# 12. Problem №16: Hardness

## *12. Solution of Korea*

### Problem №16: Hardness

Kim, Hong Kyun
Korean Minjok Leadership Academy
1334, Sosa, Anhung, Hoengsung, Kangwon, Korea 225-823

### The Problem

A steel ball falls onto a horizontal surface. If one places a sheet of paper onto the surface with a sheet of carbon paper on top of it, a round trace will be produced after the impact. Propose a hardness scale based on this method.

In this paper we examine a hardness value obtained from the carbon trace produced by a steel ball. A steel ball falls onto a horizontal surface with a sheet of paper with a sheet of carbon paper on top of it. The hardness scale based on this method is highly correlated to the Young's Modulus and elasticity of the test material and the deepest contact area before elastic restoration.

*Keywords*

Hardness, carbon trace, steel ball, hardness scale, Young's modulus, elasticity

### I. Introduction

Hardness of a material is usually defined as resistance of a material to permanent indentation, or the resistance of a material to the penetration of a second material. However, hardness is not a fundamental property such as mass, time, or length. There is no absolute value of hardness, and its definition is totally dependent on the test procedure. Thus, hardness is operational in definition, and conversion between different hardness values is possible, but again not absolute.

Thus, hardness value is not a well-defined property, for tests use different combinations of the elastic, yielding, and work hardening characteristics of materials. First, we examine the various types of hardness measurements. Brinell, Vickers, and Rockwell hardness scales are all indentation hardness test methods. Tests such as Shore hardness tests use the height of rebound to measure hardness. Because each method uses different methods, the hardness values each tell different properties of the test materal.

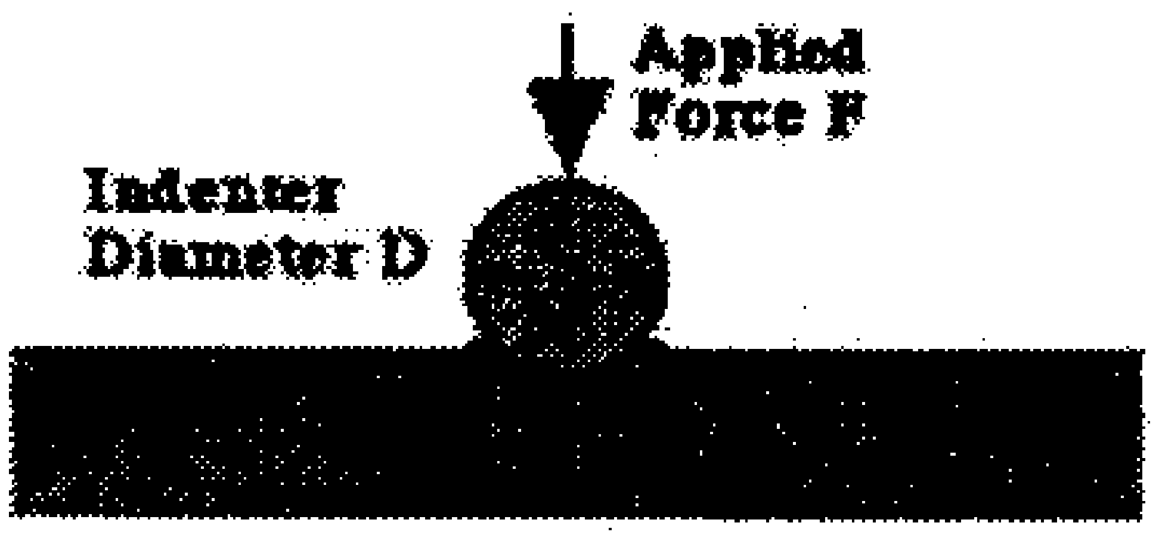


*Figure1.*

Brinell hardness testing, as shown in Figure (1), measures the size of indentation made from a steel sphere. Steel ball that has a diameter of 10 mm is gradually pressed on to the test material with a 3000 kg load. The Brinell hardness value is calculated as the applied load divided by the contact surface area of the indentation, shown in equation (1).

$$BHN = \frac{F}{\frac{\pi}{2} D \cdot (D - \sqrt{D^2 - D_i^2})} \qquad (1)$$

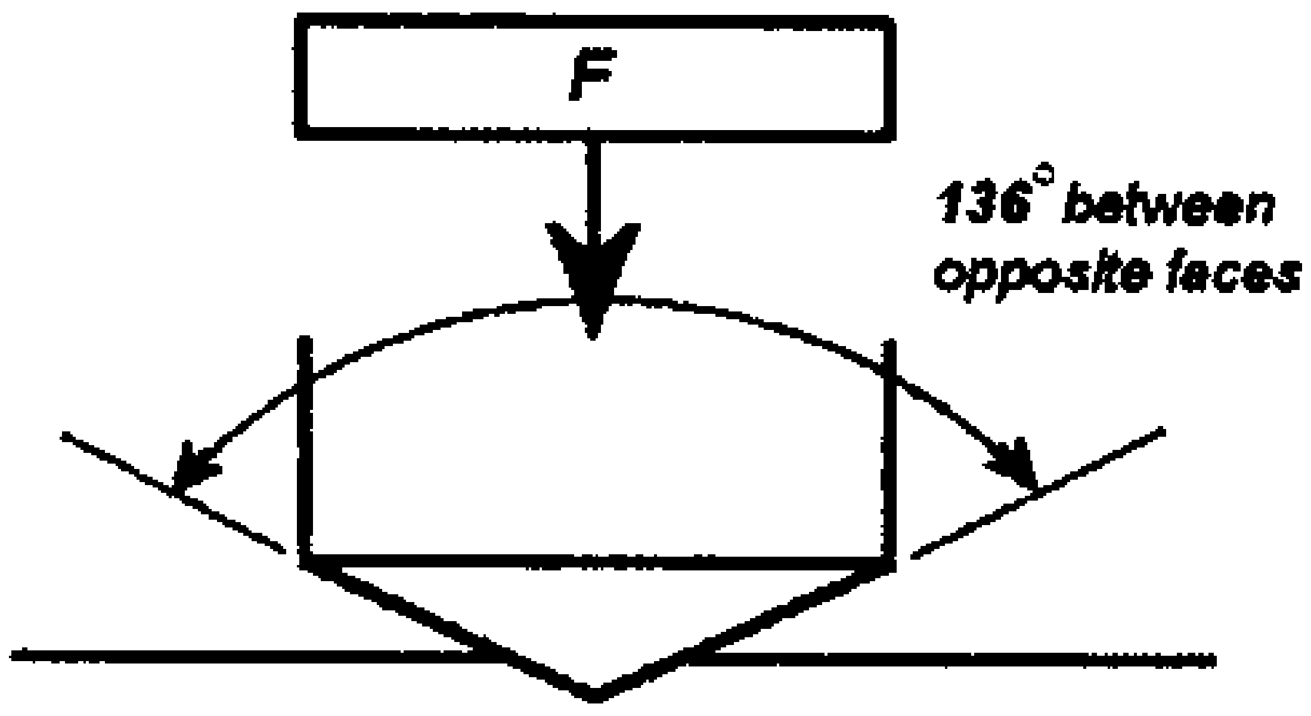


*Figure2.*

Vickers hardness testing as shown in Figure (2) uses a diamond square-based pyramid indenter. The indenter included angle is 136 degrees, and the variable load is from 120 kg to 5kg to grams. Again, the Vickers hardness value is obtained from dividing the load by the area of the indentation produced, as shown in equation (2).

$$HV = \frac{2F \sin \frac{136}{2}}{d^2} \qquad (2)$$

Examining the two hardness values, we see that although the procedures are different, both use the same concept of dividing the load by the indentation size. In the carbon paper hardness scale, we therefore, use the similar concept to calculate the hardness value of the test material. The indentation size is easily measured from the size of the carbon paper produced. The intensity of the carbon trace, how dark the trace is, is what shows the pressure or the load applied in the process of indentation.

## II. Carbon Paper Hardness Test - Procedure

The basic test equipments for carbon paper hardness test is shown in the Figures below. First, steel bearing balls were used as the indenters. Different sized balls of diameters 25mm, 19mm, 12.7mm, and 10 mm were used.

*Figure3.*

As shown in Figure4, a sheet of paper was placed on top of the test material, and a sheet of carbon paper covered the top.

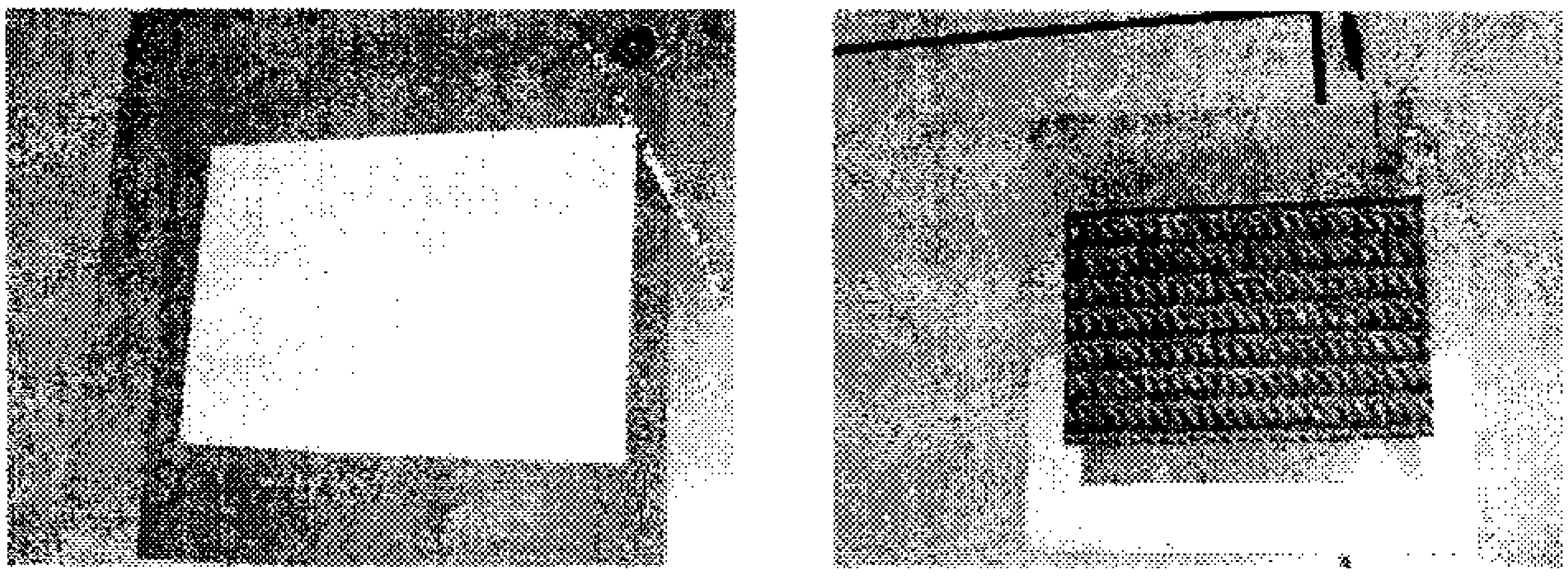


*Figure4.*

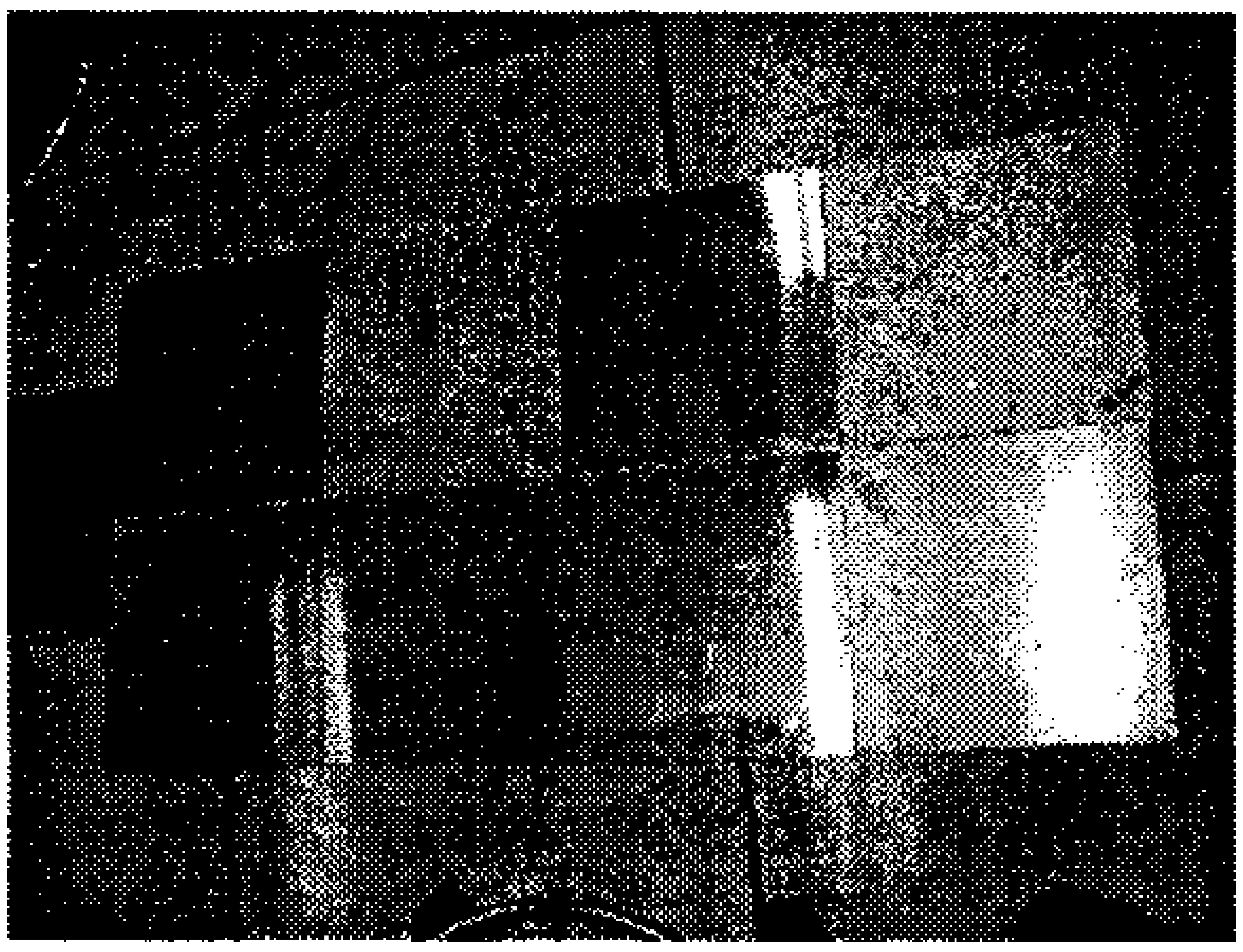


*Figure5.*

Different types of test materials were used to obtain the carbon traces to be used in constructing a hardness scale. Figure 5 shows the different types of test materials used. 4 metals, 4 plastics, and 3 other test materials were used. The 4 metals were stainless steel, brass, iron, and copper. The 4 plastics were bakelite, PVC, polycarbonate, and acetyl. Finally, the 3 others were wood, marble, and rubber.

Figure 6 shows the total equipments set together for the actual test procedure. A tripod was used to hold an electromagnet that would drop the steel ball at same initial conditions for each successive drop. The tripod was also used to adjust the falling height from 0.25m to 1m. The test material was placed below the electromagnet where impact would take place. The test material was placed above an aluminium plate that served as a bottom support just to keep conditions same for all drops.

With the carbon paper hardness testing setup above, we construct a hardness scale based on the carbon trace produced by the test method, with reference to the typical hardness values and calculation of hardness as shown in the introduction. Before constructing the hardness scale, we first examine the theoretical background into the process of the impact before analyzing the carbon traces.

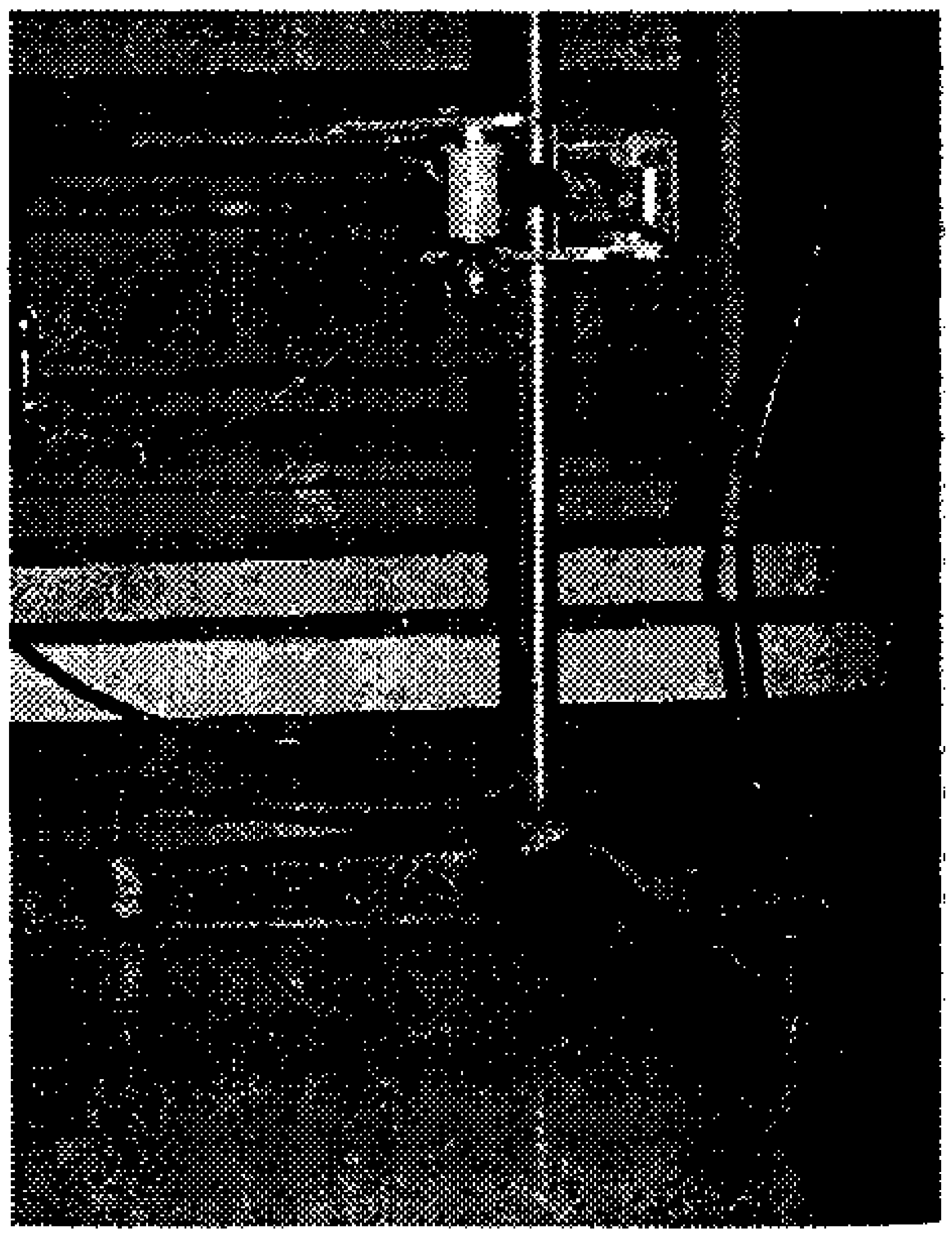


*Figure6*

## III. Theoretical Background

### *Stages of Impact: Elastic Impact*

Elastic impact is the impact between the steel ball and the test surface in which no permanent indentation takes place. That is, although the steel ball goes into the test material, deforming

the test material during the process, the test material restores to its original state. In this case, the stages of impact can be classified into 3 stages.
First is the impact between the ball and the surface

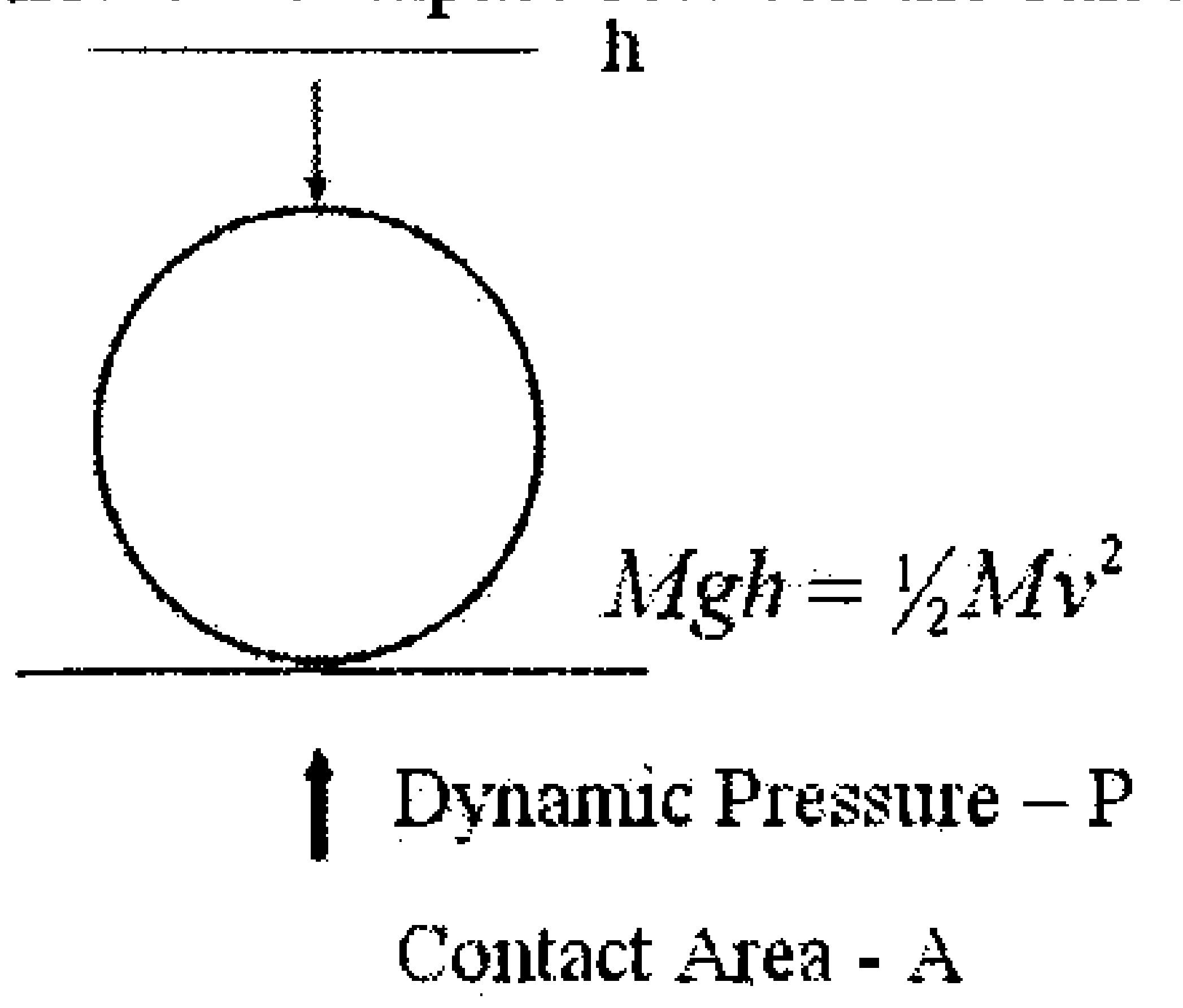


*Figure7.*

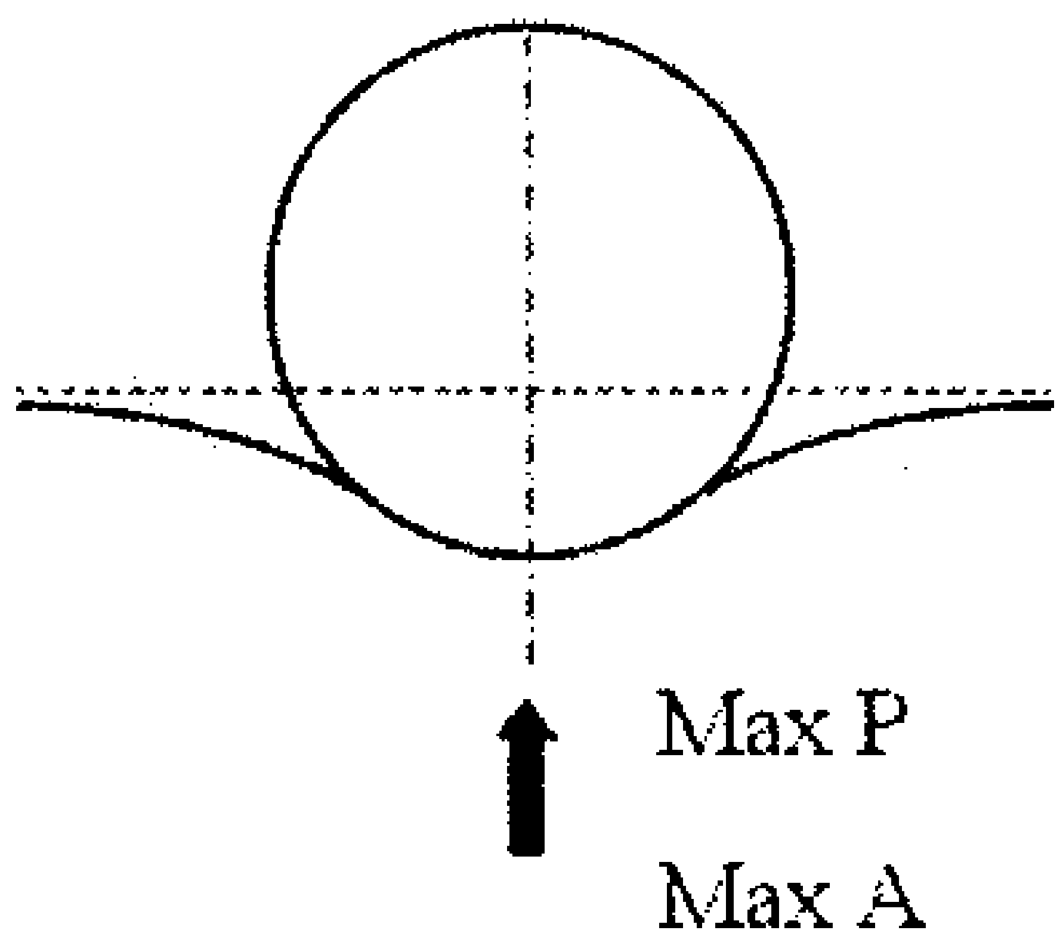


*Figure8.*

Figure 7 illustrates the initial impact stage. The ball drops from a height of h, with its potential energy transformed into kinetic energy. We define two terms the dynamic pressure P, the pressure exerted on the ball by the surface as the ball goes into the surface material, and the contact area A, which increases as the ball goes into the surface.

The second stage is the purely elastic stage, in which the surface material goes through temporary indentation. In this stage, the dynamic pressure P increases as the ball penetrates the surface material. The contact area also increases and reaches maximum when the ball stops deforming the test material, as shown in figure8.

The last stage of the elastic impact is the release of elastic stress and rebound of the ball. The deformed test surface returns to its original shape and in the process

*Elastic Contact: Spherical Indentation*

The relationship between the depth of test material's deformation and the properties of the test materials and the ball is explained by Hertz's Theory of Elastic Contact.

$$\delta = [9/8]^{1/3}[1/E^*][1/D]^{1/3}F^{2/3} \qquad (3)$$

$\delta$ is the depth of penetration, and D is the diameter of the steel ball. F is the load force that is applied to the steel ball in pressing against the test surface. Although the situation here is different in the carbon trace method that the steel ball is dropped from a certain height rather than being pressed on to the surface material, the situation is applicable. The F in the equation refers to the variable load force determined by the height of ball drop.

However, the characteristics of the test material also contributes as a variable to the penetration depth, as represented by E*.

$$\frac{1}{E*} = \frac{1-v_m{}^2}{E_m} + \frac{1-v_i^2}{E_i} \qquad (4)$$

$E_m$ refers to the Young's modulus of the test material while $E_i$ refers to the Young's modulus of the indenter, the steel ball. $v_m$ and $v_i$ refer to the poisson's ratio of the test material and the indenter, the steel ball, respectively. With the relationship between the depth of the penetration and the characteristics of the test material, we need to find the contact area between the ball and the surface. The carbon trace left by the impact represents the contact surface between the ball and the test material at its largest contact.

$$a^2 = R\delta \qquad (5)$$

The relationship between the contact surface and the depth of penetration is shown in the above equation. R represents the radius of the indenter and $a$ represents the radius of the circle of contact. To see the relationship between the contact surface and the characteristics of the material, we simply replace the depth with equations (3) and (4).

$$a^2 \propto 1/E^* \qquad (6)$$

The surface area of contact is inversely proportional to the Young's modulus of the test material, assuming that the elastic modulus, that is the Young's modulus, of the indenter is infinitely high. That is, we assume that the steel ball is very hard and does not go through any deformation during the process of impact.

The derivation of the Hertz' equation and the following equations are not thoroughly reviewed in this paper. However, a qualitative and quantitative examination of the relationship between the variables and the physical explanations through theory and experiments are given.

*Elastic Contact: Physical Explanation*

The physical explanation for the relationship between the characteristic of the test material and the contact area can be seen through figure 9.

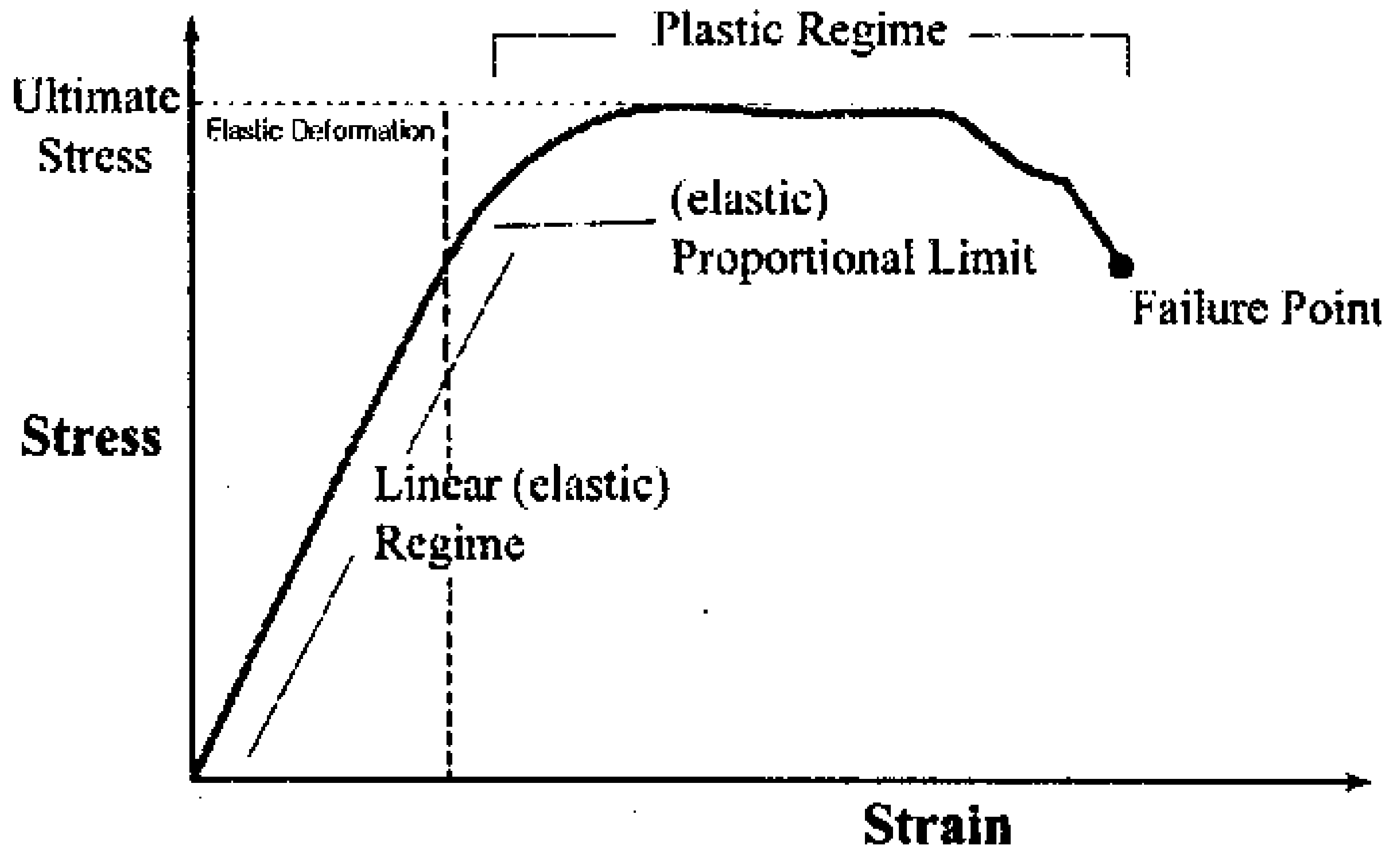


*Figure9.*

Figure 9 shows the typical strain and stress curve of a material. As shown in the graph, the process of straining the material can be divided into the linear(elastic) regime and the plastic regime. In the elastic regime, strain occurs proportionately to the stress applied. If stress is taken away in the elastic regime, the material will restore its shape to its original without any permanent deformation. In the plastic regime, however, stress and strain are no longer proportional, and stress reaches its maximum value, and permanent indentation takes place.

However, if we limit ourselves to the elastic impact between the ball and the surface material, we only need to examine the elastic regime, for plastic deformation does not take place. Young's modulus is the constant of elasticity, the slope of the stress and strain curve.

$$Y = \sigma / \varepsilon \qquad (7)$$

A higher Young's modulus means more stress per strain. That is for the same amount of deformation to take place, a higher stress need to be applied to a material with higher Young's modulus. Therefore, the contact surface area is smaller for materials with a higher Young's modulus, meaning that the two are inversely related.
The confirmation will be shown in the following contents of this paper.

*Inelastic Impact: Plastic Deformation*

The stages of impact for the impact in which plastic deformation takes place have an additional stage. The first two stages are identical with the ones of elastic impact. As the ball penetrates the material with increasing dynamic pressure and contact surface, the ball first makes an initial impact with the test material and goes through the elastic regime. However, as the stress and the dynamic pressure reach the end point of the elastic regime, the test material goes through a third stage, the plastic regime. Plastic deformation takes place at this point, and we assume that the dynamic pressure reaches its maximum and stays constant at this point, as shown in the stress and strain curve of a typical material.

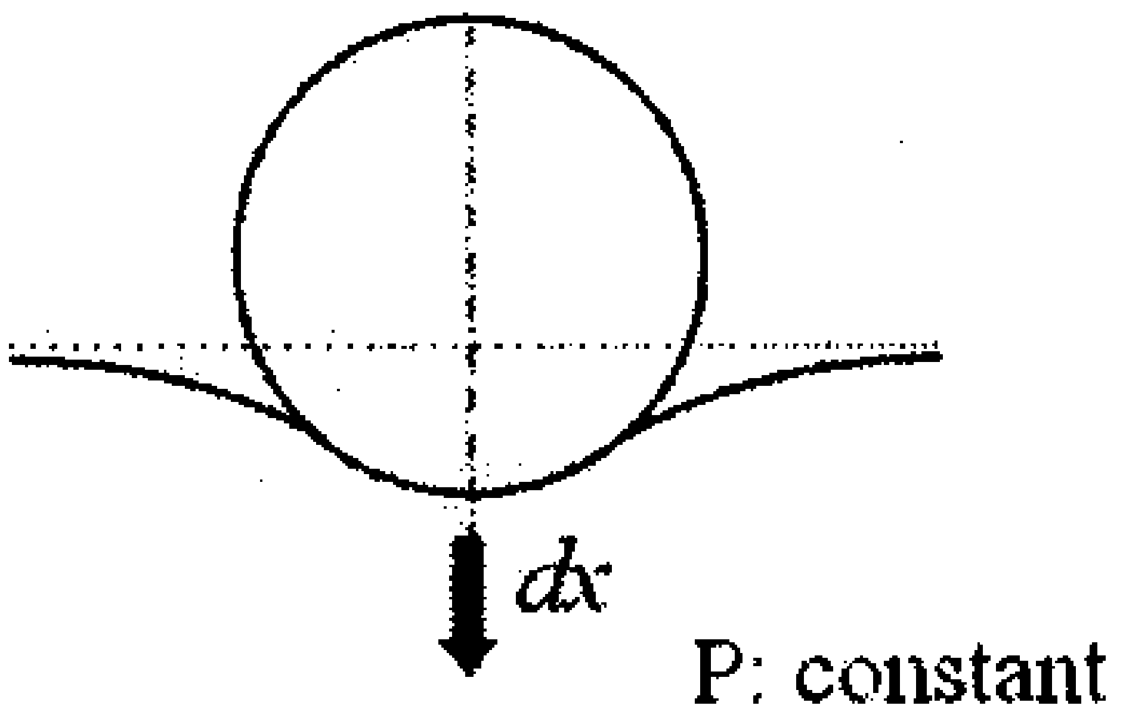


*Figure10.*

With the dynamic pressure constant, as the ball penetrates the material deeper and deeper, dx representing the minute change in depth of penetration, we see that equation (8) holds.

$$\int PAdx = P\int Adx = PV \qquad (8)$$

A represents the contact surface area, and V is the volume of the deformed region. The above equation represents the energy of impact. The dynamic pressure at each point is multiplied by the contact surface of each successive point of minutely increasing depth to obtain the final energy of impact. Therefore, we arrive at the relationship shown in equation (9).

$$P = \frac{Energy \quad of \quad impact}{V} \qquad (9)$$

Simply, the potential energy of the steel ball which had been transformed into kinetic energy was used in deforming the ball. The kinetic energy is stored as elastic energy required to make the ball go through deformation.

The final stage is the release of stress and rebound of the ball. In the process, some elastic restoration of the surface may take place. However, the surface has permanent indentation left from reaching the plastic regime during the impact.

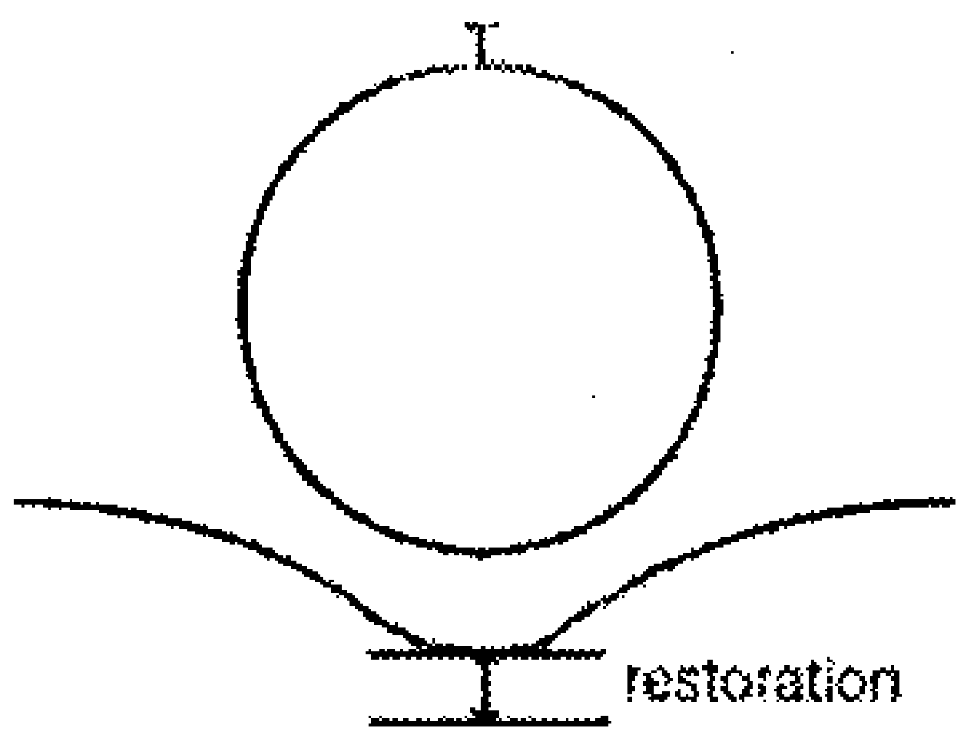


*Figure11.*

The size of the carbon trace is determined by the maximum point of contact between the ball and the test material. If we neglect the restoration that may take place, the energy that was left after being used to make the permanent indentation was transformed into the rebound energy. Thus, if more energy was put into plastic deformation, the ball would have a lower rebound.

### *Intensity of Carbon Trace: Elastic Contact*

We examined the stages of impact for both elastic and inelastic impact of the ball and the test surface and the relationship to the size of the carbon trace produced. However, we did not examine the intensity of the carbon trace that would be produced.

The intensity of the carbon trace is related to the maximum dynamic pressure that was applied during the whole process of the impact. The concept of maximum is important here, because dynamic pressure is always changing as the ball goes through the stages of impact.

In the elastic impact between the ball and the test surface, the intensity of the carbon trace is proportional to the maximum point of contact area, for dynamic pressure is largest when surface area of contact is the largest.

### *Intensity of Carbon Trace: Inelastic Contact*

In case of inelastic contact, the intensity of the carbon trace is also related to the maximum dynamic pressure that was applied during the stages of impact. The maximum pressure is the pressure that was applied when plastic regime was reached, as dynamic pressure was assumed to be steady when plastic deformation occurs.

Let d = 2a be the diameter of the circle of full contact between the ball and the surface when the plastic deformation is completed. At any intermediate instant when the contact circle has a diameter $2\alpha$ (where $\alpha$<a), the force $f$ on the indenter is given by equation (10).

$$f = F\frac{\alpha^3}{a^3} \qquad (10)$$

F is the force applied to the ball at the maximum contact, when the contact circle has the diameter of d. At this stage, because of the elastic deformation of the contacting surface, the center of the indenter has descended a distance z (Prescott, 1927) given by

$$z = \frac{3f}{4\alpha E^*} \qquad (11)$$

Then, the total elastic energy K stored in the surface is the integral $f$ dz from $\alpha$=0 to $\alpha$=a. We obtain

$$K = \int f dz = \int_0^a \frac{3}{2}\frac{F^2}{a^6 E*}\alpha^4 d\alpha = \frac{3}{10}\frac{F^2}{aE*} \qquad (12)$$

Since this process is exactly the converse of what happens when the surface recovers and the ball rebounds from the surface, the energy involved in both cases is the same. Thus, if the ball was dropped from a height of h1 to rebound up to a height of h2, the difference in potential energy is the energy used in permanent indentation of the surface material.

The energy used in making the permanent indentation, W3 is therefore

$$W_3 = W_1 - W_2 = PV_r \qquad (13)$$

W1 is the initial potential energy, and W2 is the rebound energy of the ball. Vr is the permanent indentation left in the surface. If we express Vr in terms of the radius of the sphere ball and the terms we know, we obtain

$$V_r = (\pi a^4)/(4r_2) \qquad (14)$$

Where $a$ is the maximum contact radius of contact and $r_2$ is the curvature of the indentation. Now we express $r_2$ in terms of $r_1$, the radius of the ball to get

$$\frac{1}{r_2} = \frac{1}{r_1} - \frac{3F}{4a^3E^*} \qquad (15)$$

where F is the maximum pressure and the effective Young's Modulus. This is because the indentation's curvature, $r_2$ will be somewhat greater than the curvature of the ball $r_1$ during the impact. Thus, substituting Vr in equation 12, we get

$$W_3 = P\frac{\pi a^4}{4r_1} - \frac{3}{16}\frac{F^2}{aE*} = PV_a - \frac{5}{8}W_2 \qquad (16)$$

The first term is the apparent volume of the indentation if the indentation had the same radius of curvature as the indenter. The second term is obtained by comparison with equation (12). Thus, we obtain

$$P = \frac{mg(h_1 - \frac{3}{8}h_2)}{V_a} \qquad (17)$$

This equation is obtained because assuming no energy loss, W3, the energy used for permanent indentation, added to W2, the rebound energy, is equal to the initial potential energy of the ball, mgh1. Thus, simply speaking, the maximum dynamic pressure applied depends on the energy dissipated in plastic deformation before rebound. We relate rebound height with the intensity, for later experiments were based also on rebound.

## IV. Experiment Results

The experiments were carried out as the described in the introduction of this paper. The sizes of the carbon traces were obtained mainly for constructing the hardness scale at the end.

*Size of Carbon Trace: Elastic Impact*

The first experiment was carried out with the same steel ball dropping onto different test material. The diameter of the carbon trace was measured. The materials had no permanent indentations left after the test, meaning that the procedure was all elastic.

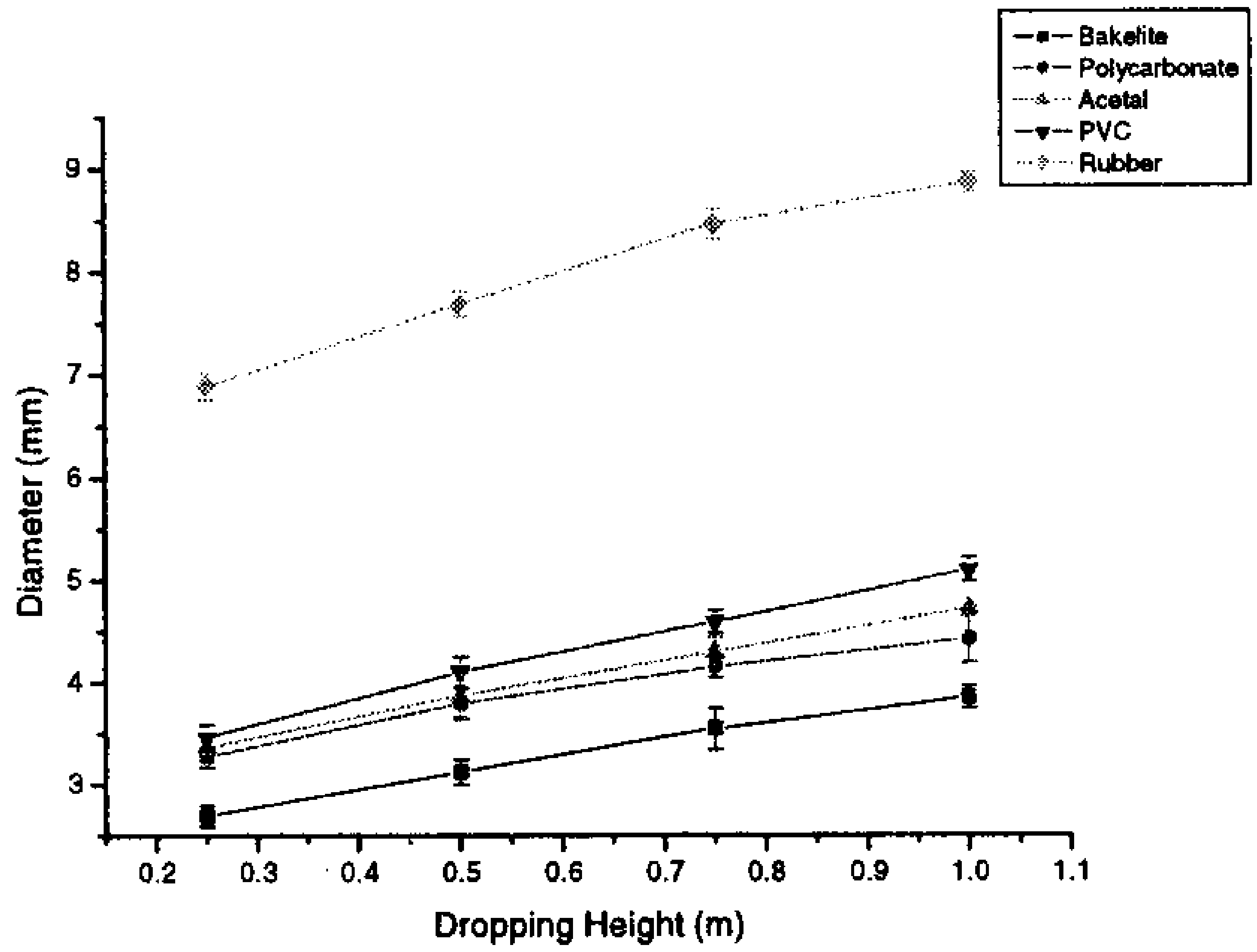


*Figure12.*

As the dropping height increased, the diameter of the carbon trace increased correspondingly. Since, the contact area is positively related to the force applied to the test material, it is obvious that the diameter of the carbon trace increases.

The next experiment was carried out using different masses of the ball at 1m height. Obviously, the increase in mass led to increase in the diameter of the carbon trace, due to increased force of the impact.

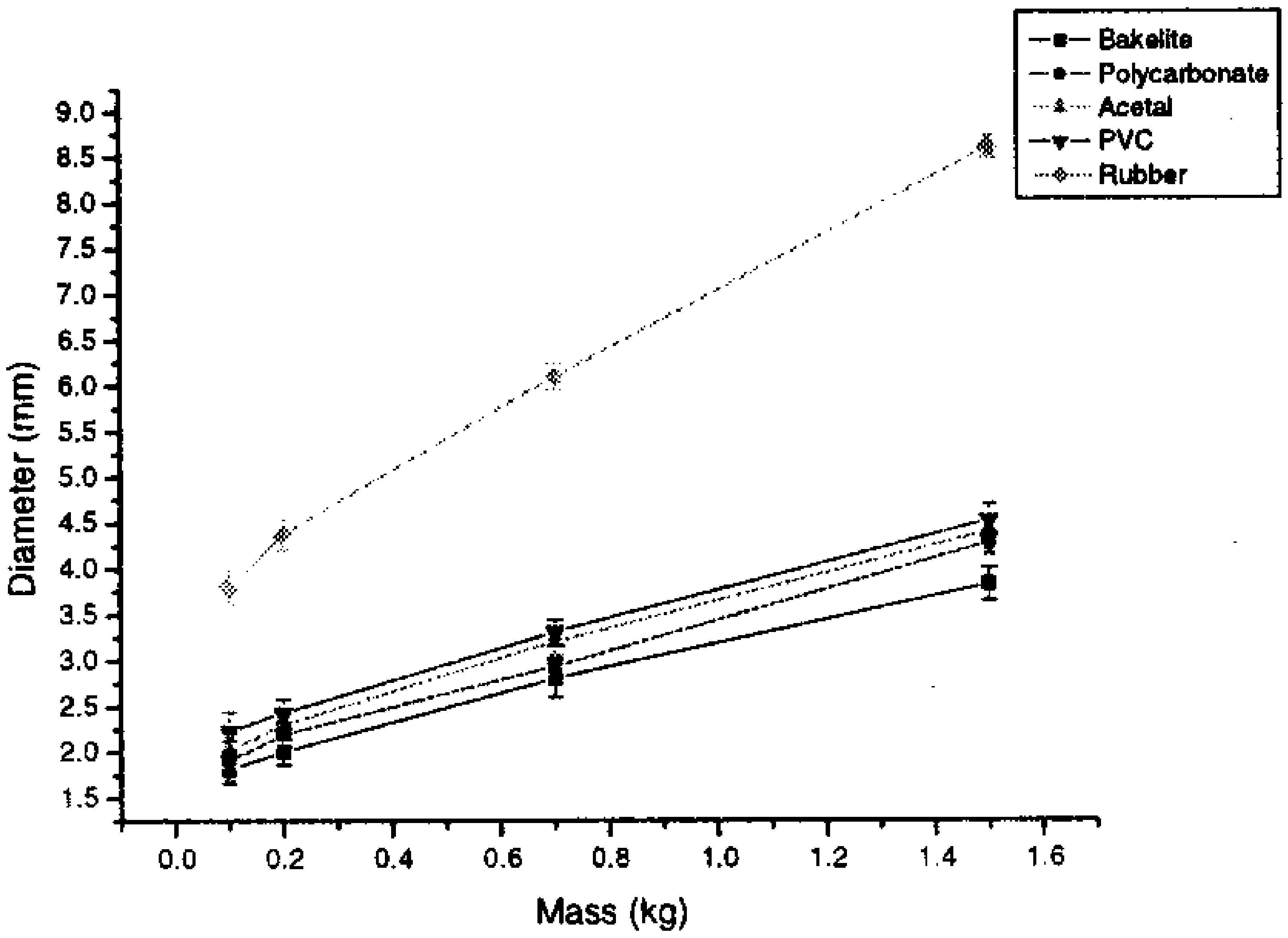


*Figure13.*

*Size of Carbon Trace: Inelastic Impact*

The same procedure was applied to the metals, in which permanent indentation were left after dropping the ball.

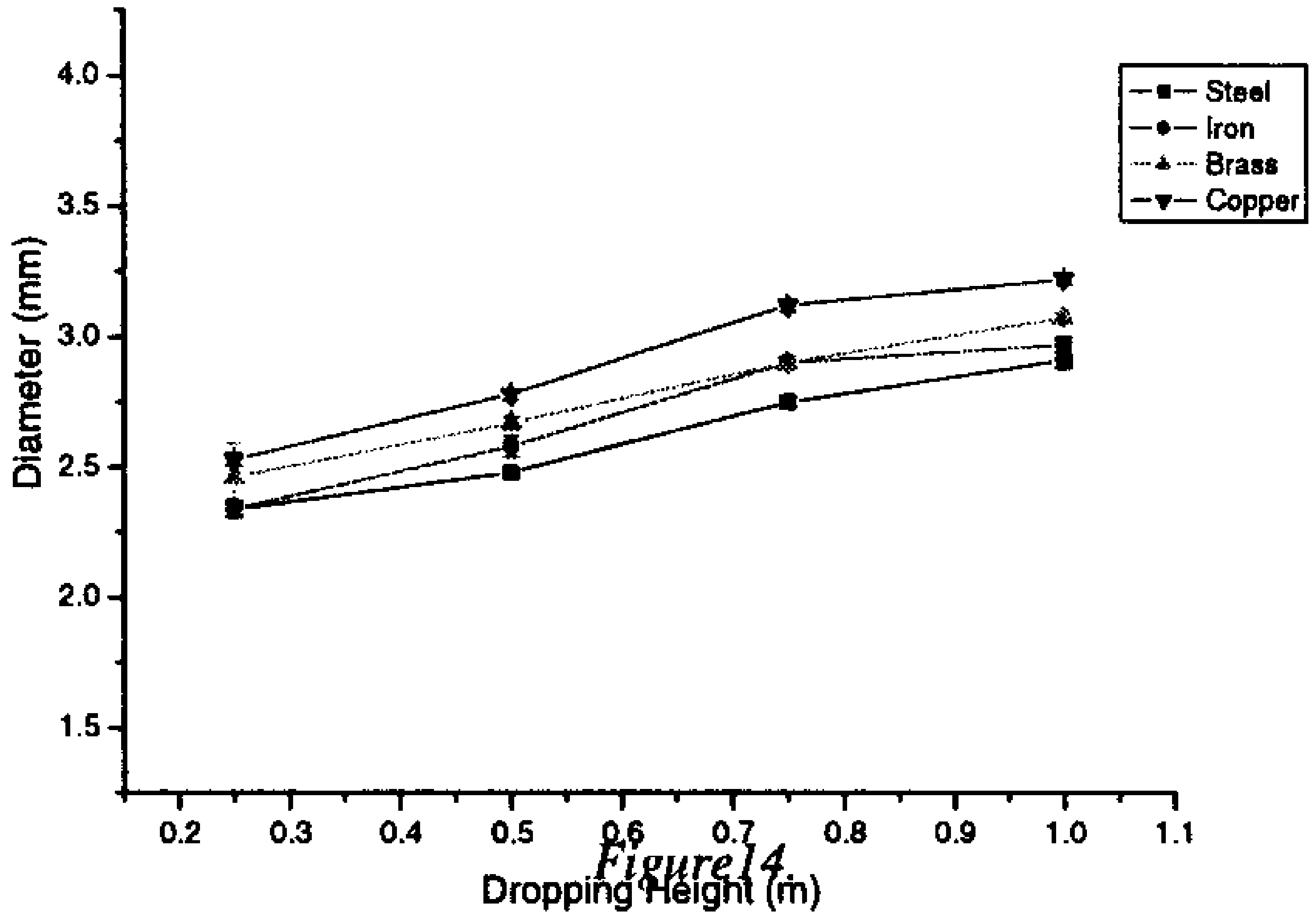


*Figure14.*

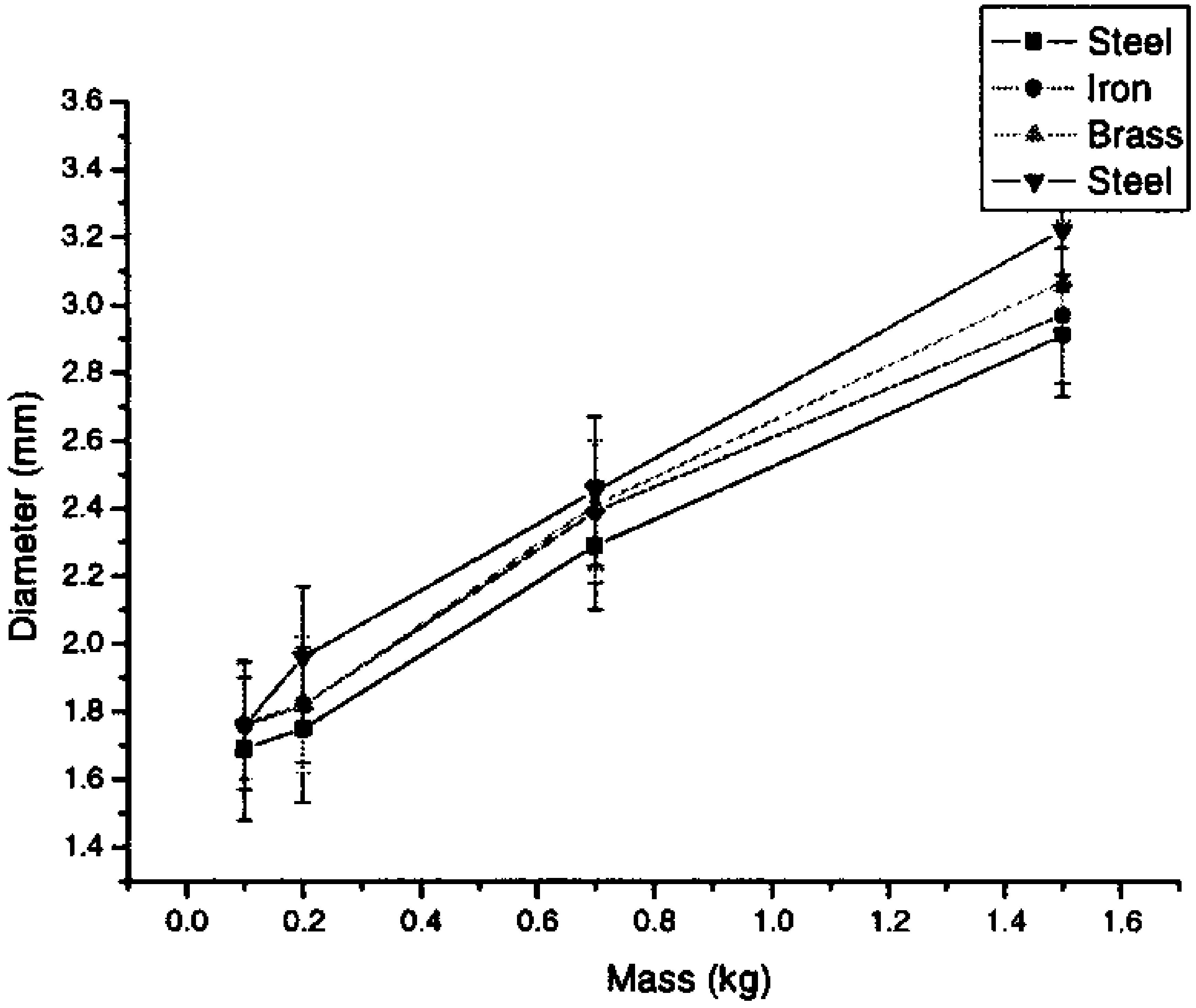


*Figure15.*

### *Intensity of Carbon Trace*

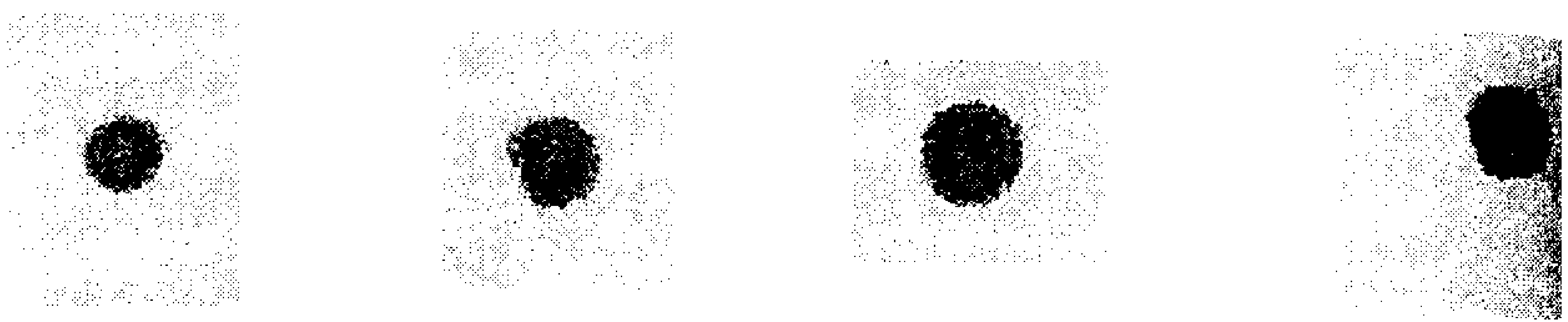


*Figure16. Metals*

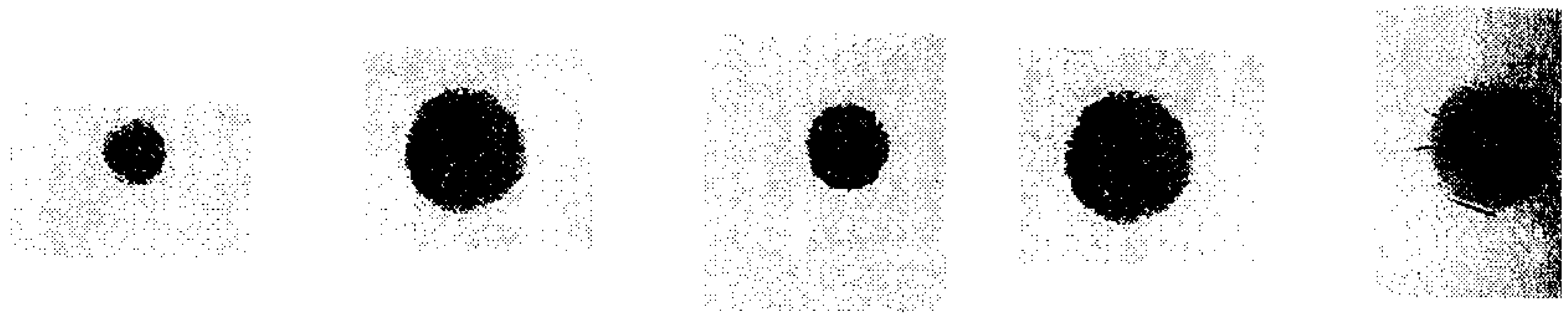


*Figure17. Plastics*

The carbon traces that were obtained showed distinguishable differences in size, but the intensities were not distinguishable with methods that were available. Therefore, in constructing the hardness scale, the rebound height and contact time were measured to fit in the variables for the hardness scale.

### *Measurement of Rebound height and Contact Time*

The measurements were taken with a piezoelectric center. The piezoelectric sensor converts pressure into voltage. For the rebound height, the different pulses produced by the impact of the ball through successive rebounds showed the time interval between. Using the free fall equation, the height of fall was measured.

The contact time was measured in the same way. When the ball strikes the ball, it presses the contact surface and then as it leaves, the pressure decreases. Using the oscilloscope, different time scales were set to measure the instant changes in pressure. The time interval of the change was measured to obtain the contact time between the ball and the surface.

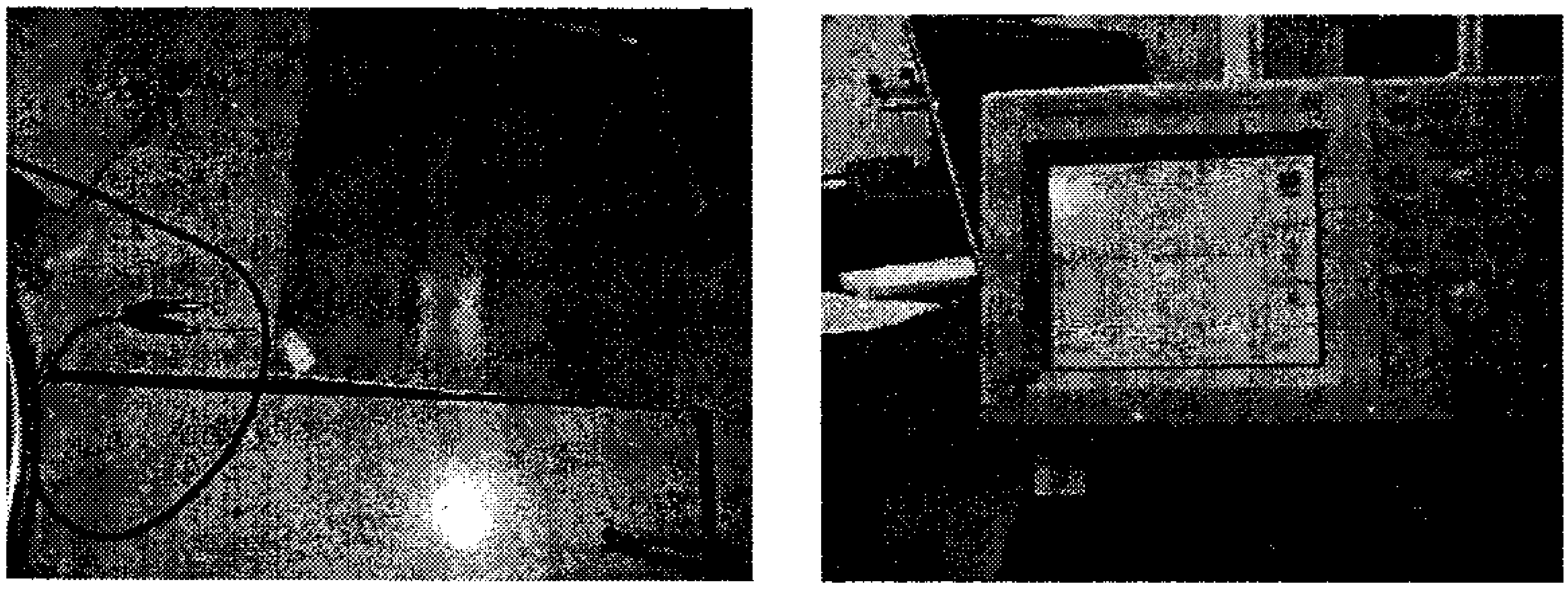


*Figure18. Piezoelectric sensor and oscilloscope*

*Rebound height and Contact Time*

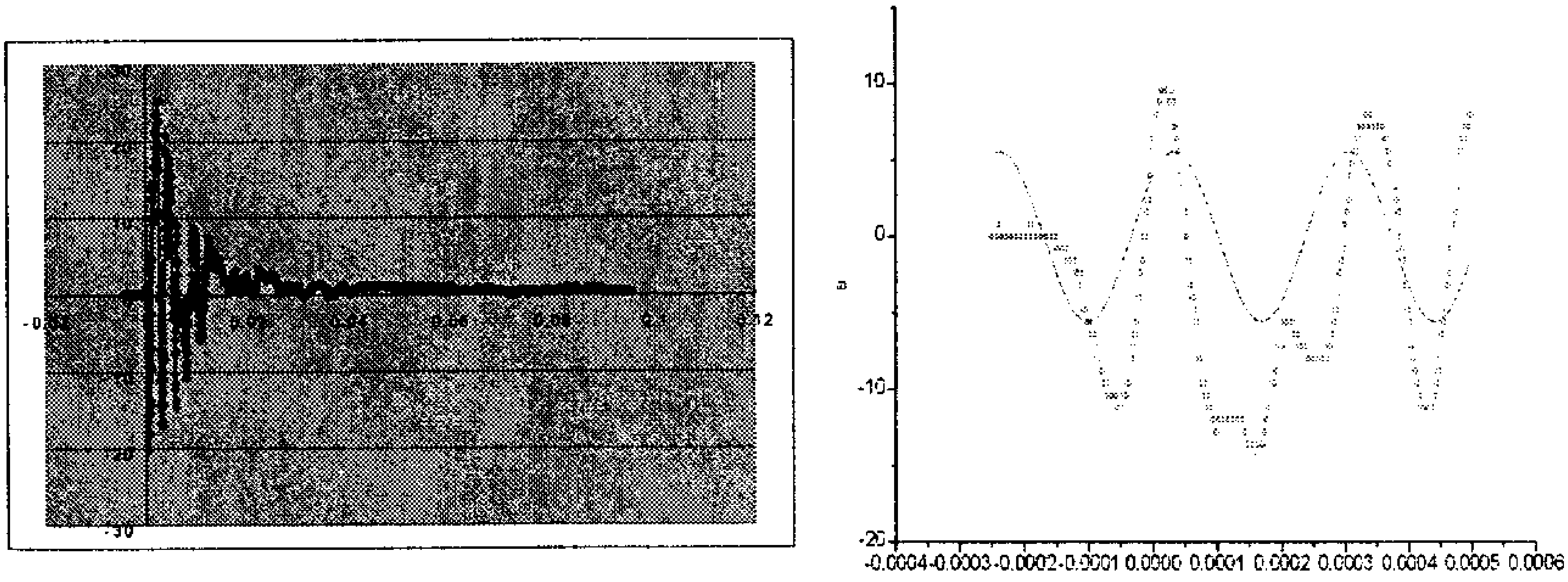
| | Iron | Stainless | Brass | Copper | Bakelite | Marble |
|---|---|---|---|---|---|---|
| **H (cm)** | 0 | 3.4 | 0.6 | 3.7 | 11.6 | 26.3 |

| | Poly | PVC | Acetal | Wood | Rubber |
|---|---|---|---|---|---|
| **H (cm)** | 27.3 | 24.6 | 28.6 | 16.5 | 5.2 |

*Figure19. Rebound height*

| Copper | Rubber | Brass | Poly | Bakelite | Acetal | Stainless |
|---|---|---|---|---|---|---|
| **0.000118** | 0.00081 | 0.00016 | 0.00020 | 0.00015 | 0.00024 | 0.00016 |

| Iron | PVC | Acryl | Foamex | Marble | Wood |
|---|---|---|---|---|---|
| **0.000134** | 0.00017 | 0.000188 | 0.000216 | 0.000184 | 0.000277 |



*Figur20. Contact Time*

## V. Analysis

### *Elastic Impact: Size and Young's Modulus*

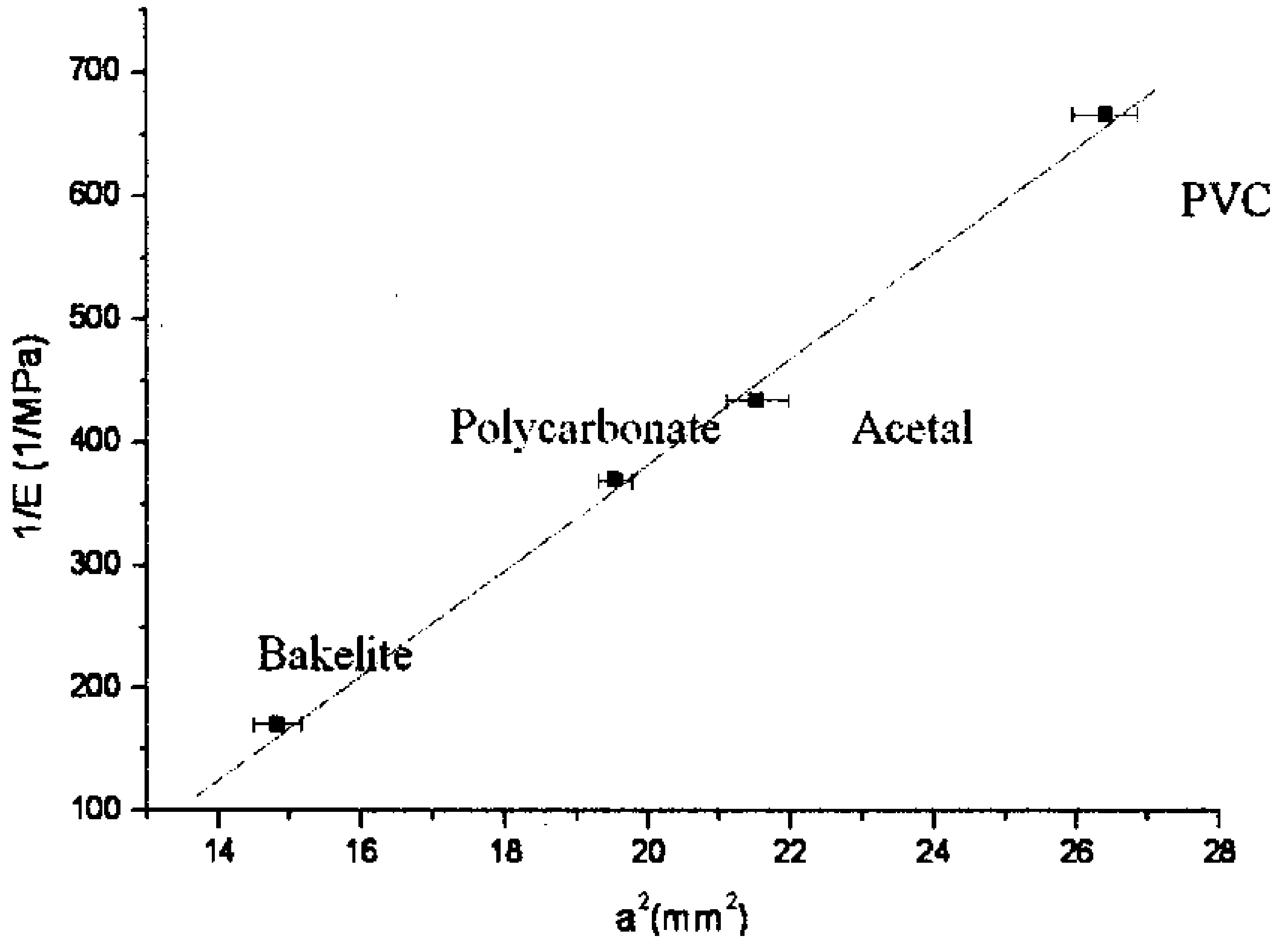


*Figure21. Different times scales of the oscilloscope*

According to equation (6), the contact surface area is inversely proportional to the Young's modulus of the material. The carbon trace sizes obtained in the experiment was used to see the relationship. It is shown that the materials that went through elastic impact well cope with the relationship. Because the size of the carbon trace of rubber was much bigger, it was left out on the above graph. The one including the trace of rubber is shown below. It also goes well with the relationship shown in equation (6).

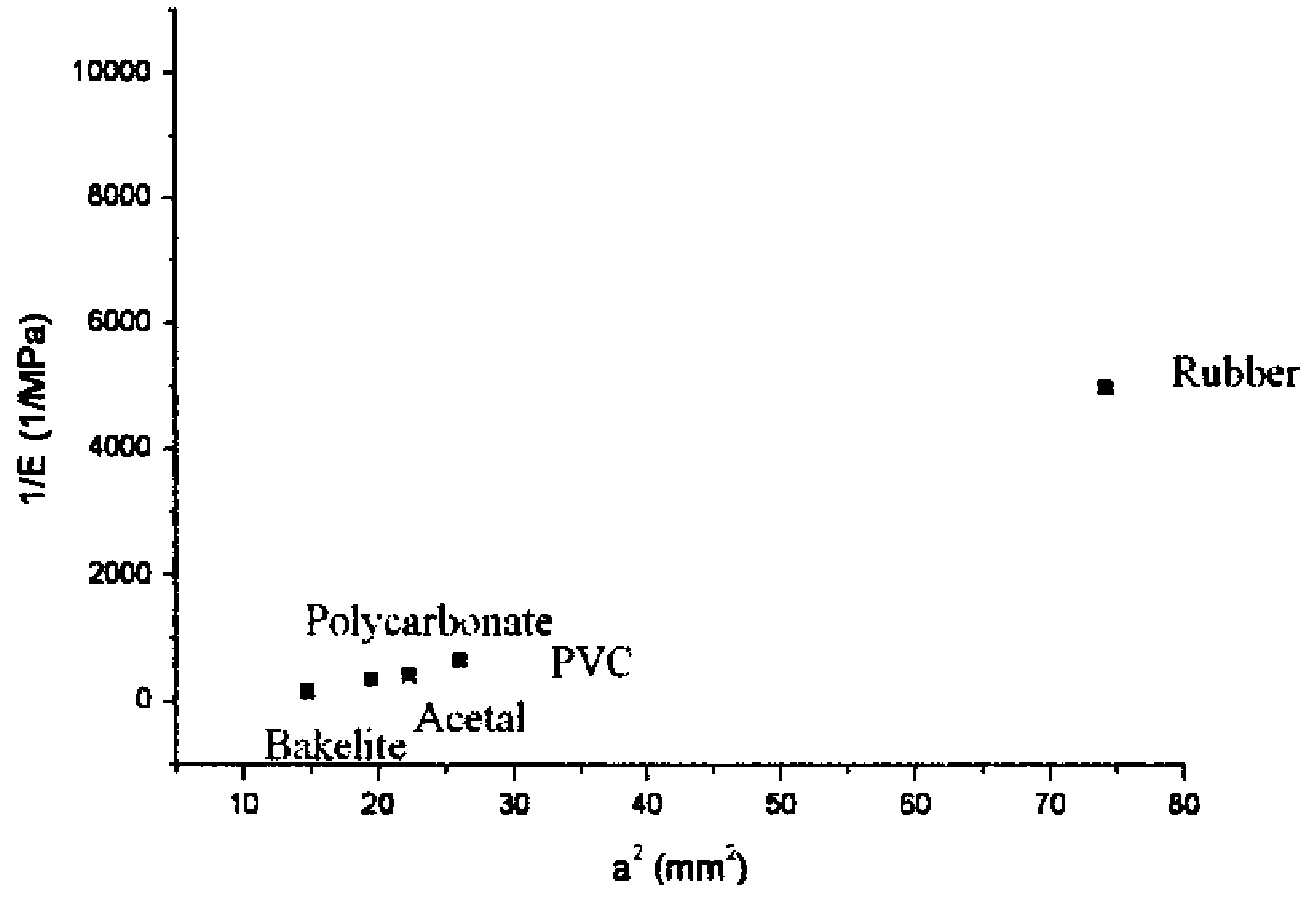


*Figure22.*

### *Inelastic Impact: Size and Young's Modulus*

The same relationship was investigated for inelastic impact that occurs in metals. As shown on the graph below, a positive relationship was not found. For inelastic impacts, the size of the trace depends more than just the elastic regime. The size increases with the plastic regime, in which the strain and stress relationship does not show linear relationship.

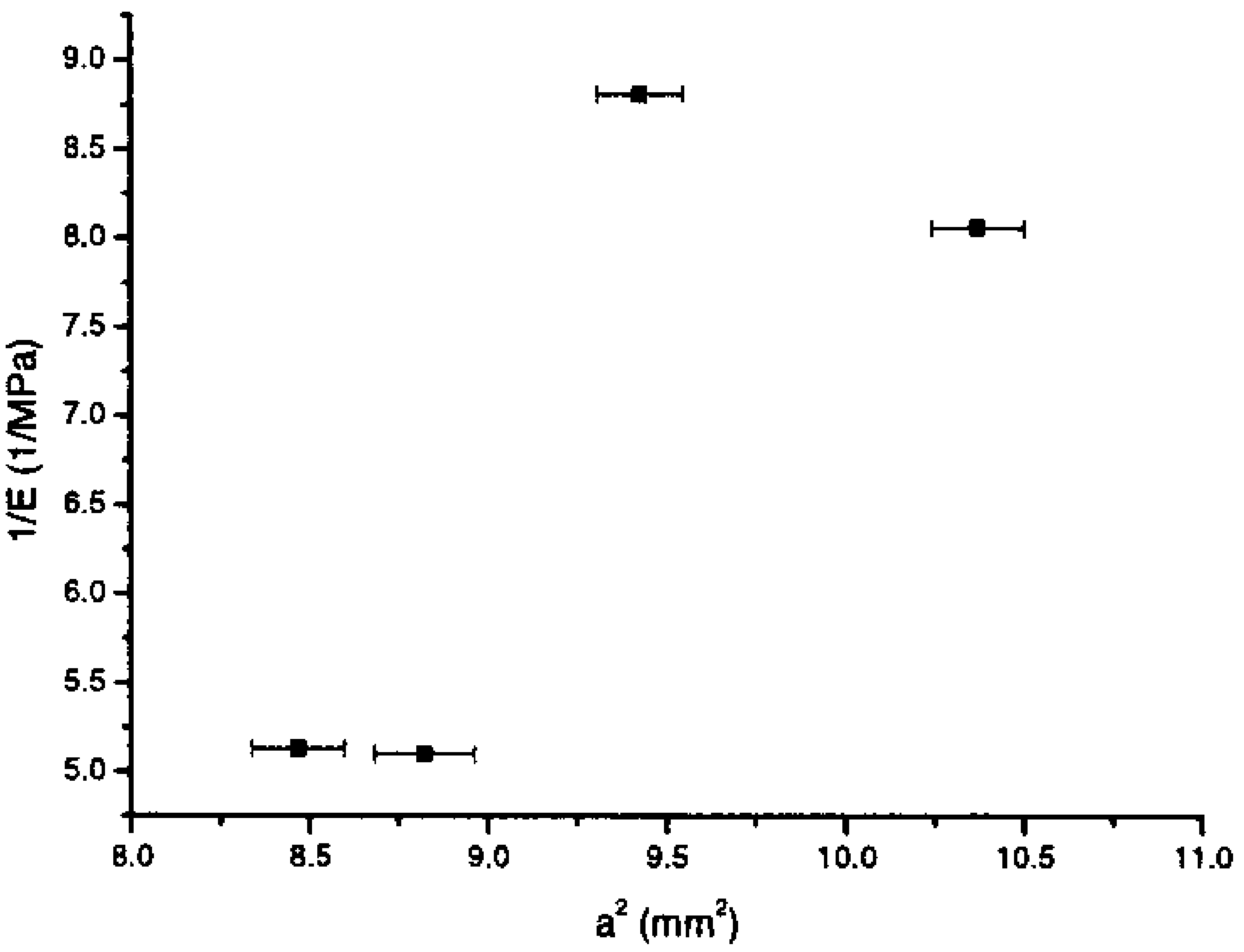
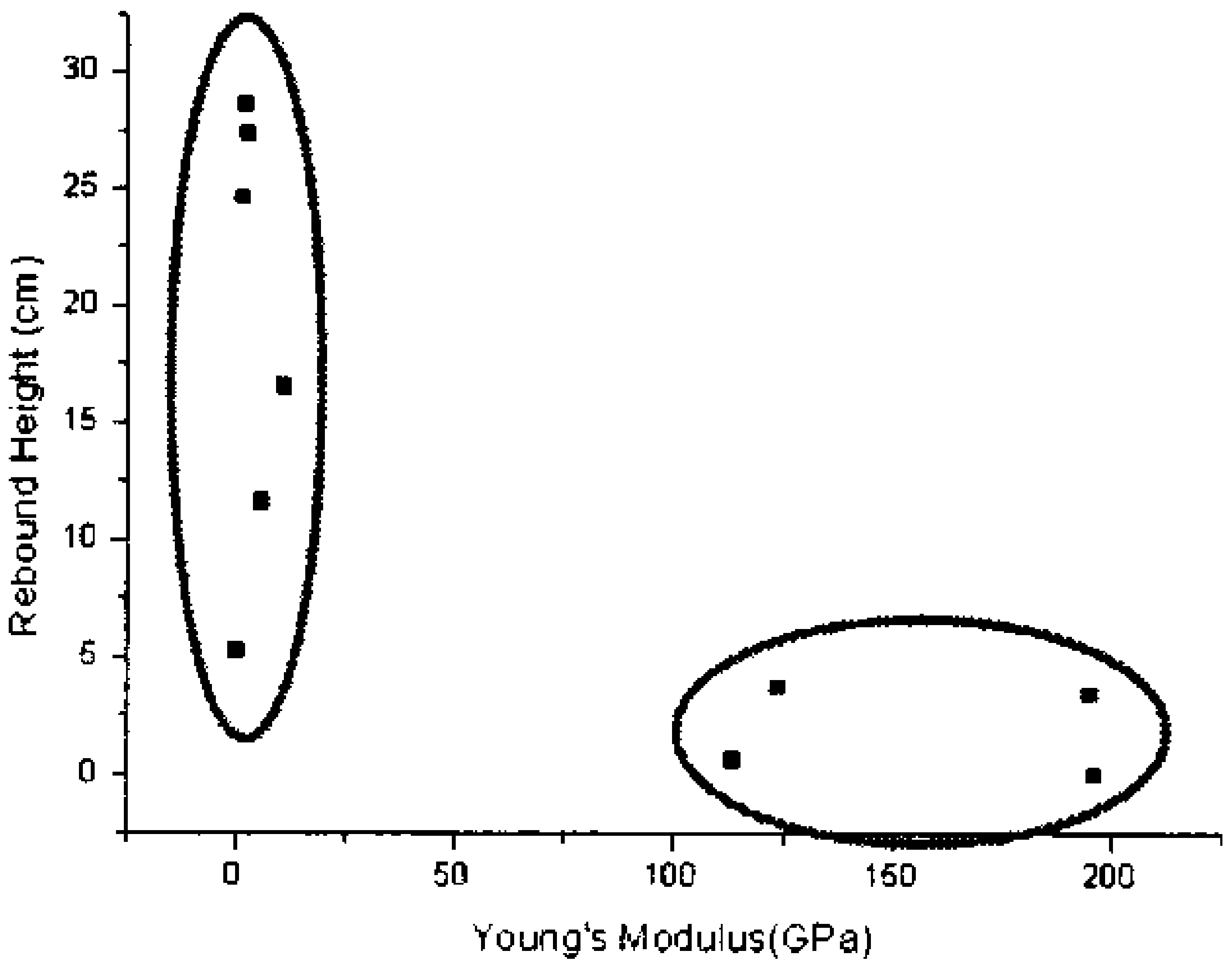


*Figure22.Steel, Iron, Brass, Copper respectively*

*Rebound Height and Young's Modulus*

The rebound height differs among the plastics inside the red circle on the left. The plastics all show elastic impact with the ball, in which no permanent indentation occurs. The metals, with high Young's modulus on the right all show low rebound, almost zero. Although a pattern is difficult to see in this graph, it can be easily seen that metals, which go through permanent indentation, have no rebounds or very little.



## VI. Construction of Hardness Scale

### *Definition and Principle*

The hardness in the carbon paper trace is defined as the average resistant pressure that is exerted on the ball for different materials. If the intensity of the carbon trace could be accurately measured, we would obtain the maximum resistant pressure. However, because the carbon trace intensities were not distinguishable, the contact time measured was used to obtain the average force applied to the material.

When the steel ball falls from a certain height, the potential energy is transformed into kinetic energy. The speed of the ball at the point of impact is easily obtained.

$$V = \sqrt{2gh} \tag{18}$$

Since the momentum of the ball is all lost when the speed of the ball is lost due to collision at the maximum point of contact, the average force applied could be calculated knowing the collision time, which was obtained through previous experiments.

The average resistant pressure could be calculated as follows.

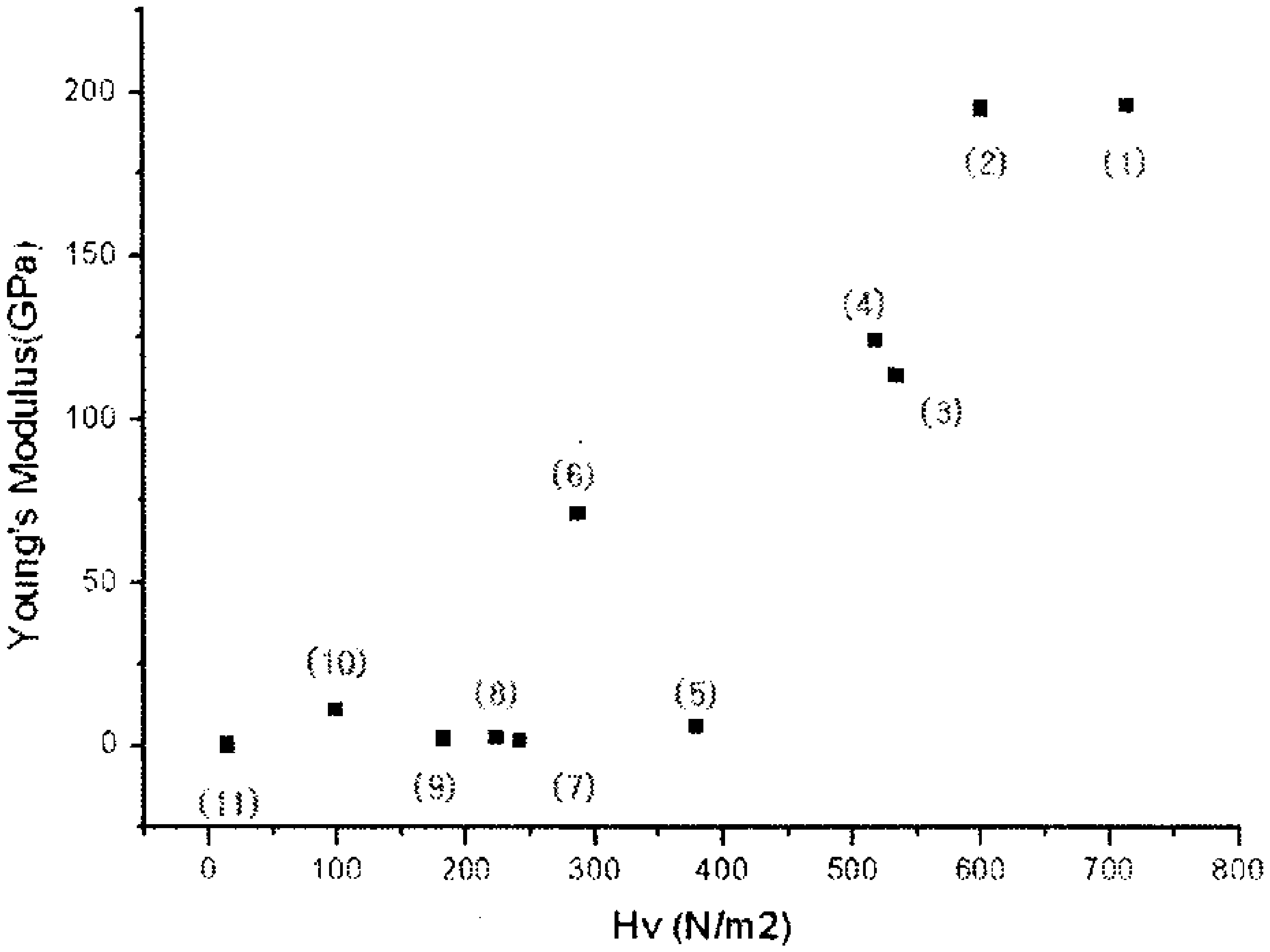
$$P = F / A = MV / A\Delta T \tag{19}$$

Knowing the value of M, the mass of the sphere ball, V, A, the contact area shown on the carbon trace, and the contact time, we could achive the average pressure which was applied.

### *Hardness Value Calculated*

| | *Iron (1)* | *Stainless (2)* | *Brass (3)* | *Copper (4)* | *Bakelite (5)* | *Marble (6)* | *Poly(7)* | *PVC(8)* | *Acetal (9)* | *Wood (10)* | *Rubb (11)* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Hv** | 715 | 601 | 534 | 518 | 379 | 287 | 223 | 242 | 183 | 98 | 14 |

The hardness values obtained above have the units of pressure. As expected, the hardness values obtained above have a strong positive correlation with the Young's modulus of the materials.



It can be seen that the hardness scale obtained from the carbon trace size and the impact time shows not only average resistant pressure of the materials, but also corresponds with the Young's modulus of the materials. Thus, the hardness scale is a good way of indirectly knowing the Young's modulus of the test material.

## VII. Conclusion

In this paper, the theoretical background behind the impact between a steel ball and a surface material was investigated. The elastic impact has three stages of impact in which the dynamic pressure exerted on the surface is max at the maximum point of contact, when the ball's speed has reached zero. The inelastic impact has an additional stage of plastic deformation.

The size of the carbon trace and the intensity of the carbon trace could be used to calculate hardness value that shows the maximum resistant pressure that was exerted by the surface. However, because the intensity of the traces could not be measured and distinguished, the impact time was measured to obtain the average resistant pressure.

The hardness values show high correspondence to Young's modulus, and are a good indirect measure of the Young's modulus of the materials.

## References

References (*Reference* style) should be listed in alphabetic order by surname at the end of the paper.

Each line of a reference, except the first, should be indented 0.5" from the left margin.

[1] D. Tabor (1951). The Hardness of Metals. Oxford University Press Inc., New York, US

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