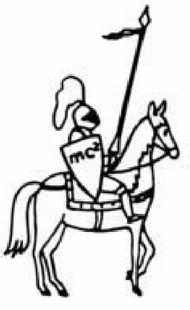
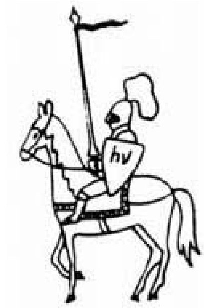




# IYPT

## The Young Physicists` Tournament

### The Physics World Cup

### Proceedings of 18th IYPT 2005

**14th- 21st July 2005**

**Winterthur, Switzerland**

# Contents

# I. Introduction

This edition is an attempt to collect and present solutions of the problems for the IYPT, prepared and most of them presented at the $18^{th}$ IYPT in Winterthur, Switzerland.

The main aim of the edition is to reveal and summarize the results of the investigations concerning the problems of the IYPT, done by the participants in the IYPT from different countries.

Another aim is the popularization of the tournament and estimating its educational effectiveness it to be used for the raising of the level of physics education.

## 1. AIMS AND TASKS OF THE IYPT

The IYPT is a competition between teams of secondary school students in their ability to solve complicated problems, to present solutions to these problems in a convincing form and defend them in scientific discussion, called Physics Fight (PF) [*Regulation of the IYPT, www.iypt.org*].

The IYPT is an organization of participants, which share its aims, mainly to provoke and enhance of the interests of the students from the secondary schools to physics and another branches of knowledge, to contribute for the realization of a connection between schools and research scientific institutions.

The concrete tasks of the tournament are:

- To develop the scientific thinking of the students and their investigation skills, communicative skills and skills to work in a team. By the problems of the tournament the students to come across with real scientific problems, giving them the possibility to become well acquainted and to make sense of the essence and stages of each investigation process, to enrich their knowledge;

- To quicken the interest of the young people and their desire to realize themselves in the area of engineering and scientific specialities and especially in the area of hi-tech industry, which is very important at the present in a global aspect;

- To create new kind of relationships between researchers, teachers and students, thanks to the specific content of the problems and the need of the consultations with scientists and specialists during the preparation and participation in the tournament;

- To popularize the tournament widely among schools and universities, seeking for new forms for the collaboration between them.

## 2. HISTORY

The tournament was established in 1979 as a Tournament of Young Physicists (YPT) and was developed at the Physics Faculty of Moscow State University for students from Moscow and its vicinity.

The important role for its foundation was played by Academician E.P.Velichov, Academician G.T,Zacepin, profesor S. M. Gudinov and Dr. Yanusov (2).

The reason of the establishment of the tournament was, by original and different from the traditional physics textbook tasks, to attract attention of the young people not only to physics, but as well as to the science and nature.

During the first tenth years the YPT was organized as a tournament of Moscow secondary school students. Since 1985 till 1988 the secondary schools students from the former Soviet Union could participate in the tournament The tenth annual tournament was then organized as a Soviet Tournament and simultaneously as an International Young Physicists` Tournament.

The first sixth IYPT took place in Russia and after that in other countries, participated in the tournament (2,4).

Now in the tournament take part teams from many European countries, from Australia, Brazil, Indonesia, Kenya, Korea, New Zealand, USA. The number of the participated countries in 18$^{th}$ IYPT is 26.

## 3. ORGANIZATIONAL STRUCTURE OF THE IYPT

The IYPT administration consists of the following units:

a) The IOC (International Organization Committee)

b) The LOC (Local Organization Committee)

c) The Executive Committee

The President of the IYPT is Prof. Gunnar Tibell (Department of Radiation Sciences, Upsala University, Sweden)

The General Secretary is Dr. Andrzey Nadolny (Institute of Physics, Polish Academy of Science, Poland

## 4. REGULATIONS OF THE IYPT

The IYPT is a competition between teams of secondary school students.

Each team, participating in the IYPT consists of five students and two team leaders.

From each country participates one team.

The participants of the IYPT can be national teams, teams of regions, towns, clubs and colleges. The decision about participation of the latter may be taken by the LOC.

The working language is English.

The IYPT is a competition, which examines and compares the abilities of the students to present, opponent and review convincingly in a scientific discussion the solutions of the preliminary put 17$^{th}$ physics problems. On these problems students have to do approximately during one school year the needed experimental activity and to give the respective theoretical explanation. The $17^{th}$ problems are prepared of the representatives of the participating countries in the International Organizing Committee (IOC)

The competition includes **selective discussions and a final**.

The basic structure unit of the competition is the Discussion Group, less formally called a Physics Fight (PF). All teams participate in Selective Physics Fight (SPF).

In one PF three teams (or four depending on entire number of teams) compete in three (or four) stages. In each stage the teams perform one of the three (or four) roles:

*Reporter, Opponent, Reviewer (Observer ).*

In the next stages the teams change their roles, according to definite scheme. On the base of the received results from the SPF three teams are in the final.

More details about the regulations can be seen on the website of the IYPT- www.iypt.org

## 5. THE PROBLEMS

The problems of the tournament posses the characteristic features of the interesting and original problems, presented to the students by R.Feyman, P.L.Kapitzza and Nobel Prise Winner (open-ended problems).

They are complex problems with scientific and technology content, from real life and nature, from different areas of physics and science and their boundary fields.

The problems reflect the role and influence of physics on many areas of human activity such as engineering, building, art, sports, etc.

The solution of the problems requires doing the necessary activities for every scientific research-experimental investigation and theoretical analysis.

## 6. EDUCATIONAL EFFECTIVENESS

### 6.1 .Development of the cognitive skills of the students

The untraditional problems and discussion character of the tournament is new and unknown challenge for the students to enter into the real world of the science. It allows many of the aims of the education of physics to be realized.

Ones of them are the development of the cognition of students, their adoption of the contemporary style of scientific thinking. The latter are in direct connection with the formation of a system of scientific terms and knowledge, with mastery of the common methodological knowledge through concrete educational content.

**Adoption of the methodological terms**

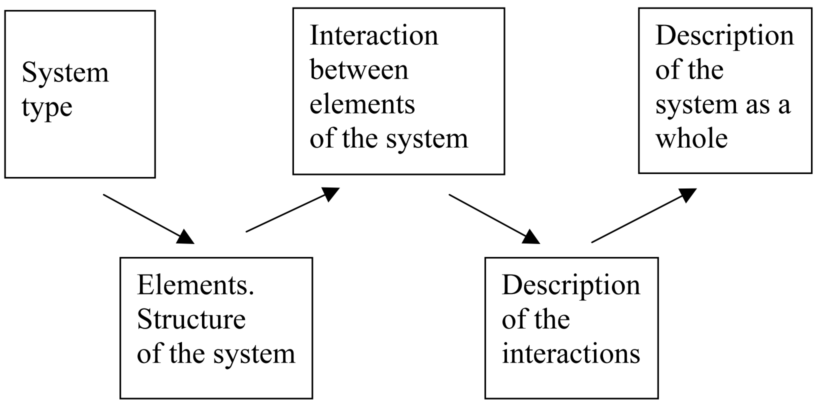
Independently of a great variety of the problems of the IYPT their solutions are based on such common scientific terms as a system, structure, state of the system, model and hypothesis.

**Making sense of key ideas and principles**
The solutions of the problems are also based on key ideas and principles (wave-particle duality, correspondence principle – “Optical Tunneling”, conservation of energy, momentum conservation, angular momentum conservation – “Hydraulic Jump”, ”Einstein-de Haas Experiment”), which as whole defines the attitude of the students towards science, systematizes and make sense of their knowledge, outlines their vision about nature, universe and human life.

**Adoption of scientific approaches and methods**
Ones of the most widely used methods for the solution of the problems are the modeling, analogy and system-structure approach.

The term with the greatest weight here is the term a physics system.

The content of the problems gives the possibility to understand and to repeat the stages of the studying of a given system in the sense of the system-structure approach, namely:




The analysis of the presented solutions in the edition shows that the above scheme is the base to do comparison and **analogies between classical and quantum systems in physics**, for example the problems “Optical Tunneling” (Brazil, Croatia, Poland, Ukraine), “Einstein de Haas Experiment” (Brazil, Bulgaria, Czech, Poland).

Analogical scheme to the above is used to do modeling of the granular system, which is far complicated than non-granular (“Obstacle in a Funnel” – Hungary). In the solution there is analogy between **different kind of systems-physics** (granular system) **and social system** (so called pedestrian escape panic).

Using the analogy **between different physics systems (**the glass is interpreted as a spring with a weight), the complex phenomenon in the problem “The Sound in the Glass” is investigated (New Zealand, Ukraine).

The common scheme of the study of the complex system and complex phenomenon, used in the presented solutions allows creating of different didactical schemes for the study of the physics phenomena, for example:

| | | | | |
|---|---|---|---|---|
| *System of objects, participating in the phenomenon* | → | *Structure of the system mutual connection between objects* | → | *State of the system. Change and evolution of the state of the system* |

### 6.2. Extension of knowledge of students, acquaintance with new ides and contemporary problems of the science

Another very important feature of the problems is that they give the possibility to convey in very easy and attractive way the study of the system with predictable behaviour (the study basic for the secondary school) to the study of the system with unpredictable behaviour and connected to them new terms and problems (chaotic motion, catastrophe theory, fold catastrophe system, etc.). By this way the students would learn more about new area of researches and acquire broad vision for the complexity of the phenomena in the nature.

### 6.3. Creating communicative and organization skills

The necessity to do serious preliminary theoretical and experimental activity to the problems and the character of their presentation in the PF is a condition to create skills with students to work in team, to distribute tasks, to take responsibilities, to organize and plan their activity and preparation.

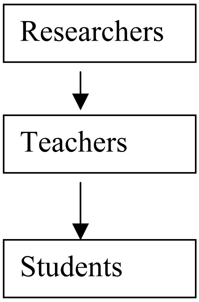
### 6.4. Influence over the relationships between the participants in the educational system

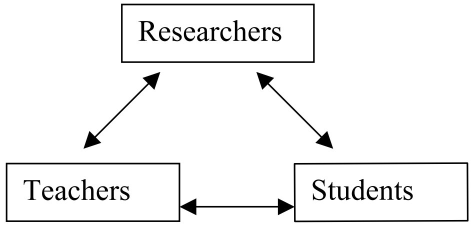
The practice of the preparation of the problems of the tournament and the realization of the latter, lead to the inference that thanks of the essence of its problems and its regulations, also thanks to the broad using of the information technologies, we can observe interesting direction of delivering of new knowledge, interesting and new relationships between students, teachers and researchers as participants in the educational system in principle.

We can say that now the relationships teacher-student doesn`t build on the presumption that the teacher is this who knows, but student is this who doesn`t know. They have to in position as partners and the teacher is this who will help student to accumulate himself the information, to classify it, to check and give it a meaning.

About the delivering of new knowledge the practice of the tournament suggests the following parallel. The first figure/fig.1/ presents the being vision for the delivering of the new knowledge, subordinated to a defined hierarchy from top to the bottom.

The second one presents the relationships between respective participants in educational system and possible obtaining of new knowledge, thanks to the open-ended character of the problems and character of the tournament. It is realized by a continuous feedback between participants in education, typical and needed for the effective function of every complicated system, including and the educational system.




Such parallel can be a basis for didactic analyses, regarding the change of learning, teaching and education, as result of new positions and functions, which gives in concrete case the participation in the tournament.

## 7. CONCLUSION

The IYPT is a school competition with great educational effectiveness.

As a domino wave the main aims of the tournament entail the realization of many other aims, concerning the upbringing and growth of the young people- to be well educated with wide spirit and mental horizons.

Maybe the most beautiful feature of the problems of the tournament is this one that they bring up with students the skills to observe, investigate and understand nature, to enjoy its beauty, to provoke and evaluate their curiosity to the things around us and seeking for answer to the questions concern simple at first glance things, such as why the color of the sky is blue, why the sunrises and sunsets are red, the young people to take the path of discovering of the deep secrets of the nature.

## ACKNOWLEDGMENTS

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I would like to thank the colleagues from Austria, Brazil, Bulgaria, Croatia, Czech, Hungary, Korea, New Zealand, Poland and Ukraine, which have supported the idea of publishing of such edition and given their solutions for this purpose.

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IOC member of the IYPT 2003, 2004, 2005
Bulgaria

# II. Problems for the 18th IYPT

### 1. Dragonfly

Propose a model of how a dragonfly flies, investigate the major parameters and validate your model.

### 2. The two balls problem

Two balls placed in contact on a tilted groove sometimes do not roll down.
Explain the phenomenon and find the conditions, under which it occurs.

### 3. Avalanche

Under what conditions may an avalanche occur? Investigate the phenomenon experimentally.

### 4. Hydraulic jump

When a smooth column of water hits a horizontal plane, it flows out radially. At some radius, its height suddenly rises. Investigate the nature of the phenomenon. What happens, if a liquid more viscous than water is used?

### 5. Mirage

Create a mirage like a road or desert mirage in a laboratory and study its parameters.

### 6. Noise

When a droplet of water or other liquid falls on a hot surface, it produces a sound. On what parameters does the sound depend?

### 7. Bouncing plug

A bathtub or sink is filled with water. Remove the plug and place a plastic ball over the plughole. As the water drains the ball starts to oscillate. Investigate the phenomenon.

### 8. Wind car

Construct a car, which is propelled solely by wind energy. The car should be able to drive straight into the wind. Determine the efficiency of your car.

### 9. Sound in the glass

Fill a glass with water. Put a tea-spoon of salt into the water and stir it. Explain the change of the sound produced by the clicking of the glass with the tea-spoon during the dissolving process.

**10. Flow rate**

Combine powdered iron/ iron filings/ with a vegetable oil. Connect two containers with plastic tubing and allow the mixture to drain through the tube. Develop an external mechanism to control the flow rate of the mixture.

**11. Water droplets**

If a stream of water droplets is directed at a small angle to the surface of water in a container, droplets may bounce off the surface and roll across it before merging with the body of water. In some cases the droplets rest on the surface for a significant length of time. They can even sink before merging. Investigate these phenomena.

**12. Ball spin**

Spin can be used to alter the flight path of balls in sport. Investigate the motion of a spinning ball, for example a table-tennis or tennis ball, in order to determine the effect of the relevant parameters.

**13. Hard starch**

A mixture of starch (e.g. cornflour or cornstarch/ and a little water has some interesting properties. Investigate how its viscosity changes when stirred and account for this effect. Do any other common substances demonstrate this?

**14. Einstein-de-Haas Experiment**

When you apply a vertical magnetic field to a metallic cylinder suspended by a string it begins to rotate. Study this phenomenon.

**15. Optical tunneling**

Take two glass prisms separated by a small gap. Investigate under what conditions light incident at angles greater than the critical angle is not totally internally reflected.

**16. Obstacle in a funnel**

Granular material is flowing out vessel through a funnel. Investigate if it is possible to increase the outflow by putting an obstacle above the outlet pipe.

**17. Ocean "Solaris"**

A transparent vessel is half-filled with saturated salt water solution and then fresh water is added with caution. A distinct boundary between these liquids is formed. Investigate its behaviour if the lower liquid is heated

# III. Solution of the problems for the 18$^{th}$ IYPT

## 1. PROBLEM №2 “THE TWO BALLS PROBLEM”

*SOLUTION OF BRAZIL*

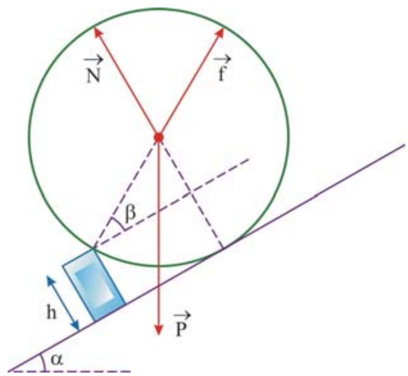
**Problem№2 -The Two Balls Problem**
*Marcelo Puppo Bigarella*, **Brazil**

**The problem**

*Two balls placed in contact on a tilted groove sometimes do not roll down. Explain the phenomenon and find the conditions, under which it occurs.*

**Physic Insight:**

First, we will treat about the case where we have just one sphere in a plan. If we want the sphere does not roll down, we need to fix an obstacle (as showed in the picture) in front of it.




**Picture 1: A ball in a ramp with an obstacle**

In this case we have $\vec{f}$ force: contact force that obstacle exert in the sphere (with $R$ radius), $\vec{P}$: sphere’s weight, $\vec{N}$ :Normal force that the titled ramp exert in the sphere; $h$ is the obstacle height fixed in the ramp, α is the ramp’s inclination and β is the angle formed between the ramp and $\vec{f}$ direction.
In the equilibrium, the external force sum at the sphere must be equal to zero (null). Thus,

$$\sum_i F_{ix} = \sum_i F_{iy} = 0 \qquad (1)$$

where $F_{ix}$ and $F_{iy}$ are the $x$ and $y$ component of the i-esimal force, respectively. In this way:

- X axis: $f \cdot \cos(\alpha + \beta) = N \cdot \sin\alpha$
- Y axis: $P = f \cdot \sin(\alpha + \beta) + N \cdot \cos\alpha$

Solving these two equations (in function of *P*) we have:

$$f = \frac{\sin\alpha}{\cos\beta} \cdot P \quad (2) \qquad\qquad N = \frac{\cos(\alpha + \beta)}{\cos\beta} \cdot P \quad (3)$$

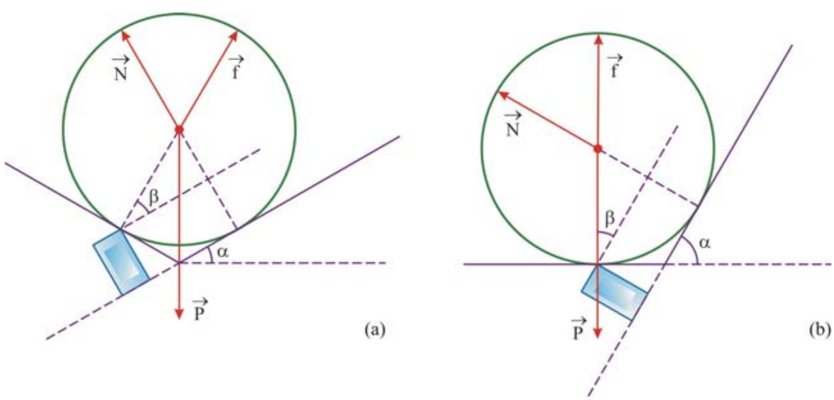
This solution above just have a physical sense, when $N \geq 0$. Thus, from equation 3 we obtain:

$$\alpha + \beta \leq \frac{\pi}{2} \quad (4)$$

With this, we can conclude that the contact point between the sphere and the obstacle must stay at the left of $\vec{P}$ vector; otherwise the sphere will roll over the obstacle.

We could thing in other way to understand the equilibrium: let's imagine that the ball is placed in a groove with V shape (picture 2a). Therefore, we can consider that the ball is in under a tilted ramp with an obstacle as the spotted lines in the drawing shows.

The equilibrium configuration happens when both walls of the groove are inclined upwards (stability). It is way, in picture 2b, the ball is under a permanent equilibrium configuration (instability), which means that the sphere can roll to left in one moment. (It is possible because the left groove wall is in a horizontal position). Geometrically, this condition happens when $\alpha + \beta = \pi/2$. In this way we also arrive in the equilibrium condition (equation 4).




**Picture 2: Equilibrium configurations**

With assistance of this theory, we can substitute the obstacle for a small sphere, placed in the same local: at the left of the superior ball. The two balls would not roll down because the friction force torque over them must be compensates by the torque of the contact force between them. Therefore, we can expect that the torques cancel themselves and do not occur any rolling movement.

We can notice that if the left sphere is bigger than the right sphere, it will be impossible maintaining the equilibrium. It would be equivalent to having the obstacle at the right side of the ball. So, we would not deal with this situation.

<u>The Two Balls Problem:</u>

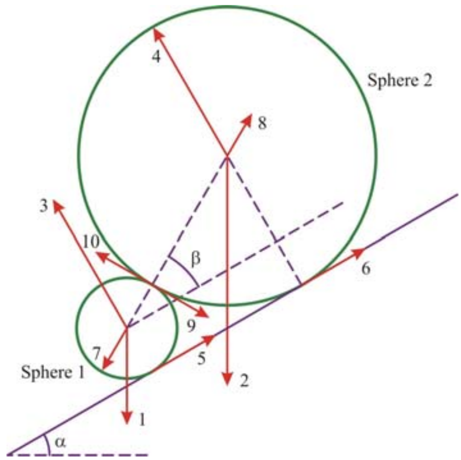
We need to analyze the condition in under which two balls in touch in a tilted groove do not roll down.

Picture 3 shows us the two balls in a tilted ramp and the external forces that are applied over each ball.

The ball at the left is *sphere* 1 and the ball at the right is *sphere* 2.

These forces are:

1. Sphere 1 weight force: $\vec{P}_1$;
2. Sphere 2 weight force: $\vec{P}_2$ ;
3. Sphere 1 Normal: $\vec{N}_1$;
4. Sphere 2 Normal: $\vec{N}_2$ ;
5. Friction force in sphere 1: $\vec{f}_1$;
6. Friction force in sphere 2: $\vec{f}_2$ ;
7. Sphere 2 contact force in sphere 1, which tends to push down the sphere e 1: $\vec{f}_3$
8. Reaction to force $\vec{f}_3$: -$\vec{f}_3$;
9. Sphere 2 contact force in sphere 1, which tends to rotate sphere 1 in a clockwise movement: $\vec{f}_4$;
10. Reaction to force $\vec{f}_4$: -$\vec{f}_4$



**Picture 3: The two balls problem**

the angle $\alpha$ represents the ramp inclination and $\beta$ represents the angle formed between the tilted ramp and the forces pair $\vec{f}_3$ and $\vec{f}_4$ actuation line

As we want that the system continues in equilibrium, the sum of the forces over each spheres have to be zero.

$$\sum_i F_{ix} = \sum_i F_{iy} = 0$$

So, such as the external forces, as well the external torques which acts in the two balls must annul themselves. First, the force equilibrium equations for each ball are:

<u>Sphere 1:</u>

- X axis: $f_1 \cos\alpha + f_4 \sin(\alpha+\beta) = f_3 \cos(\alpha+\beta) + N_1 \sin\alpha$ ;
- Y axis: $\vec{P}_1 + f_3 \sin(\alpha+\beta) + f_4 \cos(\alpha+\beta) = f_1 \sin\alpha + N_1 \cos\alpha$ ;

- Rotation: $f_1 = f_4$

  Sphere 2:
- X axis: $f_2 \cos\alpha + f_3 \cos(\alpha+\beta) = f_4 \sin(\alpha+\beta) + N_2 \sin\alpha$ ;
- Y axis: $\vec{P}_2 = f_2 \sin\alpha + f_3 \sin(\alpha+\beta) + f_4 \cos(\alpha+\beta) + N_2 \cos\alpha$ ;
- Rotation: $f_2 = f_4$

The system solution is showed below:

$$N_1 = P_1 \cos\alpha + \frac{\sin\alpha}{2\cos\beta}\left[P_1 + P_2 + (P_2 - P_1)\sin\beta\right],$$

$$N_2 = P_2 \cos\alpha - \frac{\sin\alpha}{2\cos\beta}\left[P_1 + P_2 + (P_2 - P_1)\sin\beta\right],$$

$$f_3 = \frac{\sin\alpha}{2\cos\beta}\left[P_2 - P_1 + (P_2 + P_1)\sin\beta\right],$$

$$f_1 = f_2 = f_4 = \frac{1}{2}\sin\alpha(P_1 + P_2).$$

In order not roll any ball, it is necessary that

$$f_1 \le \mu_1 N_1 \quad (6), \qquad f_2 \le \mu_2 N_2 \quad (7), \qquad f_4 \le \mu_3 f_3 \quad (8)$$

where $\mu_1$, $\mu_2$ $\mu_3$ are the static friction coefficient between the sphere 1 and the ramp, between the sphere 2 an the ramp and between sphere 1 and sphere 2, respectively. The system will be in movement imminence as soon as one of these equations becomes saturated.

Now, we must study the module of $\alpha, \beta, P_1, P_2, \mu_1, \mu_2, \mu_3$ that saturate the equilibrium's conditions (equation 6, 7, 8) with purpose of finding out the imminence conditions in which the spheres roll down.

Firstly, let's analyze the restriction 8. So,

$$\cos\beta - \mu_3 \sin\beta \le \mu_3 \frac{P_2 + P_1}{P_2 - P_1} \quad (9)$$

which solution is:

$$\cos\beta \ge \frac{\mu_3 p + \sqrt{\mu_3^{\,2} + (1-p^2)\mu_3^{\,4}}}{1-\mu_3^{\,2}},$$

where $p = \dfrac{P_2 - P_1}{P_2 + P_1}$ . We must to emphasize that is not the unique solution. There are negative solutions which refers to case in where the smallest ball is at the right side.

As we know that:

$$\beta_{\min} \leq \beta \leq \frac{\pi}{2} - \alpha \quad (10)$$

we have:

$$\beta_{\min} = \arccos \frac{\mu_3 p + \sqrt{\mu_3^{\ 2} + (1 - p^2)\mu_3^{\ 4}}}{1 - \mu_3^{\ 2}}$$

In an analog form, to the equation 7:

$$\cot\alpha \geq \frac{P_1 + P_2}{2P_2}\left(\frac{1}{\mu_2} + \frac{1 + p\sin\beta}{\cos\beta}\right) \quad (11)$$

In the same form, to equation 6:

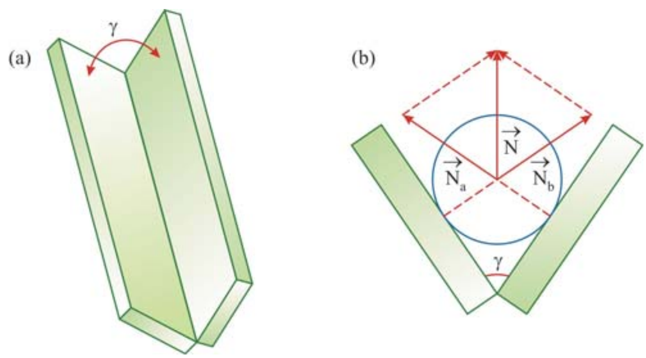
$$\cot\alpha \geq \frac{P_1 + P_2}{2P_2}\left(\frac{1}{\mu_1} - \frac{1 + p\sin\beta}{\cos\beta}\right) \quad (12)$$

The restriction ensemble (10), (11) e (12) defines α and β modules, in function of the others parameters $P_1, P_2, \mu_1, \mu_2, \mu_3$

**Groove:**

The first point we want to discuss is the tilted ramp (opening groove wall's angle = 180º). Two balls in a tilted ramp always roll down because the bi-dimensional configuration is unstable. Let's remember that we are working with the hypothesis that the spheres are confined to move along the line which defines the tilted plan (conform picture 3).

The experiment in a tilted plan would just be successful if we worked with cylinders, instead of balls. As we want to respect the problem, using spheres, we must have to use a gutter with a V format (the groove), as it is showed in picture 4.




**Picture 4: Groove's characteristic**

The difference between this model and the model utilized in the previous section is the β angle's determination. We can notice that the equilibrium force equations remain unaltered. For example, the Normal force vector which acts over a sphere in this groove (see picture 4b) remains in the rolling plan. The transversal component to the vectors $\vec{N}_a$ and $\vec{N}_b$ rolling plan cancel themselves (see picture 4b) and yet remains in the rolling plan. β angle's determination in the assembly of picture 3 is extremely simple: $\sin\beta = (R-r)/(R+r)$. In the groove, the β angle's determination should depend on $\gamma$. As illustrated in picture 4b, the distance between the groove basis (intersection point between the two walls) and the sphere centre (with radius equal to *R*) is:

$$D = \frac{R}{\sin\frac{\gamma}{2}}$$

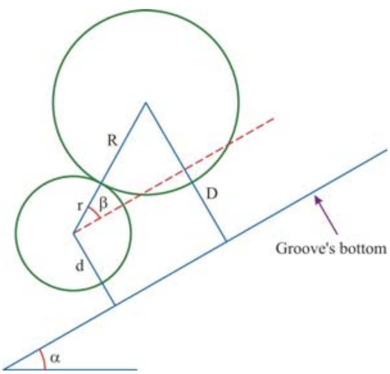
**In this way, β angle have to be determined by (see picture 5),**

$$\sin\beta = \frac{R-r}{(R+r)\sin\frac{\gamma}{2}} \qquad \textbf{(16)}$$

There is a minimum $\gamma$. It is necessary to remember condition $\alpha+\beta \le \frac{\pi}{2}$, therefore:

$$\frac{R-r}{(R+r)\sin\frac{\gamma}{2}} \le \cos\alpha, \Rightarrow \sin\frac{\gamma}{2} \ge \frac{R-r}{(R+r)\cos\alpha}$$

To finish, the experiment to be done still have seven free parameters: $\alpha, \beta, P_1, P_2, \mu_1, \mu_2, \mu_3$. In the true, we can simplify it to just six parameters because in the restriction inequations (6), (7), (8) $P_1$ and $P_2$ appears in the two sides, so, the interest quantity is the relative mass between the spheres, and not the absolute mass of each one.




**Picture 5: A lateral view of the groove**

In the attached Excel plan *two_balls* we manipulated all the formulas, where we can place the variables (theoretical values or experimental data) to obtain the maximum angle for a specific situation. The angle formed between the two groove walls is fixed (in our case 90º). We fixed this angle in order to a better experimental performance. However, as we fixed this angle, we can not apply it into a future theoretical angle's maximum model.

As we are working with an opening groove's angle equal to 90º, equation 13 becomes:

$$\sin\beta = \frac{R-r}{(R+r)\sin\frac{\pi}{4}}$$

We are going to use its plan to compare the experiment with theory and see the difference between these two parts of the problem's resolution

**Not rolling conditions**:
In this section we will determine the conditions and the inclination angle α which must satisfy the equations presented, in order to occur the phenomenon. It means that α must satisfy:

$$\alpha_{\min} \le \alpha \le \alpha_{\max}$$

where $\alpha_{\min}$ and $\alpha_{\max}$ depends on $\beta, P_1, P_2, \mu_1, \mu_2, \mu_3$.

$\alpha_{\min}$ determination is simple: in the case the groove is in the horizontal position, the system will stay in rest, hence, $\alpha_{\min} = 0$ for any of the others values.

$\alpha_{\max}$ determination is more complex: it demands that all restrictions (4), (6), (7) and (8) is satisfied. From condition 4:

$$\alpha_1 = \frac{\pi}{2} - \beta \tag{13}$$

where $\alpha_1$ is a possible $\alpha_{\max}$ value, and, by inequation 10, $\beta \ge \beta_{\min}$.

If $\beta < \beta_{\min}$, $\alpha_{\max} = 0$.

It is important to notice that condition if the sphere at the left is smaller than the sphere at the right; otherwise β would be smaller or equal to zero.

The others possible $\alpha_{\max}$ values come from inequation (11) and (12) and they are, respectively:

$$\alpha_2 = \arctan\frac{2P_2\mu_2\cos\beta}{(P_1+P_2)(\cos\beta+\mu_2+\mu_2 p\sin\beta)} \tag{14}$$

$$\alpha_3 = \arctan\frac{2P_1\mu_1\cos\beta}{(P_1+P_2)(\cos\beta-\mu_1-\mu_1 p\sin\beta)} \tag{15}$$

Therefore,

$$\alpha_{\max} = \min\{\alpha_1, \alpha_2, \alpha_3\}$$

For determining the maximum inclination that the groove can has, without the spheres rolling down, we must calculate angles $\alpha_1, \alpha_2, \alpha_3, \beta_{\min}$. If $\beta \geq \beta_{\min}$, then the maximum groove inclination will be given by the smaller value among $\alpha_1, \alpha_2, \alpha_3$. Otherwise, If $\beta < \beta_{\min}$, the phenomenon (problem) will not occur.

This conclusion ends up the study about phenomenon, but they do not give a good indication how to vary $\beta, P_1, P_2, \mu_1, \mu_2, \mu_3$ parameters in the way that it maximizes the groove inclination. It happens because the equation which determines $\alpha_1, \alpha_2, \alpha_3$ are completely non-linear equation.

First, we need to understand more about the opening wall's angle (which is always, in our case, 90º)

In the next topics we will explain the manipulation possibilities of these equations in one more simplified context.

**Simplified Situation:**

In this section we will treat about the realization possibilities of the described phenomenon in situations more simplified. Our simplification choice will be specified in each case

- Same Size Spheres

Considering two spheres with same size (same radius), is it possible that they do not roll down?

In this situation, $\beta$ is equal to zero (see equation 13). In this way:

$$f_3 = \frac{\sin\alpha}{2}(P_2 - P_1) \qquad f_4 = \frac{\sin\alpha}{2}(P_1 + P_2)$$

As $f_4 \leq \mu_3 f_3$, $\qquad \mu_3 \geq \dfrac{P_2 + P_1}{P_2 - P_1} > 1$

Hardly we will find in the nature material which have static friction coefficient bigger than 1. So we conclude that the experiment success in this situation is remote (if it is not impossible).

We confirm our hypothesis supported in the excel plan *two_balls*.

- Sphere and groove made of the same material

Now we will analyze the case in which we have both spheres and groove made of the same material (e. g. wood). This imply in $\mu_1 = \mu_2 = \mu_3 = \mu$. Defining $\Delta$ as the relation between the spheres weight ($P_2 = \Delta P_1$), the minimum $\beta$ is given by:

$$\beta_{\min} = \arccos \frac{\mu(1-\Delta) + \mu\sqrt{(1+\Delta)^2 + 4\Delta\mu^2}}{(1+\Delta)(1+\mu^2)}$$

We will test this equation experimentally and see the difference between the theoretical value and the experimental one.

**Experiment:**
Our experiment consists in reproducing the problem according to the explanation giving. After it, we intend to place the experimental data in the theory equation (previous studied) and see how much approximate is our experience.

We made a groove and fixed an angle's measure equipment at its base. Photos of the apparatus are attached. Our variables, in this experiments, were:

The surface material: cardboard paper, polystyrene paper, paper, wood or plastic.

The balls were made of: polystyrene, plastic, leather, steel, glass.

In our experiment, we choose two balls and one surface. We measure the maximum inclination angle (α). After this, we compared with the maximum alfa's angle given by equations 13, 14 or 15 (we can use this equation because we know the relation between the two ball radius, and, consequently, β). The theoretical values were taken from the Excel plan.

We repeat the experience changing the ball combinations and marking the respective angles.

As we said before, in our experiment the angle between the walls of the groove have not been changed (as we used two fixed wood board as our groove, the angle was 90º angle)

The first step in the experiment was measuring the characteristic of the ball (for substituting in the equations), like the diameter, and consequently, the radius, the mass, and, consequently, the weight

For measuring the mass, we made use of a balance (with maximum precision as a hundredth of the gram) and for measuring the diameter we used special ruler (with maximum precision as a tenth of the millimeter).

The balance's error is 0,01g (it is written in the technical characteristic protocol)

The measure instrument error is the half of the smallest measure step. So, the error in the diameter was 0,005mm

The experimental measure data as well the correspondent error is attached in the Excel plan *two_balls_measures*.

It is important to emphasis that we did not work just with full solid balls: we worked also with spherical husks (ball with air inside).

| | |
|---|---|
| Completely solid (6): | Golf, iron 1 and 2, glass 1 and 2, plastic ball |
| Spherical husk (12): | All polystyrene balls, tennis balls, blue, orange and violet balls, ping-pong balls |

The unique difference between this two kind of ball (hollow or full) is the great difference between the weight force in balls with almost the same size.

As we need to place the static friction coefficient to calculate theoretically the maximum angle, we need to know its value for each material combination in the experiment.

A complete static friction coefficient table is attached in a Word document *two_balls_friction*

Those coefficients we did not managed to find in any table, we calculate using the equation below (where θ is the movement imminence angle).

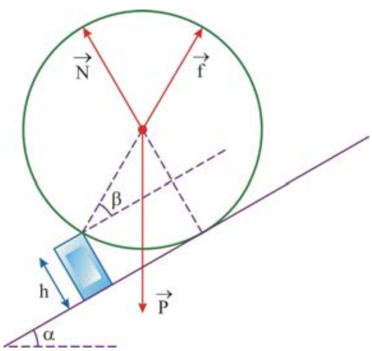
$$\mu_{static} = \tan\theta$$

Just in four material's combination we did not have a theory value we made this experience. The results are showed in the table below

| | θ | tan θ = μ (static) |
|---|---|---|
| Cardboard and polystyrene | 27º | 0,50 |
| Plastic and polystyrene | 24,5º | 0,45 |
| Paper and polystyrene | 20,5º | 0,37 |
| Leather and plastic | 32º | 0,62 |

## DEDUCTIONS

### DEDUCTION 1




**Picture 1: A ball in a ramp with an obstacle**

- X axis: $f \cdot \cos(\alpha + \beta) = N \cdot \sin\alpha$ (I)
- Y axis: $W = f \cdot \sin(\alpha + \beta) + N \cdot \cos\alpha$ (II)

As, $\sum_i F_{ix} = \sum_i F_{iy} = 0$

(I) $N = \dfrac{f \cdot \cos(\alpha + \beta)}{\sin\alpha}$

(II) $N = \dfrac{W - f \cdot \sin(\alpha + \beta)}{\cos\alpha}$

$$N = N \rightarrow \frac{f \cdot \cos(\alpha+\beta)}{\sin\alpha} = \frac{W - f \cdot \sin(\alpha+\beta)}{\cos\alpha}$$

□ $f \cdot \cos(\alpha+\beta) \cdot \cos\alpha = [W - f \cdot \sin(\alpha+\beta)] \cdot \sin\alpha$

□ $f \cdot \cos(\alpha+\beta) \cdot \cos\alpha + f \cdot \sin(\alpha+\beta) \cdot \sin\alpha = W \cdot \sin\alpha$

□ $f \cdot [\cos(\alpha+\beta) \cdot \cos\alpha + \sin(\alpha+\beta) \cdot \sin\alpha] = W \cdot \sin\alpha$

□ $f \cdot [\cos(\alpha+\beta-\alpha] = W \cdot \sin\alpha$

□ $f \cdot [\cos(\cancel{\alpha}+\beta-\cancel{\alpha}] = W \cdot \sin\alpha$

□ $f \cdot (\cos\beta) = W \cdot \sin\alpha \rightarrow \boxed{f = \frac{\sin\alpha}{\cos\beta} \cdot W}$

Placing ***f*** result in equation (I), we have:

$$N \cdot \sin\alpha = f \cdot \cos(\alpha+\beta) \rightarrow N \cdot \sin\alpha = \frac{\sin\alpha}{\cos\beta} W \cdot \cos(\alpha+\beta) \rightarrow \boxed{N = \frac{\cos(\alpha+\beta)}{\cos\beta} \cdot W}$$

# Deduction of *Alfa*$_1$

$$\boxed{N = \frac{\cos(\alpha+\beta)}{\cos\beta} \cdot W} \qquad N \geq 0 \qquad \alpha+\beta \leq \frac{\pi}{2} \qquad \boxed{\alpha_1 = \frac{\pi}{2} - \beta}$$

# Deduction of *Alfa*$_2$

For finding β minimum, we need to pick condition

$$f_1 \leq \mu_1 N_1$$

We need to substitute $\boldsymbol{f_1}$, $\boldsymbol{N_1}$ and $\boldsymbol{\mu_1}$ (static friction coefficient between the two ball's material). Therefore,

$$f_1 = \frac{1}{2}\sin\alpha(W_1 + W_2)$$

$$N_1 = W_1 \cos\alpha + \frac{\sin\alpha}{2\cos\beta}[W_1 + W_2 + (W_2 - W_1)\sin\beta]$$

□ $f_1 \leq \mu_1 N_1$

□ $\frac{1}{2}\sin\alpha(W_1 + W_2) \leq \mu_1 \left[ W_1 \cos\alpha + \frac{\sin\alpha}{2\cos\beta}[W_1 + W_2 + (W_2 - W_1)\sin\beta] \right]$

□ $$\frac{\frac{1}{2}\sin\alpha(W_1+W_2)\le \mu_1\left[W_1\cos\alpha+\frac{\sin\alpha}{2\cos\beta}\left[W_1+W_2+\left(W_2-W_1\right)\sin\beta\right]\right]}{\frac{\sin\alpha}{2}}$$

□ $$(W_1+W_2)\le \mu_1\left[\frac{2W_1\cos\alpha}{\sin\alpha}+\frac{\left[(W_1+W_2)+\left(W_2-W_1\right)\sin\beta\right]}{\cos\beta}\right]$$ and

$$\frac{\cos\alpha}{\sin\alpha}=\cot\alpha \rightarrow$$

□ $$(W_1+W_2)\le \mu_1\cdot 2W_1\cot\alpha+\frac{\mu_1(W_1+W_2)}{\cos\beta}+\frac{\mu_1\left(W_2-W_1\right)\sin\beta}{\cos\beta}$$

and $$w=\frac{W_2-W_1}{W_2+W_1}\rightarrow w\cdot(W_2+W_1)=W_2-W_1$$

□ $$(W_1+W_2)-\frac{\mu_1(W_1+W_2)}{\cos\beta}-\frac{\mu_1\left(w\cdot(W_2+W_1)\right)\sin\beta}{\cos\beta}\le \mu_1\cdot 2W_1\cot\alpha \rightarrow$$

□ $$\frac{(W_1+W_2)\cos\beta}{\mu_1\cos\beta}-\frac{\mu_1(W_1+W_2)}{\mu_1\cos\beta}-\frac{\mu_1 w\cdot(W_2+W_1)\sin\beta}{\mu_1\cos\beta}\le 2W_1\cot\alpha \rightarrow$$

□ $$(W_1+W_2)\left(\frac{\cos\beta}{\mu_1\cos\beta}-\frac{\mu_1}{\mu_1\cos\beta}-\frac{\mu_1 w\cdot\sin\beta}{\mu_1\cos\beta}\right)\le 2W_1\cot\alpha$$

□ $$\frac{(W_1+W_2)}{2W_1}\left(\frac{1}{\mu_1}-\frac{1}{\cos\beta}-\frac{w\cdot\sin\beta}{\cos\beta}\right)\le\cot\alpha \rightarrow$$

$$\frac{(W_1+W_2)}{2W_1}\left(\frac{1}{\mu_1}-\frac{1+W\cdot\sin\beta}{\cos\beta}\right)\le\cot\alpha$$

□ $$\boxed{\cot\alpha\ge\frac{W_1+W_2}{2W_1}\left(\frac{1}{\mu_1}-\frac{1+w\sin\beta}{\cos\beta}\right)}$$

$$\cot\alpha=\frac{1}{\tan\alpha}\rightarrow \frac{1}{\tan\alpha}\ge\frac{W_1+W_2}{2W_1}\left(\frac{1}{\mu_1}-\frac{1+w\sin\beta}{\cos\beta}\right)$$

□ $$\frac{1}{\frac{W_1+W_2}{2W_1}\left(\frac{1}{\mu_1}-\frac{1+w\sin\beta}{\cos\beta}\right)} \geq \tan\alpha$$ →

$$\frac{2W_1}{(W_1+W_2)\left(\frac{\cos\beta-\mu_1-\mu_1 w\sin\beta}{\mu_1\cos\beta}\right)} \geq \tan\alpha$$

□ $$\tan\alpha \leq \frac{2W_1\mu_1\cos\beta}{(W_1+W_2)(\cos\beta-\mu_1-\mu_1 w\sin\beta)}$$

□ $$\boxed{\alpha_3 = \arctan\frac{2W_1\mu_1\cos\beta}{(W_1+W_2)(\cos\beta-\mu_1-\mu_1 w\sin\beta)}}$$

# Deduction of *Alfa*₃

For finding β minimum, we need to pick condition

$$f_2 \leq \mu_2 N_2$$

We need to substitute $\boldsymbol{f_2}$, $\boldsymbol{N_2}$ and $\boldsymbol{\mu_2}$ (static friction coefficient between the two ball's material). Therefore,

$$f_2 = \frac{1}{2}\sin\alpha(W_1+W_2) \qquad N_2 = W_2\cos\alpha - \frac{\sin\alpha}{2\cos\beta}\left[W_1+W_2+(W_2-W_1)\sin\beta\right]$$

□ $$f_2 \leq \mu_2 N_2$$

□ $$\frac{1}{2}\sin\alpha(W_1+W_2) \leq \mu_2\left[W_2\cos\alpha - \frac{\sin\alpha}{2\cos\beta}\left[W_1+W_2+(W_2-W_1)\sin\beta\right]\right]$$

□ $$\frac{\frac{1}{2}\sin\alpha(W_1+W_2) \leq \mu_2\left[W_2\cos\alpha - \frac{\sin\alpha}{2\cos\beta}\left[W_1+W_2+(W_2-W_1)\sin\beta\right]\right]}{\frac{\sin\alpha}{2}}$$

□ $$(W_1+W_2) \leq \mu_2\left[\frac{2W_2\cos\alpha}{\sin\alpha} - \frac{\left[(W_1+W_2)+(W_2-W_1)\sin\beta\right]}{\cos\beta}\right]$$ and $$\frac{\cos\alpha}{\sin\alpha} = \cot\alpha$$

→

□ $$(W_1+W_2) \leq \mu_2\cdot 2W_2\cot\alpha - \frac{\mu_2(W_1+W_2)}{\cos\beta} - \frac{\mu_2(W_2-W_1)\sin\beta}{\cos\beta}$$

and $w = \dfrac{W_2 - W_1}{W_2 + W_1}$ → $w \cdot (W_2 + W_1) = W_2 - W_1$

□ $$(W_1 + W_2) + \frac{\mu_2 (W_1 + W_2)}{\cos\beta} + \frac{\mu_2 \left(w \cdot (W_2 + W_1)\right)\sin\beta}{\cos\beta} \le \mu_2 \cdot 2W_2 \cot\alpha \rightarrow$$

□ $$\frac{(W_1 + W_2)\cos\beta}{\mu_2 \cos\beta} + \frac{\mu_2 (W_1 + W_2)}{\mu_2 \cos\beta} + \frac{\mu_2 w \cdot (W_2 + W_1)\sin\beta}{\mu_2 \cos\beta} \le 2W_2 \cot\alpha \rightarrow$$

□ $$(P_1 + P_2)\left(\frac{\cos\beta}{\mu_2 \cos\beta} + \frac{\mu_2}{\mu_2 \cos\beta} + \frac{\mu_2 p \cdot \sin\beta}{\mu_2 \cos\beta}\right) \le 2P_2 \cot\alpha$$

□ $$\frac{(W_1 + W_2)}{2W_2}\left(\frac{1}{\mu_2} + \frac{1}{\cos\beta} + \frac{w \cdot \sin\beta}{\cos\beta}\right) \le \cot\alpha \rightarrow$$

$$\frac{(W_1 + W_2)}{2W_2}\left(\frac{1}{\mu_2} + \frac{1 + w \cdot \sin\beta}{\cos\beta}\right) \le \cot\alpha$$

□ $$\boxed{\cot\alpha \ge \frac{W_1 + W_2}{2W_2}\left(\frac{1}{\mu_2} + \frac{1 + w\sin\beta}{\cos\beta}\right)}$$

$$\cot\alpha = \frac{1}{\tan\alpha} \rightarrow \frac{1}{\tan\alpha} \ge \frac{W_1 + W_2}{2W_2}\left(\frac{1}{\mu_2} + \frac{1 + w\sin\beta}{\cos\beta}\right)$$

□ $$\frac{1}{\dfrac{W_1 + W_2}{2W_2}\left(\dfrac{1}{\mu_2} + \dfrac{1 + w\sin\beta}{\cos\beta}\right)} \ge \tan\alpha \rightarrow$$

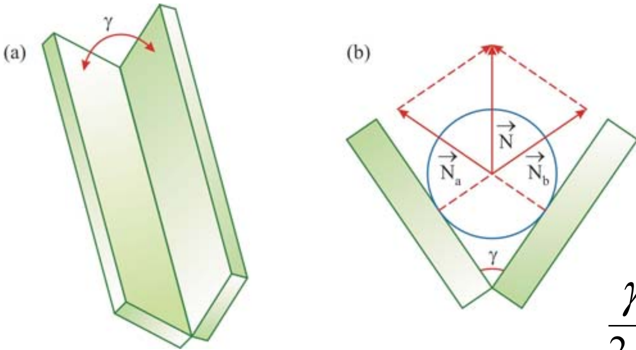
$$\frac{2W_2}{(W_1 + W_2)\left(\dfrac{\cos\beta + W_2 + \mu_2 w \sin\beta}{\mu_2 \cos\beta}\right)} \ge \tan\alpha$$

□ $$\tan\alpha \le \frac{2W_2 \mu_2 \cos\beta}{(W_1 + W_2)(\cos\beta + \mu_2 + \mu_2 w \sin\beta)}$$

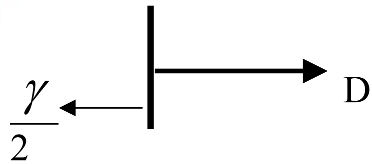
□ $$\boxed{\alpha_2 = \arctan \frac{2W_2 \mu_2 \cos\beta}{(W_1 + W_2)(\cos\beta + \mu_2 + \mu_2 w \sin\beta)}}$$

# Deduction of Beta

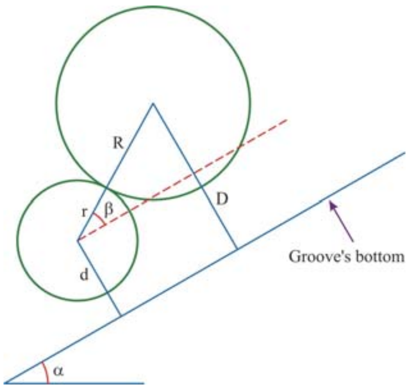
- In relation to gama ( $\gamma$ ) and to the ball's size -




**Groove:** **frontal view**




From the rectangle triangle, we have: $D = \dfrac{R}{\sin\frac{\gamma}{2}}$ and $d = \dfrac{r}{\sin\frac{\gamma}{2}}$




□ $\sin\beta = \dfrac{D-d}{R+r}$ → $\sin\beta = \dfrac{\dfrac{R}{\sin\frac{\gamma}{2}} - \dfrac{r}{\sin\frac{\gamma}{2}}}{R+r}$ →

$\sin\beta = \dfrac{(R-r)}{R+r}\left(\dfrac{1}{\sin\frac{\gamma}{2}}\right)$ → $\sin\beta = \dfrac{R-r}{(R+r)\sin\frac{\gamma}{2}}$

□ $\beta = \arcsin\dfrac{R-r}{(R+r)\sin\frac{\gamma}{2}}$

**Groove: lateral view**

# Deduction of ***Beta***$_{\mathbf{min}}$

For finding β minimum, we need to pick condition

$$f_4 \le \mu_3 f_3$$

We need to substitute $\boldsymbol{f_4}$, $\boldsymbol{f_3}$ and $\boldsymbol{\mu_3}$ (static friction coefficient between the two ball's material). Therefore,

$f_4 = \dfrac{1}{2}\sin\alpha(W_1 + W_2)$ $\quad f_3 = \dfrac{\sin\alpha}{2\cos\beta}\left[W_2 - W_1 + (W_2 + W_1)\sin\beta\right]$ $\quad f_4 \le \mu_3 f_3$

□ $\dfrac{1}{2}\sin\alpha(W_1 + W_2) \le \mu_3\left[\dfrac{\sin\alpha}{2\cos\beta}\left[W_2 - W_1 + (W_2 + W_1)\sin\beta\right]\right]$

□ $(W_1 + W_2) \le \mu_3 \left[ \dfrac{1}{\cos\beta} \left[ W_2 - W_1 + \left( W_2 + W_1 \right) \sin\beta \right] \right]$

□ $\cos\beta \le \mu_3 \left[ \dfrac{1}{(W_1 + W_2)} \left[ \left( W_2 - W_1 \right) + \left( W_2 + W_1 \right) \sin\beta \right] \right]$

□ $\cos\beta \le \mu_3 \left[ \dfrac{\left( W_2 - W_1 \right)}{(W_1 + W_2)} + \dfrac{\left( W_2 - W_1 \right) \sin\beta}{(W_1 + W_2)} \right]$ and $w = \dfrac{W_2 - W_1}{W_2 + W_1}$

□ $\cos\beta \le \mu_3 \left[ w + \sin\beta \right] \rightarrow \cos\beta \le \mu_3 w + \mu_3 \sin\beta$

□ $\cos\beta - \mu_3 w \le \mu_3 \sin\beta$ and $\sin\beta = \sqrt{1 - \cos^2\beta}$

□ $\cos\beta - \mu_3 w \le \mu_3 \sqrt{1 - \cos^2\beta} \rightarrow \left( \cos\beta - \mu_3 w \right)^2 \le \left( \mu_3 \sqrt{1 - \cos^2\beta} \right)^2$

□ $\cos^2\beta - 2\mu_3 w \cos\beta + \mu_3^{\,2} w^2 \le \mu_3^{\,2} (1 - \cos^2\beta)$

□ $\cos^2\beta (1 + \mu_3^{\,2}) - \cos\beta \cdot 2\mu_3 w + \mu_3^{\,2} (w^2 - 1) \le 0$ (second class equation)

$a = (1 + \mu_3^{\,2}) \,/\, b = -2\mu_3 w \,/\, c = \mu_3^{\,2} (w^2 - 1)$

□ If $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow \cos\beta \le \dfrac{-(-2w\mu_3) \pm \sqrt{(-2w\mu_3)^2 - 4(w^2 - 1)(1 + \mu_3^{\,2})\mu_3^{\,2}}}{2a}$

□ $\cos\beta \le \dfrac{2w\mu_3 + \sqrt{4w^2\mu_3^{\,2} - 4(w^2 - 1)(1 + \mu_3^{\,2})\mu_3^{\,2}}}{2(1 + \mu_3^{\,2})} \rightarrow$

$\cos\beta \le \dfrac{\mu_3 p + \sqrt{p^2\mu_3^{\,2} - (p^2 - 1)(\mu_3^{\,2} + \mu_3^{\,4})}}{1 + \mu_3^{\,2}} \rightarrow$

$\cos\beta \le \dfrac{\mu_3 w + \sqrt{\left[ w^2\mu_3^{\,2} - (w^2 - 1)\mu_3^{\,2} \right] + (1 - w^2)\mu_3^{\,4}}}{1 + \mu_3^{\,2}} \rightarrow$

$\cos\beta \ge \dfrac{\mu_3 w + \sqrt{\mu_3^{\,2} + (1 - w^2)\mu_3^{\,4}}}{1 - \mu_3^{\,2}}$

As we know that: $\beta_{\min} \le \beta \le \dfrac{\pi}{2} - \alpha$

we have $$\boxed{\beta_{\min} = \arccos \dfrac{\mu_3 w + \sqrt{\mu_3^{\,2} + (1 - w^2)\mu_3^{\,4}}}{1 - \mu_3^{\,2}}}$$

## 2. PROBLEM №4 -HYDRAULIC JUMP

*SOLUTION OF KOREA*

**Problem №4 -Hydraulic Jump**
**by overlapping of gravitational wave with viscous fluid**

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### The problem

*When a smooth column of water hits a horizontal plane, it flows out radially. At some radius, its height suddenly rises. Investigate the nature of the phenomenon. What happens, if a liquid more viscous than water is used?*

We investigated hydraulic jump of a radially spreading film of water originated by column-like jet that falls onto a horizontal plate. The reason of formation was suggested in terms of Froude number and overlapping of gravitational waves upstream and downstream. With volume-flux and momentum-flux constancy, some equations were made which describe the jump. The role of viscosity of fluid was explained by laminar boundary layer flow. In experimental parts, the water depth before and after the jump, and the radius of the jump were measured with the variation of water column radius, spreading speed, and viscosity. The depth could be measured with steel probe which was attached to micrometer and connected to ammeter, using the fact that when the probe touches the water surface the voltage changes suddenly. In jump radius, it increases when the efflux radius and speed increase and kinematic viscosity decreases and well matched with equations made.

### Introduction

In normal kitchen sink, we can see very interesting phenomenon called 'Hydraulic jump'.(See Fig. 1.) This phenomenon has been issued for almost one century [1][2][3], and this can be easily identified in everyday life. However the reason of formation and characteristics of the jump have not been explained fully, and still many studies are conducted on it.

In this paper, the hydraulic jump was explained in terms of overlapping of gravitational wave and especially roll of viscosity to the jump was investigated.

### Theoretical background

In theoretical part, we approached to the jump in terms of gravitational wave first, and made equations with volume-flux and momentum-flux constancy.

### The reason of formation ; Overlapping of wave

Waves in water can be divided into mainly two waves; gravitational wave, and surface wave. For our fluid, of which depth is shorter than the half of the wavelength of wave made, the wave has the characteristic of gravitational wave. That is, the speed of wave is determined only by the depth of the water. When h is the depth of water,

$$v_{wave} = \sqrt{gh}$$

Froude number Fr is the significant number for fluid, which is the ratio between wave speed and fluid speed.[4] Especially for gravitational wave,

**Fig. 1.** Sample hydraulic jump in kitchen sink.

$$Fr = \frac{v_{water}}{v_{wave}} = \frac{v}{\sqrt{gh}}$$

From now on, v is the speed of water. The fluid can be divided into 2 regions in terms of the Fr.(See Fig. 2.)

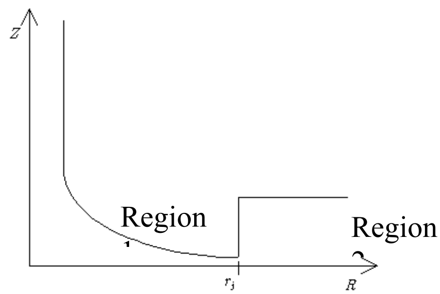



**Fig. 2.** A sketch of simplified hydraulic jump. Region 1 is supercritical and region 2 is subcritical. $r_j$ means the radius of jump.

Before the jump, in the region 1, Fr is less than 1, and the region 1 is called subcritical. In this region, because the water flows faster than the wave, the wave can go only downstream, but upstream. After the jump, in the region 2, Fr is bigger than 1, and the region 2 is called supercritical. Because the speed of water is slower than that of wave, the wave can go upstream and downstream both now and can be overlapped. At the point of jump, $r_j$, Fr is equal to 1, which is critical region, and it can be said that the jump position is the point where the wave upstream and downstream can start overlapping of wave. Therefore, the jump was made by the overlapping of gravitational wave.

**Hydrodynamic approach**

**Modeling** To make some equations to describe the jump quantitatively, the simplified model of the jump is necessary. In order to make some conditions simple, we made a model like Fig. 3. To make the $v_1$ constant, we assumed that the depth of water in region 1 decreases until the point of jump. Also, although the jump occurs with some thickness, it was ignored.

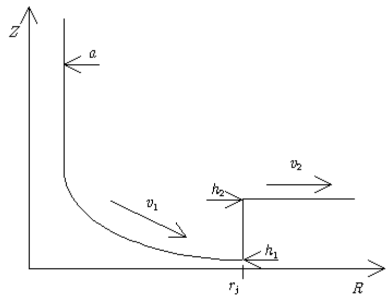



**Fig. 3.** The modeled hydraulic jump. $r_j$ is the jump radius, $a$ is the radius of the vertical water column. $v_1$ ($v_2$) is the speed of the water before the jump(after the jump) and $h_1$ ($h_2$) is the depth before the jump(after the jump).

**Volume constancy** The volume of the water will be preserved because water is incompressible fluid.

$$Q = \pi a^2 v_1 = 2\pi r_j h_1 v_1 = 2\pi r_j h_2 v_2 \quad -(1)\ [5]$$

Then,

$$h_1 v_1 = h_2 v_2 \quad -(2)$$

**Momentum constancy** Also, the momentum of the stream should be constant. Make a momentum constancy equation by finding the force of the stream in two different ways.

First, consider a cylindrical shell element of fluid from radius $r_\alpha$ to $r_\beta$. The total force to deform the element is $F_{tot} = F_\beta - F_\alpha$ $\quad F_\alpha = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$

$\frac{dm}{dt} = \rho Q$ and $\frac{dv}{dt} = 0$ at steady state

$$F_{tot} = (\rho Q v)_\beta - (\rho Q v)_\alpha \quad -(3)$$

Second, find the deforming force due to the pressure.

$$F = \int PdA = 2\pi r \int_0^h \rho g(h-z)dz = \pi r \rho g h^2 \quad -(4)$$

Equating (3) and (4), we have

$(\rho Qv)_\beta - (\rho Qv)_\alpha = \pi r \rho g h_\alpha^2 - \pi r \rho g h_\beta^2$

Using (1) $Q = 2\pi r h v$,

$2\pi r \rho v_\beta^2 h_\beta - 2\pi r \rho v_\alpha^2 h_\alpha = \pi r \rho g h_\alpha^2 - \pi r \rho g h_\beta^2$

$v_\alpha^2 h_\alpha + \frac{1}{2} g h_\alpha^2 = v_\beta^2 h_\beta + \frac{1}{2} g h_\beta^2$,

That is,

$$v_1^2 h_1 + \frac{1}{2} g h_1^2 = v_2^2 h_2 + \frac{1}{2} g h_2^2 \quad - (5)$$

**Depth relationship** With two constancy equations, the relationship between $h_1$ and $h_2$ can be known easily, and later we will check this relationship is valid with the experimental results.

Use equation (1) to change $v_1$ and $v_2$ term in equation (5).

$$v_1 = \frac{Q}{2\pi r_j h_1},\ v_2 = \frac{Q}{2\pi r_j h_2}$$

$$\frac{Q^2}{4\pi^2 r_j^2 h_1} + \frac{1}{2} g h_1^2 = \frac{Q^2}{4\pi^2 r_j^2 h_2} + \frac{1}{2} g h_2^2 \qquad \frac{Q^2}{4\pi^2 r_j^2}\left(\frac{h_2 - h_1}{h_1 h_2}\right) = \frac{1}{2} g \left(h_2^2 - h_1^2\right)$$

Since $h_1 \neq h_2$, and $q \equiv \frac{Q}{2\pi r_j}$ as the volume flux per unit width,

$$\frac{q^2}{h_1 h_2} = \frac{1}{2} g \left(h_1 + h_2\right)$$

Multiply $h_2$ to both side and change the equation in terms of $h_2$.

$$\frac{1}{2} g h_1^2 + \frac{1}{2} g h_1 h_2 - \frac{q^2}{h_1} = 0 \qquad h_2 = -\frac{h_1}{2} \pm \frac{h_1}{2}\sqrt{1 + \frac{8q^2}{g h_1^3}}$$

Since $h_2 > 0$,

$$h_2 = h_1\left(-\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{8q^2}{g h_1^3}}\right) = \frac{h_1}{2}\left(-1 + \sqrt{1 + \frac{8q^2}{g h_1^3}}\right)$$

$$h_2 = \frac{h_1}{2}\left(\sqrt{1 + \frac{8v_1^2}{g h_1}} - 1\right) \quad - (6) \quad \text{since } q = \frac{Q}{2\pi r_j} = \frac{2\pi r_j h_1 v_1}{2\pi r_j} = h_1 v_1$$

We now found the relationship between $h_1$ and $h_2$. However, let's try to simplify more. When $Fr_1 = \frac{v_1}{\sqrt{g h_1}}$, $\frac{h_2}{h_1} = \frac{1}{2}\left(\sqrt{1 + \frac{8v_1^2}{g h_1}} - 1\right) = \frac{1}{2}\left(\sqrt{1 + Fr_1^2} - 1\right) \quad - (7)$

Now we can check that the our explanation about the hydraulic jump in terms of the Froude number. In the equation (7), when $Fr_1 > 1$, $\frac{h_2}{h_1} > 1$, which means $h_2$ s bigger than $h_1$ and when $Fr_1 \leq 1$, $\frac{h_2}{h_1} \leq 1$, which means the jump is not created.

To find out the radius of the jump, use equation (1).

$$Q = \pi a^2 v_1 = 2\pi r_j h_1 v_1 = 2\pi r_j h_2 v_2 \quad -(1)$$

$$h_1 = \frac{a^2}{2r_j} \quad -(8) \qquad r_j = \frac{a^2}{2h_1} \quad -(9)$$

In the equation (5), because $v_1^2 h_1 \gg v_2^2 h_2$ and $\frac{1}{2} g h_2^2 \gg \frac{1}{2} g h_1^2$, then

$$v_1^2 h_1 = \frac{1}{2} g h_2^2$$

and by applying equation (9),

$$r_j = \frac{v_1^2 a^2}{g h_2^2} \quad -(10)$$

And here, we can make one more $h_1 - h_2$ relationship equation with equation (9) and (10).

$$h_2 = \sqrt{\frac{2h_1}{g}} v_1 \quad -(11)$$

Actually, This is the same results with equation (6) because when $\frac{8v_1^2}{g h_1}$ is big enough to ignore the 1,

$$h_2 = \frac{h_1}{2}\left(\sqrt{1 + \frac{8v_1^2}{g h_1}} - 1\right) \approx \frac{h_1}{2}\left(\sqrt{\frac{8v_1^2}{g h_1}} - 1\right)$$

$$\approx \frac{h_1}{2}\left(\sqrt{\frac{8v_1^2}{g h_1}}\right) = \sqrt{\frac{2h_1}{g}} v_1$$

and in our experimental condition, $v_1 = 1.2 m/s$ and $h_1 = 0.4 mm$, $\frac{8v_1^2}{g h_1}$ was about 3000, which is big enough.

**Roll of viscosity** In the problem, the roll of the viscosity of the liquid is asked. The theoretical explanation above does not have any concern of viscosity. It is for the inviscid liquid. For the roll of the viscosity, the boundary layer can be concerned.[6][7][8] The boundary layer means the layer of the water stream which is influenced by the friction with bottom surface and kinematic viscosity of the liquid(See Fig. 4.). Near the bottom, the liquid does not have same speed with the surface; in fact, the speed of the stream is much slower at the bottom. Because there is sudden decrease of the stream at the point of the hydraulic jump, the thickness of the boundary layer and the whole stream becomes same at the point of the jump. The thickness of the viscous laminar boundary layer is

$$\Delta = k\sqrt{\frac{\nu r}{v}} \quad -(12)$$

where $k$ can be experimentally acquired.

For $h \gg \Delta$, the deviation from inviscid flow is negligible. However, as $h \to \Delta$, the no-slip boundary condition becomes important and eventually dominate the whole flow behavior.

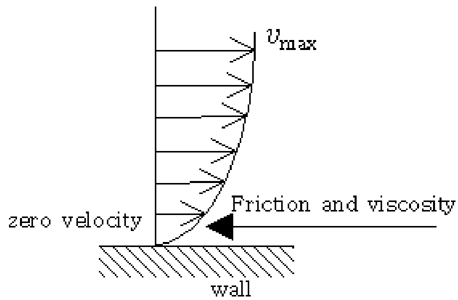



**Fig. 4.** Side shape of water flow considering laminar boundary layer. At the surface, the speed of water is the biggest.

In our experiment, the Reynolds number is smaller than the transition between laminar and turbulent flow, so we can use the laminar boundary layer.

Consider $h_1 = \Delta$ in the equation (9).

$$r_j = \frac{a^2}{2}\frac{1}{k}\sqrt{\frac{v_1}{\nu r_j}}$$

$$r_j = \frac{a^{4/3}}{(2k)^{2/3}}\left(\frac{v_1}{\nu}\right)^{1/3} = 0.63\frac{a^{4/3}}{k^{2/3}}\left(\frac{v_1}{\nu}\right)^{1/3} \quad -(13)$$

And equation (13) can be very useful because we can know the radius of the jump without the $h_1$ or $h_2$ which should be measured to be known. On following experiments, it will be the main equation to compare the experimental data with theory.

## Materials & Methods

With the wide water container, the flat board was placed upon the surface of the water and the acryl plate was put on the board.(See Fig. 5.) Between the board and plate, there was plotting paper which made it easier to measure the radius of jump. Next to the water container, there was water reservoir and by the small pump water was sprinkled onto the plate along the plastic tube. The amount of flowing water was controlled by the clamp attached to the end-point of plastic tube. The height of end-point of tube and water reservoir was able to be changed and it changed the efflux speed of water.

Near the place at which the jump was made, the sawn micrometer was set with steel stick tightly fixed by steel stand. At the end point of micrometer, steel pin was attached and it was connected to the ammeter which showed the voltage difference between the pin and the water in water container by putting the other end of ammeter into it. At normal condition the ammeter shows 0V, but by rotating micrometer when the endpoint of pin touches the water surface, the voltage changes drastically because water is not perfect insulator. The surface level could be known by measuring the scale of micrometer when the voltage changes, and the bottom level could be known by rotating the micrometer continuously until the endpoint of pin touches the bottom plate. By this method, it was able to measure the depth of water flow in 10 unit of length.

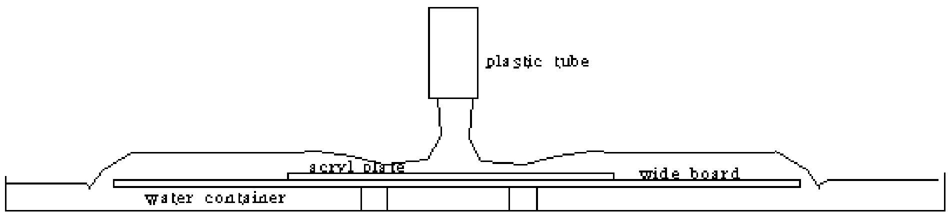
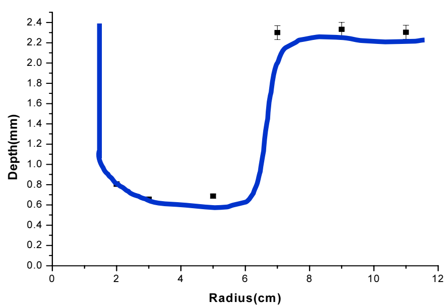



**Fig. 5.** Side shape of simplified experimental setup. Plastic tube was connected to water reservoir and pump. Between acryl plate and board, the plotting paper was put.

With three variables, efflux speed, efflux radius, and kinematic viscosity, the experiments were conducted. Kinematic viscosity was controlled by mixing the glycerin into the water. The table of dynamic viscosity of water-glycerin solution[9] was used and by measuring the density of solution for each concentration, the kinematic viscosity was obtained. With same conditions, same experiment was conducted for 5 times and the average value was used in analysis.

## Results and Discussion




### Sample hydraulic jump

One sample hydraulic jump was made, and the water depth along the radius was measured.(See Fig. 6.) Near the radius of 6cm, the jump occurred. And with this experiment, the value of $k$ could be known; 0.51.

### Changes in efflux speed

Fig. 7. is the graph of jump radius as the efflux speed changes. With the equation (13), $k$

**Fig. 6.** Sample hydraulic jump

value 0.51 and the condition $a = 2mm$, $\nu = 1\times10^{-6} m^2/s$, theoretically expected red line was made. First five dots are well matched with the line. However, when the speed becomes bigger than critical speed, about $1.6m/s^2$, the radius becomes much bigger than expected. This can be explained by the edge effect of the board. The all equations were made with the assumption that the jump was made on the infinitely wide plate. However, when the board is finite, in our experiment $50cm\times50cm$, the edge effect occurs. When water falls down to the water container, it drags other water of the plate by the cohesion of water. Therefore, the $h_2$ becomes lower and $r_j$ becomes bigger than expected.

In Fig. 8.($h_1$) and 9.($h_2$) the expected trend of $h_1$ and $h_2$ was identified; as efflux speed increases, $h_1$ decreases and $h_2$ increases. The last three dots are also by edge-effect.

**Changes in efflux radius**

Fig. 10. is the graph of jump radius as the efflux radius changes. With the equation (13), $k$ value 0.51 and the condition $v = 1.4m/s$, $\nu = 1\times10^{-6} m^2/s$, theoretically expected red line was made. We can see the red line is very well matched with the experimental data.

In Fig. 11.($h_1$) the expected trend was identified. And in Fig. 12.($h_2$), it was identified that over the critical efflux radius, the $h_2$ remains same value. This can be also explained by the edge effect.

**Changes in kinematic viscosity**

Fig. 13. is the graph of jump radius as the kinematic viscosity changes with the condition $v = 1.4m/s$, and $a = 1.328mm$. The blue line is experimentally made. With the theoretically expected line, the power of kinematic viscosity was rather different. However, the trend that the jump radius decreases as the kinematic viscosity increases.

Also in Fig. 14.($h_1$) and in Fig. 15.($h_2$), expected trend was identified; as kinematic viscosity increase, $h_1$ and $h_2$ increases both. In Fig. 16., the different shape of hydraulic jump can be identified with the eye between water and water-glycerin.

## Conclusion

First, we described what the hydraulic jump is. The overlapping of wave at the critical point was suggested as the reason of the formation of jump. For quantitative investigation, some equations were made with two useful constancy; volume and momentum. With the laminar boundary layer flow, the viscosity influences the jump. At the experimental parts, all the trend of the jump radius and the depth before and after the jump were identified. Especially for jump radius with the variation of efflux speed and radius, the theoretically expected equation was almost perfectly matched with experimental data.

**Acknowledgement**

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**Captions 7-16**

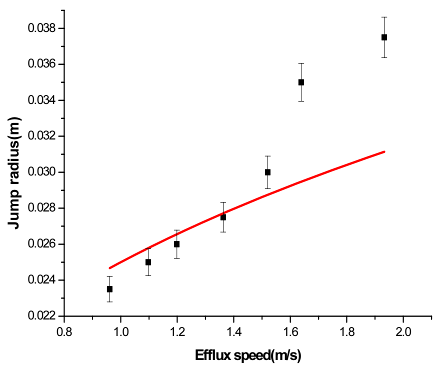



**Fig. 7.** Jump radius along efflux speed
The conditions are

$$a = 2mm\,, \nu = 1\times10^{-6}\, m^2 / s$$

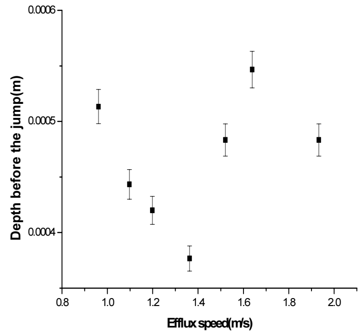



**Fig. 8.** Changes of $h_1$ along the efflux speed.
The conditions are $a = 2mm$, and $\nu = 1\times10^{-6} m^2/s$.

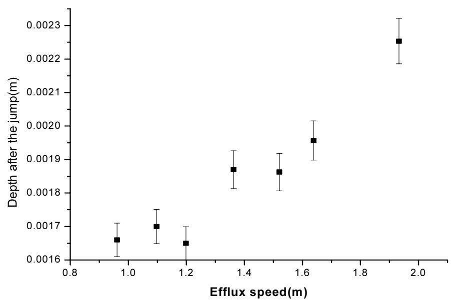



**Fig. 9.** Changes of $h_2$ along the efflux speed.
The conditions are $a = 2mm$, and $\nu = 1\times10^{-6} m^2/s$.

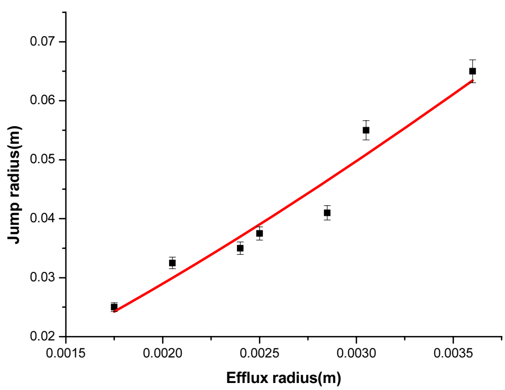



**Fig. 10.** Jump radius along efflux radius.
The conditions are $v = 1.4m/s$, and $\nu = 1\times10^{-6} m^2/s$.

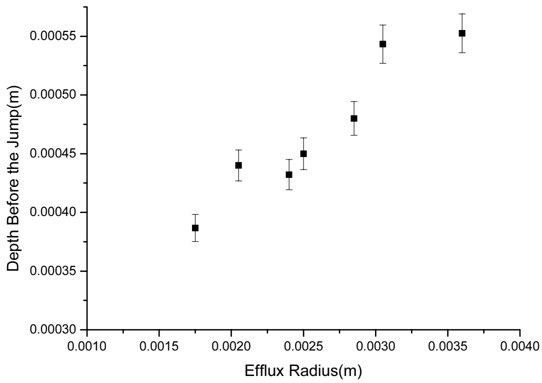



**Fig. 11.** Changes of $h_1$ along the efflux radius.
The conditions are $v = 1.4 m/s$, and $\nu = 1\times10^{-6} m^2/s$.

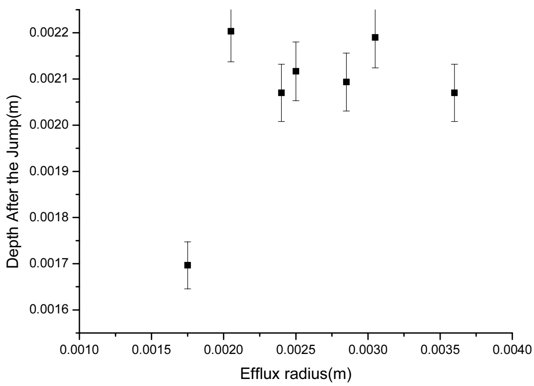



**Fig. 12.** Changes of $h_2$ along the efflux radius.
The conditions are $v = 1.4 m/s$, and $\nu = 1\times10^{-6} m^2/s$.

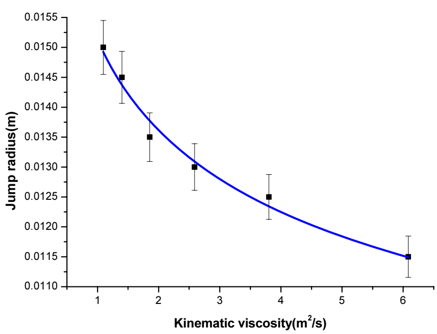



**Fig. 13.** Jump radius along the kinematic viscosity.
The conditions are $v = 1.4 m/s$, and $a = 1.328 mm$.

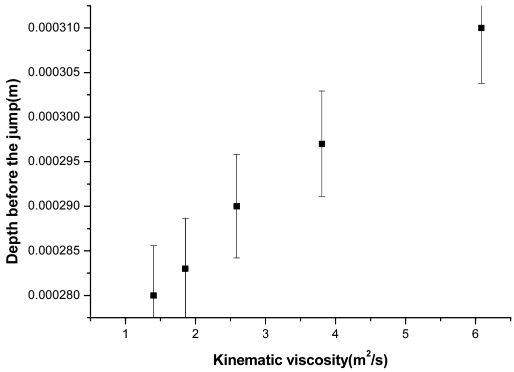



**Fig. 14.** Changes of $h_1$ along the kinematic viscosity. The conditions are $v = 1.4 m/s$, and $a = 1.328 mm$.

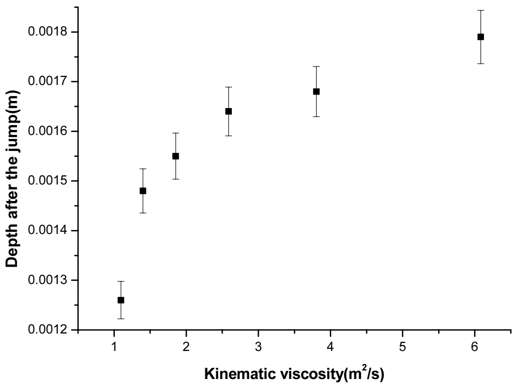
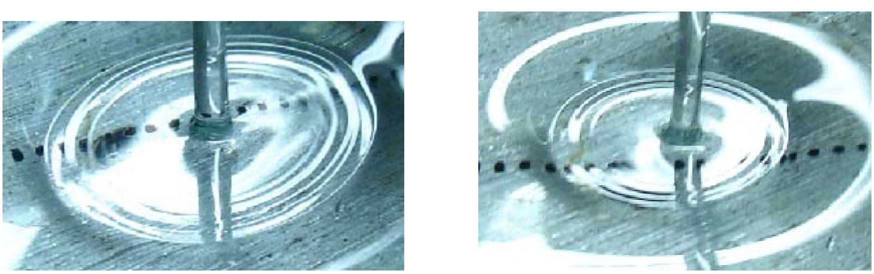



**Fig. 15.** Changes of $h_2$ along the kinematic viscosity. The conditions are $v = 1.4 m/s$, and $a = 1.328 mm$.



(a) b)

**Fig. 16.** Hydraulic jump of (a) water, (b) water-glycerin solution

# 3. PROBLEM №8 WIND CAR

*SOLUTION OF BRAZIL*

**Problem №8 WINDCAR**
*Marcelo Puppo Bigarella*, **Brazil**

**The problem**

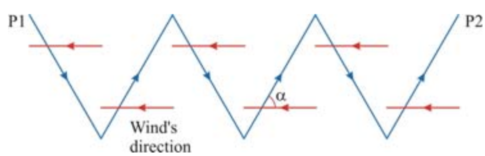
*Construct a car which is propelled solely by wind energy. The car should be able to drive straight into the wind. Determine the efficiency of your car.*

---

**1. Objective**: In this problem we are supposed to construct a car, which can be able to drive straight into the wind. The car has to be propelled just by wind energy, which means that we can not use others energy sources to move the car. The problem also asks us to determine the car's efficiency. In this part of the resolution (calculating efficiency) we have to establish the car's performance.
The experimental section of this problem is extremely necessary (indispensable). We have to construct a prototype that respect the problem's limitation, in order to make the necessary measurements to explain the car functionality and to determine its efficiency.

**2. Theoretical Background**: In this part of the resolution we will focus on each variable that could interfere with the final result, such as studies about momentum transmission as well studies about different kind of speed: scalar speed and angular speed. We will also exemplify possible models and the energy loss in each one, as well how to calculate efficiency.
At the beginning, we thought in three kinds of cars that could drive straight into the wind:

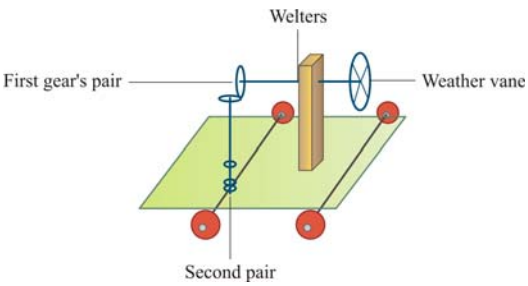
**1**- A sail's car (moved by sail): In this car we would utilize the force of the wind (blowing directly in the sail) to make the car walk. But as the car has to walk straight into the wind, this car would not respect the problem's limitation because it would have to drive in a zigzag trajectory to use the wind buoyancy.

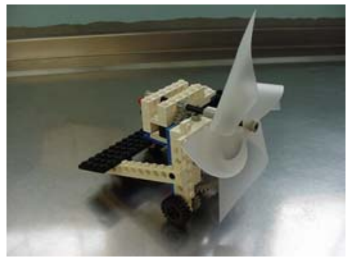



**2**- An electric car: The principle of the electric car is to transform the kinetic energy into electric energy (using a dynamo moved by a weather vane). Then, it would transform, with an electric engine, this electric energy into mechanical energy that would make the car walk.

- Loss in this car: air viscous force in the weather vane as well a considerable loss in the energy transformations (mechanic-electric-mechanic): energy dissipation.

**3-** <u>A mechanic car</u>: Using a weather vane (twirled by the wind) and pairs of gears, we would make the mechanic car move by energy transmission.
Loss in this car: friction in the axes (energy dissipation), gears heating (another example of energy dissipation) and tendency to slide in the wind direction (as the wind is blowing against the car (opposite to the problem's direction).
Considering this three models, and the energy loss of each one, we chose the third one (mechanic car) to develop the prototype. Below, a initial idea of the mechanic car

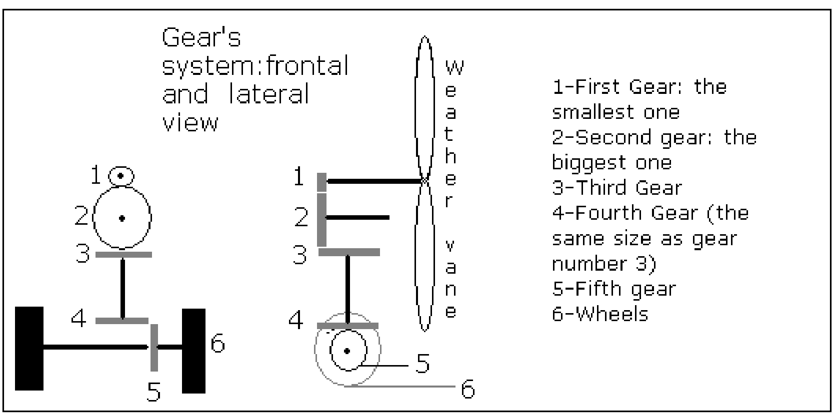


<u>2.1 Mechanic car:</u> Our mechanic car consists in a base (with four wheels) that support all the car structure, the gear's structure and its welters

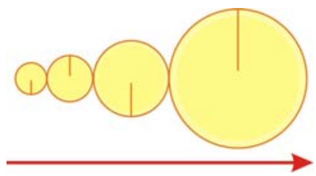
The car inner working is simple: basically the wind will twirl the weather vane, which, by gear's transmission, will roll a pair of wheels, making the car move. The transmission of momentum (wind blowing and twirling the weather vane) will be made by gears and axes.

Below, a drawing of the prototype (graphic project) and its photo:

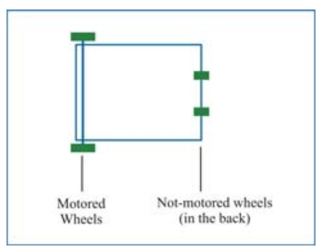


Important things for considering:

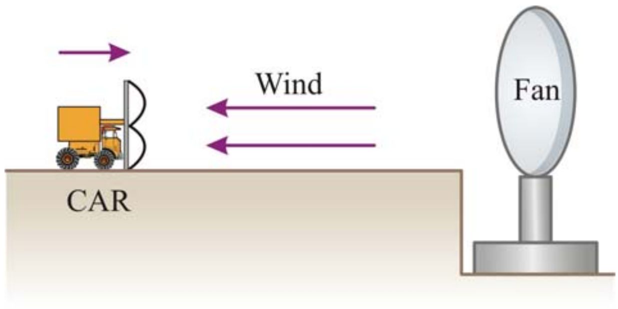
- <u>Frontal Area:</u> It is important to have the smallest frontal area (not considering the weather vane area). Air resistance force depends on the frontal area of the car. Smaller frontal area have smaller resistance (wind buoyancy), which is capable to reduces the car final speed and, consequently, the car efficiency.
- <u>The weather vane size</u>: The total torque on the weather vane determines the driving speed of the car. A bigger vane will have a larger torque on it, and, consequently, the car will drive faster into the wind. However, a larger weather vane has more contact surface area, thus it will have more resistance force, which tends to push back the car. So, we need to find a optimum size that delivers a maximum torque and a minimum resistance force.
- <u>The heating process in the gear's tooth</u>: Since we are working with gears to transmit momentum, we need to consider the heating process in the gear's tooth. The gear is heated by friction on its teeth. This) heating process can not be put aside because to twirl another gear, one gear has to hit its teeth in the other gear teeth. This process one form of energy dissipation and represents a reduction in the final performance.
- <u>Gear's position:</u> For a better performance, we have to put the smaller gear in the same axis of the weather vane and then, we need to increase the gear's size, until the wheels axis. (When we have the gears in the same axis, the gears can have the same size, because there are no force raise in the same axis). In this way we "give" a bigger torque to the wheel (responsible for moving the car).



- <u>Car's weight</u>: The car can not be too heavy, because the friction force increases directly proportional to the weight, given by the equation: $\vec{F}_{at} = \vec{N}.\mu$; where $\vec{N}$ is the compression (Normal) force between the surface and the car, which is also the weight force reaction pair; $\mu$ is the static friction coefficient between the wheel material and the surface material. With a heavier car, we will have a bigger Normal force, thus more attrition. However, the car weight is limited by the fact that too little friction increases the chance of wheels skidding (not a good "interaction" between the plan and the wheels).
- <u>Wheels:</u> There are four wheels in our mechanic car: Two are moved, indirectly, by the wind (motored wheels) and two wheels (not motored) have the function of equilibrating the car (this wheels are located at the back of the car). Below we show a drawing of the wheel's arrangement.

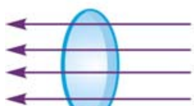


- <u>Wind Source:</u> In our experiment, we will use a fan as the wind source. Our fan has three different powers: it means that there are three possibilities of wind power (three different wind escape intensities). For a more complete resolution, we will utilize all powers to calculate the efficiency.
- <u>Wind direction:</u> The efficiency depends on the direction that the wind blows at the car frontal area. For a better performance, we used a step to put the fan, so we had a better wind utilization (picture)

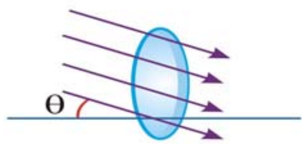



If we had used the wind in a transversal form (forming an angle (θ) with the horizontal), the air flux would change, as we show in the pictures below (because just the horizontal speed component will be utilized).

$$\text{flux} = V_n \,.\, \text{Area} \quad (1)$$



$$F = |\vec{V}|.A$$

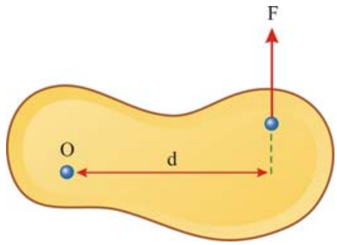



$$F = (\vec{V}.\vec{n}).A \quad (2)(3)$$

In our case we will just utilize the equation 2 because in our setup the wind will always be perpendicular to the weather vane area.

<u>2.2 Force Momentum (torque):</u> In mechanics, generally, we work with particles. But in our experimental setup it is necessary to work with objects (spatial corpus) and not just particles. Considering objects which do not deform itself when an external force is applied, we define momentum (*M*), or torque of a force $\vec{F}$ (acting in this body in relation to a axis which pass in *O*), by the relation below (where *d* is the distance between the *O* and the perpendicular projection of the force):

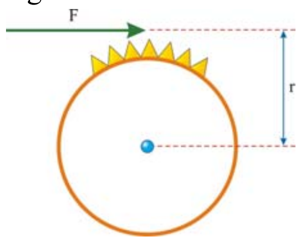
$$\boxed{M = F \cdot d}$$




(4)

In our case, we will use momentum as one form to calculate the efficiency and also to relate the transmission among the gears and the weather vane. In the gear, momentum will be calculated by the force applied in the gear's tooth (extremity of the gear) times *r*, which will represent the gear's radium.

$$\boxed{M = F \cdot r}$$



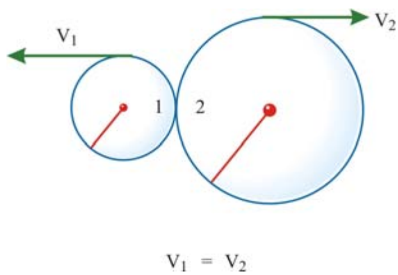

(5)

2.1.1 – Gears: In this car, the gear will be used to transmit linear and angular momentum. The gears also will help us in: (1) Changing the rotation direction, (2) Increasing or decreasing the rotation speed, (3) Changing the rotation axis and (4) synchronizing the rotation direction.

Gear's size: The relation between two gears in touch determines the rotation speed of each one. A reduction of the relative radium between to gears reduces the angular speed of the bigger gear (It happens because all the points in the extremity of the both gear have the same speed).

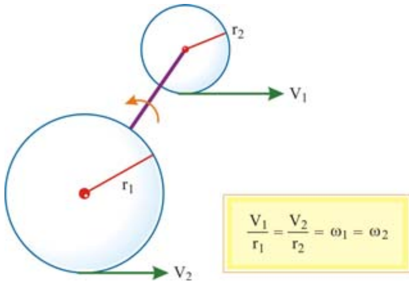
- Gears that are contact have the same scalar speed. Scalar speed is a speed defined by the equation below, where C is the length of the circle (perimeter), *r* is the radius (in our problem we just consider the radius as the maximum possible radius) and *T* is the period of gear revolution:

$$\boxed{\vec{v} = \frac{\Delta S}{\Delta T} = \frac{C}{T} = \frac{2 \cdot \pi \cdot r}{T}}$$




(6)

- Gears that are in the same axis have the same angular speed. Angular speed is a speed defined by the equation below, where 2π is the angle's variation (in our case, a complete lap), *r* is the radius (in our problem we just consider the radius as the maximum possible radius) and $T$ is the time that the gear takes to complete one revolution. Two gears in the same axis have the same angular speed and their scalar speed is proportional to theirs radius.




$$\boxed{\omega = \frac{\Delta\theta}{\Delta T} = \frac{2\cdot\pi}{T}}$$

## 2.2 Efficiency:

Classically, in general problems, we calculate efficiency as the ratio between the effective energy used to our purpose and the total energy available. Ratio equal to 1, means the system has a 100% performance.
Firstly, we thought about efficiency like being the result of the division below:

$$\eta = \frac{\overline{V}_{car}}{\overline{V}_{wind}} \tag{9}$$

But, in the wind car problem, more specifically in our experiments, this division is ambiguous. The wind speed is measured in our reference frame? Or in the car's frame? Clearly the energy available will be different. Thus we must state if we are referring to wind relative speed or to the wind absolute speed.

Therefore, let's deduce the efficiency model.

- Speed and Kinetic energy: The kinetic energy modules are related to the speed modules. The kinetic energy also depends on the body mass and it is defined by the equation below (where m is the body mass and V is its speed):

$$K = \frac{m\cdot V^2}{2} \tag{10}$$

- The done work by the car: Physically speaking, the car does not do work. The work is done by the forces which propels the car. But, we will deal with the phrase that the work is done by the car to make things easier.

The work done by the car is defined by the kinetic energy theorem, which says that the work is the difference between the final kinetic energy and the initial kinetic energy:

$$W = \Delta K = \frac{m \cdot V_F^{\ 2}}{2} - \frac{m \cdot V_I^{\ 2}}{2} \tag{11}$$

We can obtain the work modules because we have the average speed of the car (assuming that the car has a constant acceleration). With the average speed (and with constant acceleration) we can find the final speed, using the equation below (where $\overline{V}$ is the average speed and $V_F$ is the final speed):

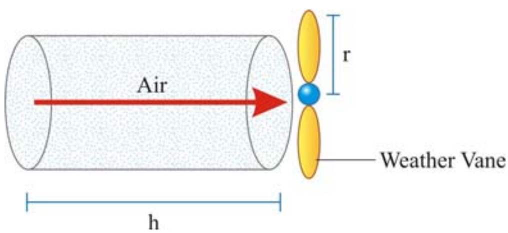
$$V_F = 2 \cdot \overline{V} \,. \tag{12}$$

Using this result and the fact that the car starts from rest, we conclude that the total car's kinetic energy is:

$$W = \frac{m \cdot \left(2 \cdot \overline{V}\right)^2}{2} = \frac{m \cdot 4 \cdot \overline{V}^{\,2}}{2} = 2 \cdot m \cdot \overline{V}^{\,2} \tag{13}$$

- The done work by the wind: Physically saying, wind does not do work, the work is done by the resistive force applied by the wind. But, we will deal with the phrase that the work is done by the wind to make things easier.

Considering a frontal air cylinder, (in front of the weather vane), like in the picture, with a fixed volume (V) and a the average wind speed ($\overline{V}$), we can say:




a) The cylinder volume is the relation between mass and density (air density):

$$\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho} \tag{14}$$

b) The cylinder volume is the same as the cylinder height multiplied by the cylinder's base (the weather vane area). So,

$$V = A \cdot h \text{ and } V = \frac{m}{\rho} \tag{15}$$

$$A \cdot h = \frac{m}{\rho} \Rightarrow m = A \cdot h \cdot \rho \tag{16}$$

$$A = \pi \cdot r^2 \tag{17}$$

$$m = h \cdot \pi \cdot r^2 \cdot \rho \tag{18}$$

$$h = \frac{m}{\pi \cdot r^2 \cdot \rho} \tag{19}$$

where $h$ is the cylinder height (length), $r$ is the weather vane radius and $\rho$ the air density.

c) The air cylinder speed is calculated by:

$$\overline{V} = \frac{h}{\Delta T} \quad \text{So,} \quad \overline{V} \cdot \Delta T = h \tag{20}$$

Comparing equation (19 and 20) we can conclude that:

$$\overline{V} \cdot \Delta T = \frac{m}{\pi \cdot r^2 \cdot \rho} \tag{21}$$

$$m = \overline{V} \cdot \Delta T \cdot \pi \cdot r^2 \cdot \rho \tag{22}$$

Using the kinetic equation (10) we substitute the mass (22) and discover that:

$$K = \frac{m \cdot \overline{V}^2}{2} \Rightarrow K = \frac{\Delta T \cdot \pi \cdot r^2 \cdot \rho \cdot \overline{V}^3}{2} \tag{23}$$

As we know the kinetic energy (work), the efficiency (performance) is defined as the division of the car work (13) by the wind work (23):

$$\eta = \frac{W_{car}}{W_{wind}} = \frac{2 \cdot (\overline{V}_{car})^2 \cdot m_{car}}{\frac{\Delta T \cdot \pi \cdot r^2 \cdot \rho \cdot (\overline{V}_{wind})^3}{2}} = \frac{4 \cdot (\overline{V}_{car})^2 \cdot m_{car}}{\Delta T \cdot \pi \cdot r^2 \cdot \rho \cdot \overline{V}_{wind}^{\ 3}} \tag{24}$$

Where:

| | |
|---|---|
| $\overline{V}_{car}$ | is the average car speed |
| $\overline{V}_{wind}$ | is the average Wind speed |
| $m_{car}$ | is the car mass |
| $r$ | is the weather vane radius |
| $\rho$ | is the average air density |
| $\Delta T$ | is the total time that the car took to cross over the fixed distance (in our case, 05, meters) |

**Efficiency →**

$$\eta = \frac{4 \cdot (\overline{V}_{car})^2 \cdot m_{car}}{\Delta T \cdot \pi \cdot r^2 \cdot \rho \cdot (\overline{V}_{wind})^3} \tag{25}$$

## 3. Experimental Setups:

1. Prototype: characteristics

Measuring:
2. Wind speeds (three different intensities wind sources)
3. Car speeds (three different intensities)
4. Car efficiency (performance

Additional Information:
5. Acceleration

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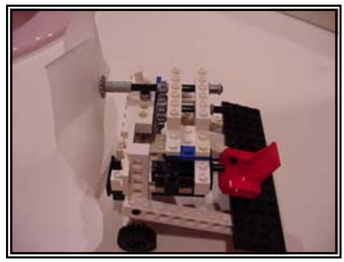
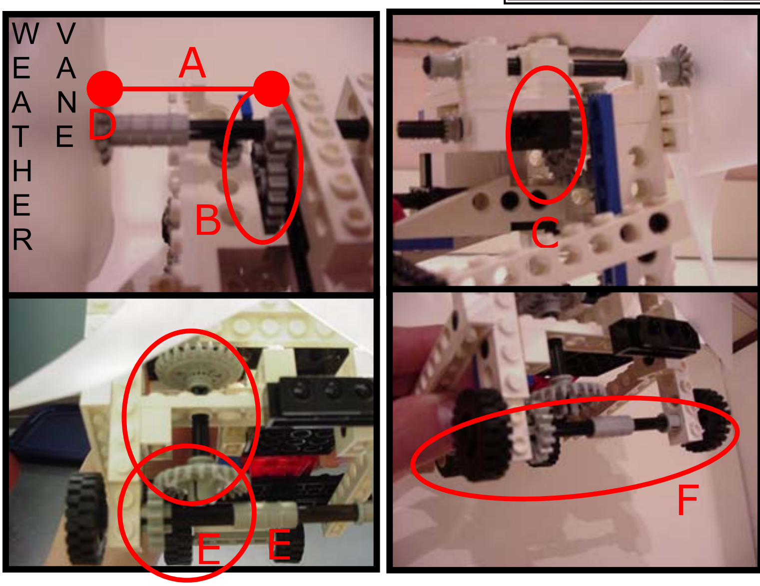
1. Prototype: characteristics



Our car has five gears, four wheels (two pairs) and one weather vane. Below, we show photos of the car and also a photo of each pair of gears (in the same axis and in contact also). It was made with pieces of LEGO™ and with a plastic weather vane.




A- First pair: gear and weather vane (same axis: equal angular speed)
B- Second pair of gear (equal scalar speed)
C- Third pair of gear (equal scalar speed)
D- Fourth pair of gear (same axis: equal angular speed)
E- Fifth pair of gear (equal scalar speed)
F- Sixth pair: gear and wheel axis (same axis: equal angular speed)

Gear's Characteristics:

Sizes: with a help of a measure instrument we measure the gears and wheels diameter and then, dividing per two, we found the average radius:

| | Diameter (cm) | Radius (cm) |
|---|---|---|
| Weather Vane | 15,49 ± 0,05 | 7,745 ± 0,025 |
| First gear (the smallest) | 0,96 ± 0,05 | 0,480 ± 0,025 |
| Second gears (the biggest) | 2,57 ± 0,05 | 1,285 ± 0,025 |
| Third gear | 2,56 ± 0,05 | 1,280 ± 0,025 |
| Fourth gear | 2,56 ± 0,05 | 1,280 ± 0,025 |
| Fifth gear | 1,77 ± 0,05 | 0,885 ± 0,025 |
| Motored wheels | 2,44 ± 0,05 | 1,220 ± 0,025 |
| Not-motored wheels | 2,10 ± 0,05 | 1,050 ± 0,025 |

Car Mass: It is important to know the car mass because we need it for knowing the kinetic energy (that involves speed and mass)

| | |
|---|---|
| Car mass | 0,150 kg |

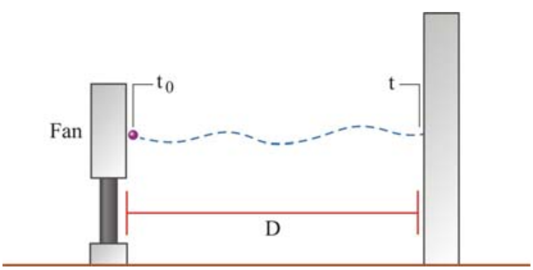
## Measuring:

2. Wind speeds (three different intensities of the same wind sources)

Wind Speed: For calculating the car efficiency, it is necessary to know the wind speed. There are a lot of methods to measure the wind speed: one method, homemade, consists in throwing small polystyrene balls in front of the fan and the, measuring the average time that the balls take to reach a fixed distance.

For measuring it, we had to do an experiment. The results for 2 meter runs are shown in the next topics, in all wind intensities. We then calculated the average time over all runs and the average wind speed for all three different wind intensities (fan's power).

First, a draw of the wind speed calculation experience:




$$\overline{V}_{wind} = \frac{\Delta S}{\Delta T} = \frac{D}{t - t_0} \qquad (26)$$

- Wind speed in the fan's third power: The time that the particle (polystyrene ball) took to cross over two meters applying the "wind third" power (the fastest/strongest one). We measured the time in 20 runs and average over them:

T(s)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0,39 | 0,40 | 0,44 | 0,42 | 0,44 | 0,47 | 0,36 | 0,35 | 0,39 | 0,32 |
| 0,39 | 0,48 | 0,44 | 0,39 | 0,45 | 0,35 | 0,47 | 0,47 | 0,28 | 0,49 |

$$\overline{M} = \frac{\sum t_n}{n}$$

Thus:

$$\boxed{\overline{M} = 0{,}4095 = 0{,}41s}$$

Wind speed: → (equation 26) $\overline{V} = \frac{2{,}0\vec{m}}{T_{medium}}$ $\overline{V} = \frac{2{,}0m}{0{,}41s}$

- Average wind speed (in the third power): $\boxed{\overline{V}_3 \cong 4{,}9m/s}$

- Wind speed in the fan's second power: The time that the particle (polystyrene ball) took to cross over two meters applying the "wind second" power (middle one). We measured the time in 20 runs and average over them:

T(s)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0,57 | 0,56 | 0,55 | 0,53 | 0,59 | 0,53 | 0,46 | 0,51 | 0,51 | 0,49 |
| 0,53 | 0,59 | 0,52 | 0,52 | 0,51 | 0,48 | 0,51 | 0,54 | 0,53 | 0,56 |

$$\overline{M} = \frac{\sum t_n}{n}$$

Thus,

$$\boxed{\overline{M} = 0{,}5295 = 0{,}53s}$$

Wind speed: (equation 26) $\overline{V} = \frac{2{,}0m}{T_{medium}}$ $\overline{V} = \frac{2{,}0m}{0{,}53s}$

- Average wind speed (in the second power): $\boxed{\overline{V}_3 \cong 3{,}8m/s}$

➢ Wind speed in the fan's first power: The time that the particle (polystyrene ball) took to cross over two meters applying the "wind first" power (the smallest one). We measured the time in 20 runs and average over them:

T(s)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0,57 | 0,66 | 0,64 | 0,61 | 0,55 | 0,62 | 0,65 | 0,69 | 0,72 | 0,54 |
| 0,74 | 0,70 | 0,66 | 0,62 | 0,59 | 0,59 | 0,64 | 0,68 | 0,71 | 0,59 |

$$\overline{M} = \frac{\sum t_n}{n}$$

In our case:

$$\boxed{\overline{M} = 0{,}6385 = 0{,}64s}$$

Wind speed: (equation 26) $\overline{V} = \dfrac{2{,}0m}{T_{medium}}$ $\overline{V} = \dfrac{2{,}0m}{0{,}64s}$

- Average wind speed (in the third power): $\boxed{\overline{V}_3 \cong 3{,}1m/s}$

3. Car speeds (three different intensities)

For measuring the car speed, important for calculating the car efficiency, we fixed a distance (in our case 0,5 meter) and then, divided for the average time that car spent to cross over this distance.

In the tables below, we show the times that the car took, in each fan's power, to cross 0,5 meters. After this, we did, for each fan's power, the calculus of the average time, to express correctly the car speed.

➢ Car speed in the fan's first power:

In the first fan power (wind intensity), we obtained these values:

T(s)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2,00 | 2,18 | 1,95 | 1,92 | 1,97 | 1,84 | 2,12 | 2,06 | 1,99 | 2,18 |
| 2,08 | 2,20 | 1,87 | 2,03 | 2,09 | 2,11 | 2,16 | 2,14 | 1,96 | 1,87 |
| 2,06 | 2,09 | 2,23 | 2,00 | 1,90 | 2,29 | 2,04 | 2,28 | 2,12 | 2,24 |
| 2,25 | 2,15 | 2,20 | 2,23 | 2,08 | 2,17 | 2,17 | 1,95 | 2,00 | 2,18 |

$$\overline{M} = \frac{t_1 + \ldots + t_n}{n}$$

In our case:

$$\overline{M} = \frac{t_1 + ... + t_{40}}{40} = \frac{2{,}00 + ... + 2{,}18}{40} = \frac{83{,}35}{40} = 2{,}0837 \approx 2{,}1\,\mathrm{s}$$

Car speed: $\rightarrow \overline{V} = \frac{0{,}5m}{T_{medium}} \rightarrow \overline{V} = \frac{0{,}5m}{2{,}1s}$

- Final car speed (in the first power): $\boxed{V_1 \cong 0{,}24m/s}$

➢ Car speed in the fan's second power:
In the second fan power we obtained these values:

T(s)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1,78 | 1,67 | 1,59 | 1,75 | 1,59 | 1,69 | 1,94 | 1,52 | 1,65 | 1,75 |
| 1,56 | 1,60 | 1,72 | 1,62 | 2,00 | 1,71 | 1,51 | 1,44 | 1,81 | 1,94 |
| 1,50 | 1,55 | 1,70 | 1,65 | 1,57 | 1,53 | 1,48 | 1,78 | 1,47 | 1,59 |
| 1,72 | 1,47 | 1,66 | 1,54 | 1,51 | 1,61 | 2,00 | 1,68 | 1,72 | 1,79 |

$$\overline{M} = \frac{t_1 + ... + t_n}{n}$$

In our case:

$$\overline{M} = \frac{t_1 + ... + t_{40}}{40} = \frac{1{,}78 + ... + 1{,}79}{40} = \frac{66{,}36}{40} = 1{,}659 = 1{,}7s$$

Car speed: $\rightarrow \overline{V} = \frac{0{,}5m}{T_{medium}} \rightarrow \overline{V} = \frac{0{,}5m}{1{,}7s}$

- Final car speed (in the second power): $\boxed{V_1 \cong 0{,}30m/s}$

➢ Car speed in the fan's third power:
In the third fan power, we obtained these values:

T(s)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1,47 | 1,44 | 1,46 | 1,65 | 1,53 | 1,50 | 1,63 | 1,53 | 1,36 | 1,38 |
| 1,33 | 1,46 | 1,50 | 1,46 | 1,43 | 1,44 | 1,44 | 1,53 | 1,53 | 1,49 |
| 1,50 | 1,39 | 1,53 | 1,42 | 1,43 | 1,56 | 1,47 | 1,50 | 1,43 | 1,34 |
| 1,51 | 1,40 | 1,50 | 1,43 | 1,39 | 1,38 | 1,53 | 1,37 | 1,69 | 1,42 |

$$\overline{M} = \frac{t_1 + \ldots + t_n}{n}$$

In our case:

$$\overline{M} = \frac{t_1 + \ldots + t_{40}}{40} = \frac{1{,}47 + \ldots + 1{,}42}{40} = \frac{58{,}75}{40} = 1{,}4687 = 1{,}5s$$

<u>Car speed</u>: $\rightarrow \overline{V} = \dfrac{0{,}5m}{T_{medium}} \rightarrow \overline{V} = \dfrac{0{,}5m}{1{,}5s}$

- Final car speed (in the third power): $\boxed{V_3 \cong 0{,}33 m/s}$

<u>4. Car efficiency:</u>

To determine the car efficiency we use equation (25). We will just substitute, in the three cases (different wind intensity) the average car speed, as also the average wind speed, the air density, the weather vane radius and the time variation (ΔT) and the car mass.

Efficiency Equation:

$$\eta = \frac{4 \cdot (\overline{V}_{car})^2 \cdot m_{car}}{\Delta T \cdot \pi \cdot r^2 \cdot \rho \cdot (\overline{V}_{wind})^3}$$

<u>Information that we need to use</u>:

| DATA | VALUE (International Units System) |
|---|---|
| Wind speed in fan's first power | 3,1 m/s |
| Wind speed in fan's second power | 3,8 m/s |
| Wind speed in fan's third power | 4,9 m/s |
| Car speed in fan's first power | 0,24 m/s |
| Car speed in fan's second power | 0,30 m/s |
| Car speed in fan's third power | 0,33 m/s |
| Average time that the car took to cross over 0,5 meter in the fan's first power | 2,1 s |
| Average time that the car took to cross over 0,5 meter in the fan's second power | 1,7 s |
| Average time that the car took to cross over 0,5 meter in the fan's third power | 1,5 s |
| Car mass | 0,150 kg |
| Weather Vane radius | 0.07745 m |
| Air density | 1,21 kg/m$^3$ |

Substituting the wind average speed the car average speed the air density, the weather vane radius, the car mass and the time that the car took to cross over 0,5 meter into equation (25), we calculate an efficiency of

- First Power: Second Power: Third Power:

$\boxed{\eta = 2{,}4\%}$ $\boxed{\eta = 2{,}5\%}$ $\boxed{\eta = 1{,}6\%}$

Presenting the car efficiency:

| | **Efficiency (%)** | **Error (%)** |
|---|---|---|
| First Fan's Power | 2,40 | ±0,04 |
| Second Fan's Power | 2,50 | ±0,02 |
| Third Fan's Power | 1,60 | ±0,06 |

3. Car acceleration in the three different wind intensities

We suppose that the car has a constant acceleration. In reality this is not the case, as when the car comes closer to the fan the torque on the weather vane can increase, and, consequently, the final car speed increases also. However, the wind speed does not vary considerably along five meters. Thus, supposing the torque and acceleration constant is a good approximation.

To measure the car acceleration we need to consider as if it is constant. We determined the car acceleration in the three fan's power using the equation below

$$V_F^{\ 2} = V_0^{\ 2} + 2 \cdot a \cdot \Delta S \tag{27}$$

Substituting $V_F$ of the car according to equation (12), ΔS equal to 0,5 (meter), we find that the acceleration is:

$$a = 4 \cdot \overline{V}_{car}^{\ 2} \tag{28}$$

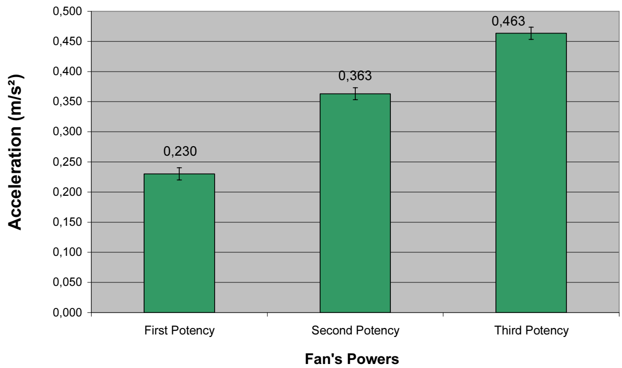
Information we will need to calculate the acceleration:

| | |
|---|---|
| Average Car speed in fan's first power | 0,24 m/s |
| Average Car speed in fan's second power | 0,30 m/s |
| Average Car speed in fan's third power | 0,33 m/s |

Substituting the average car speed and placing them in the equation (28): we have, in this fan's power, an acceleration of:

- First Power: Second Power: Third Power:

$\boxed{a = 0{,}23 m/s^2}$ $\boxed{a = 0{,}36 m/s^2}$ $\boxed{a = 0{,}44 m/s^2}$




**3.1 Materials:**

In our prototype, we used pieces of LEGO™ and a square plastic to make the weather vane, and as the wind source, we used a domestic fan. For doing the measure and calculating efficiency, we use a ruler, chronometers, a measure tape, stickers and polystyrene small balls.

**3.2 Possible errors sources:**

We did a lot of approximation, like considering the wind speed, as well the car acceleration, constant. There is also friction between the wheels and its welters that could perturb the car.

-Optimization for a next experiment: Maybe if we could work with ideal condition the experimental analyzes as well the mathematical and physical deductions, would be more precise.

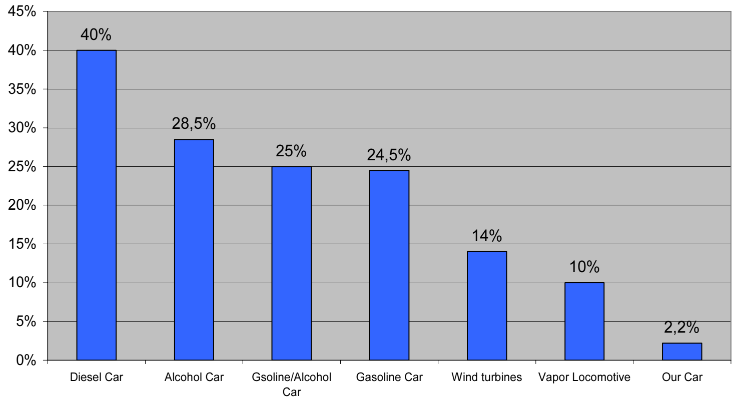
Importance:

- Future energy lapse
- Recent researches about new clean energy
- Environment preservation (pollution)

**4. Conclusion:**

In this problem, we develop a prototype and also calculated the efficiency of the prototype for different wind intensities (we discover that in the middle power the car has a better performance). The car's efficiency is high if we compare with other systems that also have wind as theirs energy sources.

Gasoline cars have an efficiency of around 25%, diesel cars have efficiency of around 40% (the highest one) and alcohol cars have efficiency of around 28%. Thermo Machines (e.g. vapor locomotive) have efficiency of 10%. Wind turbines (for energy) have an efficiency of 17%. Therefore, our car (average efficiency of 2,2%) is good if we compare it with others professionals models. Below, a graphs of others systems efficiency.

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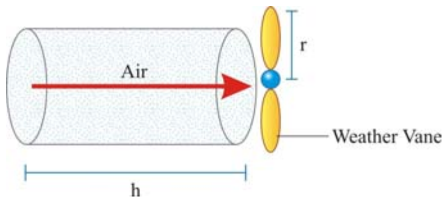
## Deduction of efficiency model

- Speed and Kinetic energy: $K = \frac{m \cdot V^2}{2}$
- 
- The done work by the car:
- 
- $W = \Delta K = \frac{m \cdot V_F^{\;2}}{2} - \frac{m \cdot V_I^{\;2}}{2}$ $\qquad V_F = 2 \cdot \overline{V}$

$$W = \frac{m \cdot \left(2 \cdot \overline{V}\right)^2}{2} = \frac{m \cdot 4 \cdot \overline{V}^2}{2} = 2 \cdot m \cdot \overline{V}^2$$




- The done work by the wind:

**a)** $\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho}$

**b)** $V = A \cdot h$ and $V = \frac{m}{\rho}$

$A \cdot h = \frac{m}{\rho} \Rightarrow m = A \cdot h \cdot \rho$ → $A = \pi \cdot r^2$ → $m = h \cdot \pi \cdot r^2 \cdot \rho$ →

$$h = \frac{m}{\pi \cdot r^2 \cdot \rho}$$

**c)** $\overline{V} = \frac{h}{\Delta T}$ So, $\overline{V} \cdot \Delta T = h$

**d)** $\overline{V} \cdot \Delta T = \frac{m}{\pi \cdot r^2 \cdot \rho}$ → $m = \overline{V} \cdot \Delta T \cdot \pi \cdot r^2 \cdot \rho$

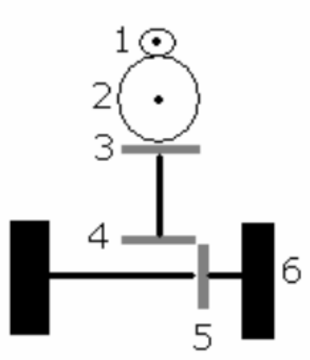
**e)** $K = \frac{m \cdot \overline{V}^2}{2} \Rightarrow K = \frac{\Delta T \cdot \pi \cdot r^2 \cdot \rho \cdot \overline{V}^3}{2}$

**f)** $\eta = \frac{W_{car}}{W_{wind}} = \frac{2 \cdot (\overline{V}_{car})^2 \cdot m_{car}}{\frac{\Delta T \cdot \pi \cdot r^2 \cdot \rho \cdot (\overline{V}_{wind})^3}{2}}$ → $\eta = \frac{4 \cdot (\overline{V}_{car})^2 \cdot m_{car}}{\Delta T \cdot \pi \cdot r^2 \cdot \rho \cdot (\overline{V}_{wind})^3}$

## Wheels, Gears and Weather Vane Revolution
## - Model -

We will make a model capable to predict how many revolutions does a gear (or also the weather vane) make with ***n*** revolutions of the wheel. After, we also can

substitute the real values to see the interdependence of the number of revolutions. Our first referential will be the "motored" wheel (number 6, in the drawing).
The number of each gear can be identified in the drawing below:




Gear NUMBER 5: As it is in the same axis than the wheel, we have:

$$n_5 = n_{wheels}$$

Gear NUMBER 4: As it is in contact with gear number 5, we have:

$$n_4 = n_5 \cdot \frac{r_5}{r_4} \rightarrow n_4 = n_{Wheels} \cdot \frac{r_5}{r_4}$$

Gear NUMBER 3: As it is in the same axis of gear number 4, we have:

$$n_3 = n_4 \rightarrow n_3 = n_{Wheels} \cdot \frac{r_5}{r_4}$$

Gear NUMBER 2: As it is contact with gear number 3 and $r_3 = r_4$, we have:

$$n_2 = n_3 \cdot \frac{r_3}{r_2} \rightarrow n_2 = n_{Wheels} \cdot \frac{r_5}{r_4} \cdot \frac{r_3}{r_2} \rightarrow n_2 = n_{Wheels} \cdot \frac{r_5}{r_2}$$

Gear NUMBER 1: As it is in contact with gear number 2, we have:

$$n_1 = n_2 \cdot \frac{r_2}{r_1} \rightarrow n_1 = n_{Wheels} \cdot \frac{r_5}{r_2} \cdot \frac{r_2}{r_1} \rightarrow n_1 = n_{Wheels} \cdot \frac{r_5}{r_1}$$

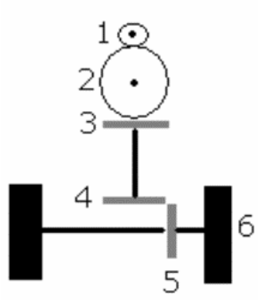
Weather Vane: As it is in the same axis of gear number 1, we have:

$$n_{Vane} = n_{Wheels} \cdot \frac{r_5}{r_1}$$

## Wheels, Gears and Weather Vane Revolution
## - Results -

We will substitute the radius of each gear in this part of the resolution to see how many revolution does each gear (or also the weather vane) make with one complete revolution of the "motored" wheel (number 6, in the drawing).
The number of each gear can be identified in the drawing below:

For each wheel complete revolution...




Gear NUMBER 5: makes:
$n = 1$ revolution

Gear NUMBER 4: makes:
$n \approx 0{,}7$ revolution

Gear NUMBER 3: makes:
makes:
$n \approx 0{,}7$ revolution

Gear NUMBER 2:
$n \approx 0{,}7$ revolution

Gear NUMBER 1: makes:
$n \approx 1{,}8$ revolutions

Weather Vane: makes:
$n \approx 1{,}8$ revolutions

## Scalar Speed

As we know the relation between angular speed of the weather vane and the wheel, and we also know the radius of both, we will now calculate the scalar speed (of the extremity of the wheel and of the extremity of the weather vane, respectively)
→ According to the relation of angular speed and scalar speed, we find that the scalar speed of the wheel and the weather vane is, in each fan's power:

$$V = \frac{2 \cdot \pi \cdot r}{T}$$

**Scalar speed**
Wheels

| | Radius (m) | Time (T) | Scalar Speed (m/s) (Error: 0,05m/s) |
|---|---|---|---|
| First Power | 0,024 | 2,1 s | 0,023 π |
| Second Power | 0,024 | 1,7 s | 0,028 π |
| Third Power | 0,024 | 1,5 s | 0,032 π |

Weather Vane

| | Radius (m) | Time (T) | Scalar Speed (m/s) (Error: 0,03m/s) |
|---|---|---|---|
| First Power | 0,077 | 2,1 s | 0,067 π |
| Second Power | 0,077 | 1,7 s | 0,091 π |
| Third Power | 0,077 | 1,5 s | 0,10 π |

## Angular Speed

As we know the relation between the number of revolutions in the Wheel and in the weather vane, we will calculate now the angular speed.
For this part, we need to know how many times does the length of the wheel fit the fixed 0,5 meter distance:

$$x = \frac{0,5}{C} = \frac{0,5}{d \cdot \pi} = \frac{0,5}{0,021 \cdot \pi} \approx 7,6$$

7,6 means the number of revolution that the wheel makes in that distance. It is, approximately, 15π Rad. Using the *models for revolution*, we find that the weather vane, make, in the same space:

13,7 revolutions (approximately, 27π Rad)

→ According to the formula of angular speed, we find that the angular speed of the wheel and the weather vane is, in each fan's power:

$$\omega = \frac{\Delta\theta}{T}$$

## Angular Speed:

<u>Wheels</u>

| | Angle's variation (Δθ) (Error: ± 0,6 π Rad) | Time (T) | Angular Speed (Rad/s) (Error: ± 0,3 π Rad) |
|---|---|---|---|
| First Power | 15π Rad | 2,1 s | 7,14 π |
| Second Power | 15π Rad | 1,7 s | 8,82 π |
| Third Power | 15π Rad | 1,5 s | 10,00 π |

<u>Weather Vane</u>

| | Angle's variation (Δθ) (Error: ± 0,4 π Rad) | Time (T) | Angular Speed (Rad/s) (Error: ± 0,5 π Rad) |
|---|---|---|---|
| First Power | 27π Rad | 2,1 s | 12,86 π |
| Second Power | 27π Rad | 1,7 s | 15,88 π |
| Third Power | 27π Rad | 1,5 s | 18,00 π |

## Statistic treatment

Average: $\bar{x} = \frac{1}{N}\sum x_i$

Error: $\sigma = \sqrt{\frac{1}{N-1}\sum(\bar{x} - x_i)^2}$

Error of the average: $\sigma_M = \frac{\sigma}{\sqrt{N}}$

## 4. PROBLEM №9 SOUND IN THE GLASS

*4.1.SOLUTION OF NEW ZEALAND*

**Problem №9 Sound in the glass**
*/.Power point Presentation/*

# The problem

●*Fill a glass with water. Put a tea-spoon of salt into the water and stir it. Explain the change of the sound produced by the clicking of the glass with the tea-spoon during the dissolving process.*

# Key definitions

●***Glass***

–Approx cylindrical container of rigid glass

•***Tea-spoon of salt***

–≈ 10 g of standard table salt. Can vary

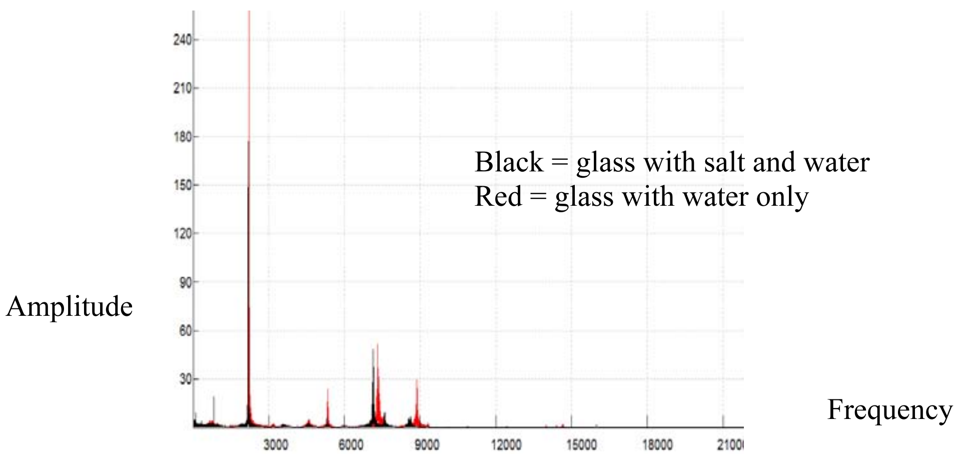
•***Change of sound***

–***Any variation observed in average frequency, amplitude and overall timbre of the clicking***

# *The Change in sound*

Please listen!!

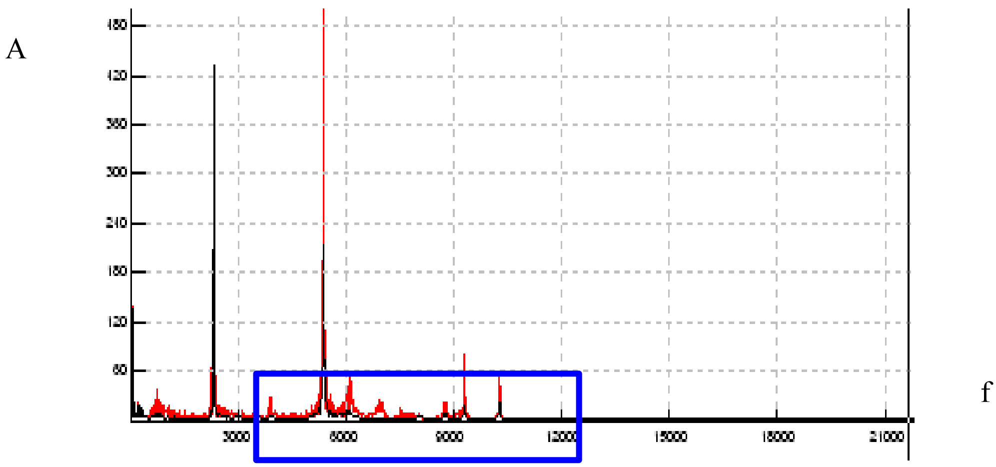
–1. Water in glass only 2. Same glass during dissolution of 1 teaspoon of salt




Not very clear, is it??

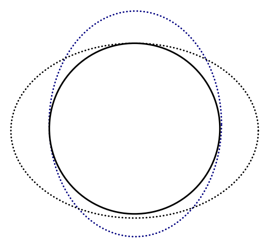
●“Sound” from glass
à Mixture of different sound components
–Cup and water vibrating
–Vibrations of spoon.**Not all components change!!**

## Compare...metal vs rubber




## Sounds produced in “clicking”

On impact...
–Glass wall flexes
–Produce vibrations
●Produce pressure variations in surrounding media
à SOUND



## Clicking Mechanism.

Clicking inside = Clicking outside !
•Clicking outside is easier to regulate

## Overview of problem

●Dissolving the salt...
–STEP A : Air bubbles released from powder
–STEP B: Salt disperses into suspension
–STEP C: Salt gradually dissolves (MAIN EFFECT)

●Changes...
–Density
–Bulk Modulus (small change)
–Attenuation

## Bulk Modulus ???Reciprocal of *compressibility (κ )*

●*Waves = rarefactions/compressions in medium!*
–*Travel of sound waves in medium will be affected*

# PRELIMINARY THEORY

## What defines a sound

●Chief sound characteristics governed by:
●Change in one parameter = change in others
–STRING VIBRATION ANALOGY
•Wavelength determined by *physical constraints of system*
●*Frequency determined by medium properties*
–Therefore, wavelength doesn't change

$$c^2 = \frac{\beta_T}{\rho_0}$$

## Speed of sound 'c' in liquids

•c = speed of sound in liquids (m/s)
●$\beta_T$ = Bulk modulus (Pa)
●$\rho_o$ = density of medium ($kg/m^3$)

## STEP A

**Air bubbles released from powder**:

**Air bubbles…**?Salt crystals not perfectly even–Air between gaps in particles
●Air escapes as bubbles
–Negligible solubility ($N_2$, $O_2$, $CO_2$ etc.)
●During bubble transition
–Density/bulk mod of medium changes

# Equation for sound speed change

$$c_{new} \approx \sqrt{\frac{1}{(1-\varepsilon)\varepsilon\rho_w\kappa_a}}$$

•$_w$ = in/of water
•κ = adiabatic compressibility ($Pa^{-1}$)
•ε = fraction of air in water

# Consequences

Assume, for example, 1% vol of air in water?
•κ for air = $7.04 \times 10^{-6}$ $Pa^{-1}$
–Speed of sound in cup of water
•Approx 120 m/s, less than in air!!!
•Unreasonable...1% is very liberal
•c has decreased à f has decreased

# Problems with practical investigation

•VERY short time span
•Effect of aeration...use aerator??
–Any aerator will introduce NOISE
–Air concentration will be very different
•Doesn't always occur
–Depends on "click" during air-escape process

# STEP B

Salt dispersal & attenuation
Salt suspension
•Before dissolution
–Salt "disperses" into suspension
–Observable during process

●Suspension
–Attenuates sound wave within water medium
–Slight timbre change, amplitude change
–More salt = more attenuation

# Attenuation due to solid suspension

$$Att \propto freq^2$$

- Higher attenuation at higher frequencies
- Combined effect of
  - Scattering
  - Absorption

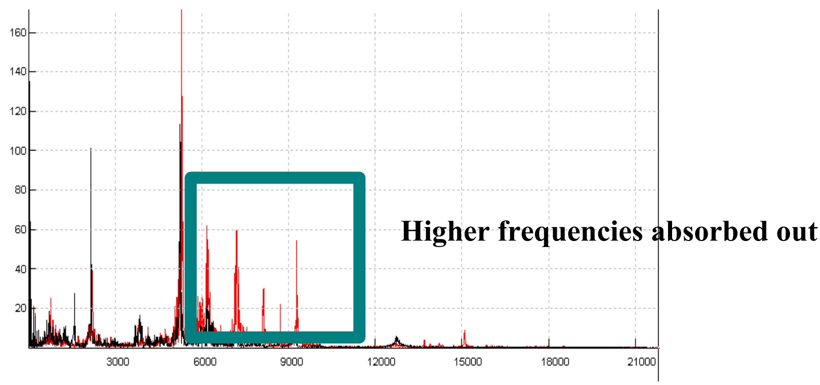
# Practical investigation

1 Use *large* quantities of salt – Clearer demonstration of effect
– Record clicking spectrum before and after suspension is achieved

# Comparison of spectrum

General disturbance of spectrum – "scattering"

- Red line – clicking sound before agitation
- Black line – clicking sound after suspension forms




# Other things to note

- As salt dissolves, attenuation dissipates
- Effect on c??
  - Minimal
  - Bulk mod/density remain approx constant
- Very small fraction of solid in suspension
- Quantitative measurements
  - Unreliable with available equipment and such a small scale

# STEP C

Density change

# Bulk modulus variation with salinity

●Increase in salinity = increase in bulk mod.
●Non-linear variation
●Increments become less with greater salinity
●Linear extrapolation can be made

# Theory: c change in water column

$$c^2 = \frac{\beta_T}{\rho_0}$$

•Density increase
Speed decreases – DOMINANT EFFECT
Bulk modulus increase
Speed increases

## Overall effect?

Recall
•If c decreases
–Wavelength remains constant
–Frequency decreases
•Negative frequency shift observed in spectrum
•More salt = greater shift

# Analogy

•Water not only acts as resonating column
–Vibrates itself, with vessel providing restorative force
–Approximated to a spring
–Natural frequency relationships determined by

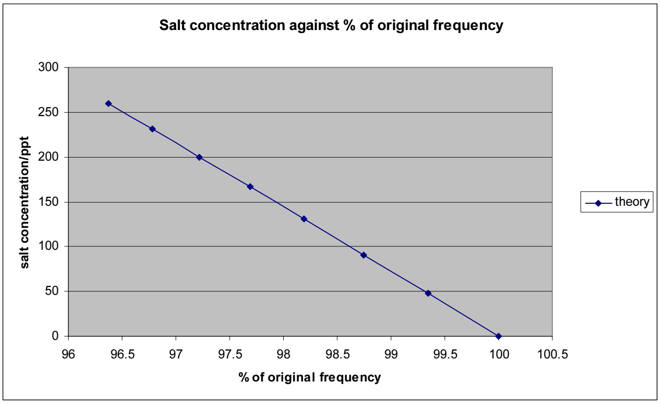
$$f \propto \sqrt{\frac{k}{m}}$$

••As salt is added, mass increases
•Therefore, negative frequency shift
•N.B. k & B, m & ρ

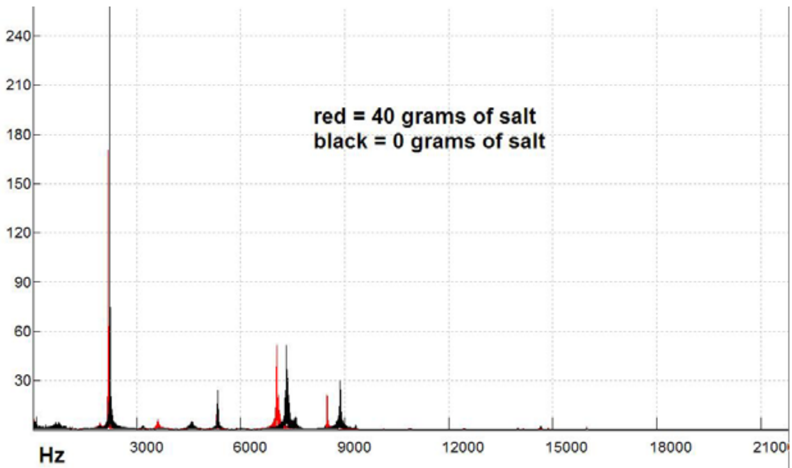
# Practical investigation 2

- Dissolve salt in water
- Record clicking spectrum
- Repeat for different amounts

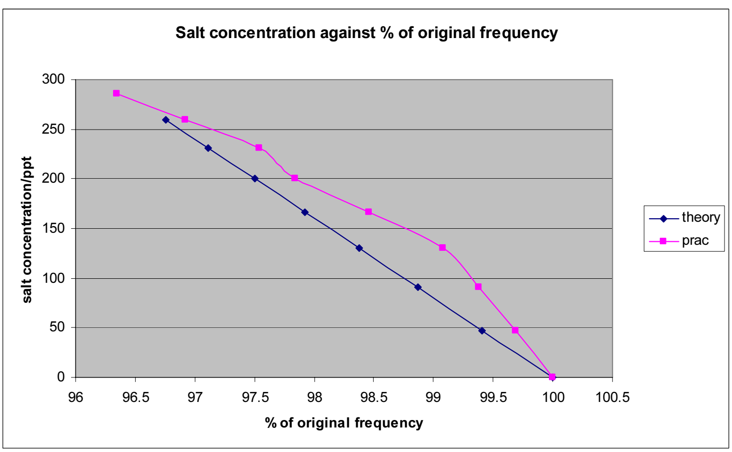
# Predicted Spectrum shift




## No salt vs. 40 grams of salt




# Spectrum shift



# Reason for variation in results

Linear extrapolation of bulk modulus used in theory

# Conclusion

Question asks: “CHANGE in sound”... “DURING dissolving process”

●Amplitude drops

–Due to attenuation, *while salt in suspension*

●Frequency drops

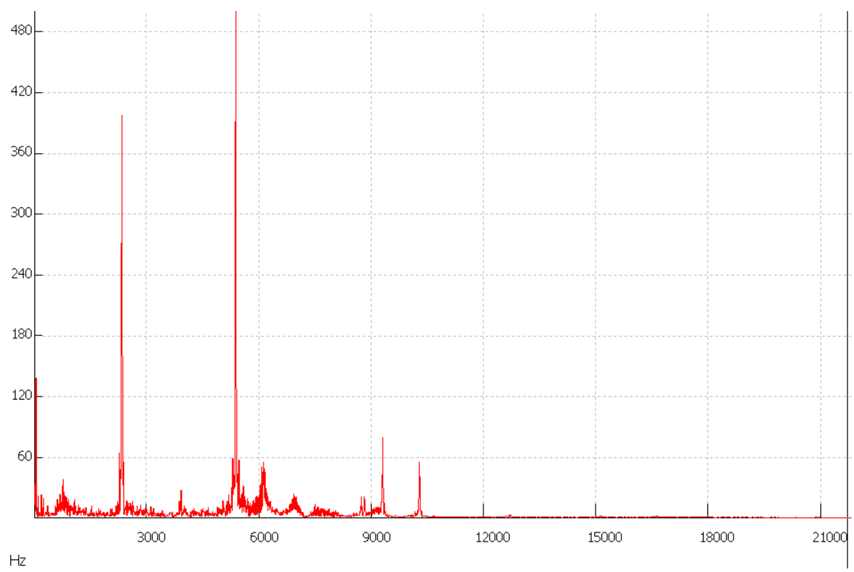
–Due to change in c, change in density (and bulk modulus)

–Lower frequencies more noticeable – due to attenuation

●(Possible large frequency drop

–Due to release of air from solid, very short time span)

# Example – Empty cup & spoon

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## 4. PROBLEM №9 SOUND IN THE GLASS

*4.2. .SOLUTION OF UKRAINE*

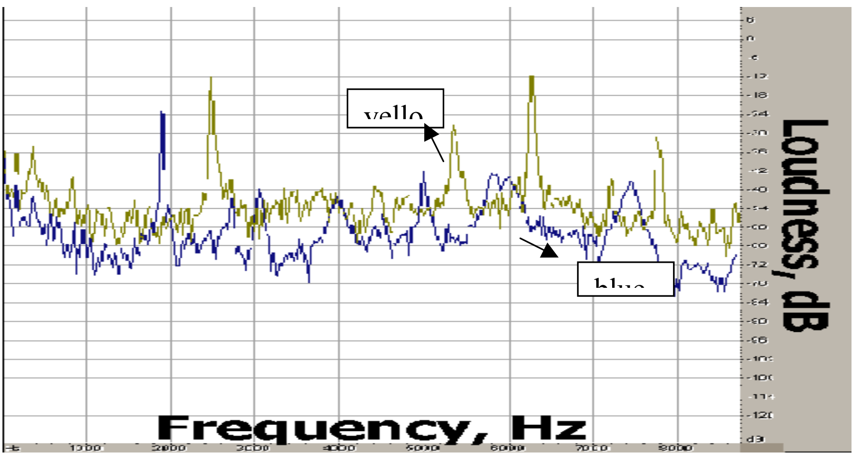
**Problem№ 9 – Sound in the glass**
*Oleg Matveichuk,, Lena Filatova, Grygoriy,Fuchedzhy, Alyeksyey Kunitskiy,Valentin Munitsa*
*Richelieu lycium, Shepkina Str 5, Odessa*

**The problem:**

*Fill the glass with water. Put a tea-spoon of salt into the water and stir it. Explain the change of the sound produced by the clicking of the glass with the tea-spoon during the dissolving process.*

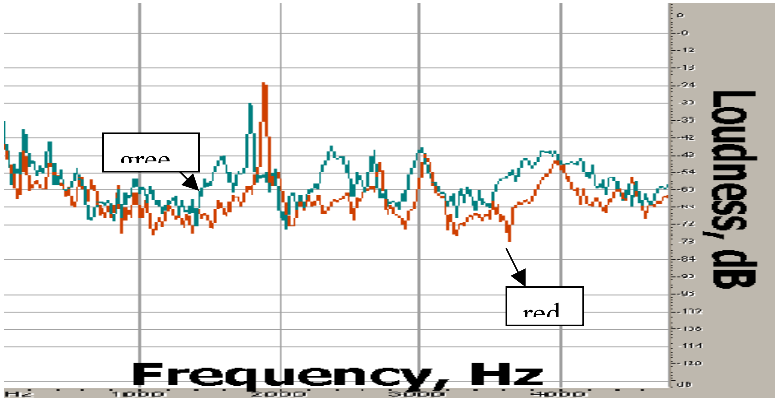
To solve the problem the aim of the work is always needed. We will look for the frequency change of clicking, which appears after stirring in the glass, experimentally and theoretically.

We wanted to observe the phenomenon so took a glass of water and recorded sounds of clicking for the empty glass, for the glass filled with water and for the glass with water and dissolved coffee in it. Using computer programms frequency analysis was done:




Here you can see **blue line** that represents frequency analysis for the glass with water and yellow line for the empty glass. It is obvious that water makes the sound duller, its frequency lower. We can use a mechanical similarity: the glass will be interpreted as a spring with a weight and it oscillates with some frequency; adding of the water corresponds to increasing of the weight.

On the next graph **red line** represents frequency analysis for the glass with water and **green line** for water with coffee.




What changes with adding coffee? When we put a tea-spoon of coffee or other soluble in water material into the glass and stir it, the dissolving starts. It causes the changing of the gas dissolubility (as you know some gas always exists in water). So the gas evolves in the form of bubbles. Because of this compressibility and density of the liquid changes consequently the speed of the sound also changes. This works for small bubbles, which don’t influence on the sound path, which we observe in the glass. Digressing into the mechanical similarity we can say that dissolving of coffee means changing rigidity for the spring and mass for the weight.

As mechanism of the phenomenon is understood, we can start mathematical investigation of the problem. In our model such assumptions were made:

1. Wave that travels in the water is longitudinal;
2. Bubbles that form in the water are small $R_b$ b $A_{oscill}$;
3. While sound travels in the glass compression of water and gas occurs adiabatically.

To find the frequency change we need new compression modulus and density of the water with bubbles.

So long as compression of water and gas occurs adiabatically:

$$pV_{gas}^{\gamma} = const \quad \Rightarrow \quad dpV_{gas} + \gamma p dV_{gas} = 0 \qquad \textbf{(1)}$$

$$\Delta p = -k\Delta V_{water} = -\frac{\gamma p}{V_{gas}}\Delta V_{gas} \qquad \textbf{(2)}$$

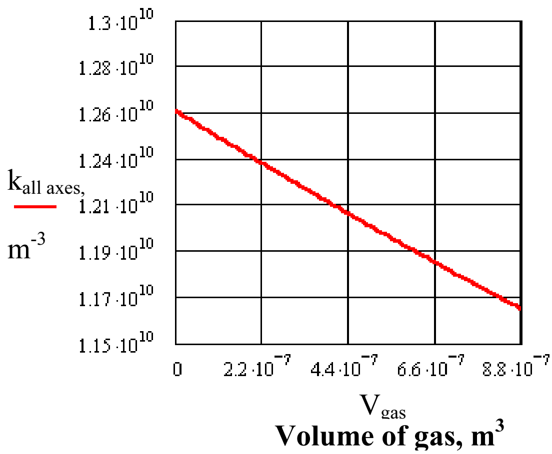
$$k = \frac{1}{V_{water}K_{water}} \qquad \textbf{(3)}$$

$$\left(\frac{\partial p}{\partial V_{all}}\right)_S \approx \left(\frac{\Delta p}{\Delta V_{gas} + \Delta V_{water}}\right) = \left(\frac{-\frac{\gamma p}{V_{gas}}}{1 + \frac{\gamma p}{kV_{gas}}}\right) = -\frac{k}{1 + \frac{kV_{gas}}{\gamma p}} \qquad \textbf{(4)}$$

$$k_{\textbf{all axes}} = \frac{k}{1 + \frac{kV_{gas}}{\gamma p}} \qquad \textbf{(5)}$$

**Compression modulus**

d is the characteristical geometrical size of the glass with water.




$$\rho_{new} = \frac{\rho_{water} V_{water}}{V_{gas} + V_{water}} \Rightarrow \frac{\rho_{old}}{\rho_{new}} = 1 + \frac{V_{gas}}{V_{water}} \quad \textbf{(6)}$$

$$C_{sound} = \sqrt{\frac{E}{\rho}} \Rightarrow \frac{C_{new}}{C_{old}} = \sqrt{\frac{\rho_{old}}{\rho_{new}\left(1 + \frac{kV_{gas}}{\gamma P}\right)}} \quad \textbf{(7)}$$

$$\nu_0 = \frac{C_{old}}{d} \quad \textbf{(8)} \qquad \nu_1 = \frac{C_{new}}{d} \quad \textbf{(9)}$$

$$\boxed{\Delta\nu = \nu_1 - \nu_0 = \frac{C_{old}}{d}\left(\frac{C_{new}}{C_{old}} - 1\right) = \nu_0\left(\sqrt{\frac{1 + \frac{V_{gas}}{V_{water}}}{1 + \frac{kV_{gas}}{\gamma p}}} - 1\right)}$$

In our experiments we saw:

| **Frequency before dissolving, Hz** | **Frequency after dissolving, Hz** | $\Delta\nu$, Hz | $\frac{\Delta\nu}{\nu}$ |
|---|---|---|---|
| 1880 | 1800 | 80 | 0.04 |
| 3036 | 2910 | 126 | 0.04 |

The only problem for theoretical solution is to find the volume of the gas that evolved. At the same time definite volume of gas corresponds to the definite frequency change. So the experiment gives us information about the frequency

$$\nu_0 = \frac{C_{old}}{d} \quad \textbf{(8)} \qquad\qquad \nu_1 = \frac{C_{new}}{d} \quad \textbf{(9)}$$

$$\alpha = \frac{1}{V_{water}} \qquad\qquad \beta = \frac{k}{\gamma p}$$

and

we can find the volume of gas theoretically. Then we'll try to estimate this value in the other way. But our lust formula is too complicated and can be simplified.

Substitution of these magnitudes gives:

$$\mathbf{K} = 4.5\cdot 10^{-10}\,\mathbf{Pa^{-1}} \qquad\qquad \mathbf{V_{water}} = 0.177\ \mathbf{m}^3$$

$$\mathbf{p} = 10^5\,\mathbf{Pa} \qquad\qquad \gamma = \frac{\mathbf{C_p}}{\mathbf{C_V}} = 1.4$$

$$\frac{C_{new}}{C_{old}} = \sqrt{\frac{1 + \dfrac{V_{gas}}{V_{water}}}{1 + \dfrac{kV_{water}}{\gamma p}}}$$

$$\boxed{\mathbf{\Delta V_{gas}} = 8.8\cdot 10^{-7}\,\mathbf{m}^3} \qquad\qquad d\nu = \nu_0\left[-\frac{1}{2}(\beta-\alpha)\right]dV_{gas}$$

$$\boxed{\Delta V_{gas} = \frac{-2}{(\beta-\alpha)}\frac{\Delta\nu}{\nu_0} = -\frac{2\Delta\nu}{\nu_0}\cdot\frac{K_{water}\gamma p}{1-K_{water}\gamma p}V_{water} \qquad \textbf{(10)}}$$

We heard that after some time sound of the clicking becomes the same as before appearance of any bubbles. It means all of them have risen to the surface. By these considerations we can estimate the average size of the bubble and consequently their volume.

We equalize buoyant and resistant forces, thinking that velocity becomes constant very quickly.

$$F_{res} + mg = F_b$$

$$\left.\begin{aligned} &F_{res} = 6\pi\eta RV, \quad V = \frac{L}{t} \\ &F_b = \frac{4}{3}\pi R^3 \rho g \end{aligned}\right\} \xrightarrow{mg \ll F_{res}\ ,\ mg \ll F_b} F_{res} = F_b$$

$\eta = 10^{-3}$ $\boxed{\text{Pa.s}}$ $\rho = 10^3 \dfrac{kg}{m^3}$

$t = 15s$ $n \simeq 10^3$

$$R = 3 \cdot \sqrt{\frac{\eta L}{2\rho g t}} \longrightarrow \Delta V_{gas} = \frac{4}{3}\pi R^3 n$$

$L \simeq 10^{-1} m$

$$\boxed{\Delta V_{gas} \simeq 6.7 \cdot 10^{-7} m^3}$$

As you see both results for the volume of the evolved gas are comparable with each other so frequencies got in the experiment and predicted by our theory are in quite good agreement.

In conclusion I would like to add that such bubbles lead to one more effect which wasn't considered here: high frequencies are dampened in the water with such bubbles. But nevertheless considerations of the effect of the frequency shift because compressibility and density of the liquid change gives good results for explanation and prediction of the phenomenon.

Special thanks to:

Oleg Matveichuk, main author of the idea.

## 5. PROBLEM № 11 WATER DROPLETS

*SOLUTION OF NEW ZEALAND*

**Problem № 11 Water Droplets**
*/Power Point Presenmtation/*

### The problem

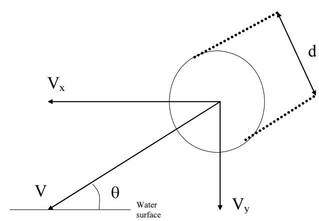
- *If a stream of water droplets is directed at a small angle to the surface of water in a container, droplets may bounce off the surface and roll across it before merging with the body of water. In some cases the droplets rest on the surface for a significant length of time. They can even sink before merging. Investigate these phenomena.*

### Definition of question

- **Droplets**; spherical balls of water
- **Bounce**; the droplets, after impact with the water surface must rebound off.
- **Roll**; the droplets don't coalesce with the water surface as they float on the surface while rotating.
- **Merging**; when the droplet coalesces with the water in the container.
- **Investigate**; Explore the nature as to why these phenomena happen and what will affect these phenomena.

### Parameters of the problem

**Diagram of a droplet**

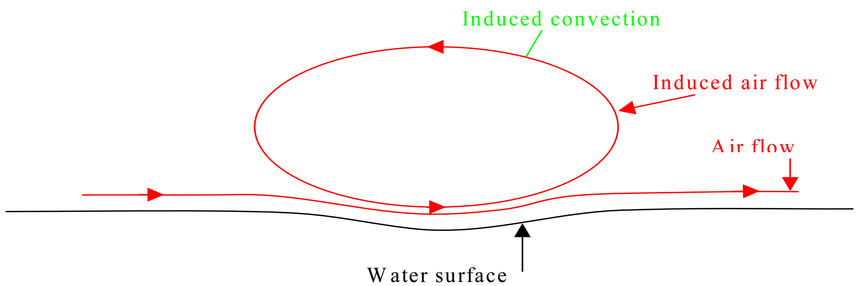



- Droplet diameter (d)
- Impact velocity (V)
- Vertical component of V ($V_y$)
- Horizontal component of V ($V_x$)
- Bounce height (b)
- Incident angle (θ)

**Theory (why the droplet doesn’t mix with the water surface)**

- The droplet doesn’t coalesce with the water surface.
- The reason for this non-coalescence is because of gas lubrication.
- This is when a very thin layer of air is trapped between the interface.

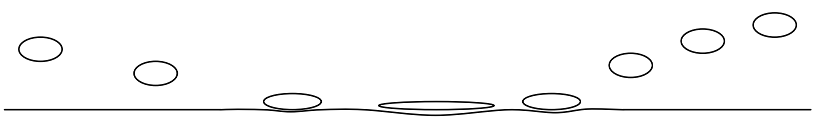
**Diagram of gas lubrication**




**Theory (bouncing)**

- The effect of gas lubrication has to be sufficient to maintain separation.
- The downwards momentum makes droplet spread out, creating a dimple in the water surface.
- They recoil to their original states, if this is forceful enough, the droplet will rebound of the water surface.

**Sequence of droplet bounce**



**Theory (rolling)**

- If $V_x$ is large enough to maintain sufficient gas lubrication, the droplet won’t coalesce.
- But if $V_y$ is too small the droplet doesn’t exert a large enough force on the water surface.
- So it will instead roll across the surface until $V_x$ becomes to small.
- Then it will coalesce.

**Theory of sinking droplets**

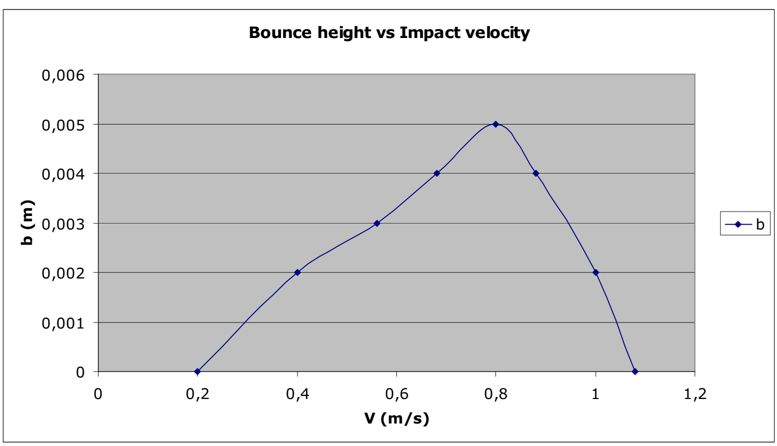
- Heat capacity of a droplet is very small.
- This causes the density to drop.
- It also increases the surface tension of the droplet.
- The droplet must break the surface.
- This can be done be weight (>4mm d) or downwards momentum.

**Theory of sinking droplets**

- For this to occur the droplet cannot be allowed to coalesce.
- This non-coalescence occurs again because of gas lubrication.
- Initial air flow under the droplet causes the droplet to rotate.
- A layer of air will fully enclose the droplet and circulate around it maintaining separation.

**Theory (V)**

- The more kinetic energy it has the more kinetic energy can be converted into potential energy.
- Therefore the higher it can bounce.
- The greater V the longer the collision time.
- The more energy loss.
- The lower the bounce height.
- There will be optimum V for any given θ.



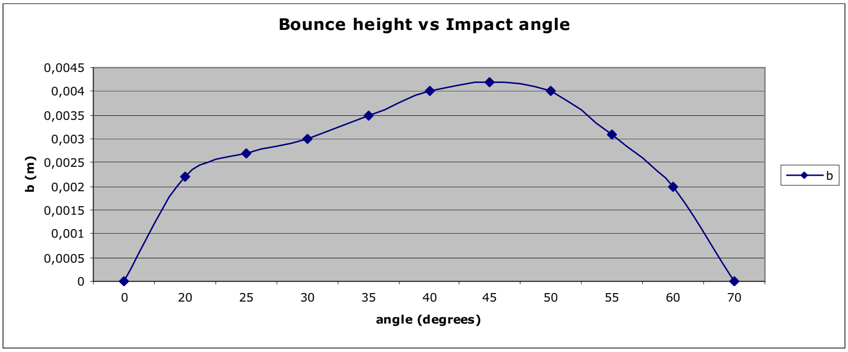
**Theory (θ)**

➢ The less θ is the greater $V_x$ is in relation to $V_y$.

➢ The greater θ is the more force the droplet can apply to the surface

➢ So there is also an optimum θ for a given V.

➢ At a lower V a greater θ is better and at a higher V a smaller θ is better.




**Theory ($V_x$)**

➢ Greater $V_x$ means better gas lubrication.

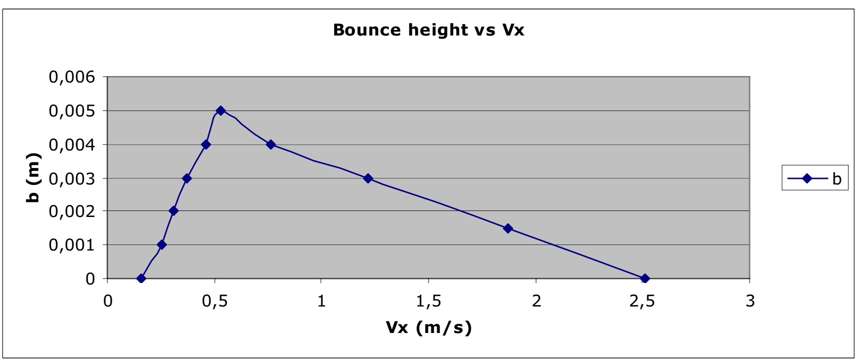
➢ Hence more downwards force it can overcome.

➢ Hence the higher it can bounce.

➢ Greater $V_x$ also means greater V hence a longer the collision time.

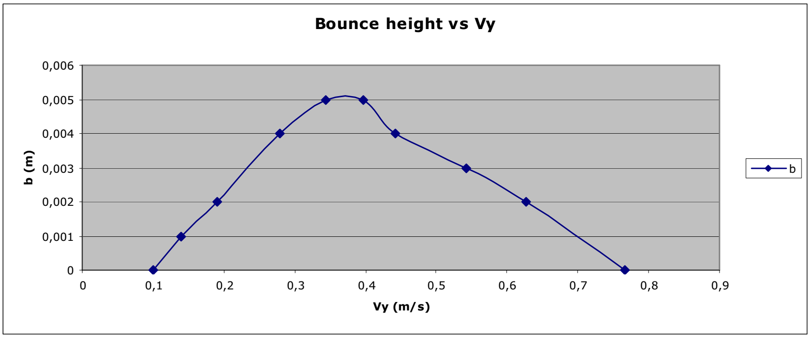
➢ Therefore the more energy

loss there is.



**Theory ($V_y$)**

- Greater $V_y$ means more downwards force exerted by the droplet.
- The effect of gas lubrication has to also increase to cope with the extra downwards force.
- Greater V also means a longer collision time.




**Theory (d)**

- Smaller d means a higher ratio of surface tension to volume.
- Therefore the more robust the droplet is.
- Therefore the higher V the droplet can withstand and not break up.
- Also the more efficient the collisions are.

**Video of small/large droplets**

**Conclusion**

- The smaller the droplet diameter the higher it bounces
- There will be an optimum impact velocity (V) for any given angle ($\theta$).
- To gain the highest bounce an increase in $V_y$ must have an increase in $V_x$ .
- For the longest roll time a large $V_x$ is needed with a small $V_y$ and if possible a forwards spin.
- For sinking the faster the spin and the larger the droplet (assuming it stays spherical) the better.

Presentation is supported by the Royal Society of New Zealand

## 6. PROBLEM № 13 – HARD STARCH

*SOLUTION OF CZECH REPUBLIC*

**Problem № 13 – Hard starch**

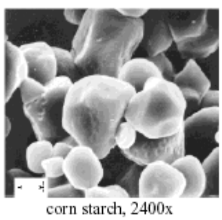
*Klára Roženková*
*Mendelovo gymnázium, Opava*

### The problem

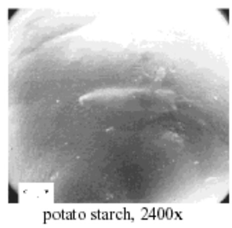
*A mixture of starch (e.g. corn flour or cornstarch) and a little water has some interesting properties. Investigate how its "viscosity" changes when stirred and account for this effect.*
*Do any other common substances demonstrate this effect?*

Starch has been used for production of papyrus and glue since 3500 B.C. In 1525 it was used for solidification of shirt's collars. There are many sources of starch in nature –bulbs and roots (potatoes, manioca), seeds (grain), fruits (chestnut, pulses). Content of starch is very different, e.g. rice 70-75 %, potatoes 12-20%.

The mixture of starch and a little water is made of small insoluble particles in liquid, which means that it is a suspension.








corn starch, 2400x rice starch, 3000x potato starch, 2400x

The interesting property of the mixture of starch and a little water is the fact, that during stirring (force application) the liquid mixture starts to behave as a solid material, but just for the duration of the force application..

After ending the force application the solid becomes liquid again and we can observe how the "solid" is melting back to liquid.

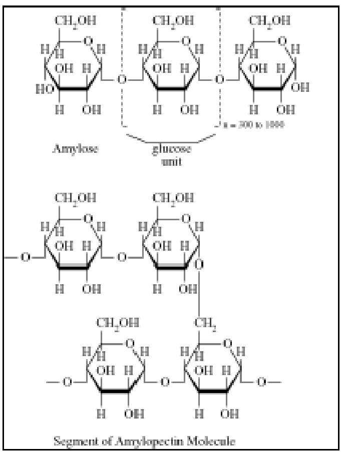
Another remarkable property to be mentioned; immerse your finger into the glass with the mixture very quickly and you will find out that it's simply impossible, because you bump into the hard upper level of the mixture. On the other hand, if you immerse your finger slowly, you can easily reach the bottom of the glass.

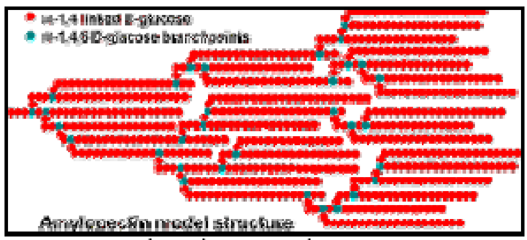
**Account for this effect?**

Water penetrates into the molecules of starch and creates hydrogen bonds with free hydroxides. Although this suspension has a high viscosity, it is possible to immerse objects with high density into it. But how is it possible that the mixture seems to be solid for a while? We can say that the starch corns in the mixture of starch and water are freely floating surrounded by water. If we apply mechanical force, the water iscrushed out, the starch corns join and create an impression of a solid material. But if themechanical force is small, the corns can freely move and the water acts as a lubricant.

It is easier to explain this effect on the microscopic structure of starch. Starch consists of two different polymeric polysaccharides – amylose (30%) and amylopectin (70%).

While amylose is linear and is made of few thousands of monomers, the structure of amylopectin isbranched and can be made of millions monomers. Molecules of water which are between the chains of amylose and amylopectin are during the force application crushed out and the chainswedge. The hydrogen links are forming and the structure of amylopectin is misshaped. It results in growth of the viscosity – this happens just for the duration of the force application.After ending the force application, the solid becomes liquid again. The bigger force we apply, the bigger viscosity and more solid properties we get




*structure of amylose and amylopectin* *amylopectin structure in space*

This property is called **rheopecticity**. The suspension owing to the movement (stirring, crumpling, shaking) becomes solid but at ease it becomes liquid again.

The better known property is **thixotrophy** – which is the opposite effect to rheopecticity. All of us know ketchup – at ease it's solid, you can't get it out of the bottle but after shaking it becomes liquid.

This means that the viscosity of starch during stirring increases so much that the mixture seams to behave as a solid material.

What is the definition of viscosity and rheopecticity? **Viscosity** is a measure of the resistance of a fluid to deformation under shear stress. It is commonly perceived as"thickness", or resistance to pouring. Viscosity describes a fluid's internal resistance to flowand may be thought of as a measure of fluid friction.

**Rheopectic fluids** are a type of non-Newtonian fluids. Rheopecticity shows a time dependant change in viscosity; the longer the fluid undergoes shear, the higher its viscosity. Rheopectic fluids are a rare type of fluids, in which shaking for time causes solidification.

How to measure viscosity using different force and intensity of the mechanical force in order to prove the increase of the viscosity?

The best how to do it is to use Stokes's figure for resistance force acting on the ball during drawing through the liquid.

$$F = 6 \pi \eta R v$$

We measured viscosity using drawing an iron ball in a mixture of starch and a little water with a constant force application, which we kept with a help of dynamometer.

Because we drew the ball in a volumetric cylinder, instead of **v** into the figure we had to institute

$$vl = vm (1 + 2,4 R/RT)$$

where vl is the velocity of drawing after Landerburg's correction and vm is the measured velocity.

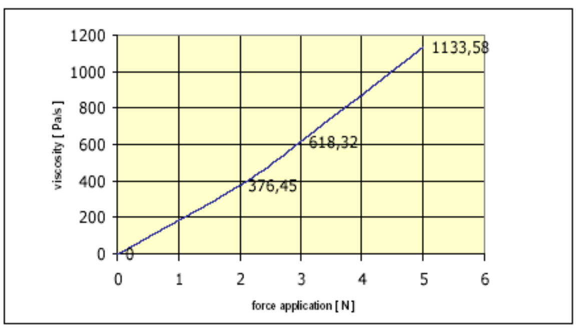
In our case::

At first we measured the velocity of starch during the force application of 2N. Average time of this measurement was t = 9, 13 s, we used this data for a calculation of the viscosity.

After calculating vl and institution into the figure for viscosity we got a viscosity 376, 45Pa/s.

During the force application of 3N and t = 10s calculated viscosity was 618, 32 Pa/s.

During the force application of 5N and t = 11s calculated viscosity was 1133, 58 Pa/s.




viscosity dependence on the force application

This experiment proved that during stirring (=force application) the viscosity of starch is increased. The bigger force we apply, the bigger viscosity we get.

Other values we used for the calculation:

RT= 0,045 m

s = 0, 13 m

r = 0,012 m

For our experiment we used a suspension of water and potato starch in weigh ratio 1: 1.

Other interesting property is that this effect occurs when the proportion of the amount of water and starch is 1: 1. If using more water for the mixture, this surplus water separates from the mixture on the bottom of the glass after some time. The arisen mixture has perfect rheopectic properties.

The same effect doesn't occur only during stirring but it also appears while shaking or crumpling. Other common substances that demonstrate this effect are all rheopectic substances, for example asphalt, gum arabic, mud or gypsum.

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# 7. PROBLEM №14 EINSTEIN–DE HAAS EXPERIMENT

## *7.1. SOLUTION OF BRAZIL*

**Problem №14 Einstein – de Haas Experiment**
*Emanuelle Roberta da Silva*
*Colégio Objetivo – São Paulo – SP*

### The problem

*When you apply a vertical magnetic field to a metallic cylinder suspended by a string it begins to rotate. Study this phenomenon.*

### 1. Objectives:

The original experiment, did in 1915, had the objective to determine the change in angular momentum which accompanies a known change in magnetic moment, or in other words, to determine the *gyromagnetic ratio.*Doing some experiments, we intend to prove our hypothesis that the cause of the cylinder movement is microscopic and interior, because there is no external torque acting on the cylinder. With experiments and theory, we will show that the movement is related with the angular momentum and magnetic moment of the electron and with the cylinder magnetization.

### 2. Theoretical Introduction:

**Torque and angular momentum:**
In order to make an object rotate, it is necessary that we apply a force. The torque is the vector product of the position vector and the force vector:

$$\vec{\tau} = \vec{r} \times \vec{F} \tag{1}$$

The angular momentum is defined as the vector product of the position vector and the linear momentum vector:

$$\vec{L} = \vec{r} \times \vec{p} \tag{2}$$

By the Newton's Second Law, we can relate the torque and the angular momentum:

$$\sum \overrightarrow{\tau_{ext}} = \frac{d\vec{L}}{dt} \tag{3}$$

When the resultant of the external torques is null, the angular momentum is constant, and we can enunciate the Conservation of the Angular Momentum Law:

If the resultant of the torques that are acting on a system is zero, the total angular momentum is conserved.

**Magnetic Moment:**
*The magnetic moment is defined as the product of the electrical current and the area of the circuit:*

$$\mu = iA \tag{4}$$

*We can arrive at the classic relation between magnetic moment and angular momentum:*

$$\mu = iA = q\left(\frac{v}{2\pi r}\right)(\pi r^2) = \frac{1}{2}qvr = \frac{1}{2}q\left(\frac{L}{m}\right)$$

$$\vec{\mu} = \frac{q}{2m}\vec{L} \tag{5}$$

The magnetic momentum of the electron, associated with its orbit is given by this equation:

$$\overrightarrow{\mu_1} = -g_l \mu_B \frac{\vec{L}}{\hbar} \tag{6}$$

where $\boldsymbol{\mu_B}$ is the Bohr magneto:

$$\mu_B = \frac{e\hbar}{2m_e} = 9,274.10^{-24}\, A.m^2 \tag{7}$$

Classically, the prediction for the *gyromagnetic factor* (*g*) is 1, but experimentally, the researches have found values near 2 , and it happens because the existence of the *spin* (the electron has an intrinsic angular momentum called spin).

So, we can calculate the magnetic moment of the electron due to the spin:

$$\overrightarrow{\mu_s} = -\frac{g_s \mu_B}{\hbar}\vec{S} \tag{8}$$

Where the *gyromagnetic fator* (*g*) is closer to 2.

**Magnetization:**
Magnetization is defined as the sum of the magnetic moments of the electrons that are in a solid.

$$\overrightarrow{M} = \sum_{i=1}^{N} \overrightarrow{\mu_i} \tag{9}$$

It is proportional to the magnetic field applied to the body.

$$\vec{M} = \chi_m \frac{\vec{B}}{\mu_0} \tag{10}$$

where $\chi_m$ is the magnetic susceptibility of the material and $\mu_0$ is the magnetic permeability. For the vacuum:

$$\mu_0 = 4\pi.10^{-7} \frac{T.m}{A}$$

**Development:**
Developing the equation that shows the relation between magnetic moment and angular momentum:

$$\overline{\mu_1} = -g \frac{e}{2m_e} \vec{L} \tag{5}$$

$$\Delta\vec{\mu} = -\frac{ge}{2m_e} \Delta\vec{L}$$

$$\sum \Delta\vec{\mu} = -\frac{ge}{2m_e} \sum \Delta\vec{L} \qquad\qquad \sum \Delta\vec{\mu} = \frac{ge}{2m_e} \Delta\vec{L}_{mac}$$

$$\Delta\vec{M} = \frac{ge}{2m_e} \Delta\vec{L}_{mac}$$

$$\chi_m \frac{\Delta\vec{B}}{\mu_0} = \frac{ge}{2m_e} \Delta\vec{L}_{mac} \tag{11}$$

Analyzing this last equation, we can conclude that a variation in the magnetic field will provoke a variation in the macroscopic angular momentum.

**Magnetic field:**
Inside a coil, the magnetic field is proportional to the electrical current:

$$B = ki \tag{12}$$

**3. Experiment:**

**Materials:**

| | | | |
|---|---|---|---|
| □2 Coils | □Iron cylinder | □Thread | □Oscilloscope |
| □Source of current | □Steel cylinder | □Banana conectors | |
| □Amplificator | □Functions generator | | □Mirror |
| □Holder□ | □Multimeter | □Laser | Rampart |

**EXPERIMENT 1:**

We plugged the source of current in the coil with the banana connectors. We moored the thread in the holder and in the cylinder, and we put the cylinder inside the coil. We switched on the source of current and start to increase the current. We could see the movement of the cylinder, when we reach 3.12 ampères. *(pictures a and b)*

**Coil:**

- Number of turns: 760

**Iron cylinder:**

- Length: (3.50 ± 0.05)cm
- Diameter: (1.30 ± 0.05)cm
- Mass: (26.60 ± 0.05)g

**EXPERIMENT 2:**

We repeated the proceeding of the experiment 1, but with a source of current that gives us it in a sinoidal function. At this time we glued a small mirror on the cylinder. We used a laser that we put between the cylinder and a rampart. When the laser incise on the mirror, we could measure the amplitude, and looking to a oscilloscope, that we also connected in our source of current, we could measured the period. We changed the frequency in our functions generator and we obtained the following data: *(pictures c and d)*

| **Time (s) *** | **2.Amplitude(cm)** |
|---|---|
| (18.0 ± 0.2) | (29 ± 1) |
| (16.1 ± 0.2) | (39 ± 1) |
| (13.8 ± 0.2) | (72 ± 3) |
| (11.1 ± 0.1) | (40 ± 2) |
| (9.9 ± 0.2) | (18 ± 1) |

* time for 5 oscillations

Distance (cylinder – rampart) = (70.0 ± 0.5)cm

R = 9.5 Ω

U = 1.3 V

**Coil**

- Number of turns: 1000

**Steel cylinder**

- Length: (3.70 ± 0.05)cm

□Diameter: (0.15 ± 0.05)cm
□Mass: (1.14 ± 0.05)g

***Calculating the gyromagnetic ratio:***

Developing the equation (11) we can arrive in an equation to calculate the *gyromagnetic ratio*:

$$\frac{\chi_m}{\mu_0}\frac{dB}{dt} = \frac{ge}{2m_e}\frac{dL_{mac}}{dt} \qquad \frac{\chi_m}{\mu_0}k\frac{di}{dt} = \frac{ge}{2m_e}\tau$$

$$\frac{\chi_m}{\mu_0}ki_{\max}\cos(\Omega t) = \frac{ge}{2m_e}\tau$$

$$\tau = \frac{\chi_m 2m_e k i_{\max}}{\mu_0 ge}\cos(\Omega t) \tag{13}$$

$$\tau = A\cos(\Omega t) \qquad \sum \tau = I\theta''(t) \tag{14}$$

$$A\cos(\Omega t) - k\theta(t) - P\theta'(t) = I\theta''(t)$$

$$\theta''(t) + \frac{P}{I}\theta'(t) + \omega^2\theta(t) = \frac{A}{I}\cos(\Omega t)$$

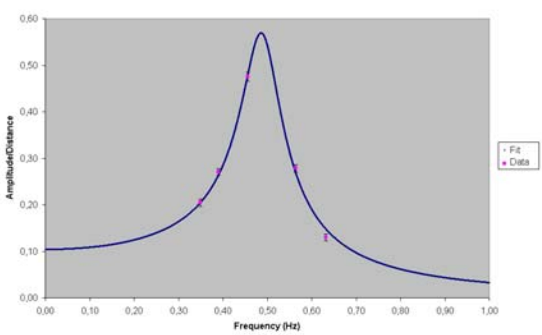
Solving this equation we have:

$$\theta(t) = \frac{A\cos(\Omega t + \Phi)}{I\sqrt{(\omega^2 - \Omega^2)^2 + P^2\Omega^2/I^2}} + \text{transient}$$

The frequency and the angle we could determine experimentally:

$$\Omega = \frac{2\pi}{T} \quad \text{and} \quad \tan\theta = \frac{ampl}{dist}$$

To determine the others parameters, we used a computer program that fit the parameters, so the distance of the curve to our points is the minimum possible. We obtained the following graphic:



**After the software determined the parameters, we could arrive in a value for the *gyromagnetic factor*:**

$$\frac{A}{I} = a$$

from the equation (13) we have:

$$Ia = \frac{\chi_m 2m_e k i_{\max}}{\mu_0 g e} \qquad g = \frac{\chi_m 2m_e k i_{\max}}{\mu_0 e I a} \tag{15}$$

Finally, we used our data:

- $a = 3.5$
- $I = 5.88\ 10^{-9}$ Kg.m²
- $e = 1.60\ 10^{-19}$ C
- $m_e = 9.11\ 10^{-31}$ Kg
- $i_{max} = 0.137$ A
- $\chi_m = 49$
- $k = 0.006$ T/A
- $\mu_O = 1.26\ 10^{-6}$ T.m/A

$\boxed{g = 17.7}$

**Error Sources:**

In our experiment, we can not affirm that the cylinder was in the middle of the coil, so the magnetic field was not constant.

Another possible error source are the perturbations on the system, because our system was not isolated of external perturbations, for example, air motion.

Finally, we have the influence of the Earth's magnetic field, that we could not minimize in our experiment.

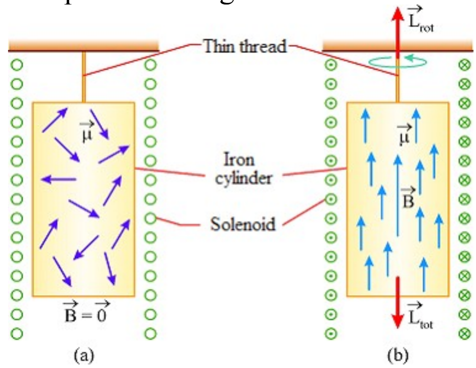
## 4. Analysis and Conclusions:

Firstly, we need to understand, why the cylinder begins to rotate when we apply a magnetic field on it.

Initially, the magnetic field is zero in the region where the cylinder is located, and the atomic magnetic moments are randomly oriented. The angular momentum have the same direction as the magnetic moment but pointed to the other side, due to this reason, there are also randomly oriented.

When we apply a vertical magnetic field on the cylinder, the atomic magnetic moments align on the direction of the magnetic field and consequently, the angular momentum align to the opposite side. So, the cylinder will have an angular momentum different of zero.

As there is no external torque acting on the cylinder, the angular momentum of it must be constant during the time. Because of the Conservation of the Angular Momentum, the cylinder begins to rotate to produce an angular momentum in order to keep the total angular momentum constant.

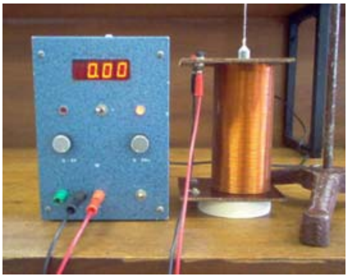



The rotation of the cylinder shows that the magnetic moment and the angular momentum are connected. Classically, the value predicted for the *gyromagnetic ratio* is 1. But doing the experiment, many researches found the numbers near to 2, and we also find a result different of 1. This results prove the theory about the existence of the *spin.*

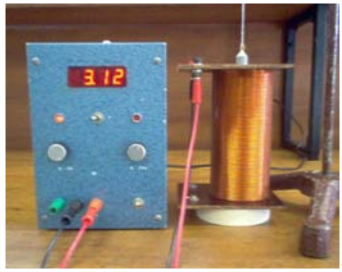
To conclude, we have a table showing the values for the *gyromagnetic ratio* calculated for different investigators, after extremely precise experiments:

| **Investigator** | **Year** | **g** |
|---|---|---|
| **BARNETT** | 1944 | 1.938 ± 0.006 |
| **MEYER** | 1951 | 1.936 ± 0.008 |
| **SCOTT** | 1951 | 1.927 ± 0.004 |
| **BARNET & KENNY** | 1952 | 1.929 ± 0.006 |
| **SCOTT** | 1955 | 1.919 ± 0.006 |
| **MEYER & BROWN** | 1957 | 1.932 ± 0.008 |
| Scott (cylinder) | 1960 | 1.917 ± 0.002 |
| Scott (ellipsoid) | 1960 | 1.919 ± 0.002 |

## 5. Pictures




*Picture a = experiment 1*




*Picture b = experiment 1*

*Picture c= experiment 2*

*Picture d= experiment 2*

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