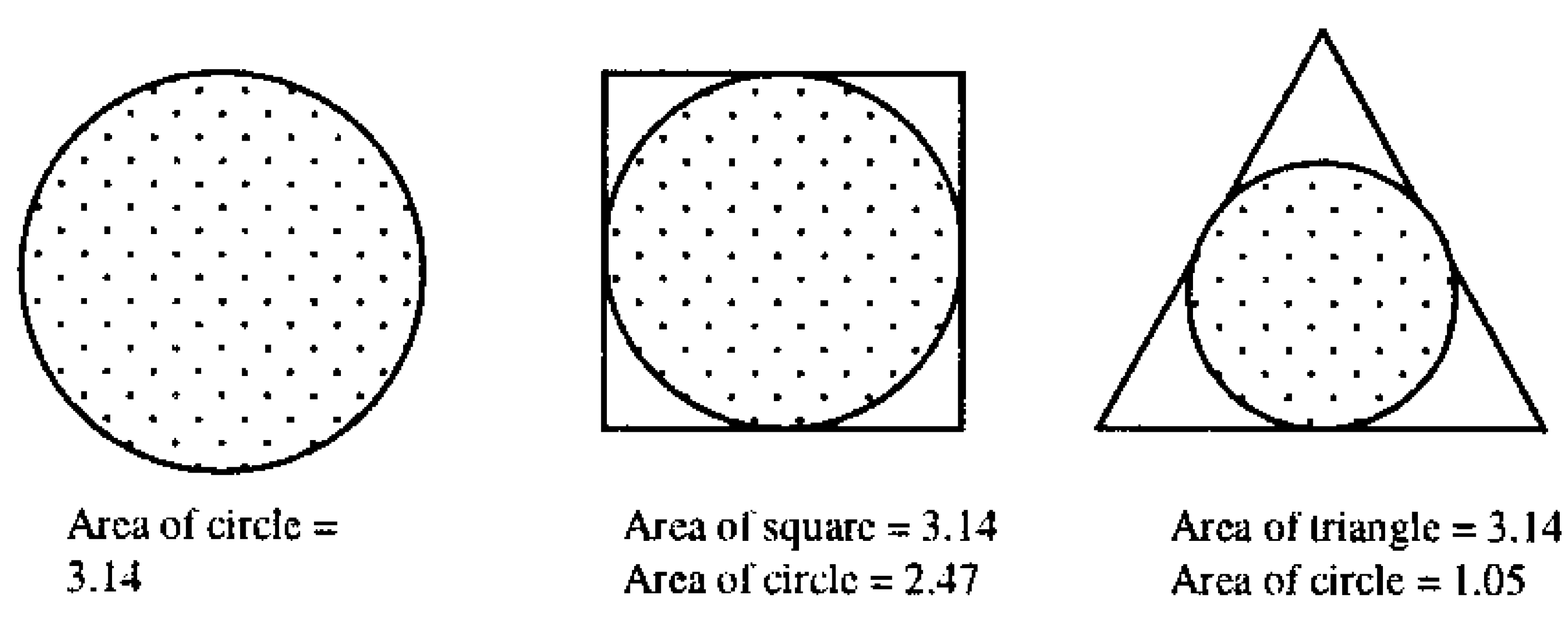


Figure 7 Although all tubes had the same area, this diagram shows that since air got caught in the corners, the shaded usable area likely decreased.

9.4.4. Conclusions and Reflections

Tube shape had little effect.

An interesting further experiment would involve electrically heating the gauze, to ensure even heating, and having an artificial airflow though the tube, this would help determine if the reasons behind the observed changes in sound intensity.

9.5. Tube Material

9.5.1. Hypothesis

We saw in section 6 that if the change in temperature of the gauze becomes less that the that required to heat the convecting air, the tube and sustain the kinetic energy of the wave, the tube stops singing, this was explained thus: when

$$\frac{dT_{gauze}}{dt} \leq \frac{1}{m_g c_g}\left(m_{air}c_{air}\frac{dT_{air}}{dt} + \frac{1}{16}\rho A v \omega^2 s_m^2 + kA\frac{dT_{tube}}{dt} \right)$$

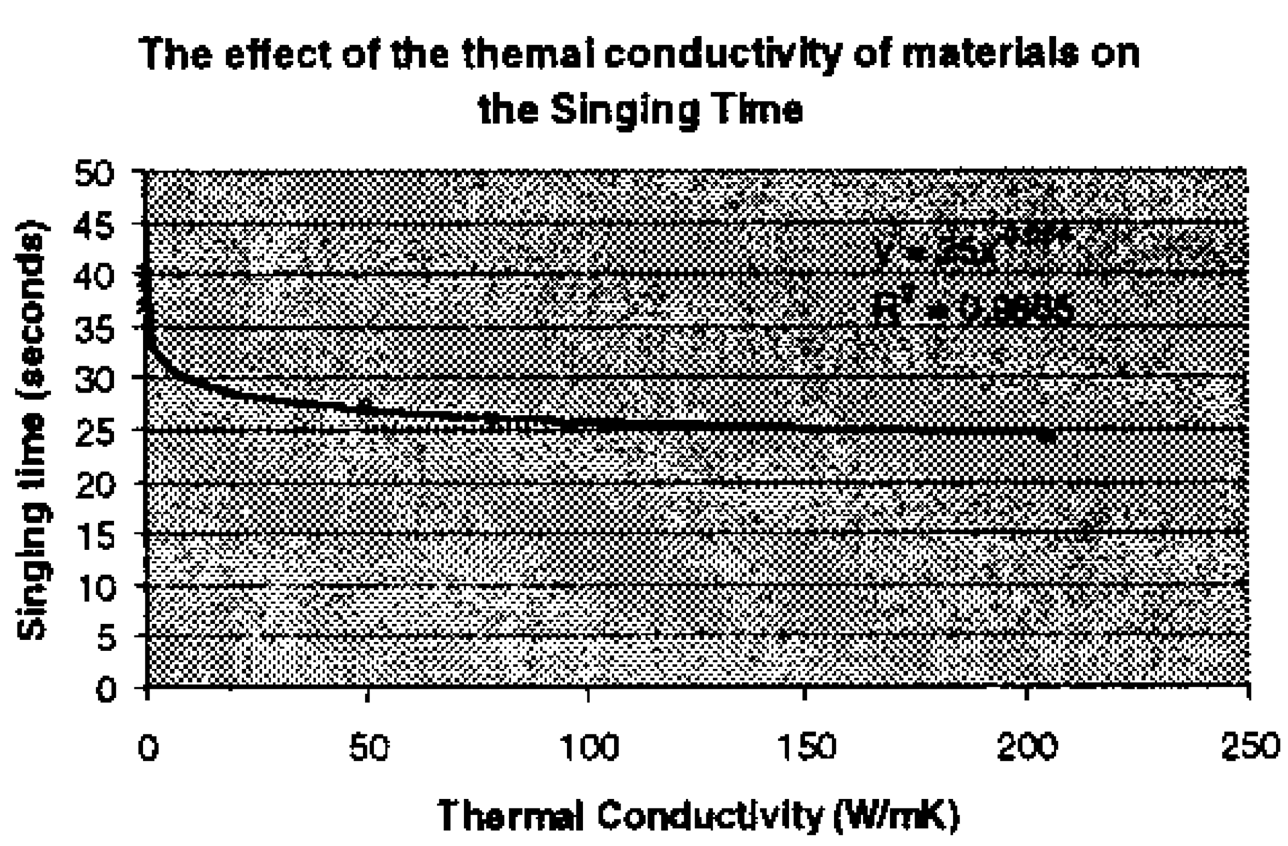
The tube stops singing. The k in the last term is the thermal conductivity of the tube. So from this equation we can see that as the thermal conductivity of the material decreases, the tube will sing for longer, as the gauze needs a lower temperature to sustain the wave. When the thermal conductivity of the tube material increases, the gauze needs more heat, and so will not be able to sustain the singing for as long. As sound is a longitudinal wave, the tube acts only as the boundary for the air column inside which vibrates, therefore the frequency should be unaffected by the tube material.

9.5.3. Results

Material	iron	steel	aluminium	PVC	wood	Glass	Paper	Plastic	cardboard
Thermal conductivity	79.5	50.2	205	0.19	0.1	0.8	0.05	0.02	0.07
frequency	218	219	215	223	223	223	-	-	223.5
Singing time	26	27	24.3	39.1	39.5	37	-	-	40

Table 6 Frequency and Singing time results for materials of different thermal conductivities

The paper and thin plastic tubes did not sing because they combusted.



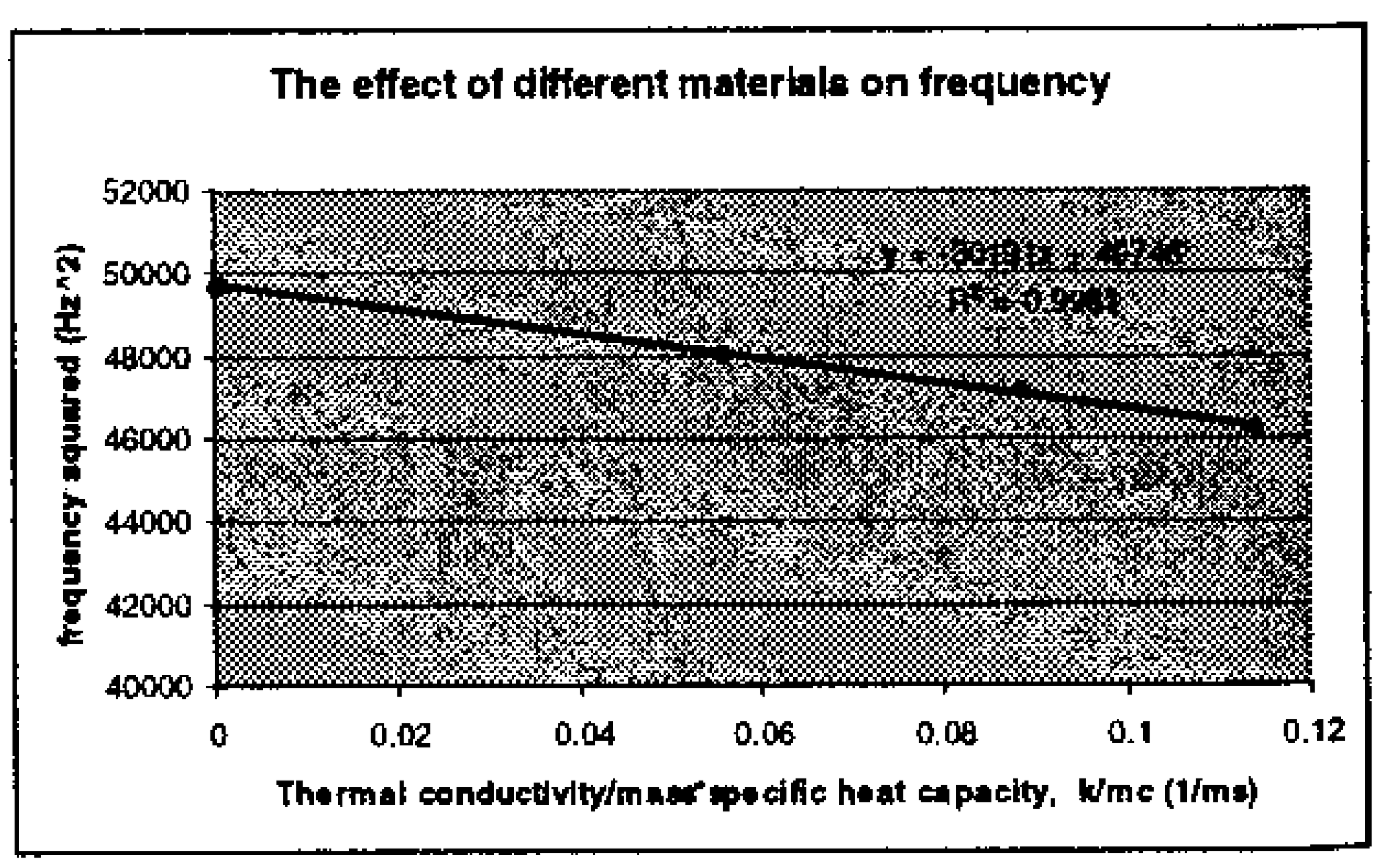


Figure 9 The effect of thermal conductivity of the tube on the Singing time and frequency

9.5.4. Discussion

The hypothesis stated that as the thermal conductivity increased, the tube should sing for less time, since more of the gauze's heat was conducted away to the tube. This was supported experimentally and an exponential relationship was found. However, the hypothesis stated that there should be no effect on frequency, however this is not what we found experimentally; there was variation in frequency for the different materials. Therefore, after brainstorming the differences in the materials, and what could cause the frequency to change, the relationship was realised. Some materials conduct heat away from the gauze and air more than others. Tubes with a high thermal conductivity will conduct heat away from the gauze easily, so the gauze is cooler and will not sing as long as it will in tubes with a low thermal conductivity. Recall from section 4.3. the equation for frequency:

$$f = \frac{v}{2L + 0.8d} \xleftrightarrow{\text{for same sized tubes, L and d are constant}} f \propto v$$

As long as the tube length and diameter are kept constant, the frequency is directly proportional to the velocity of sound. However, the speed of sound is not constant, it depends upon the temperature.

$$v = \sqrt{\gamma RT} \xleftrightarrow{\text{R and } \gamma \text{ are constants}} v \propto \sqrt{T} \qquad [4]$$

So now it is clear that frequency has a squarely relationship with temperature: $f \propto \sqrt{T} \Leftrightarrow f^2 \propto T$ However, we do not know how the different materials change the temperature in the tube. Recall the equations regarding temperature and heat from section 6, the information concerning their thermal conductivity, k, and specific heat capacity, c is available. Thermal conductivity is a measure the rate at which heat is transferred per unit area. Since all the tubes had the same dimensions, Heat lost is proportional to the thermal conductivity, $Q \propto k$.

Heat lost can also be expressed in terms of the mass and specific heat capacity of the tubes, Q=mcT. Therefore, it means that:

$$f^2 \propto T$$

$$Q = mcT \propto k \Leftrightarrow T \propto \frac{k}{mc}$$

$$\therefore f^2 \propto \frac{k}{mc}$$

So this predicts that a graph of f^2 vs. k/mc should be linear. Experimentally a linear relationship was found. This relationship is reasonable since the R^2 value is 0.9981. The relationship had a gradient -30000±1200, and intercept of 50000±4000Hz^2. This means that if the tube were a perfect insulator (k=0) then the frequency would be 223Hz=√50000

9.5.5. Conclusions and Reflections

Frequency and the thermal conductivity of the tube have the following relationship:

$f^2 = -30000\frac{k}{mc} + 50000$. It was felt that this experiment could be improved by having materials with a wider range of thermal conductivity values, as the PVC, wood, glass and cardboard tubes all had similar values. Although the hypothesis was proved incorrect, more interesting discoveries were made as a result.

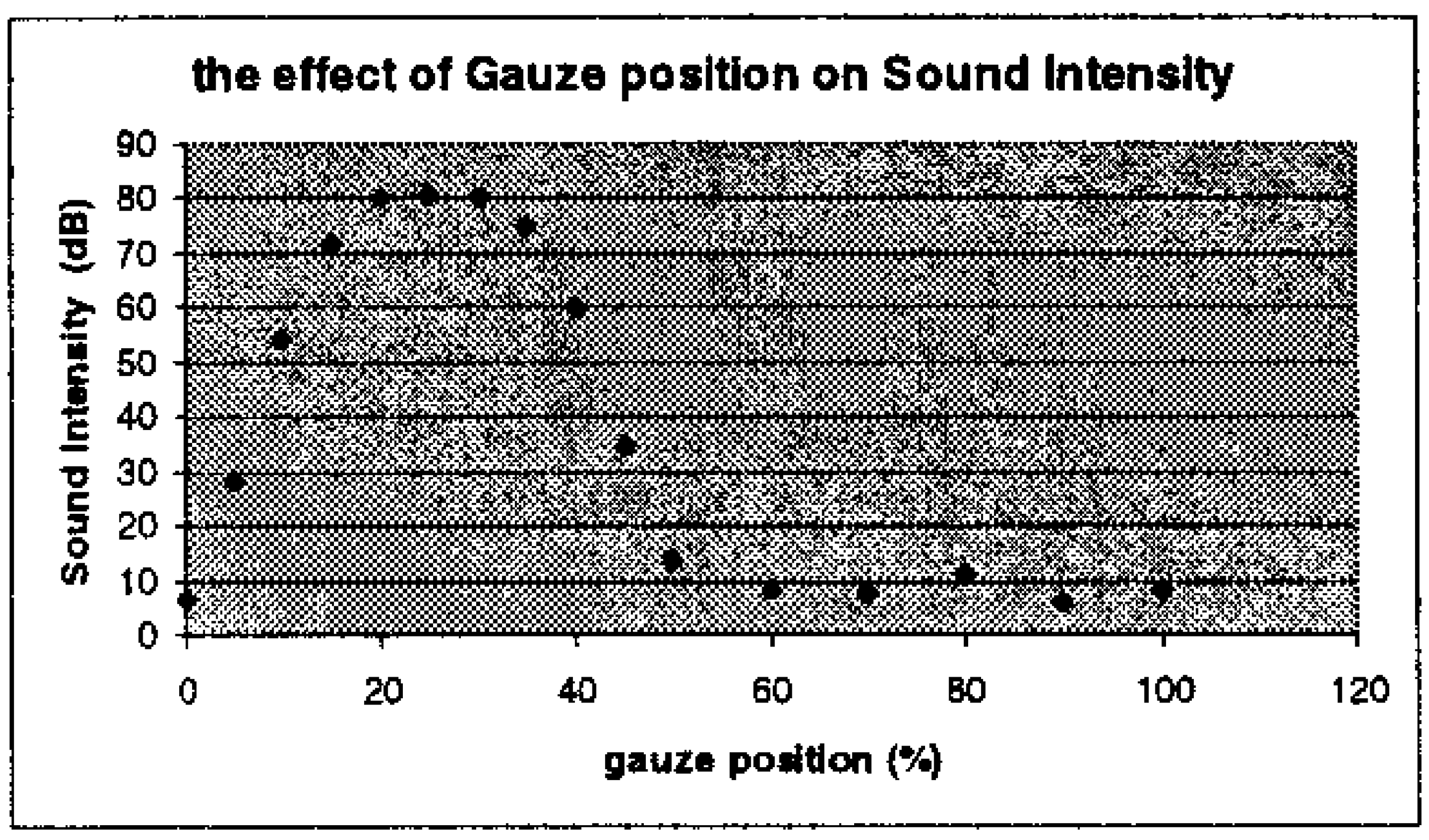
9.6. Gauze Position

9.6.1. Hypothesis

It was discovered in section 5.3. that the Rayleigh Index and therefore the sound intensity follow a sinusoidal function with respect to the position of the gauze in the tube. This sine curve has a period of the length of the tube (100%), and a maximum at 25%. The graph of sound intensity vs. the sine of the product of 3.6 and gauze position should be linear. (3.6 alters the normal period of the sine function (360°) to the length of the tube, 100 (%). 3.6= 360/100)

9.6.2. Results

The tube did not sound at all when the gauze was in the upper half. The computer program recorded background noise, so although seven was the lowest sound intensity recorded, the tube did not contribute to this, and was silent at the time.



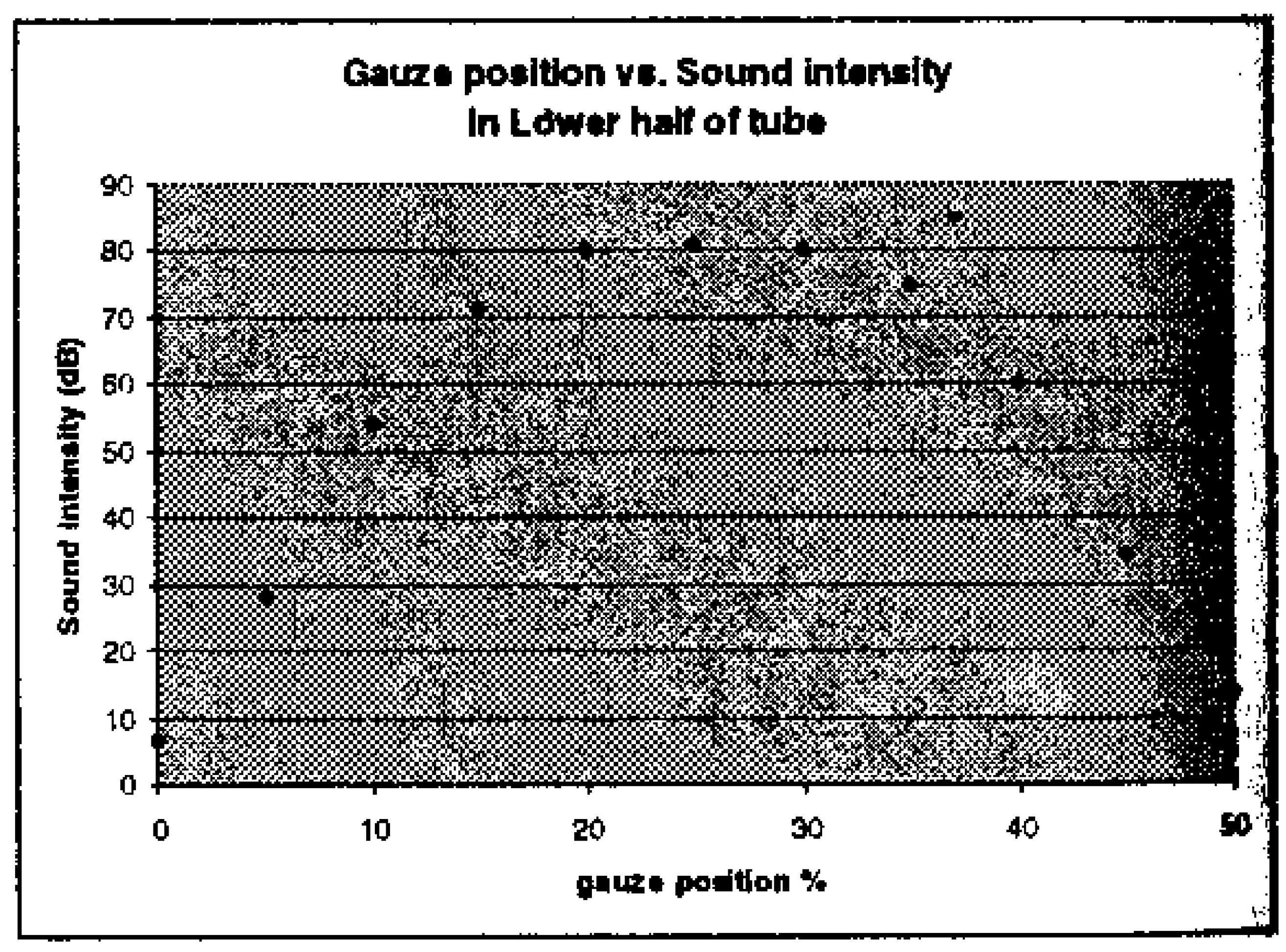


Figure 10 Graphed results for sound intensity from the Gauze Position experiments

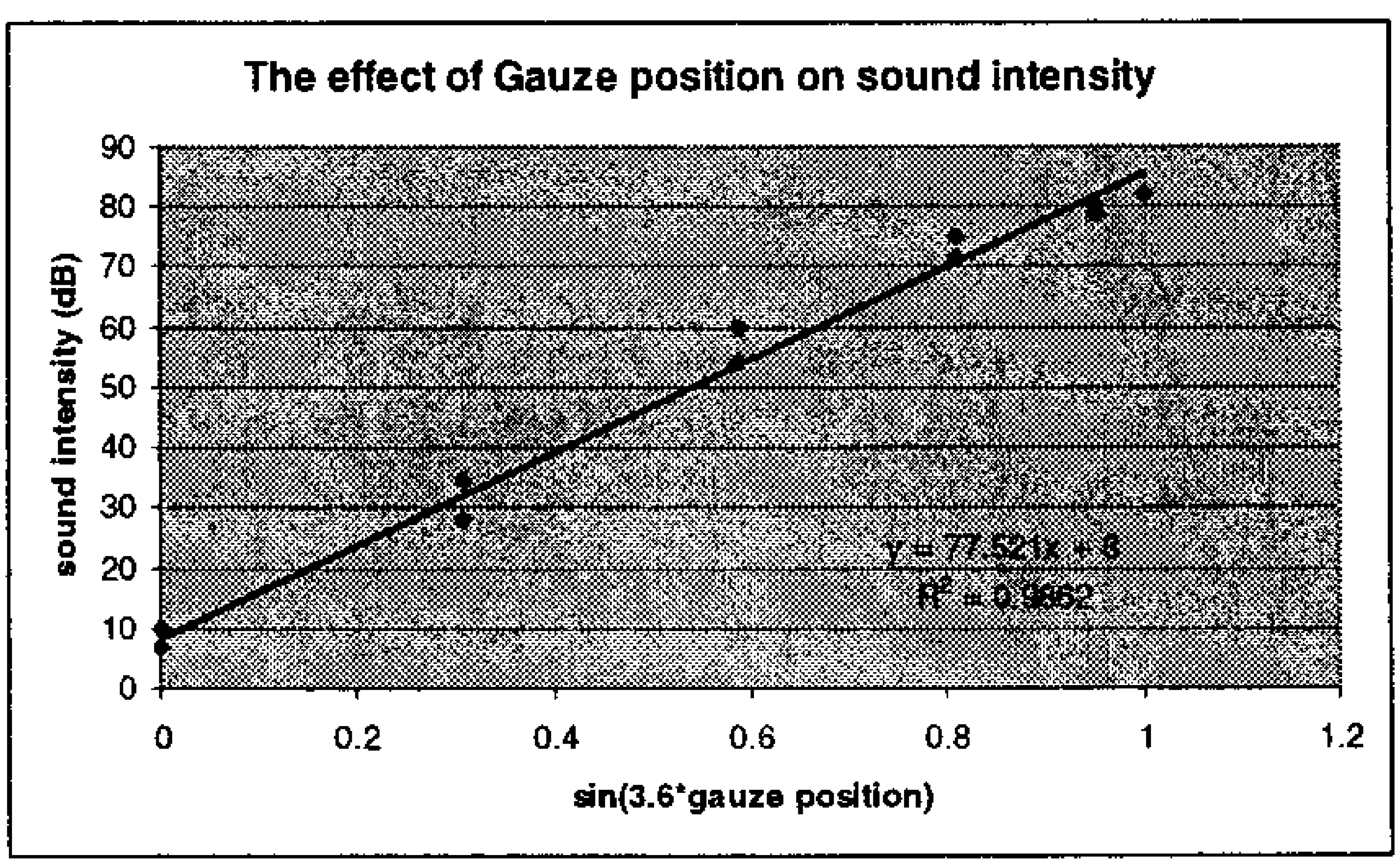


Figure 8 Linear relationship of Gauze position results

9.6.4. Discussion

As predicted, sound intensity and the sine of 3.6 times the gauze position had a linear relationship. This relationship is reasonable, as the R^2 value is 0.9862, and the data points are scattered both sides of the line. However, on this graph there were often two data points for the same x value. This is because the sine function increased then decreased so in the first two graphs the same y value is reached twice. In an ideal world, these points would always be concurrent, and the distance between them represents the error in this experiment. The equation which this linear relationship represents is:

$$I = 78\sin(3.6gp) + 8$$

Where I, is the sound intensity and gp is the gauze position. The gradient of the linear relationship was 78 ± 3, and this represents the amplitude of the sine function (of sound intensity vs. gauze position), or the maximum sound intensity achieved. The intercept is 8 ± 2 dB, and this represents the background noise picked up by the microphone.

The linear relationship did not continue when the gauze was in the top half of the tube, as the sinusoidal function predicted negative sound intensities, which are impossible.

9.6.5. Conclusions and Reflections

The hypothesis was supported; the position of the gauze in the lower half of the tube had a sinusoidal relationship with the sound intensity, the relationship is: $I = 78\sin(3.6gp) + 8$

It was felt that the errors and uncertainties in this experiment were quite large, and only just acceptable, the main reason for this was the unpredictable background noise, so the experiment could be improved by conducting the experiment in a silent environment, or one with a uniform background noise.

9.7. Heating Time

9.7.1. Hypothesis

As seen in section 6, the heat absorbed by the gauze is lost through convection to the air, conduction to the tube, and in the kinetic energy of the sound wave, so the singing time will depend on how long the heat from the gauze is greater than these loses. This is demonstrated in the following equation: when

$$\frac{dT_{gauze}}{dt} \leq \frac{1}{m_g c_g}\left(m_{air} c_{air} \frac{dT_{air}}{dt} + \frac{1}{16}\rho A v \omega^2 s_m^2 + kA \frac{dT_{tube}}{dt} \right)$$

The tube stops singing. Therefore, the singing time depends on the temperature or heat absorbed by the gauze just before it starts singing. The temperature of the gauze while heating can be modelled by:(Newton's law of Cooling) $\frac{dT_{\text{gauze}}}{dt} = kA(T_{bunsen} - T_{gauze})$ This means that initially, when the gauze is cold, it will heat very quickly so $T_{\text{bunsen}} - T_{\text{gauze}}$ is large, but as the gauze heats up, there will be a smaller temperature difference, until heating the gauze any longer won't make much difference. When you differentiate Newton' law of cooling and heating, you get a logarithmic relationship between the heating time and the temperature of the gauze. We could not derive the exact relationship between singing time and temperature of the gauze (and therefore heating time), due to the large number of variables in the heat transfer equation above, however the logarithmic relationship will still be present.

9.7.3. Results

Heating time	Singing time
2	0
3	0
4	2
5	6
10	13
30	21
60	26
120	29

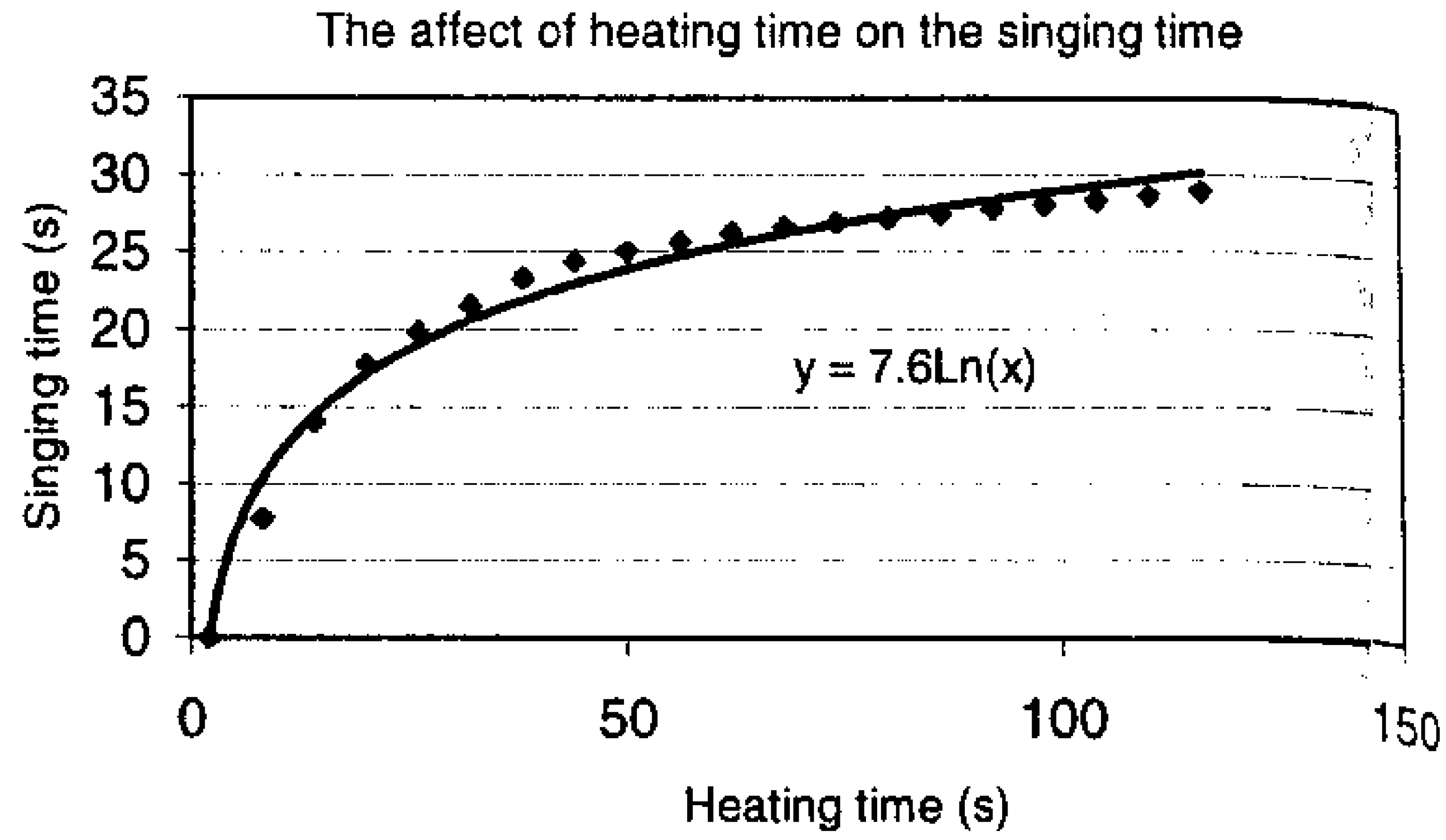


Figure 11 Graphed results from the Heating Time experiment

9.7.4. Discussion

Experimentally, there was a logarithmic relationship between heating and singing time. This relationship is reasonable, since the R^2 value is 0.9872, and the uncertainties are small, but the scatter of points indicates that there may be other factors of consideration. This may be due to the effect of heating the air aroung the gauze at the same time as heating the gauze. This supports the hypothesis that the relationship between heating time and singing time should be logarithmic because singing time is approximately proportional to the temperature of the gauze (after heating), and the temperature of the gauze after heating has a logarithmic relationship with heating time.

9.7.5. Conclusions and Reflections

As predicted, increasing the heating time increased the singing time, in the relationship. $t_{\sin ging} = 7.6\ln(t_{heating})$. It was felt that this experiment could be improved if the gauze could be electrically heated, as then the amount of heat could be calculated directly from the amount of electrical energy put in. This would simplify equations and reduce error; however, unfortunately we did not have access to the voltages required. Further investigations into measuring the temperature inside the tube, around the gauze with a thermocouple array would help to determine if Newton's law can really be applied, in that it is justified to treat the system as a closed system with just the gauze and the Bunsen burner flame.

10. Summary

The singing tube phenomenon is caused primarily by a compression drawing in cool air, which encounters the gauze, heats and increases in pressure, this adds to the pressure maximum. The optimum gauze position is determined by where the most cool air will be drawn in, and where the cool air's pressure changes will have the most effect. The Rayleigh Criterion and the Rayleigh Index model this, causing the sound intensity, I, to have a sinusoidal relationship with gauze position (gp, measured in percent): $I = 78\sin(3.6gp) + 8$ The standing wave, which this sets up, has a frequency determined by the tube length and . The frequency is also determined by the $f = 175.5\left(\frac{1}{L + 0.4d}\right)$ diameter through the expression: . The $f^2 = -30000\frac{k}{mc} + 50000$ thermal conductivity, k, of the tube material in the relationship: standing wave is longitudinal, so is mainly unaffected by changes to the shape of the tube.

The heat transferred from the hot gauze to the air, tube and acoustic wave can be modelled by the expression: $m_g c_g \frac{dT_{gauze}}{dt} = m_{air} c_{air} \frac{dT_{air}}{dt} + \frac{1}{16}\rho A v \omega^2 s_m^2 + kA\frac{dT_{tube}}{dt}$

This expression and Newton's law of Cooling lead to the relationship between the time the gauze is heated, and the time the tube sings: $t_{\text{sings}} = 7.6\ln(t_{\text{heated}})$.

This applicable and interesting problem has been thoroughly investigated, and many relationships have been found. The discovery of the relationship between gauze position and sound intensity: $I = 78\sin(3.6gp) + 8$ is of particular scientific significance. This is because if the maximum sound intensity allowable by a system (such as the vibration limit at which a jet engine would start to break up) is known, then this equation can effectively predict were and were not a heat source must be located in order to prevent these dangerous vibrations. For example, the optimum point of fuel ignition in a jet engine can be determined.

Acknowledgements

I would like to thank the whole Brisbane Girls Grammar School science department, for allowing me to interrupt their lessons with a rather loud 'singing' noise. Thanks also to Mr. Bromiley and Dr. Dancer who showed me how to manipulate differential equations. My IYPT teammates, helped with conducting most of the experiments, and I would like to thank my parents, for letting me do maths and physics all weekend. Most of all, I would like to thank my physics teacher Mr. Allinson for his guidance, support, encouragement, and answering of many long, frantic emails late at night. Thank you.

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