10. Problem №13: Sound

10.2. Solution of New Zealand

Problem №13: Sound

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The Problem

Measure the speed of sound in liquids using light.

Abstract

A standing wave is set up in a liquid using an ultra-sound transducer as the source. The compressions and rarefactions in this standing wave act as a diffraction grating. A laser is shone through this diffraction grating and from measurements taken the wavelength of the sound is determined. Knowing the frequency of the ultra-sound it was then possible to calculate the velocity of the ultra-sound. The results obtained for water at temperatures of 10°C and 20°C were both within 1% of the accepted values.

Definitions

- Liquid - amorphous matter with definite volume but no fixed shape
- Light (visible) – electromagnetic radiation with a range of wavelengths and corresponding frequencies
- Sound – a form of longitudinal wave with a series of compressions and rarefactions.

Refractive Index

- Ratio of speed of light in a vacuum and the speed of light in the particular medium

\[ n = \frac{c}{c_{\text{medium}}} \]

- The larger the difference in the refractive index the greater the degree of refraction

Density Fluctuations

- Sound waves are characterized by regions of high and low pressure
- In a liquid these pressure variations cause variations in density
• Distance of separation is the wavelength of the sound wave
• It follows that these density changes will cause changes in the refractive index of the liquid.
• The variations in refractive index cause the liquid to model a diffraction grating.

Source of Sound Waves
• Sound must be in the form of ultra sound
• Ultra sound has a frequency in the MHz range.
• Therefore it has a small wavelength.
• Small distances between density fluctuations i.e. small enough to act as a diffraction grating

Diffraction Grating
The liquid, modeling a diffraction grating will diffract monochromatic light
According to the grating equation

\[ k\lambda = d \sin \theta \]

where  
\( k \) is the order of diffraction  
\( \theta \) is the angle of diffraction  
\( d \) is the distance of separation

Procedure
• A green laser is shone perpendicular to the direction of propagation of the ultrasound

Equating Wavelength
• From the previous diagram and using the grating equation we can derive the equation in terms of the wavelength of the ultra sound

\[ \lambda \cdot \sin \theta_k = k \cdot \lambda_{medium} \]
where $k$ is the number of the order ($k = 0, 1, 2, \ldots$)

**Relationship of light wavelength**

Wavelength of light in vacuum $\lambda_{\text{vac}}$ and in medium (liquid) $\lambda_{\text{medium}}$ are related by

$$\lambda_{\text{vac}} = \lambda_{\text{medium}} \cdot n_M$$

This relation is derived from the refractive index equation.

**Snell’s Law**

- relationship between angles of incidence and refraction when a wave travels from one medium to another

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

**Diagram**

![Diagram of light and ultrasound transducer interaction](image)
• When light rays exit the glass tank they will be refracted.
• Due to change in the medium from liquid to air
• Refract away from the normal
• From Snell’s Law

\[
\sin \theta'_{k} = n_{M} \sin \theta
\]

**Diffraction Pattern**

• We view bright spots on the screen a distance \( L \) from the wall of the glass tank.
• The separation of a bright spot from the central maximum is given by:

\[
d_{k} = L \cdot \tan \theta'_{k} + \left( a/2 + g \right) \tan \theta_{k}
\]

**Angle \( \theta \)**

• Since \( \theta_{k} \) is small we can say \( \sin \theta = \tan \theta \)
• Replacing \( \tan \theta'_{k} \) and \( \tan \theta_{k} \) we get

\[
d_{k} = L \sin \theta'_{k} + (a/2 + g) \sin \theta_{k}
\]

**Recalling and Deriving**

• Recall earlier equations

\[
\lambda_{vac} = \lambda_{medium} \cdot n_{M}
\]

\[
\lambda_{s} \cdot \sin \theta_{k} = k \cdot \lambda_{medium} \Rightarrow \sin \theta_{k} = \frac{k \cdot \lambda_{medium}}{\lambda_{s}}
\]

\[
\sin \theta'_{k} = n_{M} \sin \theta
\]

• We can now replace \( \sin \theta'_{k} \) and \( \sin \theta_{k} \)
• We end up with the following equation

\[
d_{k} = k \cdot \frac{\lambda_{vac}}{\lambda_{s}} \cdot \left( L + \frac{1}{n_{M}} \left[ a/2 + g \right] \right)
\]
Relating Wavelength to Speed

\[ \lambda_s = \frac{k \cdot \lambda_{\text{vac}} \left( L + \frac{1}{n_M} \left[ \frac{a}{2} + g \right] \right)}{d_k} \]

- Relationship between wavelength and speed

\[ v_s = f_s \lambda_s \]

- The frequency of the ultra sound is known. Therefore, once the wavelength is known we can determine the speed of sound in the liquid.

**Practical – Objectives**

- Study the diffraction of light due to density fluctuations in a liquid
- Determine the speed of sound by measuring the wavelength of ultrasound

**Practical – Set up**

![Image showing practical setup with labels for liquid, ultra sound transducer, and laser light]

**Practical Water**

![Image showing front on view and diffraction pattern of practical water]

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Values known/measured

- Wavelength of green laser = 532nm
- Thickness of glass tank (g) = 4.80 ± 0.01mm
- Value of (a/2) = 36.0 ± 0.1 cm
- Distance of tank from screen (L) = 3.00 ± 0.001m
- Index of refraction of water = 1.33
- Temperature of water = 20.0°C
- Frequency of transducer = 3.0MHz

Determining wavelength

- The distance between the central maximum and the 1st maximum was measured

\[ \lambda_s = \frac{k \cdot \lambda_{iw} \left( L + \frac{1}{n_m} \left[ \frac{a}{2} + g \right] \right)}{d_k} \]

\[ \Rightarrow \lambda_s = \frac{1 \times 532 \times 10^{-9} \left( 3.00 + \frac{1}{1.33} \left[ 36 + 4.80 \times 10^{-3} \right] \right)}{3.5 \times 10^{-3}} \]

\[ \Rightarrow \lambda_s = 4.977 \times 10^{-4} m \]

Velocity

\[ v_s = f_s \lambda_s \]

\[ \Rightarrow v_s = 3 \times 10^6 \times 4.977 \times 10^{-4} \]

\[ \Rightarrow v_s = 1493 \]

\[ \Rightarrow v_s = 1490 \pm 20 \text{ms}^{-1} \]

Accepted value of speed of sound in water at 20°C is 1480ms⁻¹

Temperature and Speed of Sound

- Speed of sound in water is dependent on temperature
- Increase temperature causes an increase in speed of sound.

Water at 10°C

The distance between the central maximum and the 1st maximum was measured,

\[ \Rightarrow \lambda_s = 4.839 \times 10^{-4} \]

\[ v_s = 3 \times 10^6 \times 4.839 \times 10^{-4} \]

\[ \Rightarrow v_s = 1452 \text{ms}^{-1} \]

\[ \Rightarrow v_s = 1450 \pm 20 \text{ms}^{-1} \]
Uncertainty in Calculation

At 20°C

- Uncertainty in calculation of speed of sound was ± 1.46%
- Percentage difference between accepted value of the speed of sound and calculated value is 0.878%

At 10°C

- Uncertainty in calculation of speed of sound is ± 1.42%
- Percentage difference between accepted value of speed of sound and calculated value is 0.346%

Conclusions

- Ultrasound causes density changes in a liquid
- Result of these density changes causes changes in the refractive index of the liquid
- Acts as a diffraction grating
- Measuring distance between maxima enables wavelength of sound to be calculated
- Velocity calculated from wavelength and known frequency of the ultrasound

Extra Material

In non transparent liquids

- Non transparent liquid is added into the tank, along with water.
- Depending on the density of the liquid a layer of water will form above or underneath the layer of the liquid
- Ultrasound is projected into the non transparent liquid and as a result the liquid will experience density fluctuations and a change in the index of refraction.
- Due to viscosity of the liquid water will undergo the same effect.
- Laser is then shone through the water and diffraction pattern is observed

Bulk Modulus

- measures the substance's resistance to uniform compression.
- The ratio of the change in pressure to the fractional volume compression is called the bulk modulus of the material