13. Problem №17: Magnetohydrodynamics

13.1. Solution of Hungary

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The Problem

A shallow vessel contains a liquid. When an electric and magnetic fields are applied, the liquid can start moving. Investigate this phenomenon and suggest a practical application.

Abstract

If we apply an electric and magnetic field to an electrolyte, the charged particles will be affected by the Lorentz force and the drag force. The particles will speed up until the Lorentz force equals the drag force in magnitude. This is the mechanism through which the Lorentz force can move the fluid.

We can calculate the velocity of the fluid in certain experimental arrangements.

Experiments were concluded to investigate the relevant parameters: the structure of the electric field was changed by changing electrode shape and place. The electric current, the fluid height and the fluid itself were also changed parameters.

We found flow speed is proportional the electric current and in a solution at a given electric current the flow speed is independent of concentration.

A few practical applications will be mentioned.

1. Introduction

Magnetohydrodynamics (MHD) is generally known as the fluid model of plasma. Plasma can be modeled efficiently as a fluid with very good electric conductivity. This model is widely used in plasmaphysics for modeling applications such as experimental fusion generators and MHD accelerators. However the task was to investigate the movement of a fluid.

The fluid should be a conductor, thus containing charged particles that can move. Solutions containing ions, ion compounds and metals are all good for this purpose; however ion compounds and metals melting point is usually very high, in the following ion solutions will be discussed and other possibilities will follow.

It is an interesting experiment to only apply electric field to a solution containing anions and cations of largely different sizes with a metal plate and a pin as the anode and cathode (Fig. 1). Around the pin the electric current density will be very high and the water around the pin can start to move slowly.

However in our experiments this is a very small effect, because the oppositely charged ions are about the same size and weight thus forming similar currents in opposite directions.

This is a negligible effect in our case. A much larger effect is found when applying electric and magnetic field.

2. Theoretical analysis

J. Hartmann [1] investigated this phenomenon in 1937. In the presence of an electric and magnetic field mercury flows between two parallel planes. Voltage is applied on the planes; magnetic field is perpendicular to electric field. A similar experimental setup (Fig.: 2) with salt solutions was used to investigate the Hartman flow. Many other arrangements are possible of which a few will be discussed later. Most measurements were concluded in a circular arrangement (see Fig. 3) where the electric field is radial and the magnetic field is vertical.

If we apply an electric and magnetic field to a solution, the ions will be affected by the Lorentz force and the drag force. It is important to note that the solvent forms a structure around the ions, so not only small ions are moving through the fluid but larger structures. The ions (the structures) speed up until the Lorentz force equals the drag force in magnitude. This is the mechanism through which the Lorentz force affects the movement of the fluid.

$$\mathbf{F}_{Lorentz} = Q \cdot \mathbf{E} + Q \cdot \mathbf{v} \times \mathbf{B} \quad (1)$$

Where Q is the charge of particle, $\mathbf{E}$ is the electric field vector, $\mathbf{B}$ is the magnetic field and $\mathbf{v}$ is the velocity of a particle.

In both the Hartmann flow arrangement (Fig. 2) and the circular arrangement (Fig. 3) the an electric current forms in the direction of the electric field vector which means the cations move in direction of the electric field vector while the anions move in the opposite direction. The second term in the Lorenzt force causes both positive and negatively charged particles to be diverted in the same direction because both the charge and the speed have opposite signs. Consequently the fluid will start to move perpendicularly to the electric field vector.

Since the problem is about a shallow vessel we will make the assumption currents may only form horizontally, so only the vertical component of the magnetic field ($B_{\perp}$) can affect the fluid. The sum of the Lorentz force affecting the particles:

$$\sum F_{Lorentz} = l \cdot I \cdot B_{\perp} \qquad (2)$$

where I is the electric current and l is the length measured in direction of the electric current.

There is a friction between the fluid and the bottom of the vessel and an internal friction due to the horizontal velocity gradient. Since the vessel is very shallow compared to its radius we can neglect the friction due to the horizontal velocity gradient compared to the friction due to the vertical gradient.

According to Newton's viscosity law the internal friction is:

$$F_{friction} = \eta \cdot A \cdot \frac{v}{h} \quad (3)$$

where v is the speed of the fluid at its top, h is the height of the fluid, η is the dynamic viscosity of the fluid and A is the area where the friction affects the fluid. This assumes that the flow is laminar (our measured speeds were around a few mm/s).

Assuming a static equilibrium will form and the velocity is a linear function of the height using Eq. 2 and Eq. 3 we can calculate the velocity of the fluid assuming a homogeneous $B_\perp$ in the circular arrangement (Fig. 3) at distance r from the center:

$$v = \frac{I \cdot B_\perp \cdot h}{\pi \cdot \eta \cdot r} \qquad (4)$$

3. Experimental setup and measurements

A circular vessel was used; current and flow speed was measured *(see the experimental setup in Fig. 4)*. Changed parameters were voltage, electrode shape and place, solution concentration, and viscosity by adding glycerin.

Flow speed was measured afterwards with video analysis. Paper flakes were sprinkled on the fluid. Flake speed was measured on flakes only slightly submerged in the fluid. With this method we can measure the speed of the fluid at its top. A NaCl solution was used with copper electrodes. A strong cylindrical permanent magnet was placed under the vessel.

3.1. The circular arrangement

According to the above theoretical considerations the magnetic field was assumed to be homogenous, however this is only a valid assumption just above the magnet (Fig. 3). Flow speed was measured half way between the perimeter of the inner electrode and the perimeter of the magnet (2.65 cm from the center).

We can observe that the flow speed seems to be independent of salt concentration, and is proportional to the electric current (Fig. 5), of course, in the case of very small concentrations such electric currents can not achieved. This is in good agreement with the theory.

Adding a small amount of glycerin increased the viscosity drastically, the fluid almost stopped to circulate.

3.2. Comparing the results and theory

In the circular arrangement (Fig. 3) the flow speed can be calculated by Eq. 4. The velocity and electric current was measured; their quotient was 0.023 (m/s)/A. The solution was always poured to be 1 mm high and the magnetic field just above the magnet was estimated to be 0.002 T.

$$\frac{v}{I} = \frac{B_\perp \cdot h}{\pi \cdot \eta \cdot r} = 0{,}024 \frac{(m/s)}{A}$$

This theoretical result corresponds well with the measurements.

3.3. Changing the electric and magnetic fields structure

Changing the direction of the magnetic or electric field by turning the magnet or switching the anode and the cathode causes the flow direction to change, as expected according to the theory. The structure of the electric and magnetic field was changed by changing the experimental setup. In the following we discus a few possibilities:

a) The electrode in the middle of the vessel was moved to the side with the magnet.

This causes an increase in flow speed between the two electrodes, because the electric field is much larger here, and the cross-section is much smaller.

b) The circular electrode in the middle of the solution was changed to a drop shaped electrode (Fig. 6).

The electric field was much greater at the tip of the electrode because of the corona effect; this resulted in the flow speed being larger as well.

c) The drop shaped electrode was then placed with its tip closer to the other electrode (Fig. 7).

The electric field at the tip of the electrode was even larger than in the previous setup and the cross-section was very small, so the flow speed was even greater.

3.4. The Hartman flow experiments (fig. 3.)

Between the two electrodes and above the magnet we found that the flow speed was constant (Fig. 2). The system behaves similarly to the circular arrangement: flow speed is proportional to the electric current and independent of salt concentration.

3.5. The chemical reaction

At a given voltage the electric current was not constant: the resistance of the system increased in time due to the chemical reaction at the anode and cathode. The chemical reaction may also change the properties of the investigated (on some photos (Fig. 6, 7) we can see some green precipitation). Because of this measurements using a single solution were made only just after the start of the electrolysis and after the solution reached its maximum constant speed.

3.6. Experiments with mercury

In mercury there are delocalized electrons and positive ions. Only electrons can enter and leave the fluid at the electrodes. Because of this cations will not form an electric current. The electrons will move similarly to anions in solutions. However in our experiments the mercury did not move: there was no effect. Electrons can move almost freely through mercury, its resistance is very small. In our experiments the magnetic field is not large enough to speed up the electrons to a speed, where the drag force affecting the electrons is significant, thus the counteracting force of the drag force that affects the rest of the liquid is also very small.

4. Practical applications

In the above experiments the electric energy was turned into kinetic energy. The Hartmann flow can be used to make a pump. Some nuclear power plants use low melting point metals (for example a potassium-sodium alloy) as coolants (see Abonyi [2]). In these power plants such pumps are ideal for circulating the coolant, because they have no moving parts and are thus more reliable. The circular experimental setup (Fig. 3) can be used as an electric engine by inserting turbines.

Kinetic energy can be turned into electric energy in a similar way. An MHD generator can be made by pumping a conductive substance through the magnetic field, in a similar arrangement as the Hartmann flow; voltage will form between the two plane electrodes. This same arrangement can also be used to measure the flow speed. Such a measuring device can have a wide range of uses, for example blood flow measurement (Fig. 9).

As mentioned in the introduction MHD is the fluid model of plasma. Plasma has similar properties as a conducting fluid and can be used similarly. MHD generators can also be powered by plasma. Similarly to the pump we can make MHD accelerators: plasma jet engines.

5. Conclusions

The theoretical model shows how an electric and magnetic field can affect the charged particles, and how this affects the fluids motion. The theoretical model could predict the flow speed in certain experimental arrangements.

Experiments were concluded to investigate the relevant parameters: the structure of the electric field was changed by changing electrode shape and place. The electric current, the fluid height and the fluid itself were also changed parameters. According to our theory flow speed is proportional to the electric current and in a solution at a given electric current the flow speed is independent of concentration. The measurements confirmed these expectations.

References

[1] Hartmann, J. Hg dynamics. I. Theory of the laminar flow of an electrically conductive liquid in a homogeneous magnetic field. Det Kgl. Danske Videnskabernes Selskab. Mathematisk-Fysiske Meddelelser **15**, 1-28. 1937

[2] Abonyi I. The forth state of matter. Gondolat kiadó, Budapest, 1971

Figures

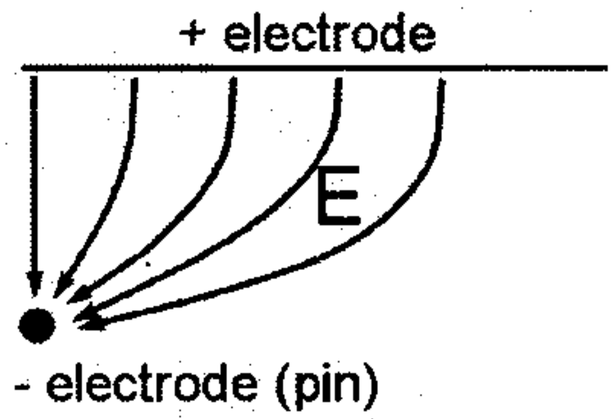


Fig.1: Pin and metal plate; only applying an electric field

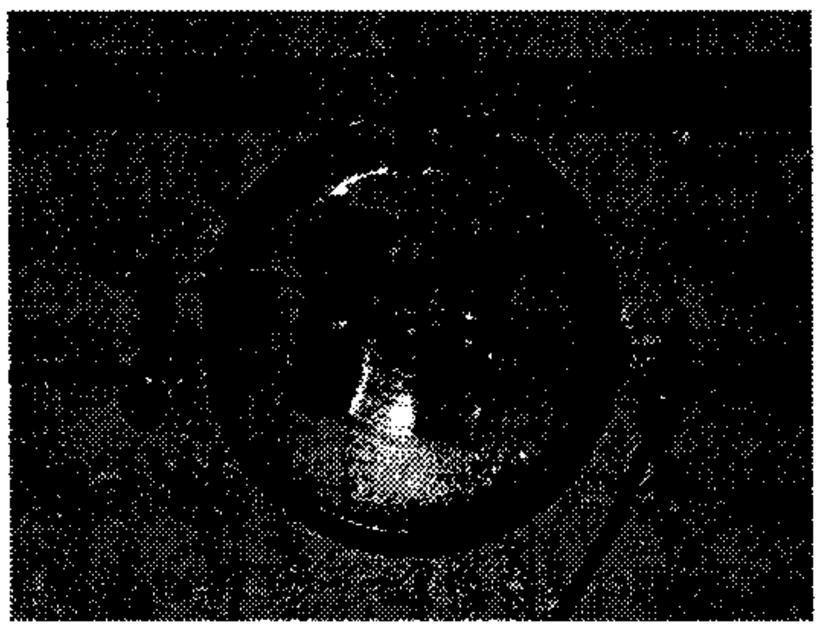


Fig. 2a: Hartmann flow

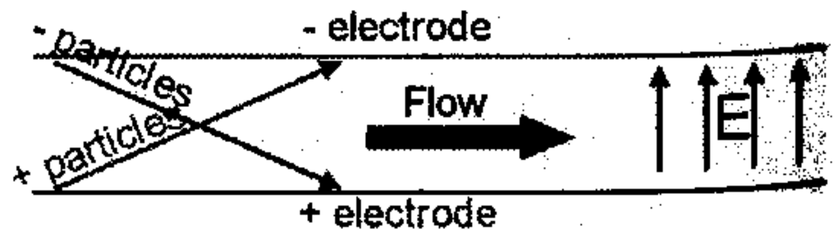
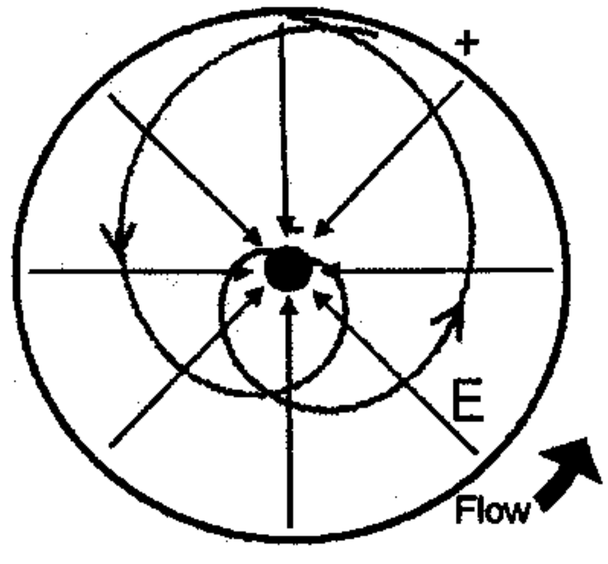


Fig. 2b: Hartmann flow; ion paths

Fig. 3a: Circular arrangement



Homogeneous B pointig outwards

Fig. 3b: Circular arrangement; ion paths

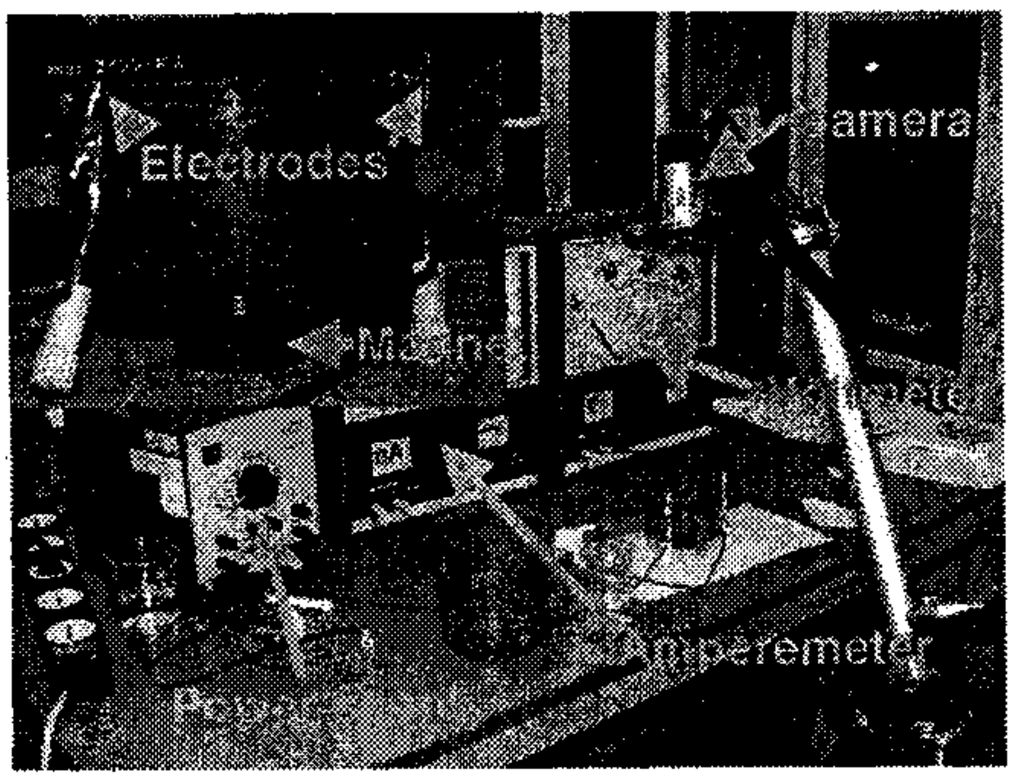


Fig. 4: Experimental setup

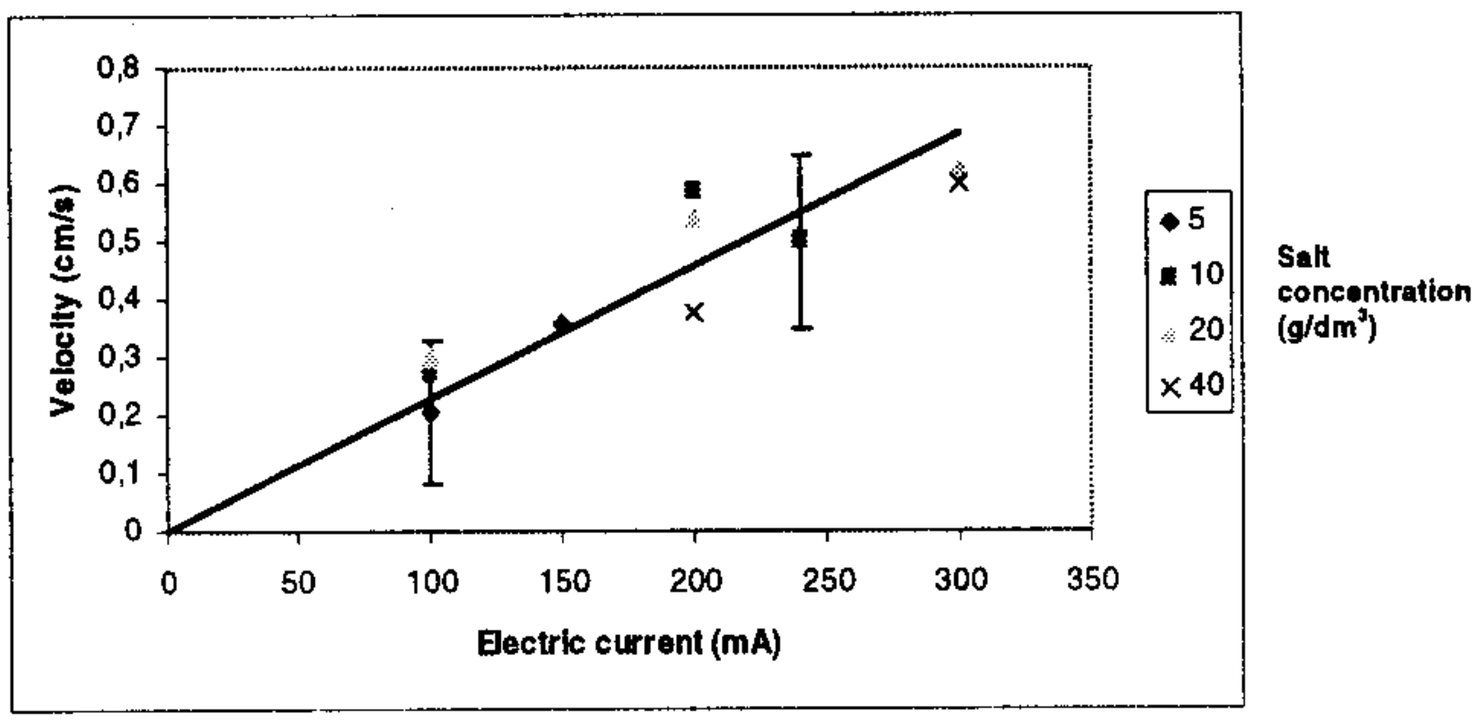


Fig. 5: Measurements

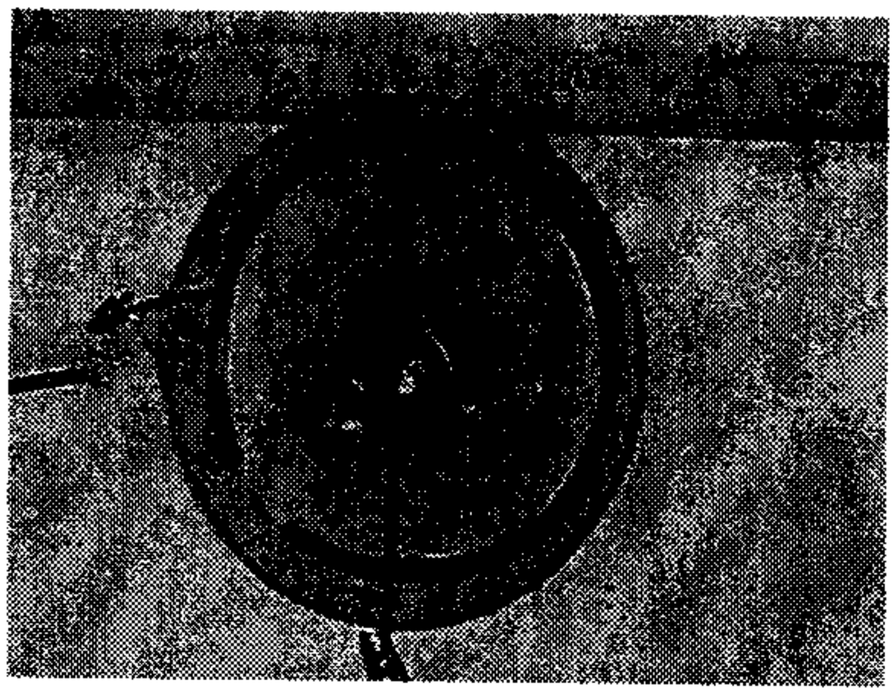


Fig. 6: Changing the structure of the electric and magnetic field

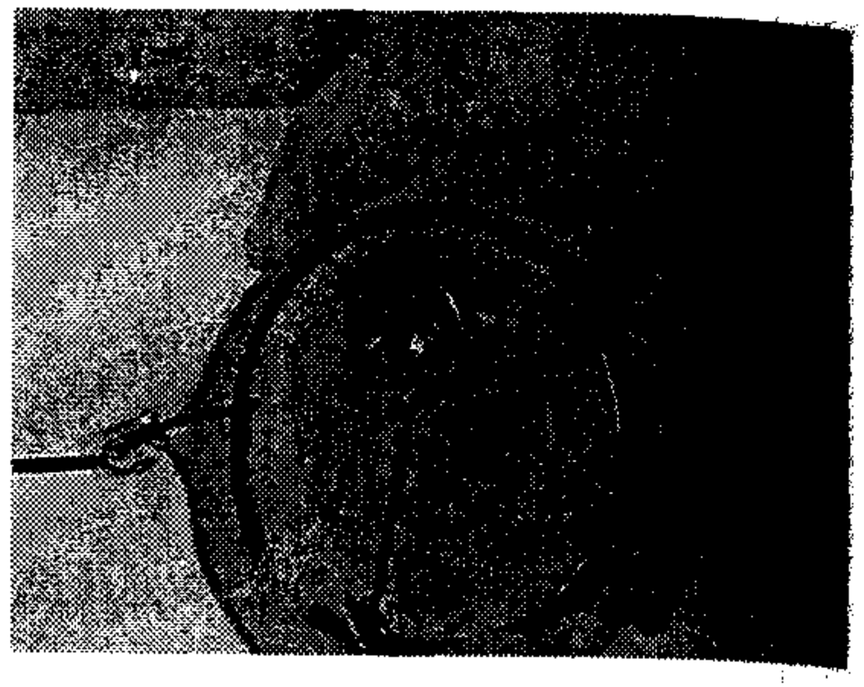


Fig. 7: Changing the electric and magnetic fields structure

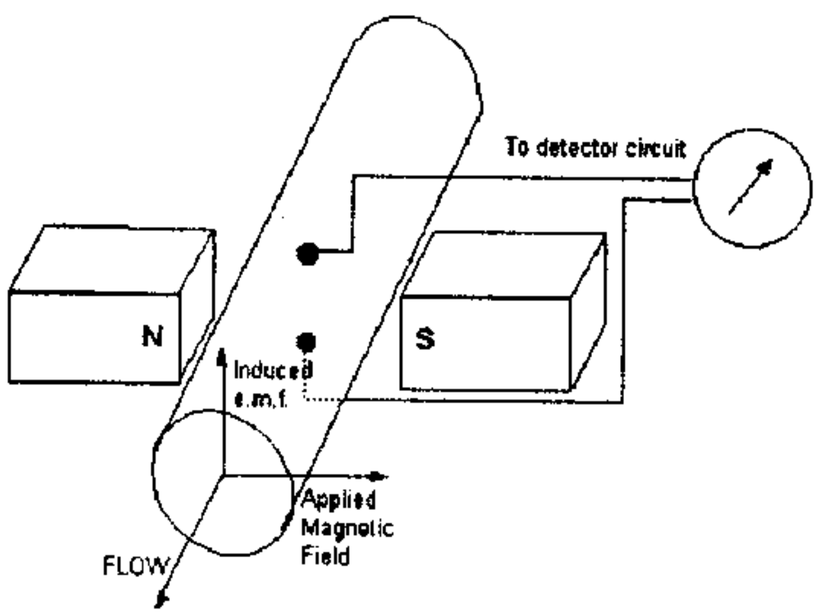


Fig. 8: Blood flow meter