# 9. Problem №12: Rolling magnet

## *9.2. Solution of Croatia*

### Problem №11: Singing tube

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### The Problem

***Investigate the motion of a magnet as it rolls down an inclined plane.***

### Abstract

We present a solution to the problem of a permanent magnet rolling down an incline, investigating two main cases, conducting and nonconducting plane. For the former, more interesting case first – order Maxwellian theory was developed and thoroughly tested experimentally, yielding good agreement, while the latter, much simpler situation was modeled considering Earth field – magnet interactions. Plate boundary effects on the conducting plate are considered qualitatively and only demonstrated experimentally due to mathematical difficulties.

### 1. Introduction.

The phenomenon of a magnet slowly sliding down a metallic incline is very familiar, being one of the most dramatic demonstrations of eddy currents and the force they produce. Many studies have been dedicated to it (eg. [1]), and we can say that it is well understood in the framework of the Maxwellian electromagnetic theory. However, this effect is by far not the only amazing consequence of eddy currents induced by moving magnets, and in this research we will study a closely related phenomenon: the *rolling* motion of a magnet on an incline. In contrast to the aforementioned matter of a magnet sliding down a metallic plane, in this case the only influence on the moving magnet is the magnetic field of the Earth. This field can provide a torque causing the magnet to change direction so as to set its magnetic moment in a direction parallel with the direction of the field. This means that, even on a nonconducting plate, the magnet will stray to the north, following a curved trajectory; thus the first part of our research will focus on the behaviour of a magnet rolling down an insulating plane under the influence of the Earth field. In the second, by far more interesting and fascinating part, we will add a conducting plate and try to explain the effects of the currents induced therein. Both cases have a number of common parameters, primarily the properties of the rolling magnets (radius, thickness, magnetization) and plane inclination.

As the main actors of our investigation were magnets, we had to be very careful when choosing them. We decided to work with three different disc magnets made of $Nd_2Fe_{14}B$, a permanently magnetic sintered ceramic material having a magnetization field of 1.4 T and a density of 7500 kg/m$^3$ (measured). These magnets have excellent magnetic properties (large magnetization, stability) while being readily available, which made them ideal for our research.

## 2. The Nonconducting Incline.

In this first, simpler case, we will explore the effects of the Earth magnetic field on a magnet as it rolls down a nonconducting plate. Unless the plate is so set up that the magnetic moment of the magnet points north all the time, the torque the Earth field is causing will deflect the magnet towards the North direction. As a very simple experiment may reveal, the curvature of the resulting trajectory will mainly depend on the plane inclination, the initial angle between magnet and Earth field direction and the parameters of the magnet, such as mass and dimensions. This makes good physical sense; the larger the plane inclination, the larger the downward – pulling gravity component will be, increasing the downward velocity component of the magnet and decreasing trajectory curvature. Also, the larger the initial angle between the magnetic moment and the Earth field, the larger the torque will be, resulting in an increased curvature. The mass influence is also quite understandable – for heavier magnets the influence of inertia will be greater and the curvature smaller. To obtain an exact analytical representation of the trajectory we need to write the equations of motion for a circular magnet (of radius $R$, thickness $D$, density $\rho$ and magnetization $\mathbf{M}$) under the influence of gravity and the magnetic field of Earth. To facilitate this task we will have to introduce some approximations. First of all, in order to always have the magnet in rolling motion we will limit the considerations to small inclination angles, which means that we may neglect the influence of the vertical component of the Earth field and consider only the constant horizontal component. Due to the homogeneity of this field we can approximate our magnet with a dipole of magnetic moment $\mu = \mathbf{M}V$ , where $V$ is its volume. Second, we must define our coordinate system. We will see that the simplest calculations are obtained if the $xy$ - plane coincides with the plane of the incline and the $z$ – axis is perpendicular to that plane; this means that initially the magnet is seated on the $x$ – axis, its magnetic moment pointing along the $y$ – axis. The rolling acceleration can easily be found:

$$a = -\frac{2}{3} g \sin\varphi \sin\vartheta \qquad (1)$$

where $g$ is the acceleration of gravity, $\varphi$ the angle of inclination and $\theta$ the angle between the magnet's direction of motion and the $x$ – axis. This acceleration we know to be equal to the second time derivative of the magnet path, $\ddot{s}$, with $dx = ds \sin\vartheta$ and $dy = ds \cos\vartheta$ . These facts will enable us to obtain the trajectory equation. The angle $\theta$ will be obtained from the second influence on the magnet, the Earth field torque:

$$I\ddot{\vartheta} = -|\mu| B_E \sin(\vartheta - \beta) \qquad (2)$$

Where $I$ is the moment of inertia of the magnet, $\mu$ its magnetic moment, $B_E$ the horizontal component of the Earth field and $\beta$ the angle between this field and the $x$ - axis. We see immediately that the further treatment of these coupled equations of motion becomes very simple if $\beta \approx 0$ , while if this is not the case matters get a lot more complicated; therefore we will first make a simple calculation for this special case. After inserting (2) into (1) and integrating we see that

$$ds = \frac{2}{3} \frac{g \sin\varphi}{\omega_0^2} d\vartheta \qquad (3)$$

where the parameter $\omega_0^2$ stands for $\frac{|\mu| B_z}{I}$, and we have taken that the initial velocity of the magnet equals zero. The trajectory equation can simply be obtained by noting that $\frac{dy}{dx} = -\cot\vartheta$ and combining this with relation (3):

$$y = -\frac{2}{3}\frac{g\sin\varphi}{\omega_0^2}\int\cos\vartheta d\vartheta + C \qquad (4)$$

where $C$ is a constant of integration we will determine from the fact that the $y$ – coordinate of the magnet position is initially equal to zero, while the $x$ – position is an initial $x_0$. In the end one obtains the trajectory equation

$$y = -\frac{2}{3}\frac{g\sin\varphi}{\omega_0^2}\left[\sqrt{1-\frac{4}{9}\left(\frac{\omega_0^2}{g\sin\varphi}\right)^2(x-x_0)^2}-1\right] \qquad (5)$$

Taking a closer look at this relation we see that the trajectory is in fact a portion of the circle $(x-x_0)^2+(y-r)^2=r^2$ with radius $\frac{2}{3}\frac{g\sin\varphi}{\omega_0^2}$! This is a fact well substantiated experimentally. However, this derivation was made in the limiting case $\beta \approx 0$; a complete treatment would call in for numerical techniques, necessary for solving the nonlinear coupled system (1) and (2) in the case $\beta > 0$. The calculation is rather cumbersome but in our opinion less physically appealing – the considered special case seems to catch all the main features of the effect. Rewriting the radius as

$$r = \frac{\pi}{6}\frac{g\sin\varphi}{|\mu|B_z}R^4\rho D \qquad (6)$$

where $R$ is the magnet radius, $D$ the thickness and $\rho$ the density of the magnet, we see that it increases as the inclination angle grows, as we have predicted before from elementary considerations; also, we see that the radius strongly depends on the inertial properties of the magnet, increasing with increasing magnet radius and density, in agreement with our previous conclusions. We might say that altering the angle $\beta$ would have no significant effects on these dependencies; it would merely change the path a bit due to the more complicated geometry involved.

The measurements on a nonconducting plane were quite simple to conduct, aside from some matters requiring greater attention. The main problem was insulating the magnet from all parasite fields due to wiring and the many iron parts in a laboratory. To minimize this influence our incline construction used no metal parts and was set as far away from iron pieces as possible. The incline itself (a wooden plate with a sheet of white paper glued to the measuring side) was standing on a wooden box so as to be separated from the wiring and metal tubing or constructions in the floor. In order to obtain the magnet trajectory we soothed the magnet we worked with so it left a trace on the paper glued to the wooden incline. The measurements were repeated for several inclination angles, yielding the predicted circular trajectories (fig. 1.).

**3. The Conducting Incline.**

3.1. Theory.

This second case is, as we have mentioned in the beginning, by far the more interesting and complicated situation. This is due to the fact that a moving magnet, sliding or rolling, causes a local change of magnetic field flux in the conducting material, which gives rise to induced currents. These currents are again time – changing, inducing secondary fields which in turn induce ternary currents and so on. The net resulting induced field in the material is the sum of all these fields. This field interacts with the rolling magnet causing a backward force (if the magnet is far from the plate edges) or even torques and lateral forces (if the magnet is close to an edge, the induced field will be asymmetric, causing lateral effects) which result in a curved trajectory. Although the given explanation based on simple induction arguments sounds quite simple, the calculation of the induced fields and forces acting on the magnet is a very difficult task, for the more complicated magnet-near-edge case almost impossible. In spite of this fact, we have managed to develop an analytical theory for a magnet on a metallic plate far from its edges, and even succeeded in calculating the asymmetric field forming in the near-edge case. However, the calculation of the magnet trajectory in this situation still proved too difficult, so we only give a qualitative description of this. We start with the simpler case of a magnet rolling down the plate far from its edges; here the induced field acts only in the direction opposite to the magnet's direction of motion, greatly simplifying the underlying mathematics. There is one important point to make, thus: this case can only be observed if *all* asymmetry is removed from the system, implying that the influence of the Earth field torque must be ruled out as well; this is done by adjusting the incline-magnet system in such a manner that the magnetic moment of the magnet points North. This is quite a source of experimental difficulties, as we shall see. So isolated from all deflecting effects, the magnet on such a „quasiinfinite“ plate will follow a linear trajectory. Furthermore, as the force due to the induced currents turns out to be quite large (and of course velocity - dependent), the magnet reaches a terminal velocity after a short period of acceleration. This means that for the largest part of the motion the induced field doesn't vary in time but for traveling along with the magnet with constant velocity! In the magnet reference system, however, the field is stationary, which simplifies calculations a great deal. In order to find the force the induced field exerts on the magnet, we must first consider the structure of the magnet. In the treatment of the motion on a nonconducting plate we were able to regard the magnet as being a dipole; this was possible because of the homogeneity of the Earth field influencing the magnet motion. In this case, however, we cannot use that approximation due to the fact that the induced field is very close to the magnet and inhomogeneous. This means that we must take a more detailed model of a disc magnet to work with. A pretty realistic model, which suits our needs very well, may be obtained by taking the magnet to be analogous to a solenoid, i.e. consisting of an infinite number of infinitesimal dipoles each having the dipole moment

$$d\mathbf{m} = \mathbf{M}R^2 \pi d\varepsilon \tag{7}$$

where **M** is the magnetization of the material, $R$ the magnet radius and $d\varepsilon$ the infinitesimal thickness of the dipole. The force exerted by the induced magnetic field on a dipole is generally

$$d\mathbf{F} = \left|\nabla(d\mathbf{m} \cdot \mathbf{B}_{in})\right|_{(x_m, y_m, z_m)} \tag{8}$$

where $\mathbf{B}_{in}$ is the induced field vector and $x_m$, $y_m$ and $z_m$ the coordinates of the dipole. The force on the whole magnet is obtained by simply summing the forces on all dipoles, i.e. integrating eqn. (8). To be able to perform this integration (and, of course, to find the induced field) we have to define a coordinate system. The most suitable will again be the one used in the first part, with the *xy* – plane coinciding with the plane of the incline and the *z* – axis pointing vertically upwards, but this time we will take it to move along with the magnet. The magnet is standing on the *x* – axis, and the magnetization vector points in the *y* – direction (fig. 2). Using this configuration we can easily deduce the coordinates of each of our tiny dipoles and perform the integration:

$$\mathbf{F} = R^2\pi|M| \int_{-\frac{D}{2}}^{+\frac{D}{2}} \left|\nabla B_{iny}\right|_{(0,\varepsilon,R)} d\varepsilon \qquad (9)$$

where $D$ is the magnet thickness and $B_{iny}$ the $y$ – component of the induced field. Now that we know how to obtain the force on the magnet from the induced field, we are left with the task of calculating this field. Now that's the real difficulty. To perform this task we have to start with the general system of Maxwell equations in a conductive medium and try to solve it for the induced magnetic field. We will take the induced electric currents to be strictly sourceless, and bear in mind that the net field in the material is in fact the vector sum of the induced field and the constant field of the magnet. The initial equations we will write in a reference system attached to the incline, denoted by primes and equivalent to the system defined in the first part of the work. The Maxwell system (eg. [2]) becomes

$$\begin{gathered}
\nabla'\times\mathbf{j} = -\sigma\frac{\partial\mathbf{B}}{\partial t} \\
\nabla'\times\mathbf{B}_{in} = \mu_0\mathbf{j} - \frac{\mu_0\varepsilon_0}{\sigma}\frac{\partial\mathbf{j}}{\partial t} \\
\nabla'\cdot\mathbf{j} = 0 \\
\nabla'\cdot\mathbf{B}_{in} = 0
\end{gathered} \qquad (10)$$

where $\mathbf{j}$ is the current density vector, $\sigma$ the conductivity of the material, $\mathbf{B} = \mathbf{B}_{in} + \mathbf{B}_0$ the net field (the sum of the induced and magnet fields), and $\mu_0$ and $\varepsilon_0$ the permeability and permittivity of the vacuum, respectively. It must be noted that this system is valid only inside the conducting material; outside it the conductivity drops to zero and no currents can flow, but an induced field caused by the currents in the material exists everywhere in space. This fact we will build into our solution, as we shall see later. The first great simplification we are to make in solving the system (10) will be transforming to the inertial system of the magnet, which is moving relative to the inclined conducting plate with constant velocity $v$. After employing a simple Galilean transformation and evaluating the time derivatives by the chain rule we get an equivalent system for the fields and currents in the (unprimed) system of the magnet:

$$\begin{gathered}
\nabla\times\mathbf{j} = -\sigma v\frac{\partial\mathbf{B}}{\partial x} \\
\nabla\times\mathbf{B}_{in} = \mu_0\mathbf{j} - \frac{\mu_0\varepsilon_0 v}{\sigma}\frac{\partial\mathbf{j}}{\partial x} \\
\nabla\cdot\mathbf{j} = 0 \\
\nabla\cdot\mathbf{B}_{in} = 0
\end{gathered} \qquad (11)$$

with the space coordinates now referring to the moving reference system. Now for metallic inclines, which we exclusively made experiments with, the conductivity is very large, the induced currents become strong as well and the velocity is small, typically a few centimeters per second. This leads to the conclusion that we may safely neglect the rightmost term in equation two of the system – it is much smaller than all other terms. The obtained simplified

(12)

system is now ready for manipulation; the current may be eliminated in a few transformations and one gets the whished-for equation for the induced field:

$$\nabla^2 \mathbf{B}_{in} + \mu_0 \sigma v \frac{\partial \mathbf{B}_{in}}{\partial x} = -\mu_0 \sigma v \frac{\partial \mathbf{B}_0}{\partial x}$$

We have written it so as to separate the terms involving the induced field from the terms involving the constant field of the magnet; in this way we can immediately see that this field in a way acts as a *source* for the induced field. This of course makes good sense because it is just the change of flux of this magnet field in the material which causes the induced currents and fields. The simplest way of solving this equation will again use the fact that velocity is quite small and that the induced field is a lot smaller than the field of the magnet (this fact we will verify a posteriori). These facts enable us to write the solution $\mathbf{B}_{in}$ as a power series in the factor $\xi = \mu_0 \sigma v$:

$$\mathbf{B}_{in} = \mathbf{B}_1 \xi + \mathbf{B}_2 \xi^2 + ... \qquad (13)$$

which will converge fast if $\xi$ is small enough or if the functions $\mathbf{B}_i$ give rapidly diminishing values for increasing $i$, in the region under consideration. These conditions aren't always simple to test, and we won't make a formal proof of the rapid convergence; as we shall see, the experimental results confirm it. Inserting this power series into the equation for the field (12) and equating the terms containing $\xi^n$ we get a system of equations for the functions $\mathbf{B}_i$:

$$\begin{aligned} \nabla^2 \mathbf{B}_1 &= -\frac{\partial \mathbf{B}_0}{\partial x} \\ \nabla^2 \mathbf{B}_2 &= -\frac{\partial \mathbf{B}_1}{\partial x} \\ &\vdots \end{aligned} \qquad (14)$$

This system is very handy because once we have obtained the first function $\mathbf{B}_1$ the higher order terms follow immediately. Each of the equations is a simple Poisson – type equation, easily solved; the net induced field is obtained by summing:

$$\mathbf{B}_{in} = \frac{\mu_0 \sigma v}{4\pi} \int_{V'} \frac{\partial \mathbf{B}_0'}{\partial x'} \frac{dx'dy'dz'}{|\mathbf{r}-\mathbf{r}'|} + \frac{\mu_0^2 \sigma^2 v^2}{16\pi^2} \int_{V''}\int_{V'} \frac{\partial \mathbf{B}_0'}{\partial x'} \frac{dx'dy'dz'}{|\mathbf{r}-\mathbf{r}'|} \frac{dx''dy''dz''}{|\mathbf{r}-\mathbf{r}''|} + \dots \qquad (15)$$

where the primed and double – primed coordinates are parameter coordinates and the spatial integration only goes over the region containing the conductor (this is easily explained; only in that region can currents exist, and they serve as sources for the field). The integrals in the series (15) are practically impossible to calculate analytically, so we will have to use numerical methods, but first we will again use our approximation of small velocity. If the magnet is slow enough the powers of $v$ will become increasingly smaller and the contribution of the higher order terms can be neglected, leaving the approximate expression

$$\mathbf{B}_{in} \approx \frac{\mu_0 \sigma v}{4\pi} \int_{V'} \frac{\partial \mathbf{B}_0'}{\partial x'} \frac{dx'dy'dz'}{|\mathbf{r}-\mathbf{r}'|} \qquad (16)$$

which is linear in velocity and conductivity. These linearities have been fully experimentally confirmed, justifying the approximation for the magnets and plates we used. To perform the integration and calculate the force on the magnet we must derive the constant magnet field first; this is again done by regarding the magnet as an assembly of very thin dipoles and adding their fields to form the resultant field. The integration for the induced field was performed numerically to give the field everywhere in space; on fig. 3. and 4. we have shown only the $y$ – component because of its importance in the force calculation (it is also the largest component; the scale is always in mT). Fig. 3. shows the field on the plane $y = 0$ and fig. 4. on the plane $z = -0.0025$ m, that is, on a plane 2.5 mm under the surface of the material; the conducting plate has no bounding planes except $y = 0$, stretching to infinity in the $x$, $-x$, $y$, $-y$

and $-z$ directions. The magnet centre of mass has the coordinates (0, 0, 5.5 mm). The shape of the field already makes sense; we see that there are two regions of maximum field under the magnet forming two „images“ of its own field, the left image pulling and the right one pushing on the magnet, creating the observed force. Also, the highest field values are of the order of 0.15 mT (for a magnet of radius 5.5 mm and thickness of 5 mm), while the field of the magnet is in the range of 0.1 T; as we supposed, the induced field is much smaller. The force on the magnet can now easily be calculated from eqn. (9) inserting the induced field (16); if will be of the form

$$F_d = -\Lambda \sigma v$$

with $v$ being the magnet velocity, $\sigma$ the conductivity and $\Lambda$ a numerically calculated constant involving the gradient of the integral in the field expression (16). The values of this parameter for our three magnets will be compared to experimental values. The calculations were performed with a 0.1 mm parameter coordinate step in a cubical region $V'$ with dimensions 7x7x7 cm, on a DOS – based platform in Turbo Pascal 7.0.

The obtained linear form of force is of course valid only for small velocities. If we apply that force to the case of the magnet rolling down an incline it becomes clear that the terminal velocity is reached when the resistant force equals the gravity component pulling the magnet downwards, what leads to

$$v_T = \frac{\zeta g}{\sigma \Lambda} \sin \varphi \qquad (18)$$

where $v_T$ is the terminal velocity, $\zeta$ the magnet mass and $\varphi$ the inclination angle. This velocity is easy to measure and will be the principal parameter explored in the experimental part.

Now that we have thoroughly analyzed the case of a magnet rolling down the plane far from its edges, we must say something about the second, more complicated situation: boundary effects. When the magnet comes near to an edge the induced currents will loose their symmetry, causing additional forces to act on the magnet; a field gradient in the direction perpendicular to the magnet direction of motion can occur, causing a lateral force (fig. 5., the magnet and its position are the same as before), and if the magnet approaches the edge with a finite angle the normally vanishing lateral field component may become noticeable and cause a torque. Both influences deflect the magnet from the edge, and the trajectory becomes curved in its vicinity. The effect is very striking, as some of our trajectory measurements will show.

## 3.2. Experiment.

The experimental work for the magnet-on-conducting-plate problem was mainly focused on the first, simpler case of a magnet moving far from the plate edges; we wanted to explore the influence of the induced field and test our theoretical model. The parameter measured was always the terminal velocity. We made several measurements varying three main parameters: plate inclination, plate conductivity and magnet size (the dependence of terminal velocity on inclination was determined for two different magnets). Our incline was an aluminum plate 1.0 cm thick and 13.5 cm wide, and its conductivity was changed by cooling it to several temperatures; we will discuss this matter in detail later in the text. Its room temperature conductivity was measured to be 29.85 MS. The magnet velocity was measured using a system of two (and sometimes three) small solenoids connected in series, a fixed distance (mainly 19.0 cm for the two-solenoid system) apart (fig. 6. and 7.). The passing of the magnet was detected by recording the induced voltage in the solenoid system. The distance in time of the two voltage peaks provided a very precise means for determining the velocity of the magnet (fig. 8.). The voltage was read with an AD converter at a rate of about

10000 samples per second and transferred to a computer. A third solenoid was occasionally added halfway between the existing two to check if the magnet was really moving with constant velocity; the time difference was less than 1%, so we were able to conclude that the velocity was indeed uniform.

The first, less complicated set of measurements consisted of determining the dependence of the terminal velocity on inclination angle for two different magnets. Figures 9. and 10. show the dependencies; the magnets had diameters of 2.54 cm and 1.00 cm and thicknesses 2.54 cm and 0.50 cm, respectively. We see that the dependence on $\sin\varphi$ is linear, just as predicted. From the measured linear functions we were able to obtain experimental values for the constant $\Lambda$ and compare them to theoretical values: for Magnet 1 the experiment yields ( 53.2 ± 0.2 )$\cdot 10^{-9}$ kgs$^{-1}$S$^{-1}$, while the theoretical value is 52.8(4)$\cdot 10^{-9}$ kgs$^{-1}$S$^{-1}$, and for Magnet 2 the experimental value is ( 1.21 ± 0.2 )$\cdot 10^{-9}$ kgs$^{-1}$S$^{-1}$, the theory giving 1.19(3)$\cdot 10^{-9}$ kgs$^{-1}$S$^{-1}$. We see that the agreement is excellent, although we believe that even better numerical results could be obtained by using finer integration steps and accounting for the higher order corrections in eqn. (15). The linearity of the obtained dependencies also fully justifies our first order approximation.

The second, more demanding part of our experimental work focused on the dependence of terminal velocity on plate conductivity. The desired conductivity change was obtained by putting the plate in a heat – insulating container, cooling it down to liquid nitrogen temperature and letting it warm up slowly, measuring its temperature. At more or less constant temperature intervals the magnet was released and the terminal velocity recorded with the solenoid system. Figs 11. and 12. show the incline in the box and solenoids schematic and photograph. The box, made of polystyrene blocks, had a removable cover with two holes in it: one containing the magnet release mechanism and one (with a plug) for taking the magnet out. The box was always closed in order to create a cold atmosphere inside and prevent the formation of „snow“ on the plate. A liquid nitrogen bath was placed underneath the incline, and the plate temperature was measured by recording the resistance of a long thin copper wire glued to the plate. Wire resistance was measured to a very high precision using a lock – in amplifier, and the plate resistance was calculated from the temperature. By using this apparatus we were able to obtain plate temperatures ranging from 73 K (nitrogen boiling point) to approximately 190 K (above that temperature the warming up of the plate became too slow for conducting measurements in a reasonable time span), corresponding to a conductivity range of 37 to 200 MS. Throughout this range the dependence of terminal velocity on $1/\sigma$ remains linear, just as the theory predicts (fig. 13.). Bearing in mind that the angle of inclination was held at a constant 28.5 degrees we can once more obtain an experimental $\Lambda$ for the magnet used in this experiment; one gets ( 2.97 ± 0.2 )$\cdot 10^{-9}$ kgs$^{-1}$S$^{-1}$. For a magnet diameter of 0.95 cm and a thickness of 0.63 cm the theory gives 2.95(2)$\cdot 10^{-9}$ kgs$^{-1}$S$^{-1}$. Again agreement is very good, although we may once more notice the slight theoretical underestimation of $\Lambda$ already encountered in the first part of the experiment. This underestimation might be caused by the imperfections of the numerical integration process or by an error in determining some of the parameters of the apparatus (e.g. solenoid distance).

All our quantitative experiments were performed with a magnet following a straight path on the plate, but due to the spectacularity of the boundary effects we decided to make some investigations concerning them too. The magnet we used had a diameter of 0.95 cm and a thickness of 0.63 cm, it was released close to an edge and photographed every 1/30 of a second from above. A scale meter was put next to the plate in order to enable trajectory determination. The obtained trajectory is a lot like a sine, due to the deflections at the boundary (fig. 14.), just as we expected. However, the exact theoretical determination of this trajectory remains out of the scope of this work due to inherent mathematical difficulties, although we believe that in principle it is possible.

## 4. Conclusion.

Our work on the problem of a magnet rolling down an incline, both theoretical and experimental, leads to several conclusions. Firstly, the problem of a rolling magnet, as opposed to a sliding motion, can be much more rewarding and interesting; as far as we know, there has been no comprehensive treatment of it up to date. We have shown in our investigation that a relatively complete analytical treatment giving results in good agreement with experiment is possible in this case (except where boundary conditions are concerned; we suspect that numerically solving the Maxwell system is the only option there). The effects on a conducting incline can serve as a dramatic and rarely seen demonstration of the influences of eddy currents on a magnet, and the motion on a nonconducting plate can, among other things, be used as a compass (the magnet always turns north!), or for measuring the Earth magnetic field.

Although our model describes the obtained experimental results rather well, there is always more to do to make things even better. For example, by calculating the higher order terms in the induced field equation, the accuracy of the theory could be improved. We have also thought of many additional experiments that could be performed in order to thoroughly test the theory, like determining the dependence of the drag force on the distance between magnet and plate. It would be very interesting as well to find a method for imaging the induced field itself; there are indications that it might be detected by our solenoid system (if the dominant signal from the magnet's field was removed), but we haven't gone into trying anything of the sort. We hope that in a future experiment we will manage to think of an effective field imaging technique as well. With the mentioned ad – ons included, we might say that the problem would be rather thoroughly solved.

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Figures:

Fig. 1. – Magnet trajectories for three different angles

Fig. 2. The coordinate system, with denoted drag force and magnetization vector

Fig. 3. The induced magnetic field on the plane $y = 0$.

Fig. 4. – The induced magnetic field on the plane $z = -0.0025$ m

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Fig. 11. – The apparatus for changing plate conductivity.

Fig. 12. – The box, incline and solenoids ready for use.

Fig. 13. – The dependence of terminal velocity on 1/conductivity

Fig. 14. – The magnet trajectory, boundary efects included.

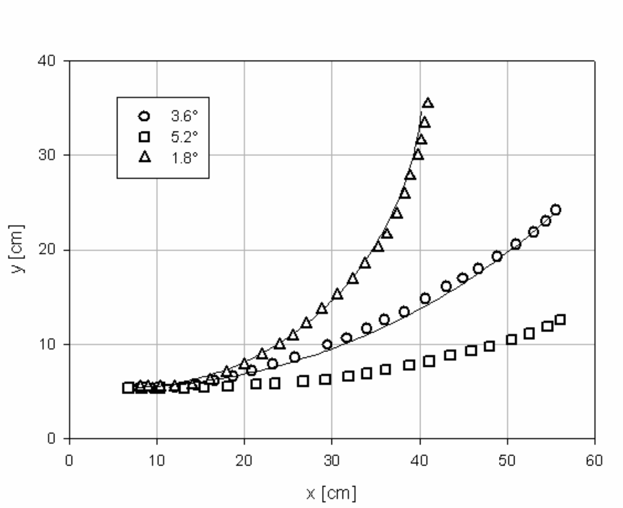



Fig.1

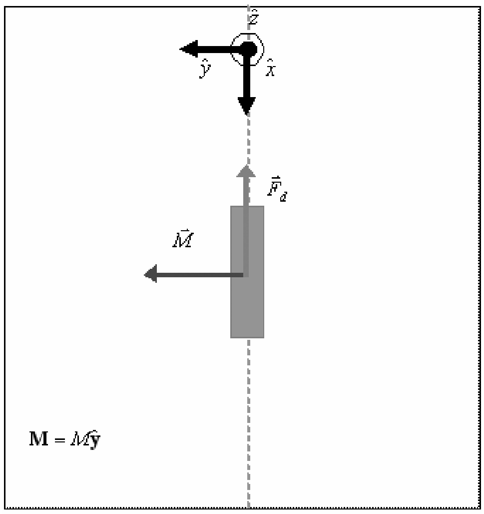



Fig2.

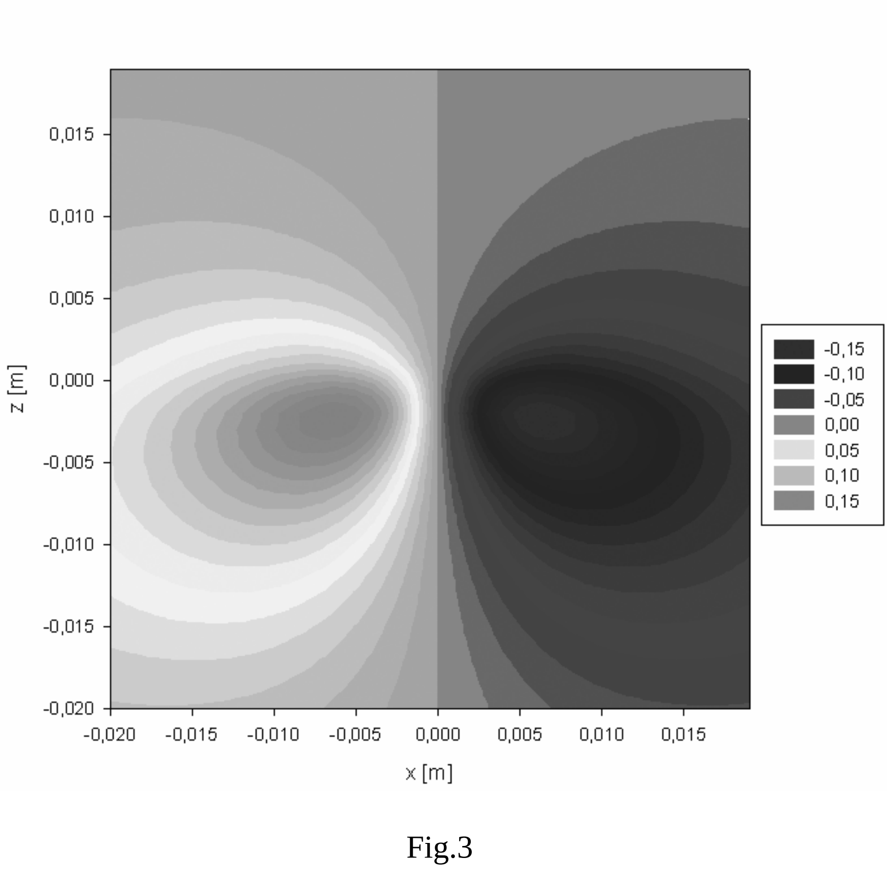
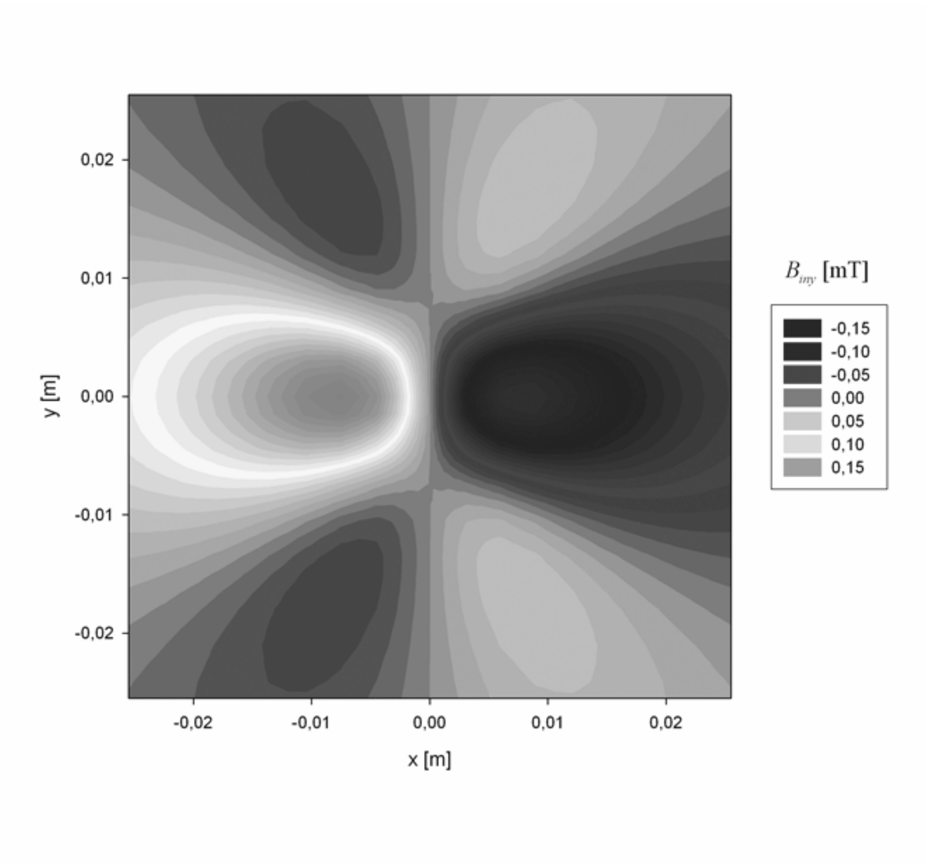



Fig.3



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-0,02
y [m]
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0,00
0,01
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x [m]
$B_{iny}$ [mT]
-0,15
-0,10
-0,05
0,00
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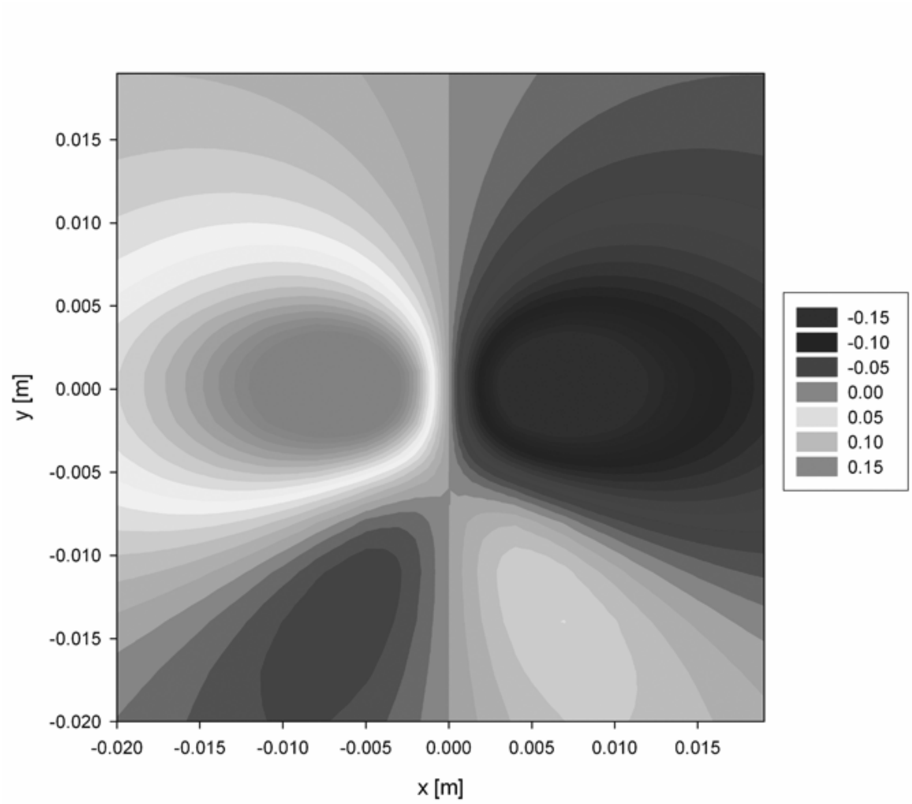
0,10
0,15


Fig.4



0.015
0.010
0.005
0.000
-0.005
-0.010
-0.015
-0.020
y [m]
-0.020
-0.015
-0.010
-0.005
0.000
0.005
0.010
0.015
x [m]
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15


Fig.5

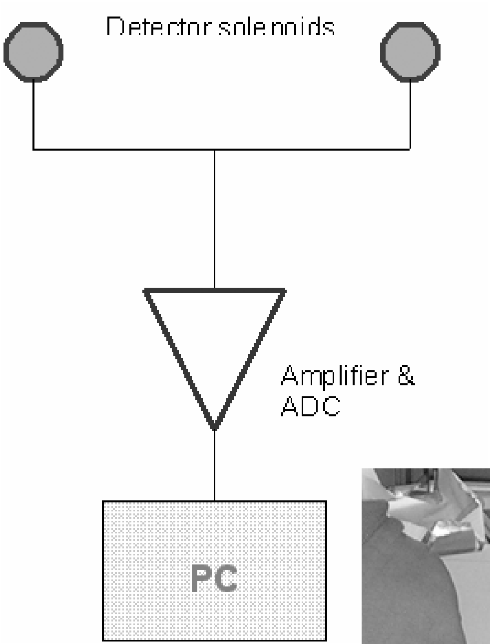



Fig.6

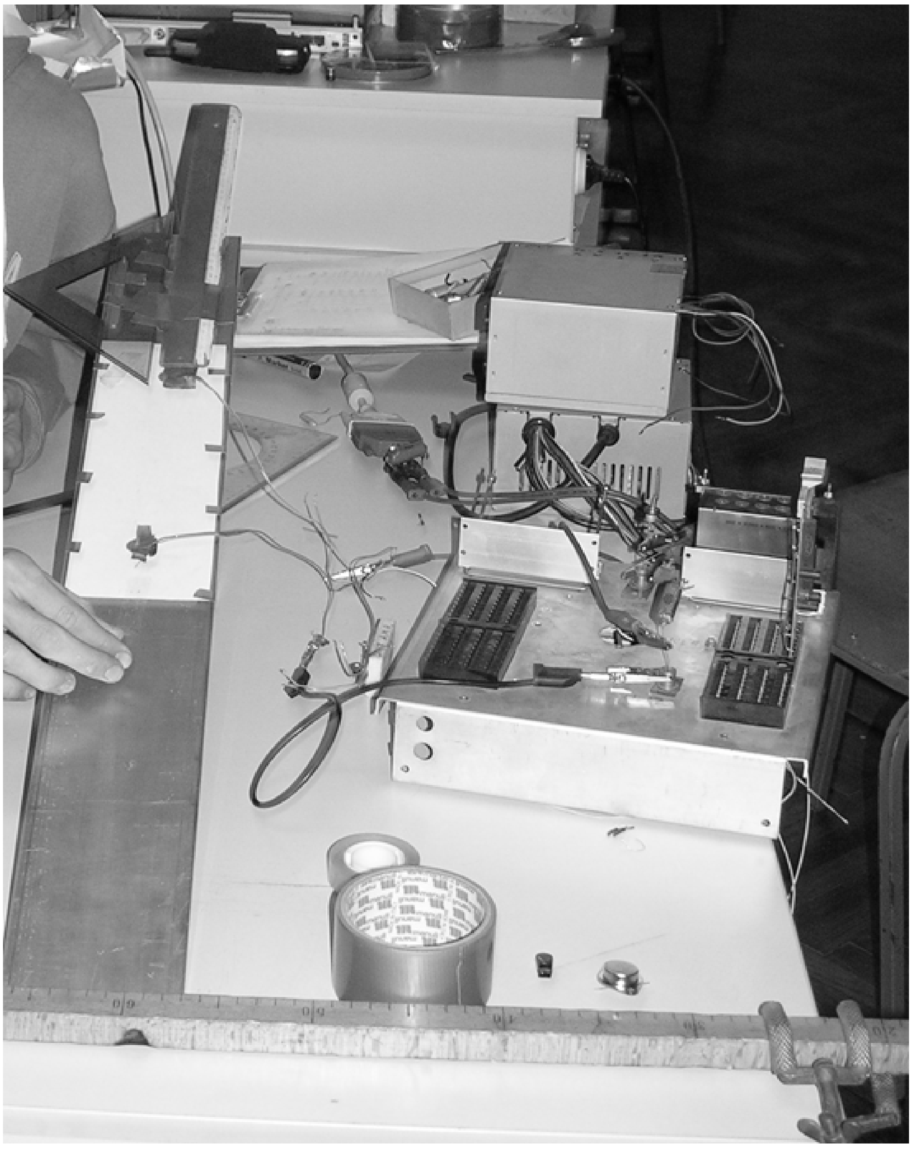


Fig.7

Fig.8

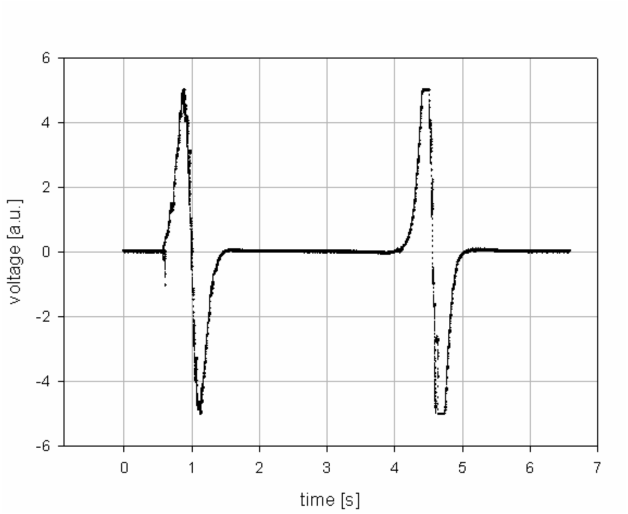
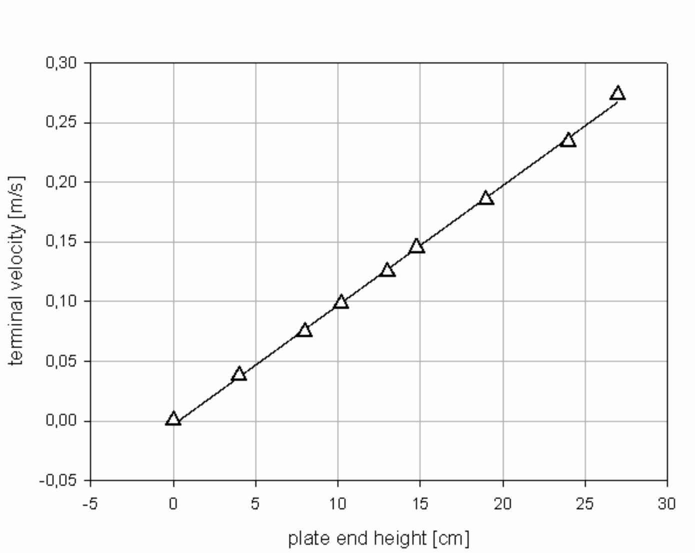



Fig.9



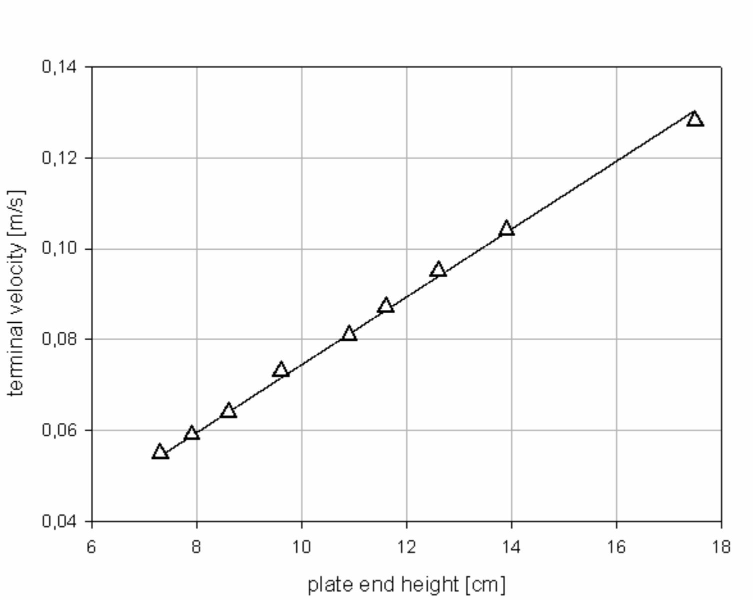




Fig.10

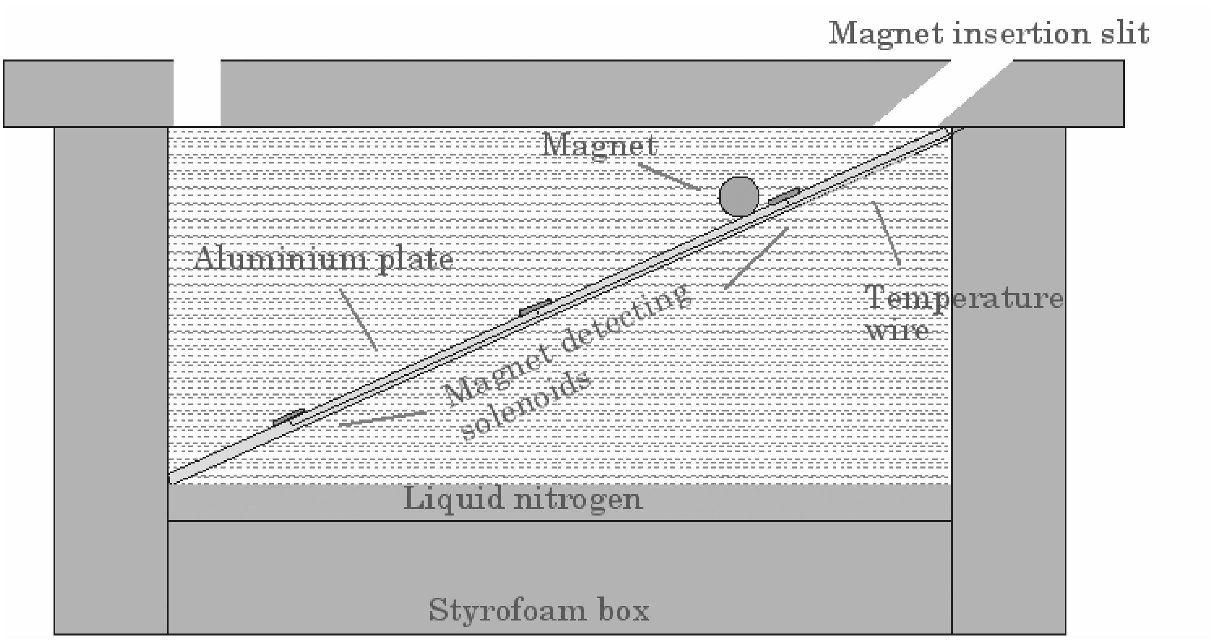

Magnet insertion slit
Magnet
Aluminium plate
Magnet detecting solenoids
Temperature wire
Liquid nitrogen
Styrofoam box

Fig.11

Fig.12

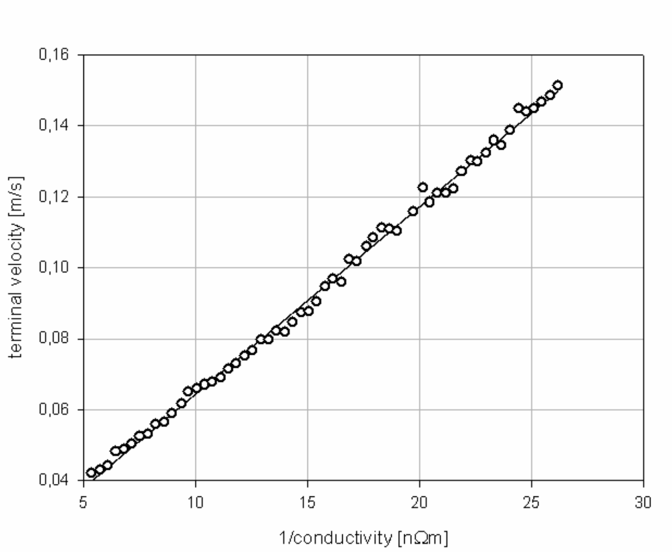

terminal velocity [m/s]
0,16
0,14
0,12
0,10
0,08
0,06
0,04
5
10
15
20
25
30
1/conductivity [nΩm]

Fig.13

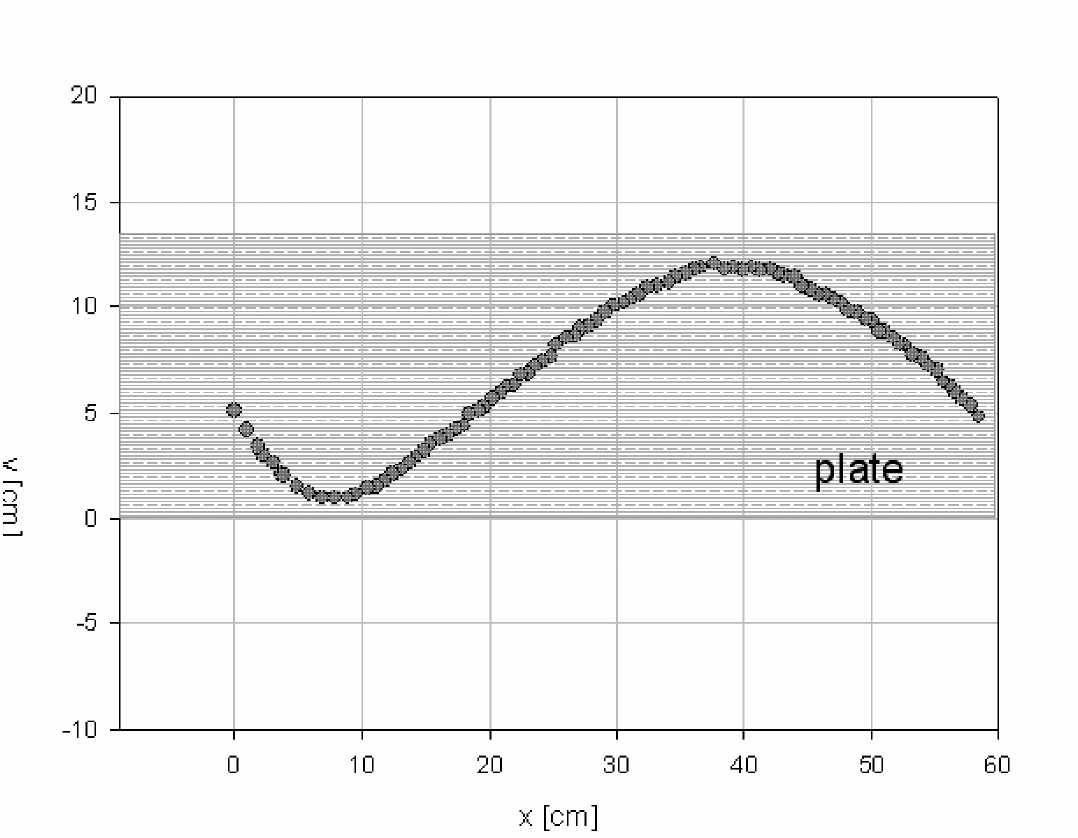

y [cm]
20
15
10
5
0
-5
-10
0
10
20
30
40
50
60
x [cm]
plate

Fig.14