10. Problem №13: Sound

10.2. Solution of New Zaland

Problem №13: Sound

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The Problem

Measure the speed of sound in liquids using light.

Abstract

A standing wave is set up in a liquid using an ultra-sound transducer as the source. The compressions and rarefactions in this standing wave act as a diffraction grating. A laser is shone through this diffraction grating and from measurements taken the wavelength of the sound is determined. Knowing the frequency of the ultra-sound it was then possible to calculate the velocity of the ultra-sound. The results obtained for water at temperatures of 10°C and 20°C were both within 1% of the accepted values.

Definitions

- Liquid - amorphous matter with definite volume but no fixed shape
- Light (visible) – electromagnetic radiation with a range of wavelengths and corresponding frequencies
- Sound – a form of longitudinal wave with a series of compressions and rarefactions.
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Refractive Index

- Ratio of speed of light in a vacuum and the speed of light in the particular medium

$$n = \frac{c}{c_{medium}}$$

- The larger the difference in the refractive index the greater the degree of refraction

Density Fluctuations

- Sound waves are characterized by regions of high and low pressure
- In a liquid these pressure variations cause variations in density
- Distance of separation is the wavelength of the sound wave
- It follows that these density changes will cause changes in the refractive index of the liquid.
- The variations in refractive index cause the liquid to model a diffraction grating.

Source of Sound Waves

- Sound must be in the form of ultra sound
- Ultra sound has a frequency in the MHz range.
- Therefore it has a small wavelength.
- Small distances between density fluctuations i.e. small enough to act as a diffraction grating

Diffraction Grating

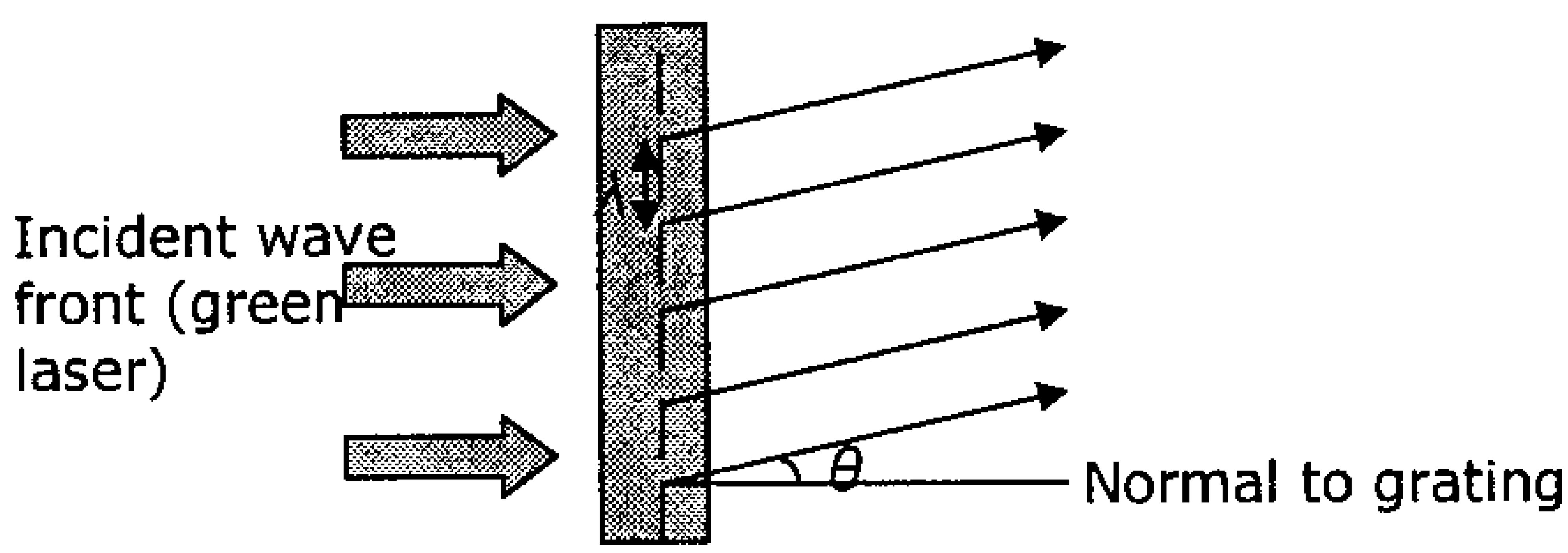
The liquid, modeling a diffraction grating will diffract monochromatic light According to the grating equation

$$k\lambda = d\sin\theta$$

where k is the order of diffraction
θ is the angle of diffraction
d is the distance of separation

Procedure

- A green laser is shone perpendicular to the direction of propagation of the ultrasound



Equating Wavelength

- From the previous diagram and using the grating equation we can derive the equation in terms of the wavelength of the ultra sound
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$$\lambda \cdot \sin\theta_k = k \cdot \lambda_{medium}$$

where k is the number of the order ($k = 0,1,2\ldots$)

Relationship of light wavelength

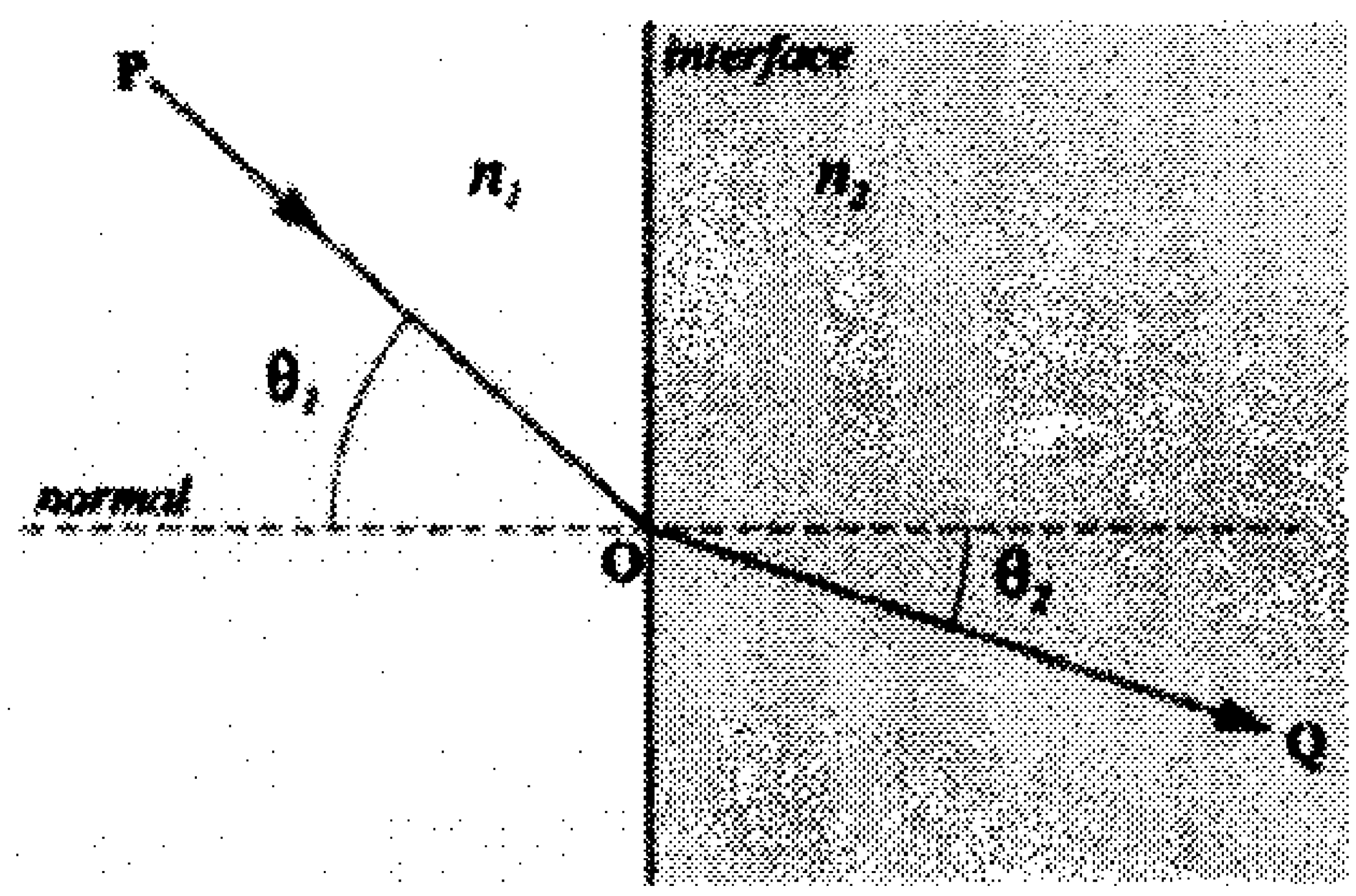
Wavelength of light in vacuum λ_{vac} and in medium (liquid) λ_{medium} are related by

$$\lambda_{vac} = \lambda_{medium} \cdot n_M$$

This relation is derived from the refractive index equation.

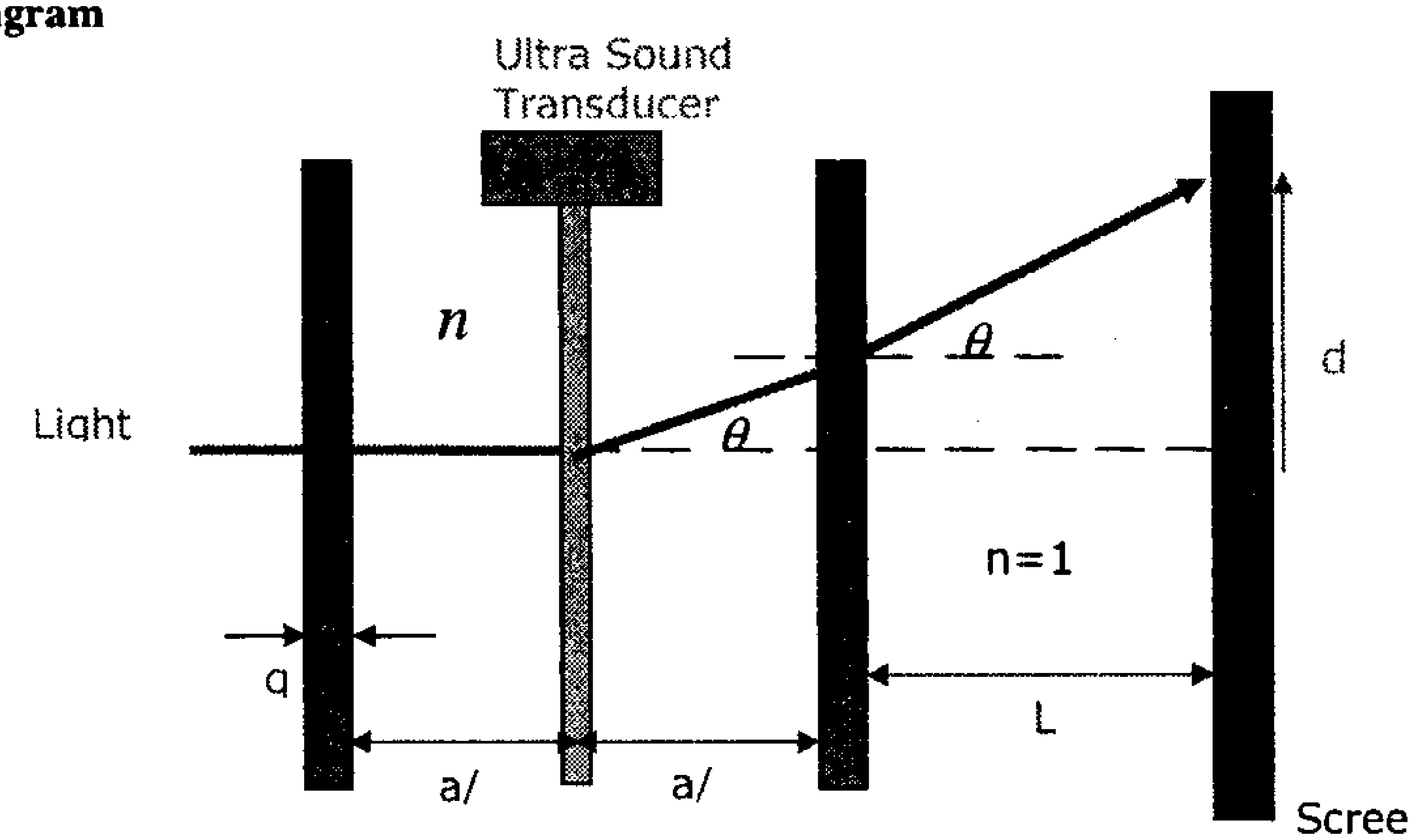
Snell's Law

- relationship between angles of incidence and refraction when a wave travels from one medium to another



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Diagram



- When light rays exit the glass tank they will be refracted.
- Due to change in the medium from liquid to air
- Refract away from the normal
- From Snell's Law

$$\sin\theta'_k = n_M \sin\theta$$

Diffraction Pattern

- We view bright spots on the screen a distance L from the wall of the glass tank.
- The separation of a bright spot from the central maximum is given by:

$$d_k = L \cdot \tan\theta'_k + (a/2 + g)\tan\theta_k$$

Angle θ

- Since θ_k is small we can say $\sin\theta \approx \tan\theta$
- Replacing $\tan\theta'_k$ and $\tan\theta_k$ we get

$$d_k = L \cdot \sin\theta'_k + (a/2 + g)\sin\theta_k$$

Recalling and Deriving

- Recall earlier equations

$$\lambda_{vac} = \lambda_{medium} \cdot n_M$$

$$\lambda_s \cdot \sin\theta_k = k \cdot \lambda_{medium} \Rightarrow \sin\theta_k = \frac{k \cdot \lambda_{medium}}{\lambda_s}$$

$$\sin\theta'_k = n_M \sin\theta$$

- We can now replace $\sin\theta'_k$ and $\sin\theta_k$
- We end up with the following equation

$$d_k = k \cdot \frac{\lambda_{vac}}{\lambda_s} \cdot \left(L + \frac{1}{n_M}[a/2 + g] \right)$$

Relating Wavelength to Speed

$$\lambda_s = \frac{k \cdot \lambda_{vac}\left(L + \frac{1}{n_M}[a/2 + g] \right)}{d_k}$$

- Relationship between wavelength and speed

$$v_s = f_s \lambda_s$$

- The frequency of the ultra sound is known. Therefore, once the wavelength is known we can determine the speed of sound in the liquid.
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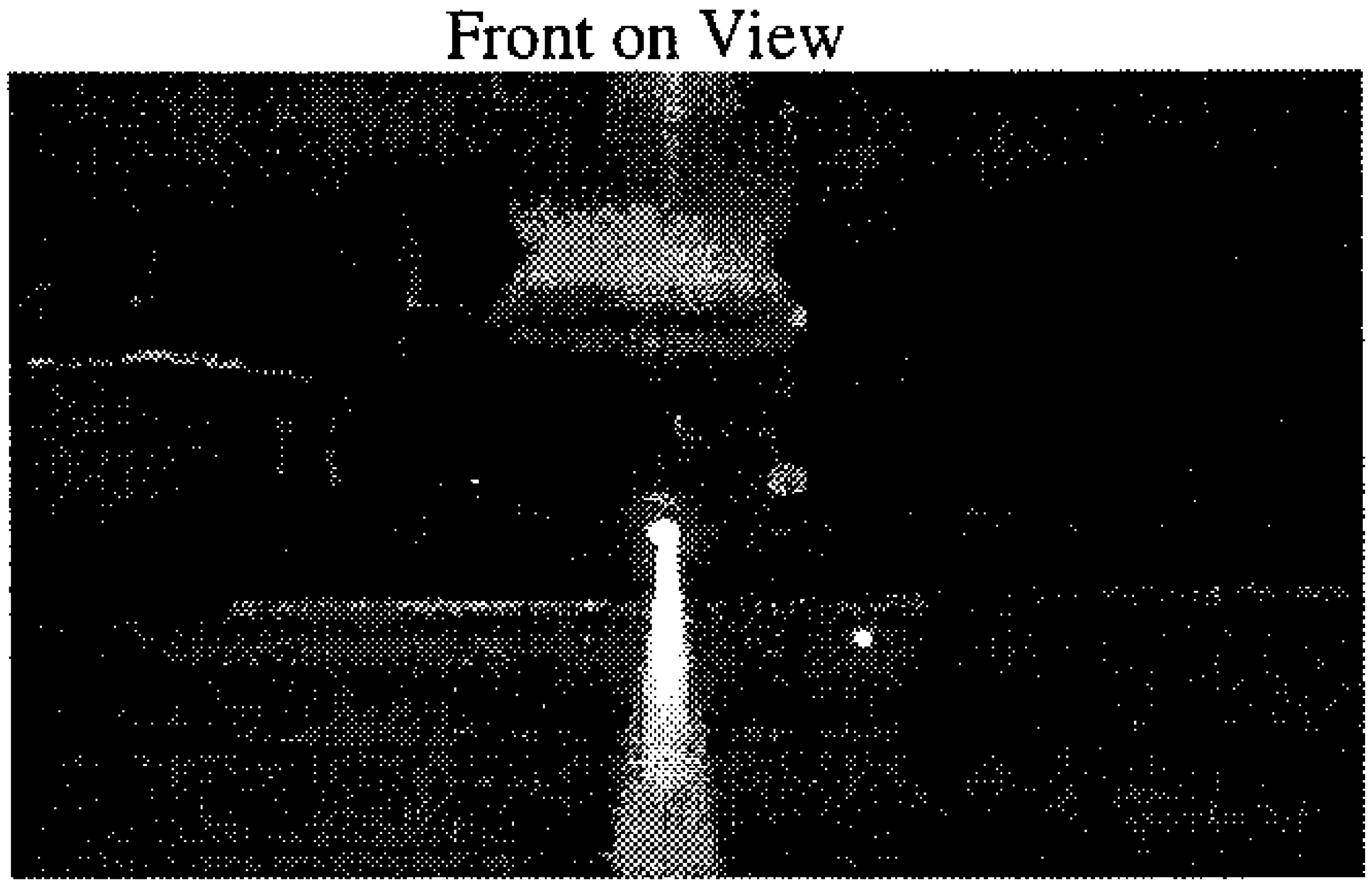
Practical – Objectives

- Study the diffraction of light due to density fluctuations in a liquid
- Determine the speed of sound by measuring the wavelength of ultrasound

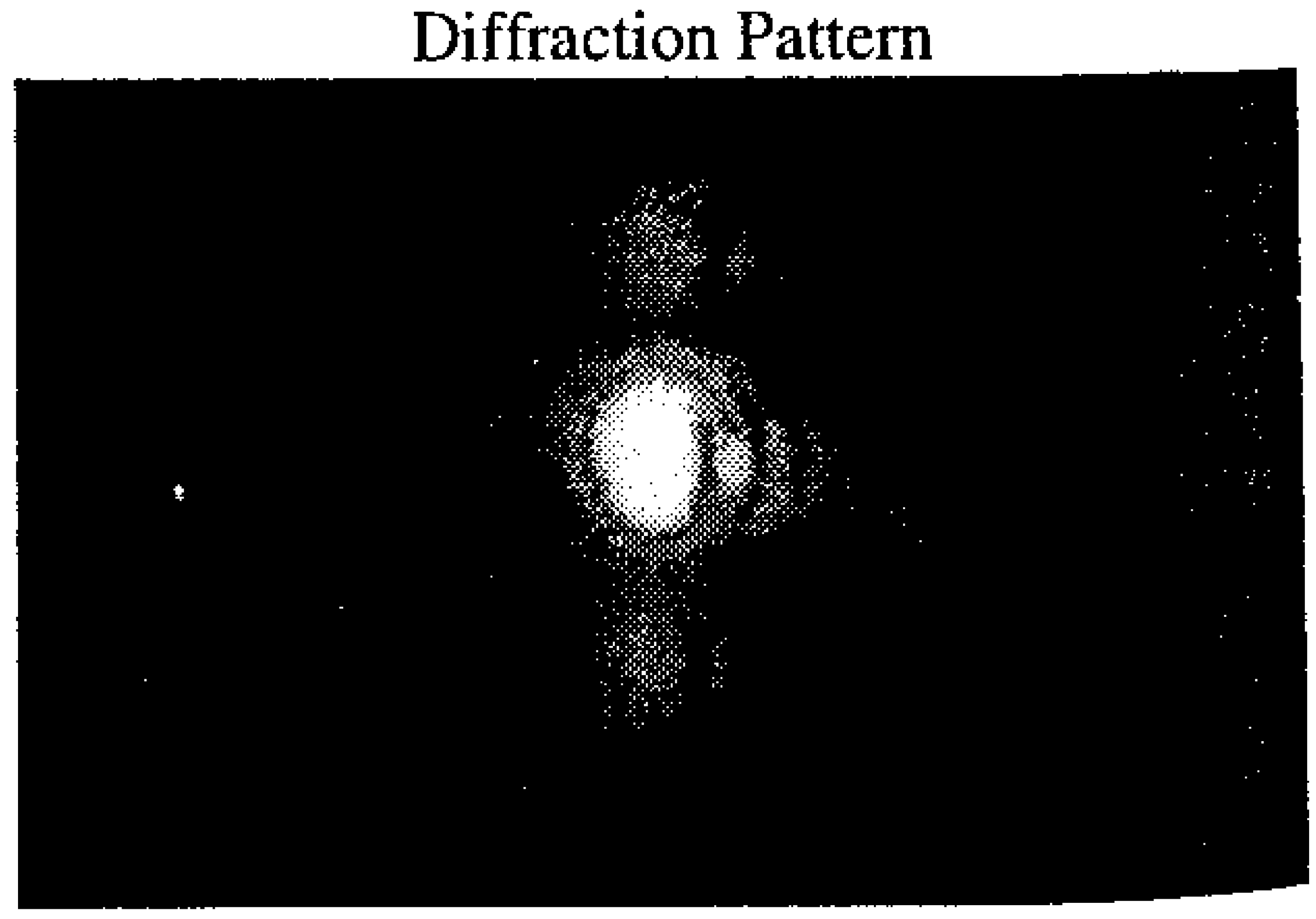
Practical – Set up

Practical Water

Front on View



Diffraction Pattern



Values known/measured

- Wavelength of green laser = 532nm
- Thickness of glass tank (g) = 4.80 0.01mm
- Value of (a/2) = 36.0 0.1 cm

- Distance of tank from screen (L) = 3.00 0.001m
- Index of refraction of water = 1.33
- Temperature of water = 20.0°C
- Frequency of transducer = 3.0MHz

Determining wavelength

- The distance between the central maximum and the 1st maximum was measured

= 3.5 ± 0.05 mm

$$\lambda_s = \frac{k \cdot \lambda_{vac}\left(L + \frac{1}{n_M}[a/2+g]\right)}{d_k}$$

$$\Rightarrow \lambda_s = \frac{1 \times 532 \times 10^{-9}\left(3.00 + \frac{1}{1.33}\left[36 + 4.80 \times 10^{-3}\right]\right)}{3.5 \times 10^{-3}}$$

$$\Rightarrow \lambda_s = 4.977 \times 10^{-4} m$$

Velocity

$v_s = f_s \lambda_s$

$\Rightarrow v_s = 3 \times 10^6 \times 4.977 \times 10^{-4}$

$\Rightarrow v_s = 1493$

$\Rightarrow v_s = 1490 \pm 20\ ms^{-1}$

Accepted value of speed of sound in water at 20°C is $1480ms^{-1}$

Temperature and Speed of Sound

- Speed of sound in water is dependent on temperature
- Increase temperature causes an increase in speed of sound.

Water at 10°C

The distance between the central Maximum and the 1st maximum was measured,
= 3.6 ± 0.05mm
Accepted value at 10°C = $1447ms^{-1}$

$$\lambda_s = \frac{1 \times 532 \times 10^{-9}\left(3.00 + \frac{1}{1.33}(.36 + 4.80 \times 10^{-3})\right)}{3.6 \times 10^{-3}}$$

$$\Rightarrow \lambda_s = 4.839 \times 10^{-4}$$

$$v_s = 3 \times 10^6 \times 4.839 \times 10^{-4}$$

$$\Rightarrow v_s = 1452 ms^{-1}$$

$$\Rightarrow v_s = 1450 \pm 20 ms^{-1}$$

Uncertainty in Calculation

At 20°C

- Uncertainty in calculation of speed of sound was ± 1.46%
- Percentage difference between accepted value of the speed of sound and calculated value is 0.878%

At 10°C

- Uncertainty in calculation of speed of sound is ± 1.42%
- Percentage difference between accepted value of speed of sound and calculated value is 0.346%

Conclusions

- Ultrasound causes density changes in a liquid
- Result of these density changes causes changes in the refractive index of the liquid
- Acts as a diffraction grating
- Measuring distance between maxima enables wavelength of sound to be calculated
- Velocity calculated from wavelength and known frequency of the ultrasound

Extra Material

In non transparent liquids

- Non transparent liquid is added into the tank, along with water.
- Depending on the density of the liquid a layer of water will form above or underneath the layer of the liquid
- Ultrasound is projected into the non transparent liquid and as a result the liquid will experience density fluctuations and a change in the index of refraction.
- Due to viscosity of the liquid water will undergo the same effect.
- Laser is then shone through the water and diffraction pattern is observed

Bulk Modulus

- measures the substance's resistance to uniform compression.
- The ratio of the change in pressure to the fractional volume compression is called the bulk modulus of the material

$$v_s = \sqrt{\frac{B}{\rho}}$$