PROBLEM № 14: EINSTEIN–DE HAAS EXPERIMENT

7.2. SOLUTION OF BULGARIA (AMERICAN COLLEGE OF SOFIA)

Problem № 14: Einstein–de Haas Experiment
(Power Point Presentation of the National Team of Bulgaria)
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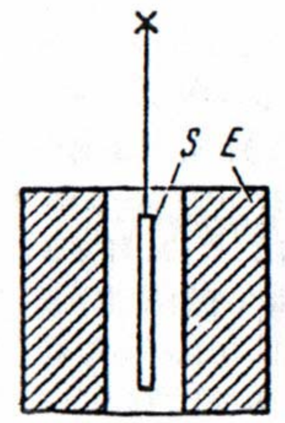
The problem*:*

When you apply a vertical magnetic field to a metallic cylinder suspended by a string it begins to rotate. Study this phenomenon.

Historical Background



- Experimenteller Nachweis der Ampereschen Molekularströme. Naturwissenschaft, 1915, **3**, 237-238
- Experimenteller Nachweis der Ampereschen Molekularströme. (Mit W.J. de Haas) Verhadl. Dtsch. Phys. Ges., 1915, **17** , 152-170

(oral report 19.02.1915 article submitted 10.04.1915)

- Berichtigung zu meiner gemainsam mit Herrn W.J. de Haas Veröffenlichen Arbeit "Experimenteller Nachweis der Ampereschen Molekularströme". Verhadl. Dtsch. Phys. Ges., 1915, **17** , 203
- Notiz zu unserer Arbeit "Experimenteller Nachweis der Ampereschen Molekularströme" (Mit W.J. de Haas) Verhadl. Dtsch. Phys. Ges., 1915, **17** , 420
- Ein einfaches Experiment zum Nachweis der Ampereschen Molekularströme. Verhadl. Dtsch. Phys. Ges., 1916, **18**, 173-177. *(oral report 25.02.1915 article submitted)*

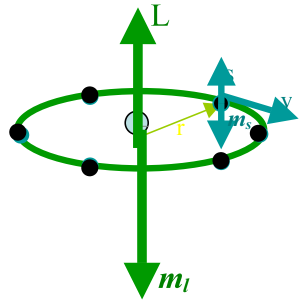
The quoted works of Einstein and de Haas, all published or reported in Germany reveal the idea and the results of an experiment to prove the existence of the so called molecular currents of Ampere, which interpret the magnetic properties of materials.

Nature of magnetism of the elements

- Atoms of the elements contain electrons
- Electrons move in orbits and "spin" around their axises
- They are the currents in the atoms (molecular Ampere's currents) => reason for magnetic properties of atoms

- Electrons possess :
    - Orbital magnetic moment and spin magnetic moment
    - Orbital angular momentum and electron spin mechanical momentum

At the time Einstein performed his experiment only the orbital motion and the relevant magnetic moment were known. The modern concept of the atom involves orbital, spin magnetic moments and their interactions.



Moments and Momenta

Orbital moments

- orbital angular momentum $\mathbf{L} = l.\hbar$
- orbital magnetic moment $\mu_l = -(e/2m).L = \gamma_l.\, \mathbf{L}$

$$\mu_l = \gamma_l.\, \mathrm{L}, \text{ where } \gamma_l = g_l\, e/2m$$

Spin moments

Spin mechanical moment

$s = ½.\hbar$

Spin magnetic moment

$\mu_s = -(e/2m).\hbar \quad \gamma_s.s$

$\mu_s = .s, \text{ where } \quad \gamma_s = g_s\, .e/2m$

Gyro magnetic ratio

$\gamma_l = \mu_l/L = -(e/2m) = 0{,}88.1011 C/kg$

$\gamma_s = \mu_s/s = -(e/m) = 1{,}76.1011 C/kg$

Mixed magnetism

$$\mathbf{j} = (\mathbf{l} + \mathbf{m_s}).\hbar$$

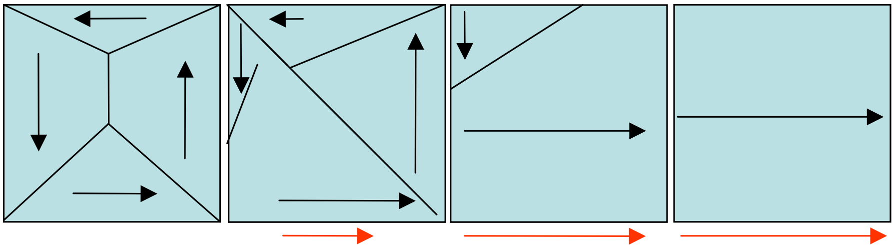
$$\mu_j = \gamma_j\, .\mathbf{j}, \text{ where } \gamma_j = g_j\, .e/2m$$

$$g_j = 1 + \frac{j(j+1) + m_s\,(m_s + 1) - l(l+1)}{2j(j+1)}$$

The theory of atomic magnetism defines the mechanical momenta of the electron (orbital and spin) and its magnetic orbital and spin momenta. The ratio between the magnetic to the mechanical moment is known as the gyromagnetic ratio, g. In the case of spin-orbital interaction this ratio is known as the Landé factor g_j

Domains

- Magnetic domains are regions in a crystal with different directions of the magnetizations
- Ferromagnets



Magneto-mechanical phenomena

- The sum of the magnetic moment vectors of the domains can be said to be the vector of the magnetic moment of the substance.
- Let $\boldsymbol{I}$ be the vector of magnetization, V is the volume of the body

$$\boldsymbol{I}.V = S\mu_d$$

- Let $\boldsymbol{Q}$ be the total mechanical momentum of the domains

$$Q = \Sigma L_d = \chi . I . V = \chi . \Sigma \mu_d$$

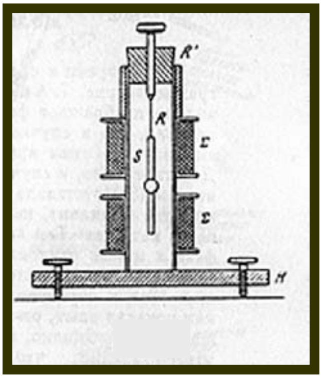
Magneto-mechanical phenomena

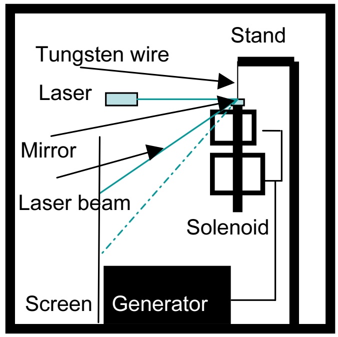
- When the body is not magnetized $\boldsymbol{I=0 \Rightarrow Q=0}$
- When the body is magnetized $\boldsymbol{I}$ is no more $\boldsymbol{0}$
- Then according to the formula $Q=\chi.I.V$, Q also changes
- According to the **law of conservation of mechanical momentum**
- $Q_{tot} = Q_D + Q_B$
- In the beginning the sum of the mechanical momenta of the domains is 0 and the body is not moving => **the total momentum is $\boldsymbol{0}$**
- $Q_D \neq 0 \Rightarrow {}_B B \neq 0$
- **So we must observe the spinning of the body**

In fact in Einstein-de Haas experiment the orientation and reorientation of domains, which are effects on a larger than molecular scale have been detected. The change of the magnetic moment of the domains causes a macro magneto-mechanical effect, which results in the rotation of the body.

Experiment Comparison

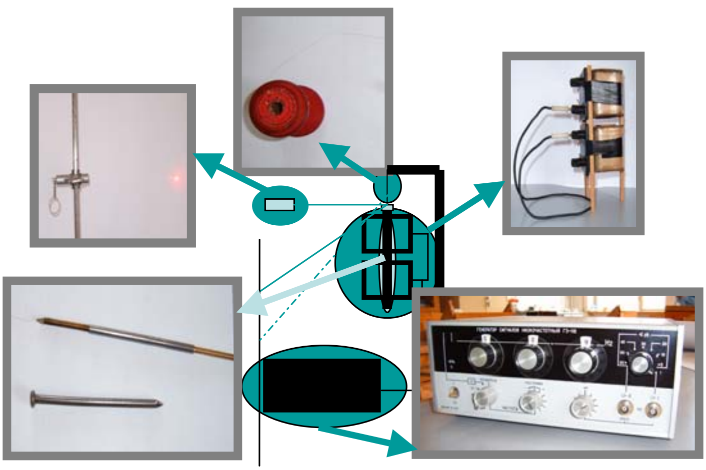
Einstein's experiment | **Our experiment**





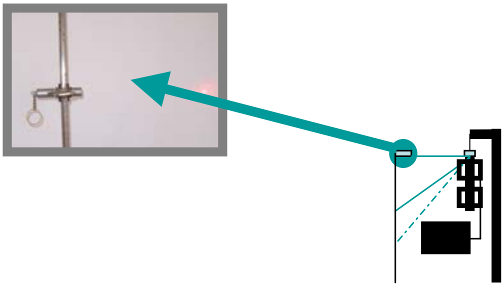
Picture from: Albert Einstein-selected scientific works, "Nauka", Moscow 1966

Experiment Setting



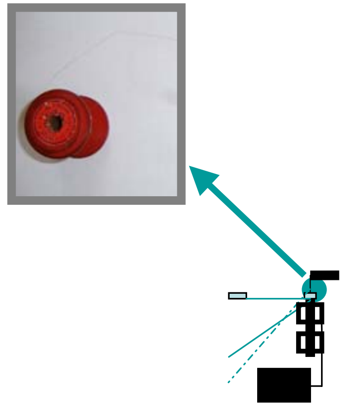
Laser

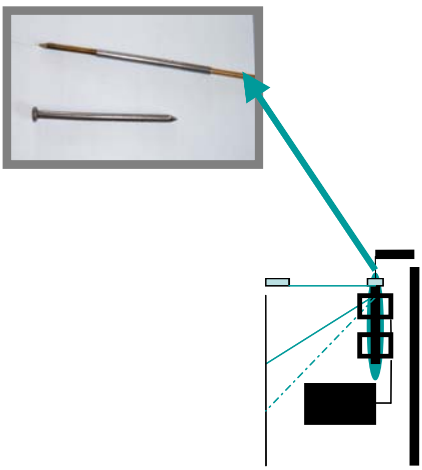
Semiconductor laser
1.5 euro from the market



- Wire
- Wire made of Tungsten with
- thickness of 15 mm
- Connected to a reel

Frequency – measured
- Torsion balance





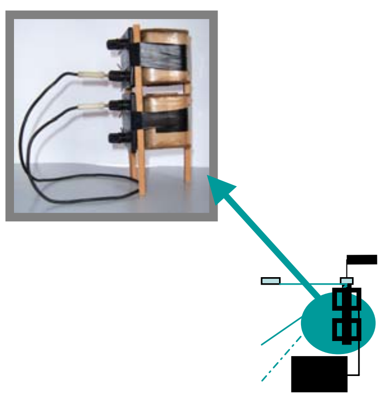
Sample

- The cylinder
- Nail (Fe)
- Two parts of brass (Cu and Zn) up and down
- Weight 7,052 g
- Length 7,44 cm
- Diameter 4mm
- Mirror – aluminum foil

The brass parts above and below the iron nail possess non-ferromagnetic properties. In our experiment they serve to damp the parasitic libration motion, which occurs at magnetization of the iron part.

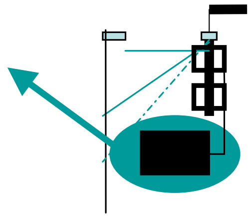
Solenoids

- Two solenoids connected in series
- 12000 turns each
- Magnetic field of about 20 G is created



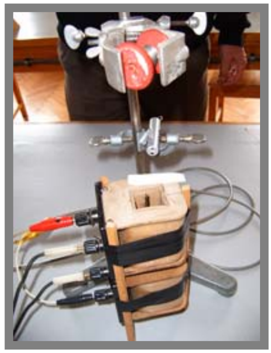
- Alternative current of **6 mA**

Generator

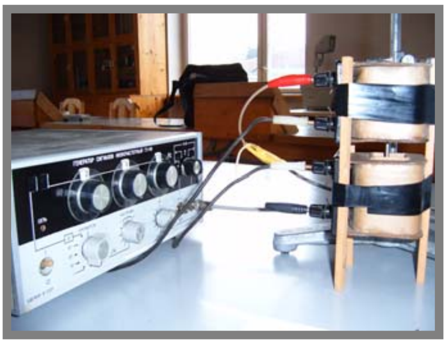


Generator of sinusoidal vibrations with changing frequency
Creates resonance to strengthen the effect
Beating effect, when slightly different frequency is set

Experiment



$$\gamma = \frac{\omega . J}{I . V}$$



Video

To run the videos that show the experiment performance, the observed oscillations and the beating effect, see www.acs.bg . Look in the link “student life”.

$$\gamma = \frac{I . V . T_e}{\varphi . J_s . \Lambda}$$

The major factor calculated is the gyromagnetic ratio of the suspended body in our experiment.

Parameters

I – vector of magnetization, its table value is **I=1,59155.106 A/m**

V – volume of the sample (only ferromagnetic part)

$$V = \pi . r^2 . h = 0,9349.10^{-6} m^3$$

$$\boxed{\gamma = \frac{I.V.T_e}{\varphi . J_s . \Lambda}}$$

- r – radius of the cylinder r=0.002 m
- h – length of the cylinder h=0.0744 m

φ – angle of declination, experimentally found:

$$\varphi \approx 180° = 3,14 rad$$

- **Te** – experimental time for measuring the angle of declination

$$T_e = \frac{1}{f} = 1.43 s$$

$$\boxed{\gamma = \frac{I.V.T_e}{\varphi . J_s . \Lambda}}$$

- f – frequency of the
- torsion balance f=0.07Hz

Js – moment of inertia of the sample

- for cylinder:

$$J_s = \frac{1}{2} \cdot m_s \cdot r_s^2$$

for our sample:

$$J_s = \frac{1}{2} . m_b . r_b^2 + \frac{1}{2} . m_f . r_f^2 = 1,7883.10^{-8} kg.m^2$$

$$\boxed{\gamma = \frac{I.V.T_e}{\varphi . J_s . \Lambda}}$$

m_b and r_b – mass and radius of the brass parts m_b=0,003359 kg and rb=0,0015 m
m_f and r_f – mass and radius of the ferromagnetic part m_f=0,007052 kg and r_f=0,002 m

Λ – decrement of decrease

$$\Lambda = \ln \frac{\varphi_n}{\varphi_{n+1}} = \ln \frac{180}{179.5} = 0.0028$$

$$\boxed{\gamma = \frac{I.V.T_e}{\varphi.J_s.\Lambda}}$$

φ_n and $\varphi_{n+1}$are two consecutive angles of declination. The difference is about 0.5 degrees

Final Calculation

$$\gamma = \frac{I.V.T_e}{\varphi.J_s.\Lambda} = \frac{1{,}59155.10^{6}.0{,}9349.10^{-6}.1{,}43}{3{,}14.1{,}7883.10^{-8}.0{,}0028} = 0{,}1353298.10^{11}$$

This will be true if :

$$g \approx 0{,}15$$

Modern experiments performed with precision much higher than that of Einstein and de Haas give a value of about 2 for the g factor. Our result differs substantially from it.

Sources of error

- The torsion balance is not centered
- Brass part – Barnett's effect
- Earth's magnetic field
- Noises and vibrations
- Domains that do not get remagnetized
- Eddie currents

Conclusions

- Einstein – de Haas experiment was successfully demonstrated
- The observed magneto-mechanical effect was predicted and explained using quantum mechanics concepts

References

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- Experimenteller Nachweis der Amperschen Molekularstrome,
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7.3. SOLUTION OF BULGARIA (SHUMEN)

Problem № 14: Einstein–de Haas Experiment

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The problem

*When you apply a vertical magnetic field to a metallic cylinder suspended by a string, it begins to rotate. Study this phenomenon***.**

Theoretical part

The experiment, done by Einstein and de Haas is connected with determination of the ratio between the magnetic moment of the electron and its angular momentum during its motion around the nucleus.

According to the known by that time the theory, the magnetization of a given magnetic material is a result of the orientation of the electrons` orbit and the summarizing of all atomic magnetic moments in the external magnetic field.

Fig.1 presents the classical understanding for the motion of the electrons around the nucleus in circular orbit.

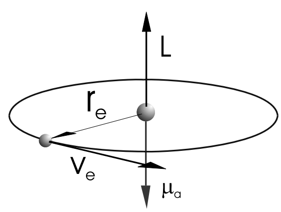


Fig. 1

For the ratio

$$\gamma_c = \frac{\mu_a}{L},$$

where μ_e is the magnetic moment of electrons and L is the angular momentum we can obtained:

$$\mu_a = \frac{e.\omega.S}{2\pi} \quad and\; L = m_e v_e r_e$$

$$\gamma_c = \frac{1}{2}\frac{e}{m_e}.$$

Einstein and de Haas realize experiment in which ferromagnetic cylinder is suspended to an elastic thread into homogeneous magnetic field in parallel to its magnetic field lines.

When the magnetic field is applied, the atoms would be orientated in the field and according to the low of the conservation of the angular momentum, the cylinder to rotate, twisting the thread.

Magnetization of the cylinder

$$J_m = \frac{P_m}{\Delta V} = \frac{N\mu_a}{\Delta V} = \chi H$$

where χ_m is the magnetic susceptibility of the cylinder, ΔV - the value of the cylinder, and H – the intensity of the magnetic field in which it is put.

The angular momentum is determined by the torsion moment of the thread D, the inertial moment of the cylinder J and the angle H, to which the thread is twisted after the applying of the field

$$L = \varphi\sqrt{J.D}\,.$$

Because of the small angle of twisting of the thread in the experiment of Einstein and de Haas, the mechanical resonance has been used and the obtained result has been two times more than expected one. Consequently the magnetization is due not only to the orientation of the electrons orbits but also and to the own magnetic moments of the electrons.

$$\gamma_q = \frac{e}{m_e}$$

The ratio: $$g = \frac{\gamma_q}{\gamma_c} = 2$$

is called gyro-magnetic ratio and in ideal case is equal to 2.

Experiment

In order to obtain the value 2 for the gyro-magnetic ratio it is needed all atoms of the ferromagnetic cylinder to be orientated into the magnetic field. Even to the intensities of the saturation of the materials, by which the cylinder is made only a part of the atoms would be orientated (interaction between domains, heat fluctuations, etc.)

In our experiment we have used the steel with low content of carbon, which have relative magnetic permeability

$$\mu = 0.988.10^4 \text{ T. m/A.}$$

In order we to be in the area of the saturation of the ferromagnetic material we have used solenoid, creating along its axis the intensity of the magnetic field:

$$H = 3880\frac{A}{m}$$

In order to be realized the condition for the homogeneity of the field, in which the cylinder is hung, the length of the solenoid is *l = 510 mm* and its diameter is *D = 65 mm* (Fig.2).

Fig.2

The experiment is divided in two parts:

The first part – qualitative part of the experiment. Establishment of the rotation of the cylinder in the homogeneous magnetic field and observing the effect of Einstein-de Haas.

The second part – quantitative measurement of the gyro-magnetic ratio.

Carrying out of the first part of the experiment

For the implementation of this part of the experiment we have used a cylinder, made from the same ferromagnetic material, with a diameter D =*14mm* and a height *h* = *9mm,* fixed on the bottom of hollow semi-sphere, leaved freely to float in a little vessel filled with water- a float (Fig. 3 and Fig. 3a).

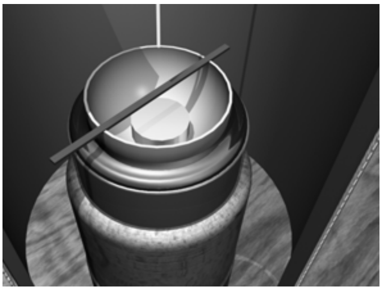


Fig. 3

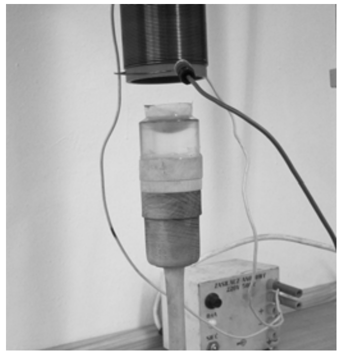


Fig. 3a

Wetting surface of the semi-sphere and not wetting material of the vessel gives a possibility of the obtaining of stable equilibrium of the semi-sphere in the middle of the vessel with the water. Because of the extremely little resistance of the liquid, the appearance of the moment of the force even with very little value would rotate the float.

When the magnetic field is applied, when the vessel with the float is inside of the solenoid, the orientation of the magnetic moments starts. In consequence of the low of the conservation of the angular momentum the ferromagnetic cylinder together with the semisphere rotate.

After the ending of the process of the orientation of the atomic moments the force of internal friction stop rotation of the cylinder. The existence of the section of the free stopping of the rotation is a proof of that the effect of Einstain-de Haas is observed. If such section missed it would mean that the rotation is provoked by magnetic forces, creating by the non-homogeneity of the field.

In order to specify the existence of such section two dependences are compared:

$\omega_L = f(t)$ - for the free stopping float and

$\omega_E = g(t)$- for the float in magnetic field.

With a digital camera the float have been photographed when it stops freely and in the magnetic field of the solenoid (Fig. 4).

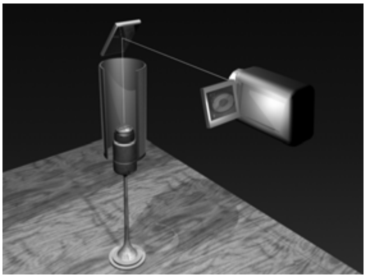
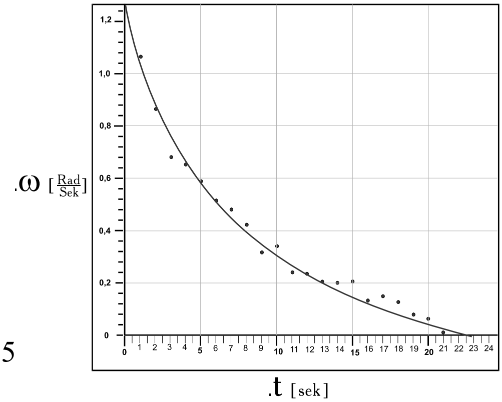


Fig.4

Fig.5



By the analysis of the slides the angle velocities have been determined at different time moments. The data are graphically presented on Fig. 5 for the freely stopped float.

On Fig.6 are presented the data for the float in the magnetic field.

On the right part above is shown a slide of the photographed float inside the solenoid. Along the diameter of the float is put amarker for the determination of the angle of rotation.

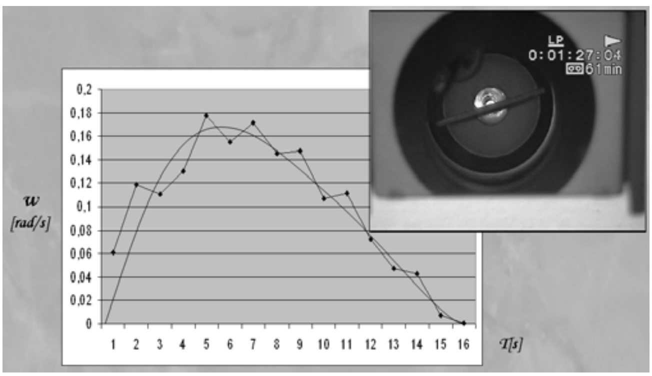
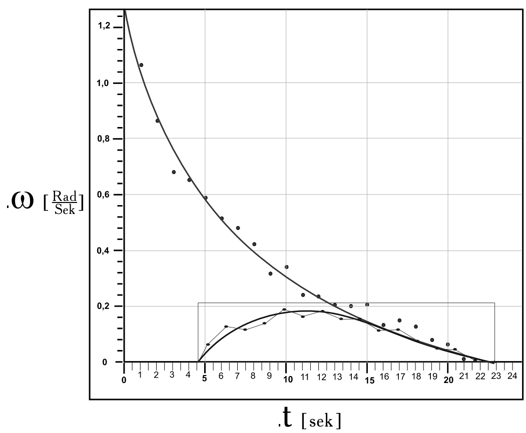


Fig6



On Fig7.the two graphs are united in order to be seen the coincidence of the two curves in the passive section (15s-22s) of the stopping of the float. It proves the effect of Einstein- de Haas.

Fig. 7

Realization of the second part of the experiment

For the quantitative determination of the gyro- magnetic ratio we have carried out the experiment of Einstein- de Haas. By this reason we have used the sample of ferromagnetic material with fixed to it a mirror (Fig. 8).

1- a sample of ferromagnetic material.

2- holder of the mirror by diamagnetic.

3- mirror.

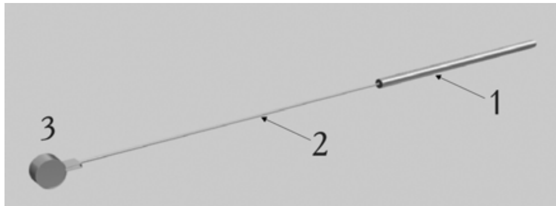


Fig.8

The sample is hung into the solenoid (3), in such way that the mirror (2) is outside it – Fig.9. The beam of the laser (1), reflected by the mirror (2) is directed to the screen (6).

The angle of rotation can be determined by direct measurement, but not by the resonance. It decreases the mistake of the final result.

The sample is hung to the thin bronze thread. In order to decrease the horizontal vibrations is used a thread (4), in the lower part of the sample, which is constantly strained (5).

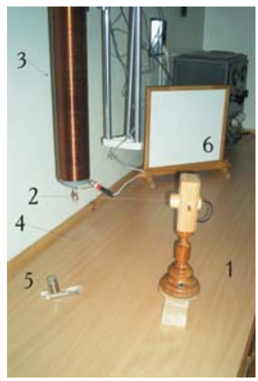


Fig. 9

Data from the experiment

1. Parameters of the setting and the sample

Sample- iron $d = 5\ mm,\quad L = 113\ mm$

Holder of the mirror –copper $d = 1.2\ mm,\ L = 220\ mm$

Mirror- glass $d = 10\ mm,\ L = 6\ mm$

Current through the solenoid – *5A*

Intensity along the axis of the solenoid – $H = 3880\ \frac{A}{m}$

Magnetic permeability of the sample – $\mu = 0.998\ 10^4$

2. Data from measurement of the angle of deviation

The distance between the screen and the mirror- $l = 547\ cm$

Measurement	1	2	3	4	5	6
Deviation of the light mark [cm]	39.5	37	38	38	37.5	38.5
Angle of displacement of the light beam φ_L $[rad]$	0,0722	0,0676	0,0694	0.0694	0,0685	0,0703

Average value of the displacement of the light beam- $\varphi_L = 69.6\ 10^{-3}\ [rad]$

Angle of rotation of the sample- $\varphi = \frac{\varphi_L}{2} = 34.78\ 10^{-3}\ [rad]$

Result

Angular momentum of the sample - $L = 2.3\,10^{-9}\ \frac{kg.m^2}{s}$

Magnetic moment of the sample – $P_m = 85.8\ A.m^2$

$$g = \frac{e/m}{P_m / L} = 4,7$$

References:

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7.4. SOLUTION OF CZECH REPUBLIC

Problem № 14: Einstein–de Haas experiment

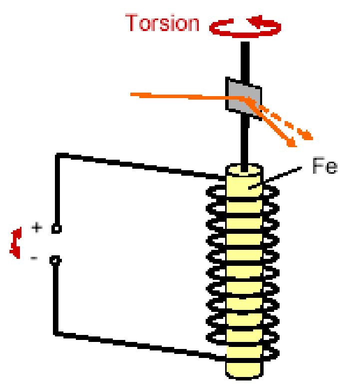
Janečka, A. – Kudela, J. – Lejnar, O. – Roženková, K. – Solný, P. – Kluiber, Z.

The problem

When you apply a vertical magnetic field to a metallic cylinder suspended by a string, it begins to rotate. Study this phenomenon.

Einstein–de Haas experiment was performed in 1915 with an iron cylinder and should clarify the cause of magnetism in ferromagnetic materials, results of their experiment were more than surprising, as the result was two times bigger than they'd expected. This 'inaccuracy' have been clarified several decades later.

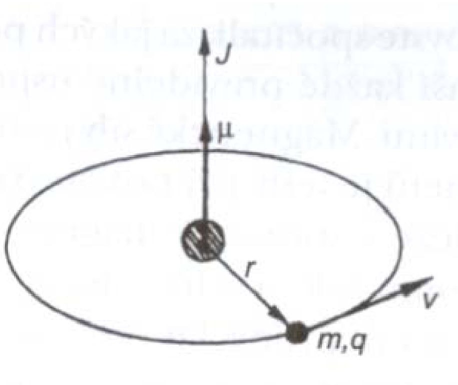
As the ferromagnetic material they used an iron cylinder hung on a thin thread. Around the cylinder is wound a coil. At the beginning, there are no moments of outer forces acting on the cylinder. Once a electric current goes through the coil, a magnetic field **B** is formed inside the coil. Due to this magnetic field, angular momentum of every single atom inside changes its direction so the total angular momentum is now not equal to zero. To preserve the total angular momentum (and to equal to the primal angular momentum, which was equal to zero), the cylinder starts rotating around its axis. If no thread was used, the cylinder would rotate as long as the magnetic field would be applied. Due to the torsion of the thread, a torsional momentum is formed (depending on the torsion modulus of the thread). The torsional momentum makes the cylinder stops rotating and makes it start rotating in a opposite direction (and thread gets straight). The thread will twist and straighten as the cylinder rotates with harmonic motion around its equilibrium like a torsional pendulum.



A mechanic analogy also exists: one stands on a rotatable pad and holds a horizontally revolving wheel (balance-wheel). When he turns the balance-wheel upside down, he starts revolving himself on the pad to preserve the total angular momentum.

First statement of classical mechanics says that if an electron moves along a circle (e.g. circulates around core due to centripetal force), its magnetic momentum and angular momentum are proportional.

Angular momentum **J** is defined as $\mathbf{J} = \mathbf{m}r \times \mathbf{v}$, where m is mass of electron; r is the distance from electron to the core and **v** is the velocity of



electron. Direction of angular momentum of the electron is perpendicular on the plane of trajectory. Radius r is perpendicular on the vector of velocity **v**, so the equation $\mathbf{J} = \mathbf{m}r \times \mathbf{v}$ can be written as:

$$J = mvr$$

Magnetic momentum μ of the electron circulating around the core is equal to $\boldsymbol{\mu} = I \cdot \mathbf{A}$, where **I** is the electric current and **A** the area inside the trajectory of the electron. Electric current is defined as charge which goes in time through any place at the trajectory, i.e. charge q times frequency of circular motion $I = q \cdot f$. Frequency is velocity divided by the length of trajectory $f = \frac{v}{2\pi \cdot r}$, so:

$$I = \frac{qv}{2\pi \cdot r}$$

The area is $A = \pi \cdot r^2$, magnetic momentum is then:

$$\mu = \frac{qvr}{2}$$

Magnetic momentum is perpendicular on the plane of trajectory, just like the angular momentum, their direction is the same, then:

$$\frac{\mu}{J} = \frac{\frac{qvr}{2}}{mvr} = \frac{q}{2m} \Rightarrow \boldsymbol{\mu} = \frac{q}{2m}\mathbf{J} \quad \text{(Orbital movement)}$$

Ratio of magnetic momentum and angular momentum is called the gyromagnetic ratio. This ration depends neither on velocity nor on the radius of trajectory. Magnetic momentum μ of every particle on the circular track is $(q/2m)$-multiple of angular momentum J. The electron's charge is negative (we'll call it $-q_e$):

$$\boldsymbol{\mu} = -\frac{q_e}{2m}\mathbf{J} \quad \text{(Orbital movement of the electron)}$$

This equation is valid also in the quantum mechanics. But we know that the orbital movement isn't the only causer of magnetism. Electron has also a spin (it's

like Earth and its rotation around its own axis) and like a consequence, there is also a spin angular momentum and related magnetic momentum.
From the quantum mechanics, we know that, the spin angular momentum of an electron is equal to:

$$J_{Spin} = \frac{n \cdot h}{4 \cdot \pi}$$

And spin angular momentum is:

$$\mu_{Spin} = \frac{n \cdot e \cdot h}{4 \cdot \pi \cdot m}$$

After substitution into the gyromagnetic ration:

$$\frac{\mu_{Spin}}{J_{Spin}} = \frac{4 \cdot \pi \cdot n \cdot e \cdot h}{4 \cdot \pi \cdot n \cdot m \cdot h} = \frac{e}{m}$$

Then:

$$\mu = -\frac{q_e}{m} J \quad \text{(electron spin)}$$

Generally, in every atom exist many of electrons and by compounding their spin and orbital movements the total angular momentum and the total magnetic momentum is formed. In spite of lack of classical mechanics explanation, in quantum mechanics is generally valid a statement that the direction of the magnetic momentum of an isolated atom is exactly opposite than the direction of angular momentum. Their ration don't have to be $-q_e / m$ or $-q_e / 2m$, though, but can be somewhere in between these values because the magnetic momentum is compound of orbital and spin portions. We can write their ration as:

$$\boldsymbol{\mu} = -g\left(\frac{q_e}{2m}\right)\mathbf{J}$$

Where g, so-called Landé Faktor, is a factor characterizing the state of atom. It's either equal to one for solely orbital movement, to two for solely spin movement and to any other number between one and two for a complicated system like an atom is. Landé factor is a non-dimensional constant. From the equation $\mu = -g\left(\frac{q_e}{2m}\right)J$ results that the magnetic momentum is parallel with the angular momentum, the size can be different though, depending on the Landé factor.
Magnetic momentum of an electron is in equation with the magnetization of the cylinder:

$$\mu = M \cdot V / N$$

Where M is magnetization, V is volume of the cylinder and N is the number of particles inside the cylinder.

Total angular momentum of the cylinder is:

$$J_{celk} = \sum_{i=1}^{n} m_i v_i r_i$$

$$J_{celk} = N \cdot J$$

After substitution μ and J:

$$M \cdot V = -g\left(\frac{q_e}{2m}\right) J_{celk}$$

In Einstein – de Haas experiment, magnetization angular momentum change within time because the system oscillates. Then, the time change of these quantities is observed:

$$\dot{M} \cdot V = -g\left(\frac{q_e}{2m}\right) D \wedge D = \dot{J}_{celk}$$

$$\Leftrightarrow g = -2\frac{m}{e} \cdot V \cdot \frac{\dot{M}(t)}{D(t)}$$

Experiment of Einstein and de Haas showed g-factor 2. So ferromagnetism is based on spin of the electron and not on the orbital angular momentum.

In Einstein – de Haas experiment, the cylinder behaves like a torsional pendulum, whose period of oscillation is:

$$T = \sqrt{\frac{2 \cdot \pi \cdot l_V \cdot m_T \cdot r_T^2}{r_V^4 \cdot \mu_V}}$$

Where l_V is the length of thread, m_T is the weight of cylinder, r_T is the radius of cylinder, r_V is the radius of thread and μ_V is torsion modulus of thread.

From the equation is obvious that period doesn't depend on current, voltage or on the used coil. If coil creates a bigger magnetic field, cylinder will revolve faster, on the other hand, it will deflect more, so the period won't change at all.

PROBLEM № 14: EINSTEIN–DE HAAS EXPERIMENT

7.5. SOLUTION OF POLAND

Problem № 14: Einstein–de Haas Experiment

Team members: Tomasz Bobiński, Pawel Debski (Captain), Maciej Lisicki, Krzysztof Wojtowicz, Maciej Zielenkiewicz
Team leaders: MSc. Stanislaw Lipinski, Karolina Kocko, Michal Oszmaniec
XIV Stanislaw Staszic Secondary School, Warsaw,

The problem

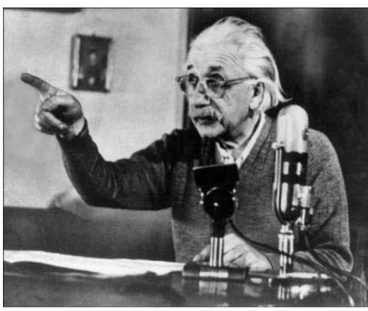
When you apply a vertical magnetic field to a metallic cylinder suspended by a string, it begins to rotate. Study this phenomenon.

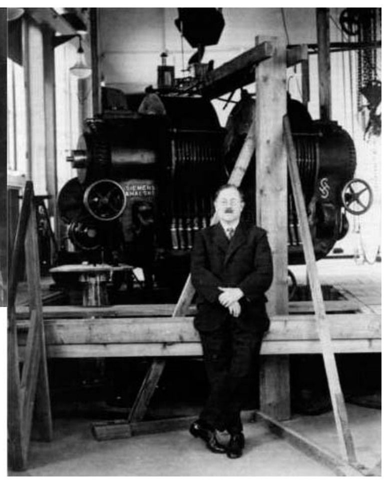
The described phenomenon was firstly investigated in 1915, when Albert Einstein, searching for a proof of existence of Ampere molecular currents, conducted along with Wander de Haas, an experiment, which would confirm his theory.

Through the late years of the XIX cent. the scientific world was searching for the reason of magnetic properties of certain materials. In 1908 Richardson was the first one to mention that orbital motion of electrons is responsible for such behaviour of magnetic materials in presence of external magnetic field.

The described phenomenon was firstly investigated in 1915, when Albert Einstein (pic.1) , searching for a proof of existence of Ampere molecular currents, conducted along with Wander de Haas, an experiment, which would confirm this theory. The results of the experiment confirmed the theoretical predictions very well. Einstein published their work and did not put more attention on the analysis of the effect.

However, in later years many experimentators (including W.J. de Haas) tried to repeat this experiment, but their results were completely different. This uncertainty of the ultimate, correct result caused a discussion on Einstein's work and in further research,





Pic. 1. Albert Eistein and Wander de Haas

lead to discovery of the electron spin as partially responsible for magnetic behaviour of materials.

Theoretical explanation

At first, let's try to analyze a simple mechanical situation, in which we shall observe the basic effect which is in fact responsible also for Einstein – de Haas effect.

Pic. 2

Let's consider a man sitting on a revolving chair, which is initially not rotating. A man holds a rotating bicycle wheel is such way, that its angular momentum vector is directed vertically (pic. 2). What happens, when we change the direction of the angular momentum vector?

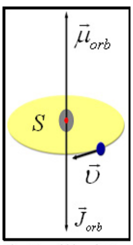
Obviously, as a consequence of the angular momentum conservation principle, the angular momentum vector of the whole system is constant, so the revolving chair starts to rotate if we change the angular momentum vector direction (pic. 3).

Pic. 3

We shall now consider situation, in which we can observe the Einstein – de Haas effect. At first, let's analyse orbital motion of an electron (pic. 4)

Orbiting electron can be considered as a small current in a loop. Such a current has a magnetic moment:

$$\mu_{orb} = IS = \nu e \cdot \pi r^2 = \frac{e\upsilon\pi r^2}{2\pi r} = \frac{e\upsilon r}{2}$$



Pic. 4 An orbiting electron (according to Bohr model)

Magnetic moment is a vector, and its direction can be noted as:

$$\vec{\mu}_{orb} = \frac{e\left(\vec{\upsilon} \times \vec{r}\right)}{2}$$

An orbiting electron has also an angular momentum vector, which can be noted as:

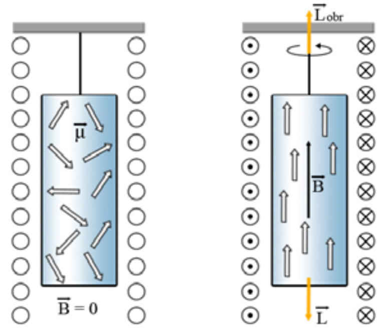
$$\vec{J}_{orb} = m\left(\vec{r} \times \vec{v}\right)$$

So the magnetic moment vector, which of course changes its direction in presence of external magnetic field, is strictly connected with the angular momentum vector of the atom. The relation is as follows:

$$\overrightarrow{\mu}_{orb} = -\frac{e}{2m}\overrightarrow{J}_{orb}$$

Basing on this relation, we can qualitatively explain the occurring effect. Let's describe it using a drawing (pic.5 and 6).

Initially, because of thermal motion and absence of external magnetic fields, the magnetic moment vectors of the atoms are randomly directed in the whole probe, so the resultant magnetic moment, as well as the resultant angular momentum, equals 0. When we apply a vertical magnetic field, the magnetic moment vectors tend to line up with the magnetic field lines, so they change their direction, so the resultant magnetic moment vector is directed vertically. We shall note, that this change is also connected with changes in angular momentum vectors, which then change their direction. The angular momentum conservation principle demands the resultant angular momentum vector to be 0, so the probe gains angular momentum directed oppositely to the resultant angular momentum vector of the atoms. This angular momentum causes the rotation of the probe.



Pic. 5.6. Magnetic moment vectors in a probe.

We shall remember, that this is only a qualitative explanation. After an electron spin has been discovered, an important lacks in the experimental results could be explained. It occurred, that an electron has its own, internal angular momentum and magnetic moment, but the relation between those vectors is as follows:

$$\vec{\mu}_S = -\frac{e}{m}\vec{J}_S$$

We shall note that the proportionality factor between those vectors is 2 times bigger than when considering an orbiting electron.

This discovery revealed new facts connected with theoretical explanation of the behaviour of magnetic materials. It caused a discussion about the exact reason of the magnetic behaviour. Caused questions, like: what is more important in this phenomenon – orbital motion or electron spin? To solve this problem, precise quantitative experiments had to be conducted.

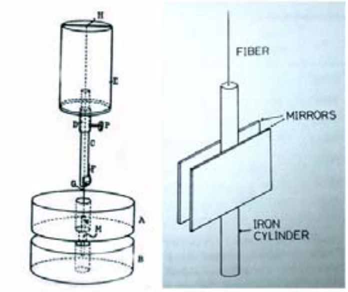
The idea was to measure the proportionality constant, called the g-factor or Lande factor:

$$\vec{\mu}_{metal} = -g\left(\frac{e}{2m}\right)\vec{J}_{metal}$$

If the g-factor value was closer to 1, it would confirm the hypothesis, that crucial for this effect are the orbiting electrons, if it was closer to 2, it would prove, that elector spin has the key role for this phenomenon.

Experimental Approach

We wanted to use the same method, which Einstein and de Haas described in their work in 1915. Their idea was to apply an external field and measure the deflection of an iron cylinder. Their setup consisted of coils, generating the magnetic field and a cylinder suspended on a glass fibre, with a mirror attached to it (pic. 6 and 7).



Pic. 7.8. Einstein's original scheme of experimental setup

The cylinder can be treated as a torsional pendulum – using a resonance phenomenon, we can amplify the oscillations. Therefore, the frequency of current in the coils, generating the magnetic field equals the natural frequency of oscillations of the cylinder, the oscillations will be amplified and easier to observe.

To measure the deflection we have used a laser and a screen.

To calculate the value of the g-factor, we have tried to use Einstein's method, but to make the measurements easier, we have modified it, as did the Berlin Technical University students. The formula for calculating g, given with no derivation, is as follows:

$$g = \frac{m_e}{q} \cdot \frac{U_{ind} \cdot V}{N_2 \cdot \mu_0 \cdot A \cdot X \cdot \omega \cdot \beta \cdot \alpha_{\max}}$$

m_e – electron mass

q – electron charge

U_{ind} – inducted voltage

μ_0 – magnetic premeability of vacuum

ω – resonant angular frequency

X – moment of inertia

β – damping constant


Pic. 11 First experimental setup.

So we had to measure various parameters of the whole system.

We have constructed our first experimental setup (pic. 11), hoping to measure necessary parameters. As it occurred, it was very imprecise and allowed us only to estimate g as about 0.002.

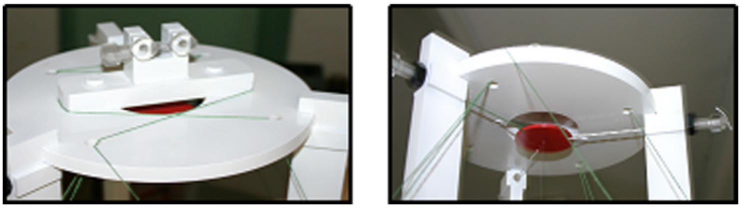
These wooden constructions, which can be seen in this photography are Helmholtz coils, used to compensate the nonvertical components of earth magnetic field.


Pic. 12 Second experimental setup.

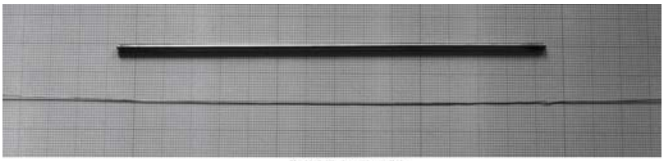
We have decided to build much more precise experimental setup, which would allow us to conduct measurements (pic.12).

This time the setup was very stable, symmetric, with special devices used to center the cylinder inside the coil and regulate the tension of the suspension (pic.13).

Our cylinder was long (150 mm) and very thin (2.4 mm), to avoid any nonuniformities of the magnetic field inside the coil. It was made of ARMCO (contained 99% of iron) We have suspended it on a CuNi wire (with diameter of 0.2 mm) (pic. 14).

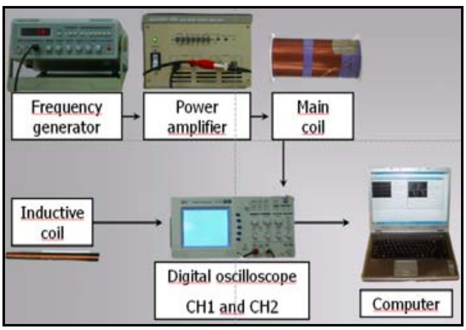


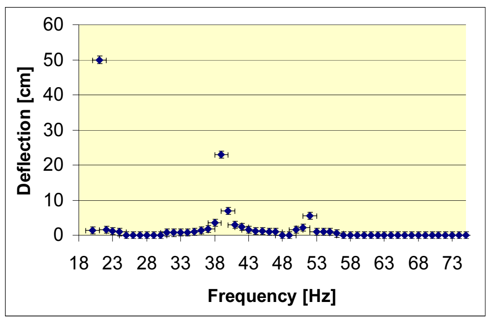
Pic. 13 String tension controller and centering device



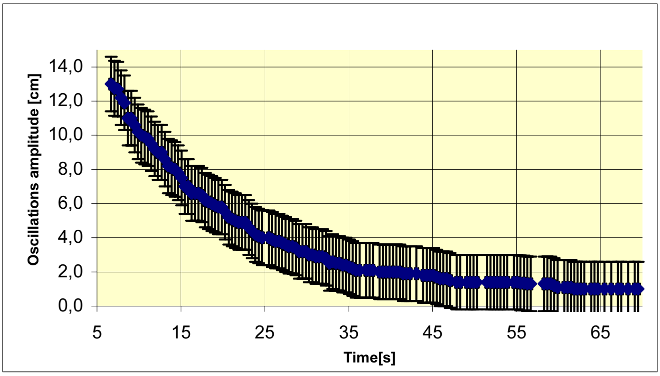
Pic. 14. CuNi wire and iron cylinder

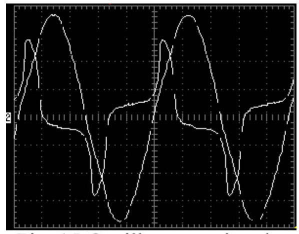
To generate the external magnetic field, we have used a frequency generator, connected to a power amplifier and a digital oscilloscope. The signals generated by presence of an iron cylinder inside the coil were gathered by an inductive coil, and this signal was also analyzed using a computer.





First of our measurements was the estimation of the resonant frequency of the cylinder. Obtained results are presented in the above graph:
We have measured the deflection on the screen while changing the external magnetic field change frequency (our generator). Three harmonics can be clearly visible. We therefore assume the lowest and strongest one, at about 19 Hz, to be the resonant frequency of the cylinder.
Another important parameter was the damping constant. We could measure it by turning off the generator at the resonant frequency and measuring the deflection decrease in time. Then, using GNUPlot, we have fitted a curve, showing the expected dependence, as it is shown below:





Pic. 15 Oscilloscope signals

The expected dependence was:

$$f(t) = Ae^{-\beta t}$$

After estimating curve-fitting parameters, we could estimate the damping constant as:

$$\beta = 0.060 \pm 0.005$$

The inducted voltage was measured using the digital oscilloscope, as it can be seen in the illustration (pic. 15).

The two signals in this screen are signals from main coil (sinusoidal) and from the inductive coil (in this signals, peaks mark the demagnetisation of the cylinder, as the current in main coil changes its direction).

$$\begin{aligned}
m &= 9.1 \cdot 10^{-31}\ \mathrm{kg} \\
q &= 1.6 \cdot 10^{-19}\ \ \mathrm{C} \\
\mu_0 &= 1.257 \cdot 10^{-6}\ \mathrm{T^2 m^3 J^{-1}} \\
V &= 6.79 \cdot 10^{-7}\ \mathrm{m^3} \\
U_{ind} &= 2.2\ \mathrm{V} \\
N_2 &= 600 \\
A &= 4.52 \cdot 10^{-6}\ \mathrm{m^2} \\
X &= 3.744 \cdot 10^{-9}\ \mathrm{kg{\cdot}m^2} \\
\omega &= 131.95\ \mathrm{Hz} \\
\beta &= 0.06 \\
\alpha_{\max} &= 5.22 \cdot 10^{-2}\ \mathrm{rad}
\end{aligned}$$

$$\begin{aligned}
&\frac{\Delta V}{V} \approx 16.5\% \\
&\frac{\Delta X}{X} \approx 16\% \\
&\frac{\Delta \omega}{\omega} = \frac{\Delta f}{f} \approx 5\% \\
&\frac{\Delta \alpha_{\max}}{\alpha_{\max}} \approx 14\% \\
&\frac{\Delta \beta}{\beta} \approx 8.4\% \\
&\frac{\Delta U}{U} \approx 4.5\%
\end{aligned}$$

After conducting necessary measurements, we were ready to estimate the value of g and analyse possible sources of error in our measurements.

The total error of our measurements was therefore:

$$\sqrt{\left(\frac{\Delta V}{V}\right)^2 + \left(\frac{\Delta X}{X}\right)^2 + \left(\frac{\Delta \omega}{\omega}\right)^2 + \left(\frac{\Delta \alpha_{\max}}{\alpha_{\max}}\right)^2 + \left(\frac{\Delta \beta}{\beta}\right)^2 + \left(\frac{\Delta U}{U}\right)^2} = 0.296$$

The calculated value of g:

$$\boxed{g = \frac{m_e V U_{ind}}{q N \mu_0 A X \alpha_{\max} \omega \beta}}$$

$$\boxed{g = 1.61 \pm 0.48}$$

Therefore we can draw some conclusions on this effect:

1. Einstein – de Haas effect is connected with atoms and electron having angular momentum and magnetic moment. In fact, professional laboratory measurements estimate g as about 1.8, so it proves that electron spin and magnetic moments have bigger influence on the behaviour of magnetic materials in presence of external field.
2. Change in direction of magnetic moments vectors causes change in angular momentum of the probe; rotation is the effect of the angular momentum conservation principle.
3. This effect has a wide historical background and had many implications on the concept of magnetism of matter (from molecular currents to spin of the electron).
4. Although quantitative analysis is very hard, but it is possible to do it in school conditions. In our case, we’ve obtained value, which is completely sufficient, even surprisingly close to professionally measured value.

Acknowledgement:

We’d PhD. Piotr Kossacki from Warsaw University for valuable discussion discussion and theoretical help. We’d like to thank Mr Krzysztof Bobiński from Warsaw Univeristy of Technology, for help in experimental setup construction and PhD. Jan Grabski fromWarsaw Univeristy of Technology, for advice, theoretical support and providing us with electronic equipment. We’d also like to thank MSc. Anna Mazurkiewicz from Jozef Poniatowski Secondary School for help in search for theoretical materials.

8. PROBLEM № 15: OPTICAL TUNNELING

8.1. SOLUTION OF BRAZIL

Problem № 15: Optical Tunneling

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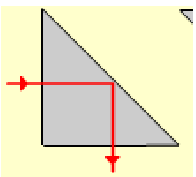
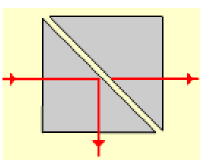
The problem

"Take two glass prisms separated by a small gap. Investigate under what conditions light incident at angles greater than the critical angle is not totally internally reflected."

Introduction

Total internal reflection is a well known optical phenomenon. It occurs when light propagating in a medium of index of refraction *n* reaches a separation boundary between this medium and one of index of refraction smaller than *n* at an angle of incidence greater than a critical angle θ_c. All light is reflected back into the first medium (of greater index of refraction). Total internal reflection can be well visualized if we have a triangular 90° prism (of, for example, glass) and a light source (for example, a laser pen). If we simply make the light enter the prism and reach the separation boundary with air on its hypotenuse at an angle greater than the critical angle, we will easily see total internal reflection. View top picture.



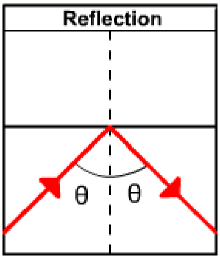


If a second prism or piece of glass is approached to the hypotenuse of the one in which total reflection is taking place, making the two prisms separated by a small gap, an unexpected phenomenon might occur (bottom picture) . A normal total internal reflection would be expected, since air is still surrounding the prism. However, if the gap is sufficiently small, part of the light that would suffer total internal reflection is unexpectedly transmitted into the second prism, leaving a smaller amount of light to suffer reflection:

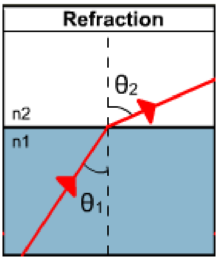
This phenomenon is given the name of Frustrated Total Internal Reflection (which we shall now call FTIR). It was first observed by Isaac Newton, about 300 years ago, and reported in his famous book *Optics*. Newton brought a convex lens close to the region into contact with the reflecting surface of the prism and realized some light started to travel through the lens. Although reported, the phenomenon could not be successfully explained by Newton. In fact, the phenomenon is not predicted by geometric optics. In this problem, we shall investigate FTIR and the conditions in which it occurs.

Theory

Geometric Optics: it is important for the understanding of FTIR knowledge of basic concepts in geometric optics.



Reflection:
Light suffers reflection when it reaches the boundary of a reflecting surface. The angle between the incident ray of light and the direction normal to the reflecting surface is equal to the angle of the reflected ray with this normal. This is the law of reflection.



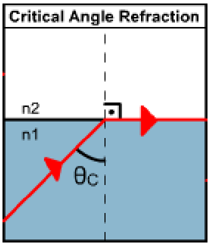
Refraction:
Is what happens to a wave when it changes its medium of propagation and consequently its propagation speed. In optics, light suffers refraction when it changes its propagation medium into one of different index of refraction. A medium's index of refraction, *n,* is given by:

$$n = \frac{c}{v}$$

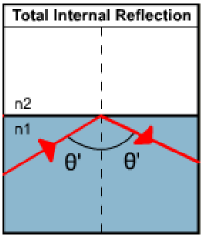
In which c is the speed of light in vacuum (aprox. 3.10^8 m/s) and v is the speed of propagation in the considered medium. The index of refraction is also referred as optical density. When a light ray suffers refraction, its speed and direction changes. The law of Snell – Descartes relates the sine of the incident and refracted angles and the index of refraction of both mediums:

$$\sin \theta_1 \,.\, n1 = \sin \theta_2 \,.\, n2$$

We can conclude from the equation that if light is traveling initially in a medium of smaller index of refraction and refracts into a medium of greater index, the angle of the refracted ray will be smaller than the angle of the incident ray. If light is traveling initially in a medium of greater index of refraction and refracts into a medium of smaller index, the angle of the refracted ray will be bigger. It is important to add that not all incident light is refracted: part of it can be reflected, returning back into the first medium. If we take the case of the initial medium of propagation be of bigger index, we will find that at a certain angle the refracted ray will be perpendicular to the normal direction and therefore parallel to the surface. The incident angle in which this occurs is the *critical angle*. From Snell – Descartes, we find that the sine of the critical angle θ_C is given by:



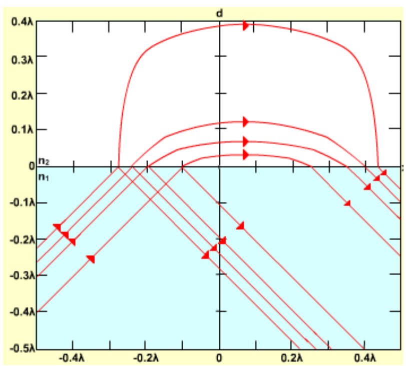
$$\sin \theta_C = \frac{n2}{n1}$$



If, in these conditions, the incident angle becomes greater than θ_C, total internal reflection will occur, in which all light is reflected and none is refracted:

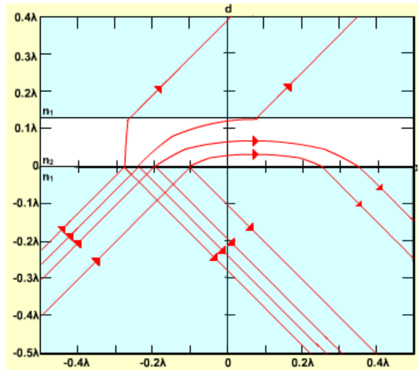
Evanescent wave in Total Internal Reflection:

Frustrated Total Internal Reflection could only be properly explained with 19th century Maxwell's electromagnetic theory. A deeper look into total internal reflection was possible. In total internal reflection, we have the penetration of the electromagnetic wave in the region beyond the totally reflecting interface, into the second medium. This penetrating wave is called the **evanescent wave** (see pic.).



This picture was taken from the third reference. It shows total internal reflection of light incident at 45° on the interface, in which n1 = 1,5 and n2 = 1,0. The flow lines are represented, showing that when light is incident in an angle greater than the critical angle, part of it is reflected back into the first medium and part, surprisingly, actually penetrates into the less optically dense medium, creating an existence of electromagnetic energy in the region beyond the interface, which travels according to the flow lines represented. This is a strange behaviour. The wave that penetrates into the second medium runs along the direction parallel to the interface, and, after a distance of the order of the wavelength λ, returns to the first medium, parallel to the reflected rays. This is what actually happens in total internal reflection. The picture also shows that, increasing the distance d from the interface, we have a smaller concentration of flow lines, which means the amplitude of the evanescent field drops if the distance from the interface is increased, so that, at some distance, the amplitude would be too small to be considered.

If we were to approach a second piece of glass to the first, at a distance in which the amplitude of the evanescent wave is appreciable, we would have that some of the electromagnetic energy would enter the second glass in the form of a light wave. This would result in a smaller amount of light returning to the first medium:

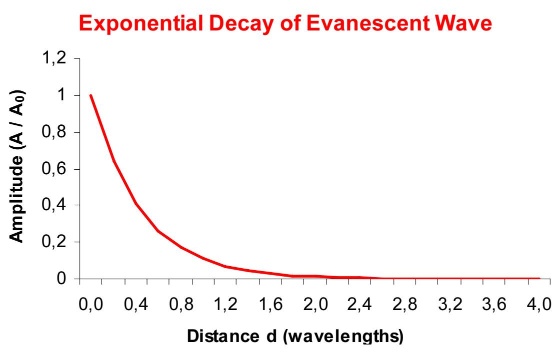


This partially explains frustrated total internal reflection. If the second prism is approached at a distance small enough, it will be able to capture the electromagnetic energy in the evanescent wave. Due to conservation of energy, less light returns to the first prism. If the distance between the prisms is too great, the amplitude of the evanescent wave will be practically zero and no light would be frustrated. The drop of the amplitude of the evanescent wave with the distance from the interface in the direction normal to this interface is exponential:

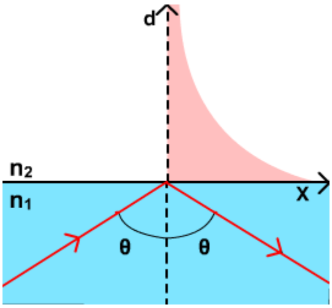
$$A = A_0 e^{-d\alpha}$$

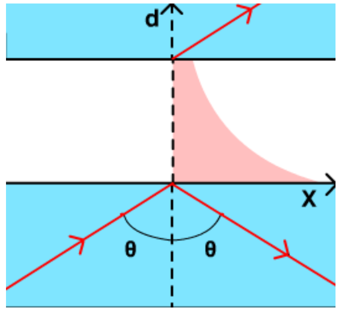
$$\alpha = \frac{2\pi}{\lambda}\sqrt{\left(\frac{n1}{n2}\right)^2 \sin^2\theta_1 - 1}$$

Where A_0 is the amplitude at distance 0, d is the distance in the direction normal to the interface, λ is the wavelength of the electromagnetic wave and θ_1 the incidence angle. The deduction of this equation may be viewed by the curious reader in the **appendix**. From the exponential equation, we can observe that the amplitude of the evanescent wave is considerable at distances smaller to or of the order of λ. The graphic below shows how the amplitude of the evanescent wave (in relation to A_0) varies with distance from the interface (in units of wavelength) when the incidence angle is 45° and n1 and n2 are, respectively, 1,5 and 1:

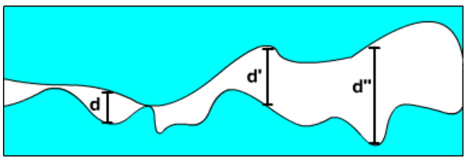


Below we have another representation for the evanescent wave and FTIR, in which the exponential decay is represented by the reddish curve:





So, in order to obtain the phenomenon, we must place our prism at a distance no much greater than 2 wavelengths. The wavelength of visible light varies from approximately 400 to 800 nanometers. We must then place our prisms at a distance of the order of 10^{-7}m from each other. The best way to try to do this is pressing one prism against the other, or at least putting them into maximum contact possible. This is because most prisms are still irregular in microscopic terms.



So we have great differences in distance between the prisms along their surfaces, if compared to the wavelength of visible light. This makes it impossible to obtain FTIR with prisms which don't have a very good surface quality.

Experiment

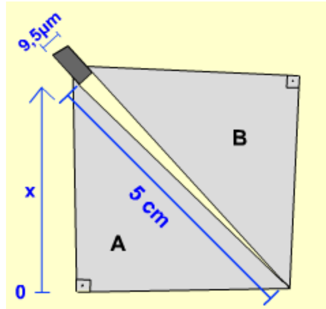
The Objective of the experiment was to verify and measure the dependence of the transmitted light (in frustrated total internal reflection) on the separation air gap d between the prisms.

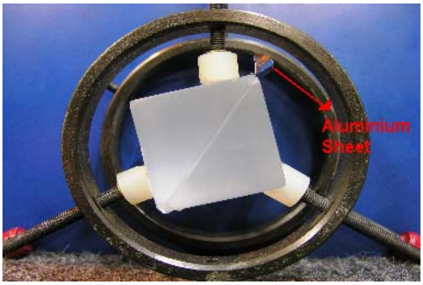
Utilized Materials:

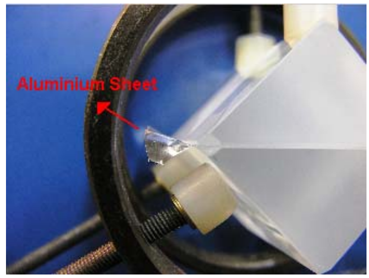
- 2 BK7 45° Prisms (n = 1,515 for 635nm)
- Red laser (λ = 635 nm)
- Aluminium Sheet (9,5 ± 0,5 μm width).
- Photocell
- Voltmeter
- 2 lens
- Micrometer
- 2 Fasteners
- Voltmeter

Experimental Methods:

With the use of the very thin aluminium sheet, the prisms were arranged so that the air gap (or distance) between them changed uniformly. The prisms used had a very good surface regularity:







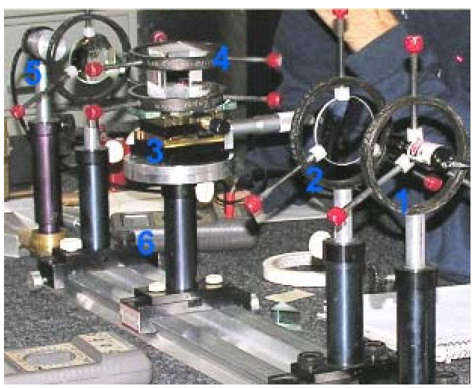
2 Fasteners were used to press the prisms against each other in order to assure they were really in contact at one end. The prisms used had a very high optical quality. Along a distance of 5 cm, the separation gap between the prisms changed continuously from 0 to 10µm. It is important to know the linear relationship between the separation gap d and the position in the direction x (shown in figures above). The relationship was obtained using basic trigonometry. The major error source is the error of the width of the aluminium sheet:

d = Width of air gap

$$d = x\,(0{,}27 \pm 0{,}01)\,.\,10^{-3}$$

$$d(\mu m) = x(mm)\,(0{,}27 \pm 0{,}01)$$

Setup:



1. Laser (λ = 635 nm): The laser was placed so that light entered the first prism at an angle of incidence of 0^o, without changing direction of propagation. Consequently, the light encountered the hypotenuse of the first prism at a 45^o angle of incidence, which is slightly greater than the critical angle between the used glass and air ($41{,}3^o$).

2. Lens: One lens was placed in front of the laser to focalize the light, in order to make the beam thin enough to be considered punctual. Another lens was placed after the prisms for the same reason, avoiding the possibility of the beam becoming too large to be detected by the photocell.

3. Micrometer: All experimental setup was maintained still, except for the prisms. The prisms were placed on top of the micrometer, which made it possible to move them in the direction of x, causing light to reach the hypotenuse of the first prism at points of different position in x, and to measure this movement. Using the relationship between x and d, it is possible to calculate the width of the air gap at the point in which light is incident on the hypotenuse of the first prism.

4. Prisms: were put on top of the micrometer. The ones used (BK7) had a high optical quality.

5. Photocell: Used to capture the transmitted light.

6. Voltmeter: Attached to the photocell, this was used to measure the intensity of the transmitted light.

The prisms were moved in the x direction, and the intensity of the transmitted light beam was measured once each half millimetre moved. The experiment was realized in the dark, so that no external light influenced the measurements.

Results:

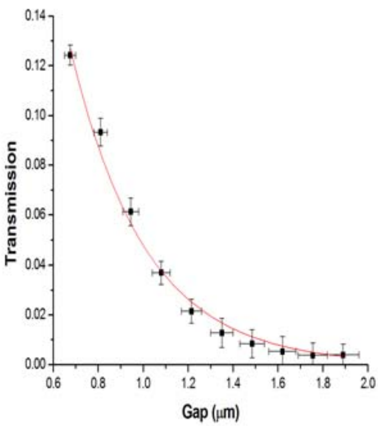
The table below shows the values obtained for each position in x:

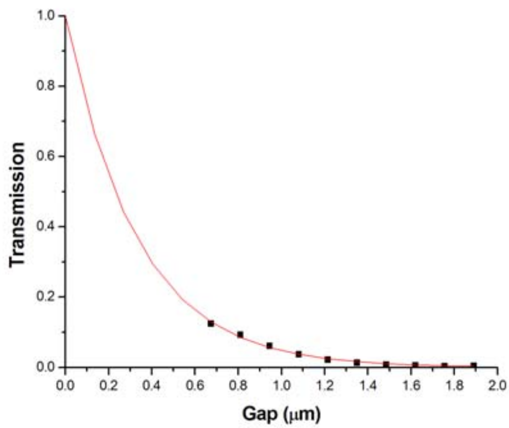
Position in X (mm)	Measure 1 (mV)	Measure 2 (mV)	Measure 3 (mV)	Average (mV)	Error (mV)
2,5	26,3	26,1	27,7	26,7	0,9
3,0	19,6	19,1	21,4	20,0	1,2
3,5	13,0	12,1	14,4	13,2	1,2
4,0	7,8	7,0	9,0	7,9	1,0
4,5	4,0	4,0	5,8	4,6	1,0
5,0	2,0	2,0	4,2	2,7	1,3
5,5	1,1	1,1	3,2	1,8	1,2
6,0	0,6	0,2	2,6	1,1	1,3
6,5	0,4	0,0	2,0	0,8	1,1
7,0	0,3	0,3	1,9	0,8	0,9

Measurements before 2,5 mm were not included due to their uncertainty, since the light incident at these regions was greatly scattered, so the intensity of transmission could not be measured in positions before 2,5 mm. With the data above, it was possible, using the program Origin 6.0, to obtain the equation which best describes the relationship of the voltage accused (I) as a function of the position in x. It is the equation which best adjusts to our experimental data and the error:

$$\mathbf{I = (210 \pm 30)e^{-x/(1,24 \pm 0,07)}}$$

where x is given in mm and I in mV. So, according to the equation, the voltage accused at x = 0 mm would be 210 ± 30 mV. Knowing this, the intensity of the transmitted beam was normalized so that this maximum intensity (at x = 0) would be equal to 1. It was possible to plot a graphic of the transmission (which can go from 0 to 1) versus the size of the gap:



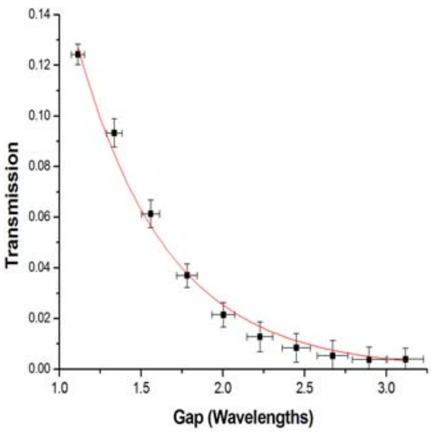


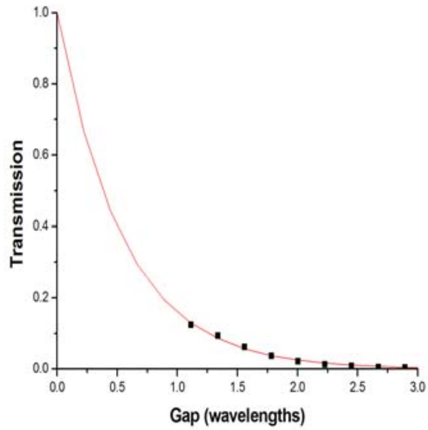
The dots represent the experimental values, and the line represents the exponential equation which best fits these results. The equation that best fits the experimental results is:

$$T = T_0 e^{-d/\,(0,33 \pm 0,02)}$$

where T stands for transmission and d is the gap in μm. The value **(0,33 ± 0,02) μm** would be our experimental value for 1/α (of the equation at the end of page 5), so the experimental value for α would be **(3,0 ± 0,2) μm^{-1}**. The expected value for α under the conditions of the experiment would be **3,79 μm^{-1}**, and 1/α would be **0,26 μm**. A possible explanation for the small difference between experimental and theoretical α would be that the width of the aluminium sheet would get smaller because it is being compressed by the two prisms. This is a non-quantifiable error source in the experiment. In fact, if we consider the width changed to 8 μm, we would have 1/α equal to (0,27 ± 0,2) μm accused by Origin.

Knowing the wavelength of the incident light used it was possible to build graphics with the gap distance in units of wavelength:





The last graphic proves the fading of the evanescent wave within a few wavelengths, which is predicted in theory. We can see that at a distance of about or greater than 2,5λ, the transmitted light is practically null.

Conclusions

- The phenomenon will occur if the distance between the prisms is of the order of the wavelength λ.
- Very well polished prisms are needed in order to perform the experiment with visible light.
- An application of the experiment would be the determination of a medium's index of refraction, because we can determine α.

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APPENDIX

1. Resolution of electromagnetic wave equation.

The following equation comes from solving Maxwell's equations of electromagnetism. It describes the wave function of an electromagnetic radiation of frequency *f* propagating in a medium of index of refraction *n* in one dimension (x):

$$\frac{d^2\psi_{(x)}}{dx^2} + \left(\frac{2\pi f}{c}\, n\right)^2 \psi_{(x)} = 0$$

It can be rewritten as:

$$\frac{d^2\psi_{(x)}}{dx^2} + k^2\psi_{(x)} = 0 \qquad k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}\, n$$

In which k is called the *wave number*. Two possible solutions for this equation are:

$$\psi_{(x)} = \psi_{(0)} e^{\pm ikx}$$

It can be proved that these two solutions are possible. First, the proof for the positive exponential:

$$\frac{d^2\psi_{(x)}}{dx^2} = \psi_{(0)} i^2 k^2 e^{ikx} = -\psi_{(0)} k^2 e^{ikx}$$

$$k^2\psi_{(x)} = \psi_{(0)} k^2 e^{ikx}$$

Proceeding in the same manner for the negative exponential:

$$\frac{d^2\psi_{(x)}}{dx^2} = \psi_{(0)} i^2 k^2 e^{-ikx} = -\psi_{(0)} k^2 e^{-ikx}$$

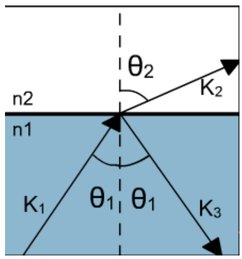
$$k^2\psi_{(x)} = \psi_{(0)} k^2 e^{-ikx}$$

A more general solution, for three dimensions, can be used to represent the spatial variation of an electromagnetic wave:

$$\psi = \psi_{(0)} e^{\pm i(\mathbf{k.r})}$$

where **k** is the wave vector or propagation vector, and **r** is the vectorial position.

2. Proof of Evanescent wave in total internal reflection.



Here we have an example of wave vectors **k** in refraction and reflection.

The equation describing the spatial variation of the electromagnetic wave is:

$$\psi = \psi_{(0)} e^{\pm i(\mathbf{k.r})}$$

The law of Snell can be applied whenever light encounters a boundary between two mediums:

$$\sin\theta_1 \,.\, n1 = \sin\theta_2 \,.\, n2$$

$$\sin\theta_2 = \frac{n1}{n2}\sin\theta_1 \longrightarrow \sin\theta_2 = \frac{\sin\theta_1}{\sin\theta_C}$$

$$\sin^2\theta_2 = \frac{\sin^2\theta_1}{\sin^2\theta_C} \longrightarrow 1 - \sin^2\theta_2 = 1 - \frac{\sin^2\theta_1}{\sin^2\theta_C}$$

$$\cos^2\theta_2 = 1 - \frac{\sin^2\theta_1}{\sin^2\theta_C} \longrightarrow \cos\theta_2 = \sqrt{1 - \frac{\sin^2\theta_1}{\sin^2\theta_C}}$$

Total internal reflection occurs when:

$$\theta_C < \theta_1 < 90^o \qquad \frac{\sin^2\theta_1}{\sin^2\theta_C} > 1$$

$$\cos\theta_2 = \sqrt{-1\left(\frac{\sin^2\theta_1}{\sin^2\theta_C} - 1\right)} \longrightarrow \cos\theta_2 = i\sqrt{\left(\frac{\sin^2\theta_1}{\sin^2\theta_C} - 1\right)}$$

$$\cos\theta_2 = i\sqrt{\left(\frac{n1}{n2}\right)^2 \sin^2\theta_1 - 1}$$

Having determined the cosine of θ_2 in total internal reflection, we may apply the wave equation to describe the wave that penetrates into the second medium in total internal reflection. To determine the behaviour of this wave in the direction d (normal to the surface) we must include in the wave equation a scalar product between **k** and the position in **d**:

$$\psi = \psi_{(0)} e^{\pm ikd(\cos\theta_2)}$$

$$\psi = \psi_{(0)} e^{\pm i(\mathbf{k}.\mathbf{d})}$$

$$ikd(\cos\theta_2) = d\, i^2 \frac{2\pi}{\lambda}\sqrt{\left(\frac{n1}{n2}\right)^2 \sin^2\theta_1 - 1} = -d\alpha$$

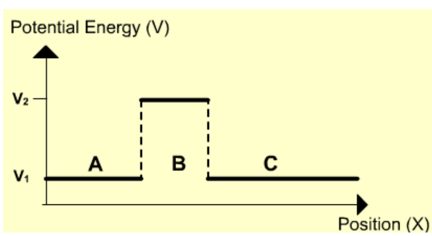
$$\alpha = \frac{2\pi}{\lambda}\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_1 - 1}$$

We have then 2 solutions, a positive and a negative exponential:

$$\psi = \psi_{(0)} e^{\pm d\alpha}$$

The positive exponential is discarded because it is physically impossible due to conservation of energy. The evanescent wave, therefore, decays exponentially.

3. Analogy to Quantum Physics: Every optical phenomenon has an analogue in quantum physics. Frustrated Total Internal Reflection is an optical analogue to particle potential barrier penetration (Quantum Tunnelling Effect). If a quantum particle (ex: an electron) encounters a potential barrier ahead of it during its movement, it will have a probability of being reflected from this barrier and a probability of passing through the barrier and therefore be encountered in the region beyond it.



Supposing our electron is moving in the sense of a growing position, in region A, and reaches a region of greater potential, B. We say it has encountered a potential barrier. It will have a probability either of returning back into region A and a probability of tunnelling into region C. This behaviour is different from macroscopic physics, in which we could predict if a body is or not to trespass a potential barrier. In quantum physics however, we must work with probabilities. The higher the potential barrier in quantum physics, the greater the probability of the particle being reflected, and the smaller the probability of tunnelling to occur. The length of the barrier also influences: the "longer" the potential barrier, the greater probability of reflection and the smaller the probability of tunnelling. In quantum mechanical optics analogies, the particles are the so – called photons that make up light. A beam of light can be well interpreted as a stream of photons. The quantum potential of a medium analogue to light will depend on the medium's index of refraction *n*.

$$\frac{d^2\psi_{(x)}}{dx^2} + \left(\frac{2\pi f}{c}\,n\right)^2 \psi_{(x)} = 0$$

This is the equation that describes electromagnetic radiation of frequency f propagating in a medium of index of refraction n. An equation applied in electromagnetism. Here ψ is the electric or magnetic field, x is position and *c* is the speed of light in vacuum.

$$\frac{d^2\psi_{(x)}}{dx^2} + \frac{2m}{\hbar^2}\left(E - V_{(x)}\right)\psi_{(x)} = 0$$

This is Schrödinger's non time-dependent equation. It comes from quantum mechanics. Here ψ is the wave function, x is position, m is the mass of the photon (which is $h\,.f / c^2$, in which h is the Planck constant = $6{,}626\ .\ 10^{-34}$ J.S), ħ is h/ 2π. E is the energy of the photon (h.f) and V is the quantum potential in the position x. The two equations express the same mathematical relation, so we can make an analogy between them. Comparing the equations, we obtain:

$$\left(\frac{2\pi f}{c}\,n\right)^2 = \frac{2m}{\hbar^2}\left(E - V_{(n)}\right)$$

So we have the relationship between a medium's quantum potential analogue to light (V) as a function of it's index of refraction *n*:

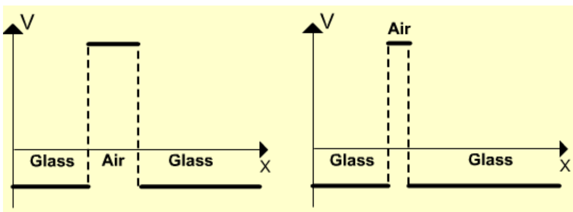
$$V_{(n)} = E - \frac{\left(\frac{fnh}{c}\right)^2}{2m}$$

By looking at the equation we can see that the bigger the medium's index of refraction *n*, the smaller the potential this medium offers to photons. Here we have some quantum potentials for red light ($f = 4{,}48.10^{14}$ Hz):

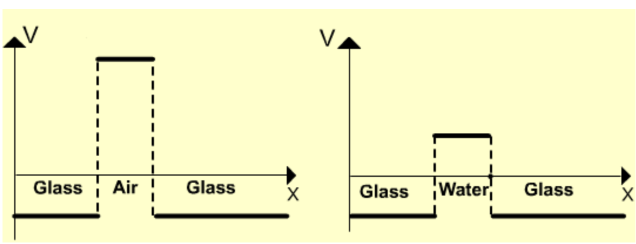
Air (n = 1) = $1{,}45 \cdot 10^{-19}$ J
Water (n=1,3) = $3{,}21 \cdot 10^{-20}$ J
Glass (n = 1,5) = $-3{,}61 \cdot 10^{-20}$ J

These values depend on the frequency of the light considered. However, the **difference** between the potential offered by two mediums is the same for all frequencies.

The greatest potential is that of air. So we can say that the photon, when reaching the separation surface between glass and air in total internal reflection, encounters a potential barrier. If a second prism is placed at a distance of the order of λ, the length of the potential barrier will be small enough for quantum tunnelling of the photons to occur. The smaller the distance between the two prisms, the greater the probability of tunnelling of a single photon. So, the smaller the gap, the greater the intensity of light frustrated, because more photons tend to tunnel:



If the distance between the prisms is maintained, but the medium between them changes, the probability of tunnelling will also change because the height of the potential barrier will change. If we spread a fluid of greater index of refraction than air's, for example, water, on the surface of the prisms and put them macroscopically in contact, the distance between them will be the same as if there were air between them. However, the photons would encounter a potential barrier far smaller. Tunnelling would be made easier, because a same distance between prisms would offer a greater probability:



Problem № 15: Optical Tunneling

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The problem

Take two glass prisms separated by a small gap. Investigate under what conditions light incident at angles greater than the critical angle is not totally internally reflected.

Introduction

If light is entering a prism at an angle greater than the critical, the ray will be totally reflected. However, if we place another prism close to the reflection plane of the first prism, some light will make it across the gap between the prisms. This effect, which is in fact the optical analogy of the quantum – mechanical potential barrier tunnelling of particles, is called optical tunnelling. As we shall see, it can be explained both classically with the aid of the Maxwell equations and quantum – mechanically, both explanations giving the same results; but as the classical picture is somewhat closer to us we will use the Maxwell explanation.

In the beginning, before we make any quantitative analyses, we will mention some properties of the tunnelling process observed experimentally; the conclusions we shall draw from the observations will be of great aid in constructing the mathematical model:

- The intensity of the tunnelled light falls off exponentially when the gap is increased linearly
- The passing light can be observed as long as the gap is a few wavelengths wide; if the gap is further increased, the light is too faint to detect, which means that the intensity falls off very rapidly
- The intensity of the tunnelled light depends on the refractive index of the medium between the prisms
- Naturally, the effect cannot be seen if the refractive index of the medium is larger than the refractive index of the prisms because total reflection doesn't occur either

The explanation of all these effects is in fact very simple: when the wave is reflected off the prism surface, the electromagnetic field can't be discontinuous at the boundary between the prism and the medium beyond, it has to extend a little further into the medium, decaying rapidly. That field can indeed be detected, and in the classical picture it presents the smooth transition between the field in the prism and the ″no – field″ in the medium beyond the reflection plane. The most understandable explanation of the occurrence of this transition uses Huygens' principle; it is known that a light wave in a crystal (here the prism) or medium is the result of the interference of all waves scattered on the atoms of the medium. During total reflection there is only one principal interference maximum, the reflected ray. However, in the vicinity of the reflective surface ″tails″ of the light that didn't manage to interfere completely are formed, decaying fast. These tails are referred to as the evanescent wave. If the second prism is placed in that region a new wave will be formed in that prism because of the ″tails″ shaking its atoms generating secondary emissions. Only in that case does energy leave the first prism; the evanescent wave itself carries no energy whatsoever because the electric and magnetic fields are in counterphase. The more beautiful quantum – mechanical explanation uses the Indeterminacy Principle: a photon cannot be localized with 100% accuracy, so there is always a finite probability that some of them are beyond the reflection surface, in the ″forbidden″ region. The classical, as well as the quantum theory, gives very reliable and simple mathematical results, although the interpretations are somewhat different. As we already mentioned, we will follow the classical theory, using the boundary conditions for the light waves on the reflection surface[1,2]. In the first part of the article we will present a short quantitative description of this theory and the determination of the dependence of the tunnelled light intensity on the main parameter – the gap width – in order to proceed to the experimental results and their evaluation. The quantum – mechanical theory won't be examined in detail because it gives the same numerical results as the classical[3,4].

Theory

To gain an exact relation between the tunnelled light intensity, the gap width and the refractive index of the medium between the prisms, we have to solve the Maxwell equations implementing the boundary conditions for reflection. As the prism material and the medium between them are dielectrics, we will work with Maxwell equations for a dielectric medium:

$$\nabla\mathbf{E} = -\frac{1}{\varepsilon_0}\nabla\mathbf{P}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\mathbf{B} = 0$$

$$c^2\nabla\times\mathbf{B} = \frac{1}{\varepsilon_0}\frac{\partial\mathbf{P}}{\partial t} + \frac{\partial\mathbf{E}}{\partial t}$$

where **E** is the electric field, **P** the polarization, **B** the magnetic field induction, ε_0 the permeability of vacuum and c the speed of light *in vacuum*. We know that plane sinusoidal waves of the electric and magnetic fields are particular solutions of these equations; the electric field is

$$\mathbf{E} = \mathbf{E}_0 e^{i(\omega t - \mathbf{kr})}$$

with $\mathbf{E}_0$ the amplitude, ω the circular frequency, t time, $\mathbf{k}$ the wavenumber vector (which points in the direction of wave propagation and has a length equal to $\frac{\omega n}{c}$, with n being the refractive index of the medium through which the wave is propagated) and $\mathbf{r}$ a radius vector. For the magnetic field one readily obtains

$$\mathbf{B} = \frac{1}{\omega}\mathbf{k} \times \mathbf{E}$$

The magnetic field is thus perpendicular to the electric field, a well – known result. What concerns us now are the amplitudes of the fields in the three regions: the first prism, the gap, the second prism. To obtain them we must find the boundary conditions at the region boundaries. First we must define the geometry and polarization of the incident light, because the relation between the direction of the electric field vector and the plane of propagation is quite important. The coordinate system will be such that light propagates in the xy – plane, and as a special case we will take the light to be linearly polarized in the z – direction. The reflecting plane of the prism lies in the yz – plane, and due to the ray being practically infinitely thin we assume the face to be of infinite size. The face of the second prism is parallel to the first, the distance between them being d. The wave vector of the incident light we have denoted by $\mathbf{k}_1$, the vector of the reflected light is $\mathbf{k}_r$ and the vector of the transmitted light $\mathbf{k}_t$. However, as we shall see, this vector is complex in the space between the prisms, becoming real in the second prism; that real vector is denoted by $\mathbf{k}_t'$. The notation is analogous for the electric and magnetic fields. The angle of incidence or reflection (it is clear that those two angles are equal) is φ (Fig. 1). Now we can write the waves in the first prism in the exponential form;

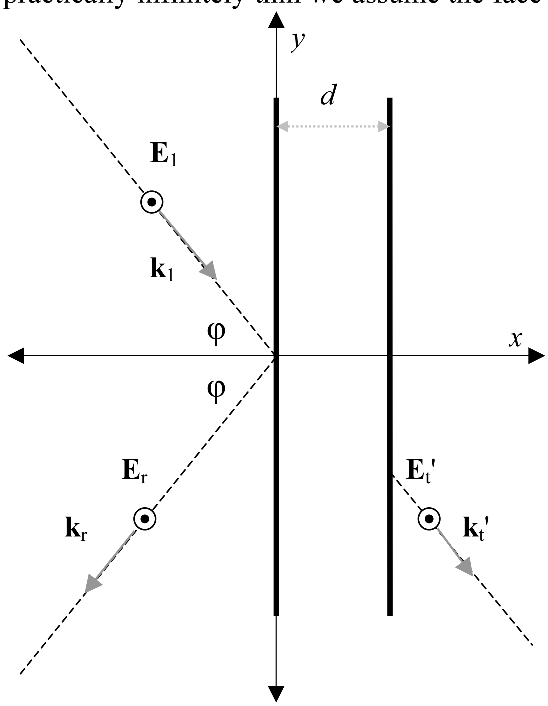

Fig. 1. Geometry of the problem

$$E_1 = E_{10} e^{i(\omega t - k_{1x}x - k_{1y}y)}$$

is the incident electric field, and

$$E_r = E_{r0} e^{i\left(\omega t - k_{rx}x - k_{ry}y\right)}$$

the reflected field. In the space between the prisms the wave will be

$$E_t = E_{t0} e^{i\left(\omega t - k_{tx}x - k_{ty}y\right)}$$

Now we want to find the relations between the wave vector components of the incident, reflected and transmitted waves. Due to energy conservation we can readily conclude that

$$E_1 + E_r = E_t$$

for all times and coordinates; that induces:

- The frequencies of the incident, transmitted and reflected waves are the same
- For the amplitudes we get

  $$E_{10} + E_{r0} = E_{t0}$$

  and

  $$k_{1x}E_{10} + k_{rx}E_{r0} = k_{tx}E_{t0}$$

  because the derivatives have to be continuous at the boundary (or the wave equation would break down at the edges which is physically impossible). Knowing that $k_{1x} = -k_{rx}$ (the incident angle is equal to the reflected) we obtain

  $$E_{r0} = \frac{k_{1x} - k_{tx}}{k_{1x} + k_{tx}} E_{10}$$

  $$E_{t0} = \frac{2k_{1x}}{k_{1x} + k_{tx}} E_{10}$$

  Of course, if the wave vector of the transmitted wave is complex, the amplitude is equal to the modulus of the complex amplitude obtained with the above relation.
- Due to light speed constancy, one obtains for the wavenumbers of the incident and transmitted waves

  $$k_{ty} = k_{1y}$$

  $$k_{tx} = \left(\frac{n_1}{n_0}\right)^2 k_1^2 - k_{1y}^2$$

  where n_0 is the refractive index of the prisms and n_1 the refractive index of the medium between them.

Now we are in possession of all ingredients necessary for finding the equation of the wave in the gap. For total reflection, the angle φ must be greater than the critical angle defined by

$$\frac{n_0}{n_1}\sin\varphi_c = 1$$

where φ_c is the critical angle. That relation is of course just a special case of Snell's law. As $k_{1y} = k_1 \sin\varphi$, the x – component of the wave vector of the transmitted wave becomes

$$k_{tx}^2 = k_1^2\left[\left(\frac{n_1}{n_0}\right)^2 - \sin^2\varphi\right]$$

If the angle of incidence is larger than the critical angle, $\sin^2\varphi > \left(\frac{n_1}{n_0}\right)^2$ and the x – component of the wave vector is a pure imaginary; inserting it into the wave equation one gets the predicted exponential drop of amplitude (the positive solution, corresponding to exponential growth, makes no physical sense):

$$E_t = E_{t0} e^{-\frac{n_1}{n_0}k_1 x\sqrt{\left(\frac{n_0}{n_1}\right)^2\sin^2\varphi - 1}}\, e^{i(\omega t - k_{1y}y)}$$

If there is only a medium of refraction index n_1 the field in this medium will decay very fast and the reflected wave will show no energy losses; energy only oscillates to and fro at the boundary but doesn't leak into the medium. That can be easily shown considering the Poynting vector; the cross – product (and accordingly the vector itself, which represents the energy carried by the wave) of the electric and magnetic fields is zero duo to their counterphase oscillations. However, if we put a second prism near to the first, the evanescent wave will shake the electrons in the second prism and cause another emission, with the fields in phase this time, and carrying energy. The energy is taken from the reflected wave, causing it to faint as the prisms are drawn closer, vanishing when they touch. Following these arguments (or referring to the expressions for the amplitudes found above) we arrive at the formula for the wave intensity in the second prism:

$$E_t' = E_{10} e^{-2\pi n_1 \frac{d}{\lambda}\sqrt{\left(\frac{n_0}{n_1}\right)^2\sin^2\varphi - 1}}$$

with λ the wavelength *in vacuum.* Due to simplicity we introduce a parameter Θ:

$$\Theta = 2\pi n_1\sqrt{\left(\frac{n_0}{n_1}\right)^2\sin^2\varphi - 1}$$

which will be measured and compared to theoretical values. To sum it all up, the presented Maxwell theory arrived at an easily investigable relation between the intensity of the tunnelled wave and the relevant parameters (gap width, refraction indices) considering only boundary conditions at the interfaces. As we shall see, the obtained expression also quite well agrees with the experimental data we produced.

Experiment

In order to obtain a larger precision and reliability, the measurements were conducted in two different frequency ranges: microwaves and visible light. With the microwaves the investigation of the phenomenon posed no problem because the gap can be up to a few centimetres wide; however, the procedure is more complicated in the optical range due to the very small gaps necessary for the effect to be measurable. The prisms must be very smooth and clean, and the measurement of the gap size is rather difficult[5]. That's why we conducted the measurements in time – the intensities of the tunnelled light were logged at fixed time intervals while the prisms were uniformly moving towards each other pushed by a slow motor. That method gives quite good results but the fast logging necessary for such small distances (a few microns!) can be a problem. But using a very slow motor (1/2 rpm) and large sampling we succeeded in measuring the effect.

The setups for the optical and the microwaves experiments look rather similar (Fig. 2.). The prisms for the microwaves were made of paraffin, with dimensions 150x150x100 mm. The gap between the prisms was varied with a simple wooden translator moved by a long screw. The source of the necessary radiation was a

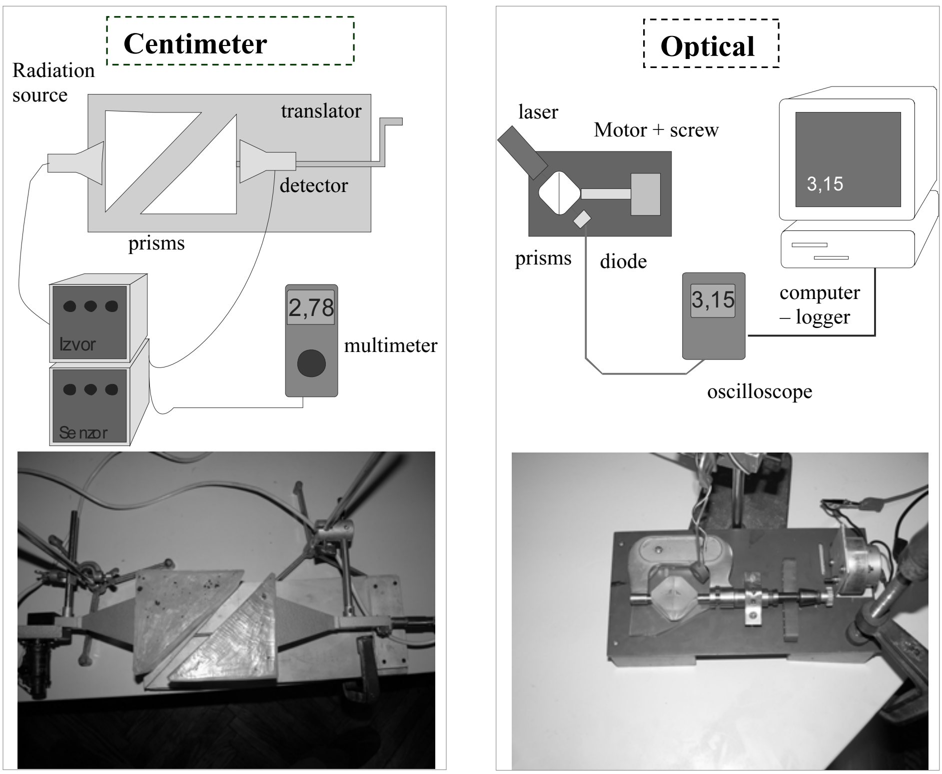

Fig. 2. Experimental setups. The setup for microwaves is on the left and the optical setup on the right

magnetron pentode with an amplifying horn which was placed close to the prism during measurement. The detector for the radiation which was used in tunnelled wave measurement consisted of an amplifying horn with a resonant space containing a 100mH coil which was placed at a node of the wave in the tube. Thus the voltage read from the coil is directly proportional to the field, not the intensity (like when working with photodiodes). The wavelength of the waves was 3.0 cm, making little scratches and defects on the prisms unnoticeable. The optical setup was quite similar, only that the prisms were made of glass, mounted in the screw driven by the motor. The velocity of the prisms was 0.025 cm/min (0.6 seconds for a micron), and the intensity was logged every fiftieth of a second. The data had later to be smoothed due to imperfections (and dust) on the prisms and noise, resulting in a 10 measurements per second sampling. A 780 nm laser diode with polarizer was used as the light source. The transmitted light was detected by a photodiode which gave signal proportional to the intensity (the *square* of the electric field) of the light.

The results of the measurements are in relatively good agreement with the theoretically predicted curves, the agreement being better for the microwaves due to much greater precision and a larger number of points (Fig. 3.). The slight deviations from the theoretical curve at low intensities are probably caused by the unfocusedness of the source or voltage variations on it; anyway, the deviations were completely random throughout the measurements so they are probably some indeterminate noise. In the optical range (Fig. 4.) the disturbances were very large, making high – sampling measurements impossible, and the discrepancies are larger due to smaller precision and defects (for example dust particles, the size of about a micron, can influence the results badly).

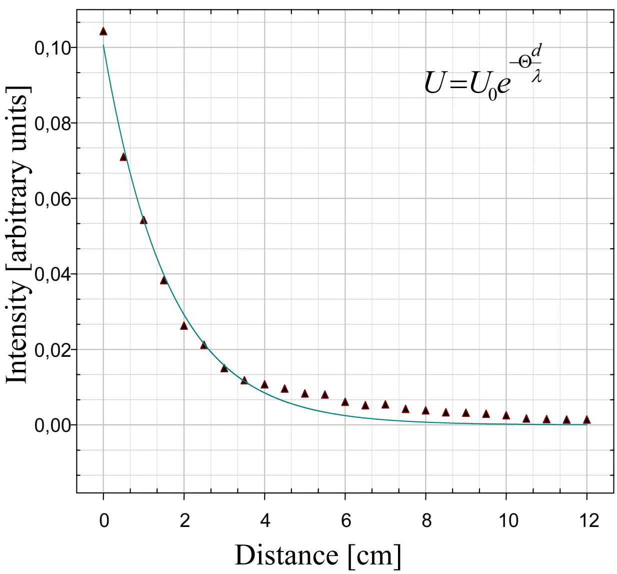

Fig. 3. The experimental curve and fit for the microwave experiment

A complete comparison between experiment and theory wasn't possible because we didn't know the relation between the measured detector voltage and the real field, but as the functions look the same it was enough to compare the decaying factors in the exponents. In the optical case we have to perform a little transformation due to the time – measurements; the gap width, d, becomes $d_0 - vt$, with d_0 being some starting value (not necessary for the comparison), v the prism velocity and t time, the measured variable, and of course we had to account for the fact that the measured output voltage of the photodiode was propo-tional to the square of the field. The refractive index of the glass prisms was 1.48 (measured) and the paraffin had 1.50 for our wavelength. The refractive index of the air between the prisms was taken to be 1.00 and the angle of incidence was 45° . That leads to the following results for the decay factor Θ (all the data have been given again for sum – up):

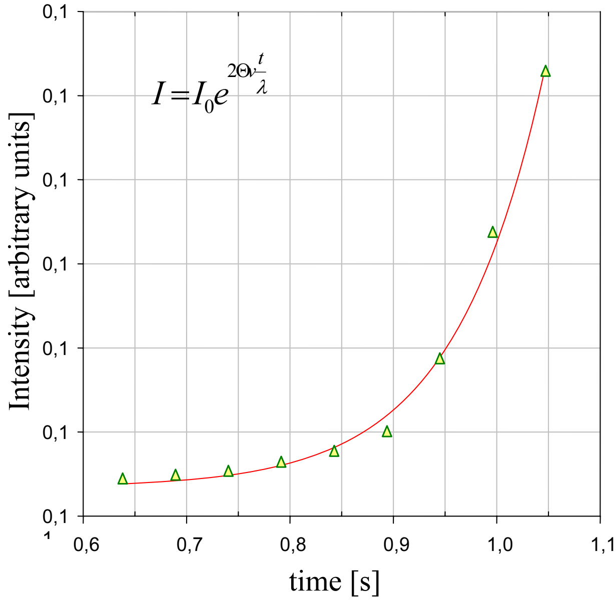

Fig. 4. The experimental curve and fit for the optical experiment in time mode.

Prism refractive index, n_0	1.48
Wavelength	780 nm
Angle of incidence	45°
Theoretical Θ	1.9
Experimental Θ	1.1±0.1

Prism refractive index, n_0	1.50
Wavelength	3.0 cm
Angle of incidence	45°
Theoretical Θ	2.2
Experimental Θ	2.5±0.1

We see that the agreement is rather good, especially in the centimetre range; the measurement was, as mentioned, rather more precise there; the agreement to a factor of 1.7 in the optical range is quite satisfactory considered all errors.

Conclusion

To sum things up, we can conclude that we have approached the problem of optical tunnelling and solved it to a certain depth, appropriate to our resources. We have obtained the fundamental relations between the intensity of the tunnelled light and the gap width using elementary classical theory and checked the theoretical results in experiment, with two different wavelengths in two different ranges of the spectrum. The microwave measurements enabled us to perform high – precision tunnelling measurements and obtain valuable quantitative data thanks to the macroscopical size of the gap, while the optical experiment served a more demonstrational purpose due to the difficulty of obtaining reliable measurements, though we tried to do that too. The parameters entering our intensity formula are the gap width as the most obvious and the prism and medium between prisms refractive indices. In our investigations we mainly focused oh the gap, somewhat neglecting the refraction indices. However, we are of the opinion that the gap is indeed much more important and maybe more fundamental to the effect itself than the refraction indices – their variation in fact only induces a change of the critical incidence angle which is not as important. On the other hand, measurements with varying refractive indices could provide an even more firm confirmation of the theory. Also, our formulas were obtained using classical theory; a more complete investigation of optical tunnelling might include a more detailed treatment of the quantum, photonic picture, linking it to the classical formalism. We have chosen the classical theory in our work due to its simplicity, in spite of the beauty of the quantum model, not having the space or time to make more detailed theoretical investigations.

With the suggested adds, for which we didn't have the time or equipment, included, we could say that the problem of optical tunnelling would be rather completely solved. And in the end we may briefly answer the question posed by the problem itself: light incident at angles greater than the critical angle is not totally internally reflected if the second prism is as close to the first as a few wavelengths of the light used.

Acknowledgements

We thank Silvije Vdović, Ticijana Ban and Goran Pichler from the Institute of Physics for help and advice in conducting the optical experiment, the workshop staff of the IF for help with the optical apparatus, Ivica Aviani and Željko Marohnić for discussions, Tihomir Surić for corrections and suggestions in the theory, Hrvoje Mesić for help with the microwave experiment, Želimir Miklić for the magnetron and horns and our mentor Dario Mičić for support and help with everything.

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8.3. SOLUTION OF UKRAINE

Problem № 15: Optical tunneling

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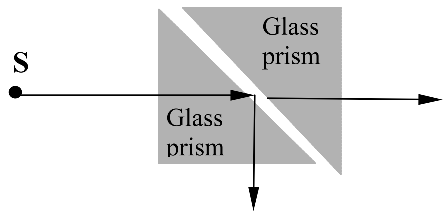
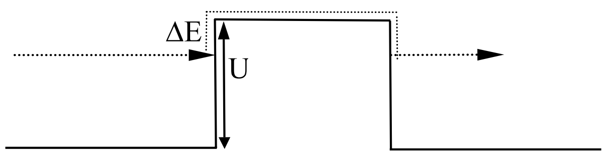
The problem:

Take two glass prisms separated by a small gap. Investigate under what conditions light incident at angles greater than the critical angle is not totally reflected.



At first we should understand what is tunneling. Tunneling is the phenomenon when a particle gets through the potential barrier if its energy is less than barrier's height.



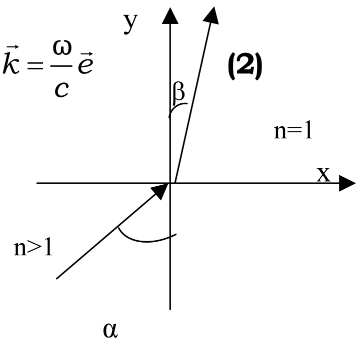
We will investigate this phenomenon from two different sides of view: optical and quantum mechanical.

Let's start with the optical approximation. The electromagnetic wave is traveling in the first prism. Than it comes into the air and starts to dampen. If there is no other prism then it disappears very fast but in our case it continues to go in the second glass prism and we see two rays: reflected and transmitted.

$$E(\vec{r},t) = E_0 \cdot \exp\left(i(\omega t - \vec{k}\cdot\vec{r})\right) \quad \textbf{(1)}$$

At first, lets write the equation of the running wave:
where k is the wave vector of the ray, r is the radius vector of the point at considered moment of time.

$$\vec{k} = \frac{\omega}{c}\vec{e} \qquad \textbf{(2)}$$



After the refraction of the light ray we get:

$$\vec{e} = (\sin\beta; \cos\beta) \qquad \textbf{(3)}$$

$$\vec{k}\cdot\vec{r} = \frac{\omega}{c}\left(x\sin\beta + y\cos\beta\right) \qquad \textbf{(4)}$$

By the Snellius law of refraction:

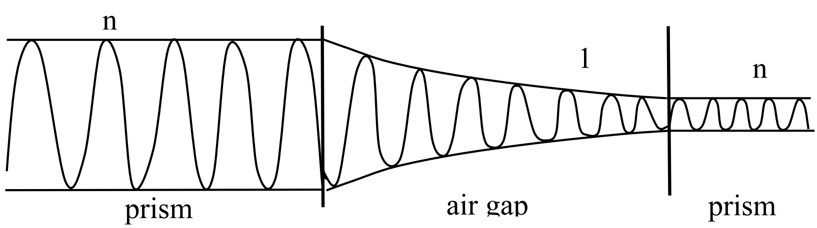
$$\frac{\sin\beta}{\sin\alpha} = n \qquad \textbf{(5)}$$

Condition for the critical angle:

$$n\sin\alpha > 1 \qquad \textbf{(6)}$$

$$E(\vec{r},t) = E_0 \exp\left(-\frac{\omega}{c}y\sqrt{n^2\sin^2\alpha - 1}\right)\exp i\left(\omega t - \frac{\omega}{c}x\sin\alpha\right) \qquad \textbf{(8)}$$

A little manipulation yields:

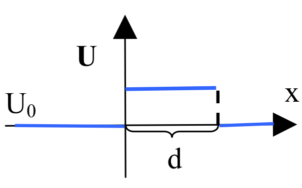


$$\frac{\omega}{c}L_{typical}\sqrt{n^2\sin^2\alpha - 1} = 1 \qquad \textbf{(9)}$$

$$\frac{\omega}{c} = \frac{2\pi}{\lambda}a \Longrightarrow \boxed{L_{typical} = \frac{\lambda}{2\pi\sqrt{n^2\sin^2\alpha - 1}} \qquad \textbf{(10)}}$$

It is obvious that for nsinα<1 light will just reflect and we won't see transmitted ray.

Now we come to the quantum mechanical solution. U(x) is the function of the energy distribution, d is the length of the gap between two prisms.



$$\mathbf{U}(\mathbf{x}) = \begin{cases} 0, \mathbf{x} > 0 \\ \mathbf{U}_0, \mathbf{x} \in [0; \mathbf{d}] \\ 0, \mathbf{x} > \mathbf{d} \end{cases}$$

Let’s write Schrödinger's equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi + U\Psi \qquad \textbf{(1)}$$

But in our case U doesn’t depend on time so we are going to look for the stationary solution:

$$\Psi(x,t) = a(x)\exp(-i\omega t) \qquad \textbf{(2)}$$

where

$$\omega = \frac{E}{\hbar}$$

$$\Rightarrow \quad \Psi(x,t) = a(x)\exp\left(-\frac{iEt}{\hbar}\right) \qquad \textbf{(3)}$$

$$U(\mathbf{x}) = \begin{cases} 0, \mathbf{x} > 0 \\ \mathbf{U}_0, \mathbf{x} \in [0;\mathbf{d}]\mathbf{a} \\ 0, \mathbf{x} > \mathbf{d} \end{cases}$$

$$a(x) = A\exp\left(\pm\sqrt{\frac{2m(U-E)}{\hbar}}x\right) \qquad \textbf{(4)}$$

$$a(x) = A_1\exp\left(-\sqrt{\frac{2mE}{\hbar}}x\right) \qquad \textbf{(5)}$$

$$a(x) = A_2\exp\left(-\sqrt{\frac{2m(U-E)}{\hbar}}x\right) \qquad \textbf{(6)}$$

$$a(x) = A_3\exp\left(-\sqrt{\frac{2mE}{\hbar}}x\right) \qquad \textbf{(7)}$$

$$A_1 = A_2 \qquad \textbf{(8)}$$

$$A_3 = A_2\exp\left(-\frac{\sqrt{2m(U-E)}}{\hbar}d\right)\exp\left(i\frac{\sqrt{2mE}}{\hbar}d\right) \qquad (9)$$

$$\frac{\sqrt{2m(U-E)}}{\hbar}L_{typical} = 1 \qquad \textbf{(10)}$$

$$\boxed{L_{typical} = \frac{\hbar}{\sqrt{2m(U-E)}} \qquad \textbf{(11)}}$$

Now, when the problem is solved from two points of view we can bring to confrontation optical relevant parameters with mechanical.

$$L_{typical} = \frac{\lambda}{2\pi\sqrt{n^2 \sin^2 \alpha - 1}} \iff L_{typical} = \frac{\hbar}{\sqrt{2m(U-E)}}$$

$$m = \frac{E}{c^2} = \frac{h\nu}{c^2} \qquad c = \lambda\nu \qquad h = 2\pi\hbar$$

$$L_{typical} = \frac{\lambda}{2\pi\sqrt{n^2 \sin^2 \alpha - 1}} \iff L_{typical} = \frac{\lambda}{2\pi\sqrt{2(\frac{U}{E} - 1)}}$$

$$\boxed{n^2 \sin^2 \alpha \iff \frac{2U}{E} - 1}$$

Maxvel's equations have the same mathematical sense as the stationary Schrödinger's equation so their solutions for the corresponding initial and final conditions have to be the same from the mathematical approach. In this problem we see such similarity between relative index of refraction with angle of incident ray and ratio of potential energy of the barrier and of the particle.

Acknowledgments:

Special thanks to:
Oleg Matveichuk, main author of the idea.

Problem № 16: Obstacle in a funnel

Reporter: Nóra Horváth

The problem

Granular material is flowing out of a vessel through a funnel. Investigate if it is possible to increase the outflow rate by putting an 'obstacle' above the outlet pipe.

Introduction

Granular materials are more common than one would guess; let us just think about agriculture, building-trade or plastic industry. For handling so many times with these in some aspects extraordinary materials, people have thoroughly studied their properties in order to find a more efficient way of storage and usage.

Background

A granular material consists of many macroscopical (i.e. above 10 μm) particles. In this range of dimension there are three main forces acting inside the system: the gravity force, the repulsive force between touching particles and the friction force between particles at contact points.

Probably the most interesting property of granular systems is that if applying stress onto the aggregation, above a certain threshold of stress the particles may jam up. The cause of this jamming is that particles form so called force chains in compressional dimensions. These chains can be modeled as linear strings of rigid particles in point contact. Chains only support mass along their own axis so they are strictly collinear. They end on the walls of the container (if there is any, in other cases they end on the bottom of the aggregation), thus there is a significant pressure on the wall and/or on the bottom. The force applied to the granules is mediated to the walls by the force chains. If the force in a chain is too large or its direction changes, then the force chain is broken. This can happen when the granular material is stirred or moved, and afterwards a network of new force chains is formed.

We investigated the phenomenon referred to experimentally. For our measurements glass funnels of three different sizes were used (diameters 0.6 cm, 0.8 cm and 1.2 cm). As granular materials we chose sand, semolina, lead-balls (diameters 0.25 cm and 0.5 cm) and plastic 'balls' of an irregular shape. And finally, we used a wide range of obstacles. The funnels were attached to an

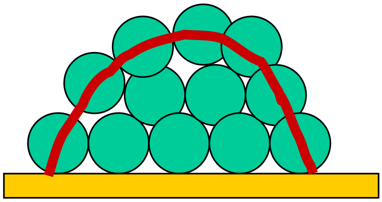


Fig. 0 Simplified conception of a force chain building up in an aggregation

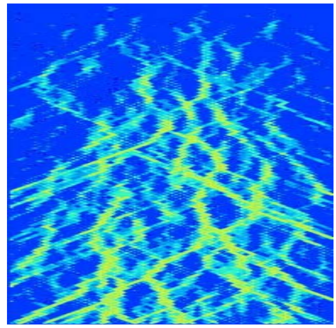


Fig. 2 Experimental observation of force chains building up in a granular system (Ref. [1])

eprouvette-stand. Material of given volume (125 cm^3) was poured into the funnel (while the outlet pipe was kept closed)

Fig. 3 Granules, funnels and obstacles used

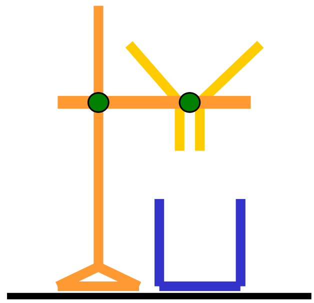


Fig. 4 Schematic figure of the way of attachment

As our first experiment, we measured the flow time of each material in the funnels without an obstacle (t_o). Afterwards as further experiments we measured the flow rate by placing an obstacle inside the funnel (t). We varied some parameters: shape of the obstacle (blunt (dull), peaked and spherulitic), diameter of the obstacle (0.3 cm, 0.8 cm, 1.2 cm) and the distance of the obstacle from the mouth of the funnel (h).

We assumed namely, that following parameters may be relevant:

- the size of the particles (d)
- the shape of the particles
- the material of the funnel – may influence friction
- the material of the obstacle – may also influence friction
- the diameter of the obstacle (d_{obs})
- the diameter of the funnel mouth (Φ)
- the distance between the obstacle and the mouth of the funnel (h)

Our observations Three different cases were analyzed. In the first case the obstacle acted really as an obstacle: the flow was hindered. In the second case the obstacle surprisingly increased the flow rate – this was the main point of our study. We also had cases in which the obstacle had no influence on the flow rate at all.

During experiments we noticed the fact that the outflow time reducing effect only appears if $d_{obs} > 3d$, so we had to take this into account in our investigations. This means that we did not go through all measurements in cases like pouring big lead-balls (d = 0.5 cm) into a medium-sized (Φ = 0.8 cm) funnel.

1. Firstly, *concerning the obstacle*s see Table 1.

Obstacle	Average flow time (s)
None	3.8
Blunt (all sizes)	3.8
Peaked (0.8 cm)	3.4
Peaked (1.2 cm)	3.6
Spherulitic	*3.26*

Table 1 Comparison for one material (small lead-ball, d = 0.25cm) in a given funnel (Φ = 1.2 cm) at given mouth-obstacle distance (h = 1 cm)

As it is clearly noticeable, the obstacle with a spherulitic end had the largest influence on the flow rate.

2. It is also important to observe, that there is a *certain region* ($\Delta h = h_{max} - h_{min}$) in which the time of outflow is reduced. This value depends strongly on the granules investigated.

	h_{min} (cm)	h_{max} (cm)	Δh (cm)
small lead-balls	1	2.3	1.3
big lead-balls	1	2	1
Semolina	0	1.6	1.6

Table 2 The region (Δh) in which reduced time of outflow can be found in case of spherulitic obstacle in a funnel (Φ = 1.2 cm)

3. Let us now investigate what happens if we change the *mouth-obstacle distance*!

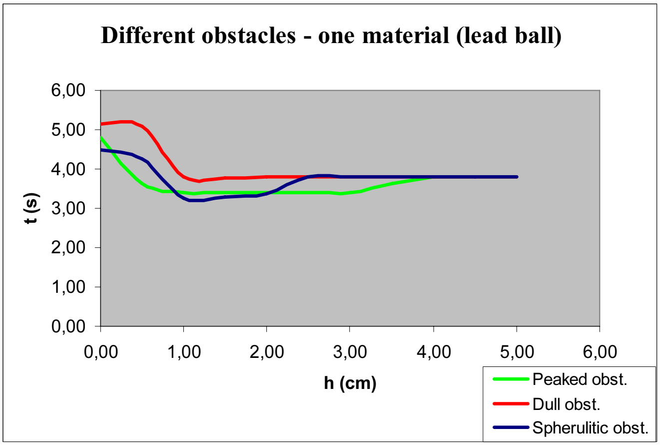


Fig. 5 Comparison of the obstacles by changing the mouth-obstacle distance (material was given)

As one can see there is a well-defined height at which the reduced time of outflow reaches its maximal value. It is also very remarkable that the previously recognized schema (see figure 5) remained: the spherulitic obstacle had the largest outflow time reducing influence on the granules, followed by the peaked obstacle and finally the blunt one. However the peaked obstacle is special in some way: the reducing effect appears even at relatively small mouth-obstacle distances. This may be explained with its conical shape

Explanation

As already told, force chains building up in a granular aggregation end on the walls and bottom of the vessel (in our case there is no 'bottom' of the vessel, for we investigate funnels).This is not the case if there is some kind of obstacle inside the system: the obstacle hinders the formation of the 'basic' chains, a different force chain network forms where some chains will end on the obstacle itself. This means of course a smaller compression of the bottom particles, which allows them an easier motion or flow in optimal cases (see figure 8). However, what do we mean by optimal case? By optimal case we mean that if placing the obstacle too near the mouth, it will really act like a physical obstacle, and the rate of outflow is reduced. The other extreme case is when we place our obstacle too far from the mouth of the funnel. In this case force chains build up even in regions below the obstacle, so the compression on the bottom particles becomes significant again and jamming appears.

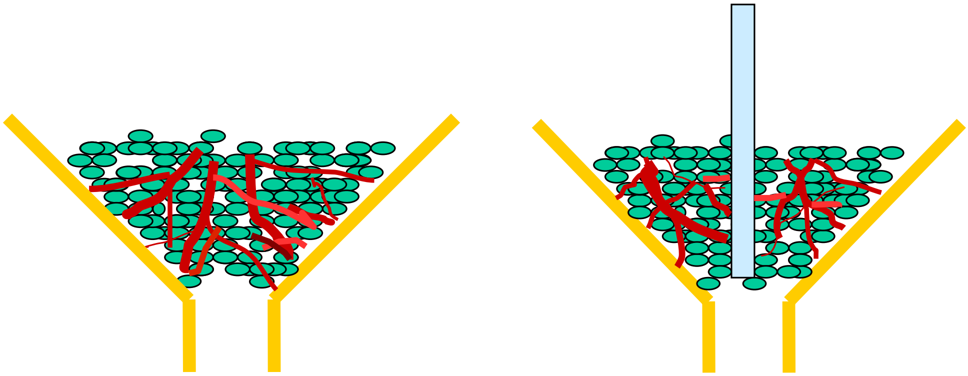


Fig. 6 The schematic diagram of a granular system in a funnel without an obstacle and with an obstacle. The red lines indicate force chains

The model

Probably the most difficult part of our investigation was to set up an appropriate model. Granular systems are namely far more complicated than non-granular ones, and thus the physical description is very complex.

But before introducing our model, let us make a detour to another, in some aspects surprisingly similar phenomenon, called pedestrian escape panic. That means, if a room is crowded with people and somehow they are forced to leave the place (for example if flames come up for some reason), just like granular particles, people may jam up at the exits. However it is a known fact that columns placed near the gates help people to get out. This observation is very similar to our original topic. That means a crowd can be modeled as a self-driven many-particle system.

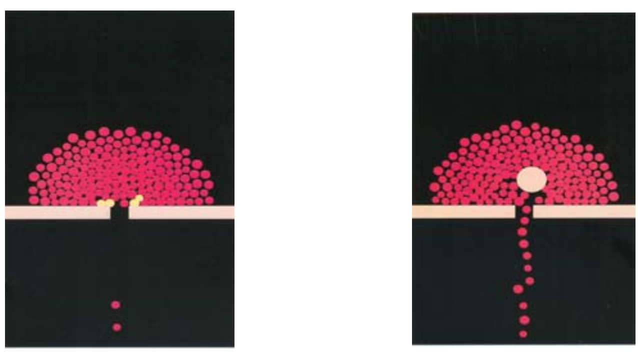


Fig. 7 Influence of a column placed in front of an exit (Ref. [4])

Escape panic can be correctly modeled. In Ref. [4] the following equation is suggested for the description:

$$m_i \cdot \vec{a}_i = m_i \cdot \frac{d\,\vec{v}_i}{dt} = m_i \frac{v_0 \cdot \vec{e}_i - \vec{v}_i}{\tau} = \sum_j \vec{F}_{ij}^{rad} + \sum_j \vec{F}_{ij}^{tg}$$

where **a** stands for the acceleration of the particles, $\mathbf{v_0}$ for the desired speed, $\mathbf{e_i}$ the desired direction, $\mathbf{v_i}$ the adapted actual velocity divided by a certain characteristic time τ. The interaction forces shown in the equation mean the radial (F^{rad}) and the tangential (F^{tg}) components of the force rising between two particles (see figure 10).

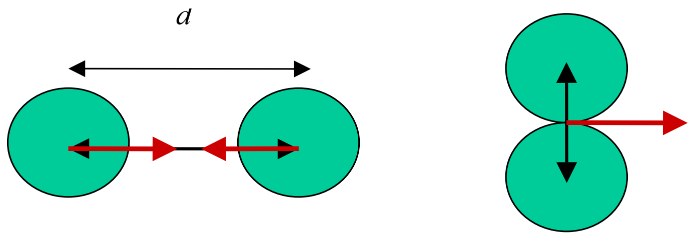


Fig. 8 Radial and tangential interaction force between two pedestrians

We applied this model to our problem, as well.

Although the main idea was the same, there are some remarkable differences between the two models. Let us now compare granular flows and panicking crowds.

The main difference is that granular flows are always accelerated by gravity instead of varying factors like in crowds of people. It is also extremely important to emphasize that while the tangential force (F^{tg}, friction force) is proportional to the force compressing the surfaces in both systems, the radial force cannot be given by such an idea. Namely the repulsive force rising for keeping off of each other is a long-range interaction ($d > r_1+r_2$) which does not exist in granular systems. Additionally, the repulsive force rising when colliding is a short-range interaction ($d = r_1+r_2$) which does not appear in a panicking crowd.
Despite these remarks the difference between the granular system model and the escape panic model (Ref. [4]) is relatively small, so most of the calculations and simulations made for an escaping crowd are also valid for granular systems investigated here.

Finally, we got the following equation as our mathematical model for the problem:

$$m_i \cdot \vec{a}_i = m_i \cdot \frac{d\,\vec{v}_i}{dt} = m_i \frac{v_0 \cdot \vec{e}_i - \vec{v}_i}{\tau} = \sum_j \vec{F}_{ij}^{rad} + \sum_j \vec{F}_{ij}^{tg} \quad \underbrace{\left(+m_i \cdot \vec{g}\right)}_{gravity}$$

Conclusion

As you could see the movement of granular materials differs from the motion of liquids as well as the moving of single particles, and this was what made our job hard for it is very difficult to describe this extraordinary movement quantitatively. However with the help of another model we could draw our own one, as well.
Finally, to answer the basic question, yes, we clearly found that it is possible to increase the outflow rate by using a kind of obstacle in the funnel. The cause of this phenomenon is that the obstacle influences the building up of force chains in our granules the stability of which thus decreases drastically.

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Problem № 17: Ocean Solaris

Janečka, A. – Kudela, J. – Lejnar, O. – Solny, P. – Roženkova, K. – Kluiber, Z.

The problem

A transparent vessel is half-filled with saturated salt water solution and then fresh water is added with caution. A distinct boundary between these liquids is formed. Investigate its behaviour if the lower liquid is heated

Ocean Solaris is very unique problem that is mainly because of two reasons. The first is that the setting of this task is not absolutely clear; second no parameters under which you should work out the solution are mentioned, so you have to choose from wide spectrum of possible solutions.
Setting says: "A transparent vessel is half filled with saturated salt water solution and then fresh water is added with caution. A district boundary between these liquids is formed. Investigate its behaviour if the lower liquid is heated."

You are given a hint how to prepare this experiment but you also have to solve how to put the fresh water on the saturated salt water solution (SSWS) first. And the mainly thing is that you have to work out the entire problem experimentally without any reasonable parameters given by the setting. So I had to do all the work with great diligence in order to do all necessary experiments.

(Used shortcuts: SSWS – saturated salt water solution, DW – distilled water, US – upper solution, LS – lover solution, OSE – Ocean Solaris effect)

I will try to describe my main founds in this article and I would like to mention the real Ocean Solaris effect (OSE) also.

Technique of the experiments

In order to prevent mixing fresh water with SSWS I used paint brush and burette fixed in a holder so tat I could easy module flow and lifting force applied in the brush broke speed of the flowing water that means the boundary was not disturbed. I revised the clearness of the added water by measuring its electro conductivity before and after water addition. Difference between measured data using this procedure was insignificant (average difference was less than 25μs micro siemens, for better measurements was used distilled water). In reality the OSE is not so influenced by the little gap between clearness' of the added water but if the difference is big enough, than the OSE is worse observable. Most of the experiments I did, was made for principal temperature 20ºC (US, LS, temperature of the air) but I also tried to change this so that I used warmer water. But nearly nothing was changed and the higher the principal temperature was, the worse could be the experiment prepared because ions went through the boundary faster.

How the boundary is formed

Boundary between SSWS and DW is like boundary between oil and water but oil doesn't do stable solution with water (standard conditions). Different densities prevent fast mixing so that the boundary appears. After about an hour you can see how the boundary is changed. Its sharpness has disappeared and now you can see SSWS gentle changing in to the upper solution which now contains hundred times more ions. After six hours you get "wary salt water" and here is no boundary any more.

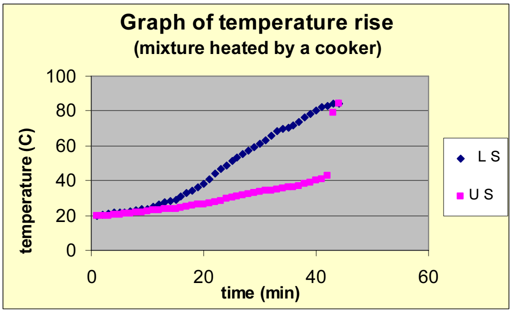
This mixing is caused by oscillating molecules of the water which disturb the boundary and allow more ions get through it. Boundary by itself is not able to prevent anything, which has higher density than the lower solution has, fall down but if you drop something with the right density (smaller than density of LS and bigger than density of US) it seems that this thing stand on the boundary. You can use it for demonstration of changing density during heating but it can influence the effects than.

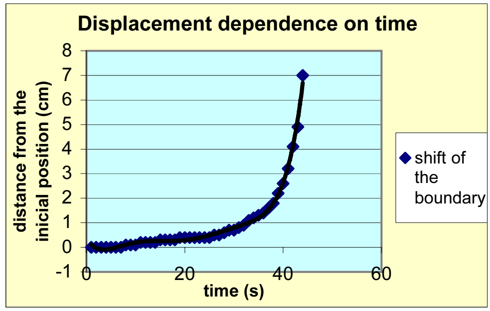
During heating the boundary starts to wave and after some time it disappears. This is the point which nearly all the experiments have common but nothing else. Because of many varieties of the experiments I will mention only two most interesting.

"Insulation" and shift of the boundary

(Main parameters of the experiments: $t_{(SSWS)} = t_{(DW)} = 20$ºC, $V_{(SSWS)} = V_{(DW)} = 180$ml dw = distilled water) When prepared system is heated on a heater you can find really interesting thing if you compare measured temperatures from both solutions. The temperature of the lower (SSWS) liquid is higher and rise faster than the temperature in upper solution. After some time the temperature difference is stabilised on about 40ºC than the lower solution reach about 85ºC (it can even boil! But bubbles destroy the boundary than) and the boundary started to lift fast. In reality the boundary has been lifting since the temperature of the lower solution reach about 60ºC but it hasn't been well observable before. With increasing temperature boundary moves faster and faster (due to the measured data shifting velocity rises exponentially in dependence on time, the power of the heater is constant).

This phenomenon is caused by salinity of the solution which tries to keep the boundary and decreasing density which tries to destroy the boundary (density of the lower solution gets near to the density of the upper solution).

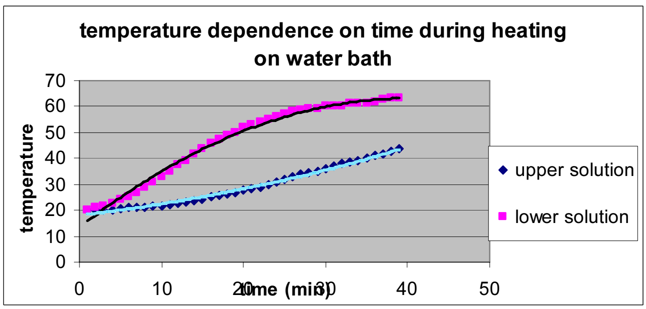




When densities (better say the solutions) are balanced than the only thing which keep them separated is the boundary. But it is not right to say that the densities are the same because density of the US is not constant in whole volume and density of the LS also is not constant in whole volume.

In the graphs you can see data from one of the measurements. Step temperature change in US means that the boundary went around the thermometer.

For heating on the water bath you don't reach high temperatures and the boundary doesn't shift but the temperature difference is also observable.



Main part of the heat added to the system is "store" in the lower solution so that the kinetic energy of the ions is bigger because of that they can go through the boundary faster and salt concentration in US rises. But the waving boundary is still well observable and the US still keeps its lower temperature. You can also see eddy currents in the LS because the added heat causes that the density is not the same in whole volume of the LS and they mixes LS in order to change the density step by step. Sometimes you can see small bubbles, which consist of salt water with different concentration of salt in them than the concentration in the mixture around them is. They appear on the bottom and go up where are stopped by the boundary (they cause bigger waves) and disappear after a while. I didn't study this effect much but I mention it in the part about "real" OSE at the end of this article because I explain OSE by it.

Shifting of the boundary means that molecules of water are transported trough the boundary down to the solution with higher salinity in order to keep the boundary in system and they provide more “space” for the added heat (temperature in US is really much more lover than the LS).

Ocean Solaris

Real Ocean Solaris effect is observable in quite big vessel with volume more than two litters and radius must be bigger than fifteen cm. It is very important how fast you are able to prepare the system for this experiment because is disturbed during the time. The absolutely same volume of the solutions is not necessary but volumes should get near each other. You also need quite large heater ensuring uniform hating of the bottom. The boundary starts to wave at first quite gently but than the waves become bigger and bigger and when they are big enough the boundary is destroyed and this effect is really gorgeous. I didn’t overwhelmingly prove that the waving was caused by bubbles formed by less concentrated salt water solution but style of the destruction of the boundary indicated that something like huge bubbles arising at the bottom could really made this effect.

I think this is possible because the larger bottom provide more space and small bubbles which at first stay at the bottom can interweave and make themselves in one really huge bubble which still stay at the bottom until its density is so low that this bubble can easy break away from the bottom and it can hit the boundary with a great bump which is able to smash it. The main problem is the detection of these bubbles because it is hard to observe the smaller ones and I except densities aren’t so different from the rest of the solution so it is really hard to observe them.

The second reason why I thing this effect is caused by bubbles is that I didn’t realise eddy currents in the lower solution as it was usual in the smaller vessels. Eddy currents mix the lower solution because temperature transport is not ideal so temperature at the bottom and near the boundary could be different. If the bottom is very small or quite large eddy currents don’t appear or appear after longer time. If there are not eddy currents in (heated) LS than is more probable you can observe the “bubbles”. If there are not eddy currents than there have to be bubbles but you can’t actually see them, the only thing you can observe is waving of the boundary and its destruction.

In fine I would like to say, that OSE how it was described in official solution of this task, appears to be less important if it is compared with the other effects which was observed during solving this task.

Problem № 17: Ocean Solaris

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Problem

A transparent vessel is half-filled with saturated salt water solution and then fresh water is added with caution. A distinct boundary between these liquids is formed. Investigate its behaviour if the lower liquid is heated.

As you see we have some conditions which we have to create in laboratory conditions to observe some specific and unexpected behaviour of the 'distinct boundary'. Experiments made by our team in our laboratory conditions gave such results:

While heating process, the boundary was rising and at the same time, the waves that occur in it disturbing the surface of the boundary. Let's explain the cause of this phenomenon.

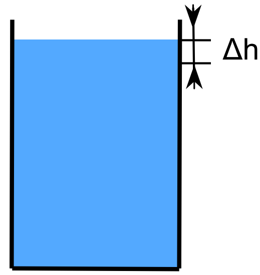
The causes the waves to occur are the convection flows create as the heating process begun. These convection flows exist because of temperature gradient, inside the salt solution. But there's one problem: these waves are impossible to be described using any method. This is because of the place of the convection flow pushes the boundary and other parameters are almost random, or factors that define these parameters are random, anyway they are impossible to be described in our model.

So let's look at another effect – rising of the boundary's height. Here everything is a bit easier. Here is the list of effects we consider in order to describe this raising:

1. Thermal expansion
2. Surface tension
3. Bubbles of gas
4. Convection flows

Thermal expansion

Thermal expansion is an effect of changing of the volume of the body (in our case it's salt solution) with increasing of its temperature. Using some approximations we got a result of changing height (Δh) of the boundary due to thermal expansion about 1mm. The same value was on experiment – about 2mm



Surface tension

Surface tension is not very important in our conditions, because of surface tension coefficient is very small. We have made both theoretical and experimental research on this question and had as a result surface tension coefficient is about $0.01 N/m^2$.Comparing with water this coefficient is very small.

We used method of wire detachment for measuring surface tension coefficient (and this is the equipment we used fir this).

Bubbles of gas

Bubbles of gas have big influence on height of the boundary only when the liquid is near the boiling point. In this case big bubbles that can form in the case of narrow vessels like a piston that pushes water out. In wider vessels this effect is not worth considering (in our work we used quite wide one). So we consider case when bubbles of gas have no strong influence on behaviour of the boundary.

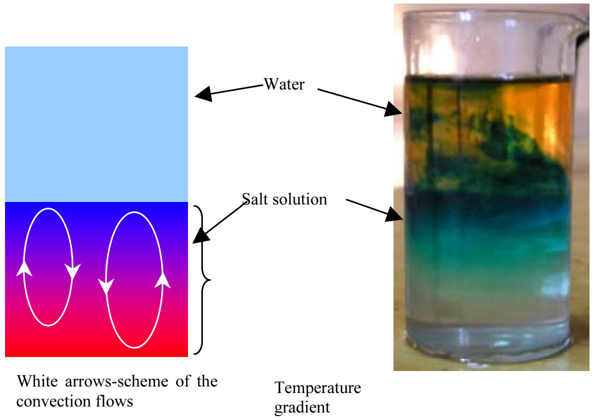
Convection flows

In our opinion convection flows have the biggest influence on the raising of the boundary. But this is right only when the conditions for the convection flow to occur are fulfilled, These conditions are: existence of the temperature gradient between different layers of liquid(in our case it is salt solution) and vessel shouldn't be very narrow(in this case bubbles and thermal expansion are important).Let's consider wide vessel(its cross section parameter is comparable with its height) and try to calculate some characteristics of the convection flow(this would be its speed).This will be done by dimension considerations:

Here's the list of values on which speed of the convection flow depends

- ✓ ΔT-temperature difference between upper and lower layers of the salt solution

- ✓ η-dynamic viscosity of the salt solution
- ✓ ρ-density if the salt solution
- ✓ β-thermal expansion coefficient



Here is the formula for the speed of the convection flow depending on the parameters, mentioned below:

$$V = \sqrt[3]{\frac{\eta \cdot g}{\rho}} \cdot f(\beta \cdot \Delta T),$$

where $f(\beta \cdot \Delta T)$ – dimensionless function, which we approximate with linear function, because $\beta \cdot \Delta T$ is quite small (for real values of β and ΔT, their product is also <<1), so in expansion into a Tailors series, we can neglect all terms, which power is more than 1. This means that $f(\beta \cdot \Delta T) \approx f(0) + A \cdot \beta \cdot \Delta T$, where A - is some coefficient and $f(0) = 0$ – because if the temperature difference is zero convection flows doesn't occur.

At last we get $f(\beta \cdot \Delta T) \approx A \cdot \beta \cdot \Delta T$.

Next step. Consider a small volume of salt solution, moving up from the bottom. Its density is smaller than surrounding liquid. Writing the Second Newton's law for this small volume, we can get formula for its acceleration when it gets the boundary:

$$a = \frac{\Delta\rho}{\rho} \cdot g,$$

where $\Delta\rho$ is the density difference between water and salt solution, it is negative. Then knowing acceleration and initial speed we can calculate the height on which this small volume can "jump" above the boundary, and thus, thus the whole boundary (because there are a lot of convection flows at the same time). This looks this way:

$$h = \sqrt[3]{\frac{\eta^2 \cdot \rho}{g`} \cdot \frac{(A \cdot \beta \cdot \Delta T)^2}{2\Delta\rho}}`$$

This formula isn't very precise, because we neglect the changing of viscosity, while liquid flows from salt water to fresh, and the velocity of the flow is estimated by the dimensions considerations.

Except of these problems, we have to remind that during the time all parameters are changing.

Also we have to remember that in truth to say – relevant parameters (such as density, temperature, viscosity) are continuously changing from point to point, so we have to find distribution of these values. This problem is very difficult from mathematical point of view.

So we preferred the other way. We made an experiment, where the boundary height measured on time, and using the results selected the possible dependencies for relevant parameters on time. Than we made experiments with other initial density difference (different salt concentrations) and determined that the found dependencies work.

We got the next empirical formula:

$$h(t) = \sqrt[3]{\frac{\eta^2 \cdot \rho}{g} \cdot \frac{(A \cdot \beta \cdot \psi \cdot t)^2}{2 \cdot (\Delta\rho_0 - \xi \cdot \sqrt{t})}}`$$

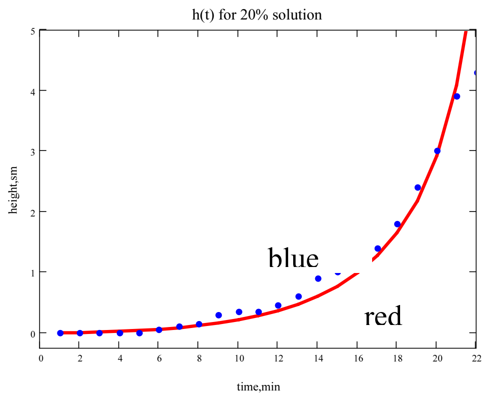
Coefficients ψ, ξ, A - are different for each kind of vessel and depend on the power of heating and quantity of fresh and salt water in the vessel. So we made experiments taking these requirements to regard, while stating previous formula.

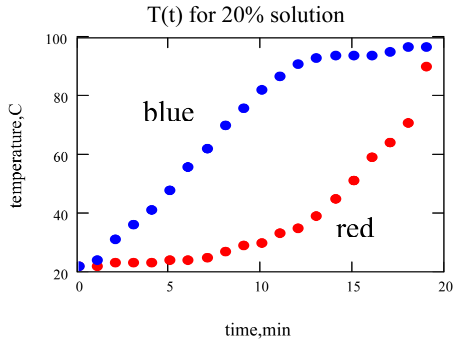
Here are the graphs we got using this formula and comparison between them and our experimental results

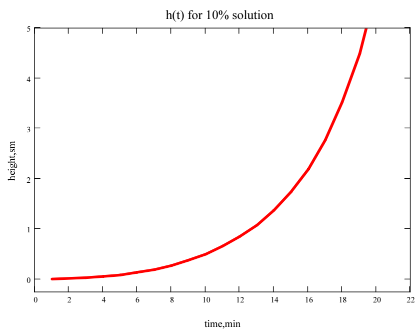
All graphs given for three different concentrations of salt in the solution-20%, 10%, 5%(the highest concentration that can be gained is 25% (for temperature $20\ ^0C$))

On T(t) graphs blue points-temperature of the water, red – of the salt solution

On h(t) for 20% solution blue points – our experimental results. The height is measured from initial position.


h(t) for 20% solution
height,sm
time,min
blue
red


T(t) for 20% solution
temperature,C
time,min
blue
red


h(t) for 10% solution
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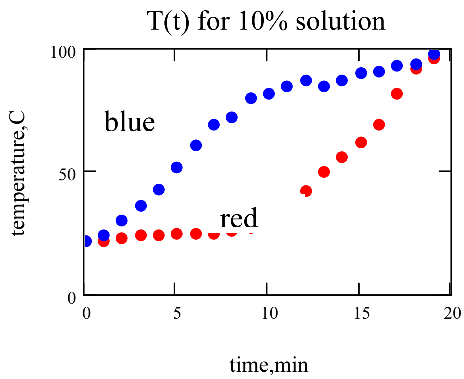
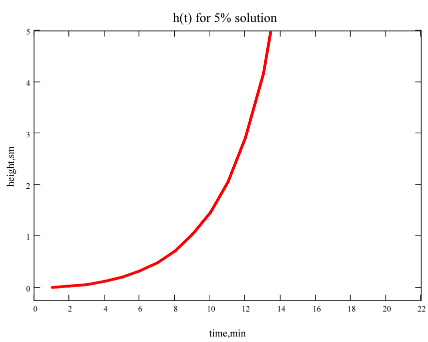
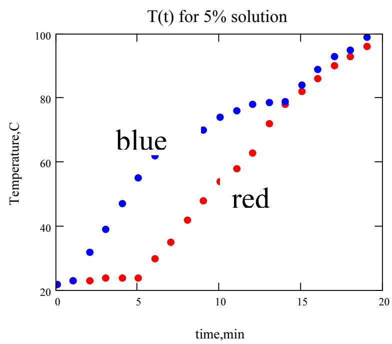
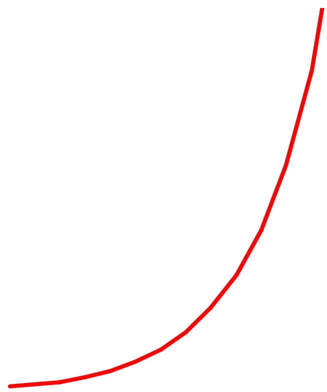
time,min


T(t) for 10% solution
temperature,C
time,min
blue
red


h(t) for 5% solution
height,sm
time,min


T(t) for 5% solution
Temperature,C
time,min
blue
red

And here three graphs h(t) for three concentrations on one plot



As you see the less concentration, the bigger the acceleration of the boundary. Why you will ask. Because of density difference rely on acceleration of the boundary-the less concentration, the easier for convection to get to the water upwards, and further.

If you look at the graphs T(t) you see that at some moment temperature of the liquids becomes equal, this means that temperature at any point inside of the vessel is equal, convection stopped, but it is possible for the limit to exist at such conditions, if we heat all system very slowly. At this temperature boiling begins, so here bubbles begin to play more important part.

So we've made some theoretical research about the behaviour of the boundary between water and saturated salt solution when heating this system.

And we made experiments to compare them with our theoretical research. As you can see these results (I mean graphs h(t) and T(t)) in experiments and theory are very near and can be comparable.

Acknowledgement:

Special thanks to:
Oleg Matveichuk, main author of the idea.

IV. Problems for the IYPT from previous years

1. PROBLEMS FOR THE 17TH IYPT – BRISBANE, AUSTRALIA, 2005

1. Misty
Invent and construct a device that would allow the size of a droplet of a mist to be determined using a sound generator.

2. Stubborn Ice
Put a piece of ice (e.g. an ice cube) into a container filled with vegetable oil. Observe its motion and make a quantitative description of its dynamics.

3.Electric Pendulum
Use a thread to suspend a ball between the plates of a capacitor. When the plates are charged the ball will start to oscillate. What does the period of the oscillations depend on?

4. Dusty Blot
Describe and explain the dynamics of the patterns you observe when some dry dust (e.g. coffee powder or flour) is poured onto a water surface. Study the dependence of the observed phenomena on the relevant parameters.

5. Sea-Shell
When you put a sea-shell to your ear you can hear 'the sea'. Study the nature and the characteristics of the sound.

6. Seebeck Effect
Two long metal strips are bent into the form of an arc and are joined at both ends. One end is then heated. What are the conditions under which a magnetic needle placed between the strips shows maximum deviation?

7. Coin
Stand a coin on its edge upon a horizontal surface. Gently spin the coin and investigate the resulting motion as it settles.

8. Pebble Skipping
It is possible to throw a flat pebble in such a way that it can bounce across a water surface. What conditions must be satisfied for this phenomenon to occur?

9.Flow
Using a dc source, investigate how the resistance between two metallic wires dipped into flowing water (or water solution) depends upon the speed and direction of the flow.

10.Two Chimney
Two chimneys stand on a box with one transparent side. Under each chimney there is a candle. A short period after the candles are lit one flame becomes unstable. Examine the case and present your own theory of what is happening.

11. String Telephone
How do the intensity of sound transmitted along a string telephone, and the quality of communication between the transmitter and receiver, depend upon the distance, tension in the line and other parameters? Design an optimal system.

12. Kundt's Tube
In a 'Kundt's Tube' type of experiment the standing waves produced can bemade visible using a fine powder. A closer look at the experiment reveals that the regions of powder have a sub-structure. Investigate its nature.

13. Egg White
White light appears red when it is transmitted through a slice of boiled egg white. Investigate and explain this phenomenon. Find other similar examples.

14. Fountain
Construct a fountain with a 1m 'head of water'. Optimise the other parameters of the fountain to gain the maximum jet height by varying the parameters of the tube and by using different water solutions.

15. Brazil Nut Effect
When a granular mixture is shaken the larger particles may end up above the smaller ones. Investigate and explain this phenomenon. Under what conditions can the opposite distribution be obtained?

16. Small Fields
Construct a device based upon a compass needle and use your device to measure the Earth's magnetic field.

17. Didgeridoo
The 'didgeridoo' is a simple wind instrument traditionally made by the Australian aborigines from a hollowed-out log. It is, however, a remarkable instrument because of the wide variety of timbres that it produces. Investigate the nature of the sounds that can be produced and how they are formed.

1.1. PROBLEM № 2: "STUBBORN ICE" – IYPT 2004

SOLUTION OF AUSTRIA

Problem № 2: Stubborn Ice

Harald Altinger, Bernhard Frena, Eva Hasenhütl, Christina Koller, Camilla Ladinig
Reporter: Harald Altinger

(Power Point Presentation)

The problem

Put a piece of ice (e.g. an ice cube) into a container filled with vegetable oil. Observe its motion and make a quantitative description of its dynamics.

Structure

- Supposition
- Experimental Setup
- Observation
- Interpretation
- Quantitative Estimation
- Conclusions
- Literature

Supposition

1. Dynamics dependent on the following parameters:
⇒ difference between the densities of ice, oil & water → buoyancy
⇒ temperature of the oil
⇒ friction
⇒ surface energy
⇒ height of the container
2. Not considered:
⇒ different temperatures/densities in the ice
⇒ layers of different densities in the oil
⇒ boundary effects at the walls of the vessel, etc.

Experimental Setup

- large container (height: 13cm, diameter: 7cm)
- ice (average density): ρ_{ice} = 917kg/m3
- water: ρ_{water} = 1000kg/m³
- vegetable oils (room temperature): ρ_1 = 925kg/m³

  ρ_2 = 923,3kg/m³ (olive oil)

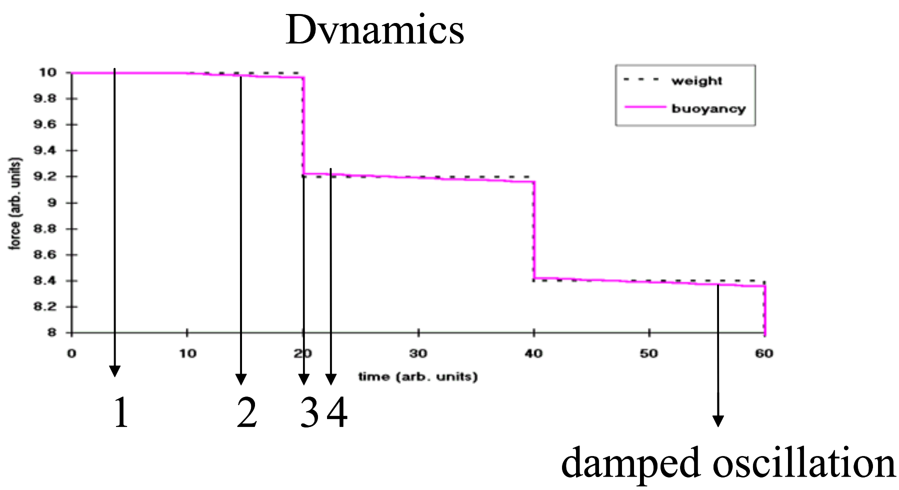
  (ρ of vegetable oils varies usually between 910kg/m³ and 928kg/m³)
- ice cubes of different shape and volume

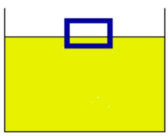
Observation

ρoil < ρice: ice descends to the bottom accelerating until the accelerating force due the weight of the cube equals the retarding force due to the friction (Stokes friction)

ρice < ρoil < ρwater : the cubes show vertical oscillation – several up and down motions may occur

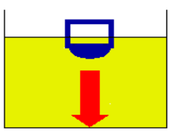
No dependence on the shape and no qualitative dependence on the volume of the cubes can be observed.





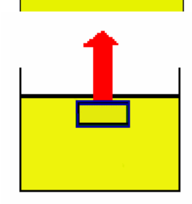
Interpretation

1 cube swims due to buoyancy – ice starts melting and forms water drops until weight is bigger than buoyancy



2 molten water sticks to the cube (surface energy) average density of the combined bodies increases and becomes larger than the density of the oil: the assembly moves down

3 surface tension does not hold the drop attached: the drop seperates and moves downwards



4 the buoyancy increases (less water) – the cube slows down – the cube might rise again until damping stops the process

Quantitative Estimation (1)

Newtons Law: $M\,a = F - \beta\eta v$

$F = M\,g - FB$

M … total mass
a … acceleration
$M\,g$…weight of the ice cube plus weight of the water
F_B …buoyancy force due to the dispelled oil

The buoyancy force results to: $F_B = g\,\rho oil\,(Vice + Vwater)$ (1)

Quantitative Estimation (2)

Friction force βηv (linearly depending on velocity) :
To good approximation: mass of the ice cube mice linearly decreasing with time due to melting

$M_{ice} = M - \alpha\, t$

$\dot{Q} = kA\Delta T \qquad \Delta T = T_{oil} - T_{icewater}$

$M_{water} = \alpha\, t$

A … surface area of the cube

$$F = g[-M(\frac{\rho_{oil}}{\rho_{ice}} - 1) + \rho_{oil}(\frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}})\alpha t] \tag{2}$$

The first term is the buoyancy reduced weight of the ice cube (negative) and second term is positive since ρ*water* is bigger than that of oil and is linearly increasing with time.

Quantitative Estimation (3)

The acceleration for the ice/water combination is

$$a \equiv \frac{dv}{dt} = g\left[-\left(\frac{\rho_{oil}}{\rho_{ice}} - 1\right) + \rho_{oil}\left(\frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}}\right)\alpha t\right] - \frac{\beta\eta}{M} v \tag{3}$$

and gives the velocity

$$v(t) = \frac{M}{\beta\eta} g\left[\left(\frac{\rho_{oil}}{\rho_{ice}} - 1\right) + \frac{\alpha}{\beta\eta}\rho_{oil}\left(\frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}}\right)\right]\left(e^{\frac{-\beta\eta}{M}t} - 1\right) + \\ + g\frac{\alpha}{\beta\eta}\rho_{oil}\left(\frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}}\right)t \tag{4}$$

Conclusions

- The drops may oscillate because of the interaction between the increasing density of the combined water/ice –system and the buoyancy force
- The phenomenon will stop when the ice cube is so small that its buoyancy force cannot withstand the weight of the new water drop formed.
- Such movement results from the theory and can be observed as shown in the movie.

Literature:

- Density of cooking oil
  http://hypertextbook.com/facts/2000/IngaDorfman.shtml
- Density of ice
  http://hypertextbook.com/facts/2000/AlexDallas.shtml
- Density of water
  http://www.ucdsb.on.ca/tiss/stretton/chem2/data19.htm

HANDOUT "STUBBORN ICE"

$$F_B = g\,\rho_{oil}\,(V_{ice} + V_{water}) \tag{1}$$

$$F = g\left[-M\left(\frac{\rho_{oil}}{\rho_{ice}} - 1\right) + \rho_{oil}\left(\frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}}\right)\alpha t\right] \tag{2}$$

$$a \equiv \frac{dv}{dt} = g\left[-\left(\frac{\rho_{oil}}{\rho_{ice}} - 1\right) + \rho_{oil}\left(\frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}}\right)\alpha t\right] - \frac{\beta\eta}{M}v \tag{3}$$

$$v(t) = \frac{M}{\beta\eta}g\left[\left(\frac{\rho_{oil}}{\rho_{ice}} - 1\right) + \frac{\alpha}{\beta\eta}\rho_{oil}\left(\frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}}\right)\right]\left(e^{\frac{-\beta\eta}{M}t} - 1\right) +$$

$$+ g\frac{\alpha}{\beta\eta}\rho_{oil}\left(\frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}}\right)t \tag{4}$$

1.2. PROBLEM № 3: ELECTRIC PENDULUM – IYPT 2004

SOLUTION OF AUSTRIA

Problem № 3: Electric Pendulum

Harald Altinger, Bernhard Frena, Eva Hasenhütl, Christina Koller, Camilla Ladinig
(Power Point Presentation)

The problem

Use a thread to suspend a ball between the plates of a capacitor. When the plates are charged the ball will start to oscillate. What does the period of the oscillations depend on?

Structure

- Basic consideration
- Experimental Setup
- Observation (1)
- Assumption
- Quantitative Estimation
- Observation (2)
- Results
- Special Arrangement
- Conclusions
- Literature

Basic consideration

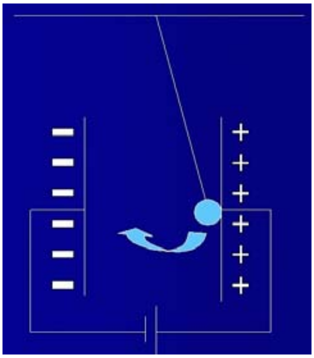
The cause of the oscillation will be the fact that whenever the ball touches a capacitor plate it will be charged as the plate and therefore be repelled afterwards. Simultaneously it will be attracted by the oppositely charged plate and the process will carry on.

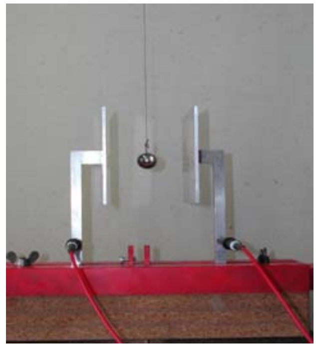
The issues of the investigation are:

⇒ Why does the oscillation start?

⇒ What are the parameters which influence the oscillation?

Experimental Setup

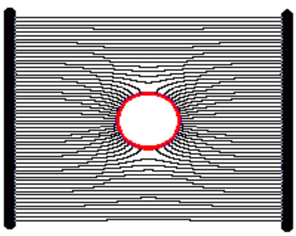




- two capacitor plates in variable distance (in the order of 10-1m)
- thread of 2,5m
- balls of iron (d = 1,91cm, m=28,7g), wood (d1= 2,02cm, m1= 3,8g; d2= 3,32cm, m2 = 15,1g), aluminium (hollow; d = 2,52cm, m = 12,3g) and table tennis (d = 3,75cm, m = 2,8g)

Observation (1)

The ball symmetrically situated between the capacitor plates would not move:



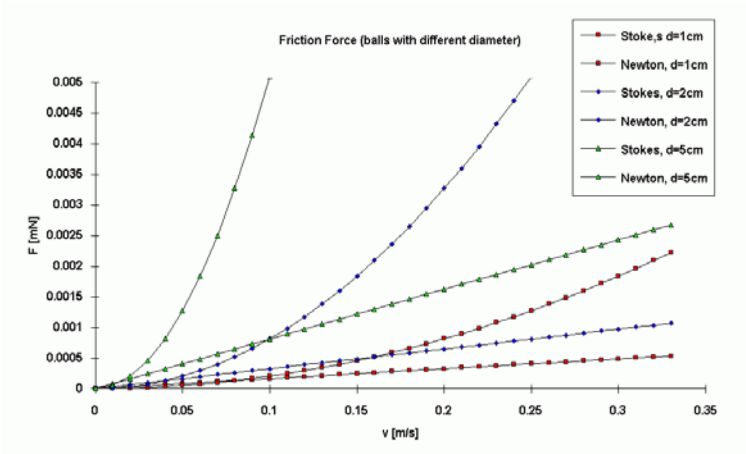
(fixed homogeneous field assumed)
Little asymmetry starts the process:
2004\ElectricPendulum\Video\DSCN3273.MOV

Assumption

- infinite length of the suspending thread
- Newton's friction law $F_{Newton} = c_D \rho A v2/2$ (1)
- accelerated motion of the charged ball until the gained energy in the electric field equals the energy loss due to friction
- total elastic reflection at the plates

Quantitative Estimation (1)

Newton's friction:



Newton's friction more likely than Stokes' friction since for air viscosity Newton's friction outweighs Stokes' friction for velocities > 0.05m/s.

Quantitative Estimation (2)

Equation of motion: $$m\frac{dv}{dt} = F_{el} - F_{Newton}$$ with $F_{el} = QE = Q\frac{U}{D}$

$$Q = \alpha dU$$

D … distance of the plates
D … diameter of the ball

(2) $$m\frac{dv}{dt} = \alpha\frac{dU^2}{D} - \beta d^2 v^2$$

β … constant variables in Newton's friction law

$\alpha = 2\pi\varepsilon 0$

Steady state: $$\frac{dv}{dt} = 0 \qquad v = U\sqrt{\frac{\alpha}{\beta Dd}}$$ (3)

Quantitative Estimation (3)

Period of the oscillation: $$T = 2\frac{(D-d)}{v} \equiv \frac{2(D-d)}{U}\sqrt{\left(\frac{\beta Dd}{\alpha}\right)}$$ (4)

2004\ElectricPendulum\Video\DSCN3224.MOV

With a thread of finite length: $T_{max} = 2\pi\sqrt{\frac{l}{g}}$ l … length of the thread
g …gravitational acceleration

Thus
⇒ velocity proportional to the voltage

⇒ the period of the oscillation is inversely proportional to the voltage
⇒ the upper limit for the period is given by the length of the thread

Quantitative Estimation (4)

Total reflection of the balls: 1 iron ball

2004\ElectricPendulum\Video\DSCN3230.MOV

2 table tennis ball

2004\ElectricPendulum\Video\DSCN3242.MOV

Energy loss:

K $\Delta E = \frac{mv^2}{2}$ proportional to v^2 as Newton's friction → no change of the dependence of v from U

⇒ Dependence on the mass (total elastic) :
→ no analytical solution of the equation of motion
→ with evidence shown in the experiments

Quantitative Estimation (5)

Fully inelastic case:

- predominant energy loss at the reflection
- uniformly accelerated motion
- full stop at hitting the electrodes

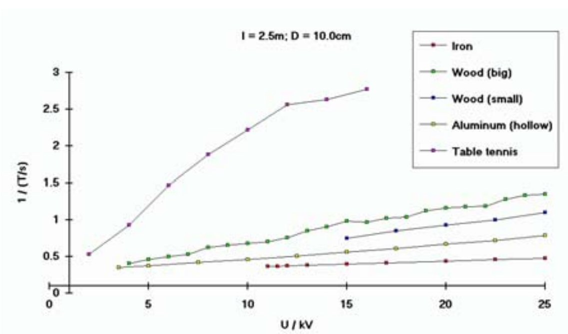
Then follows

$$m\frac{dv}{dt} = QE \quad \text{with the solution} \quad D = \frac{\alpha d U^2}{2mD} T_\sigma^2$$

and the period

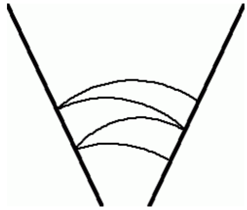
$$T_\sigma = \frac{D}{U}\sqrt{\frac{2m}{\alpha d}} \qquad (5)$$

Results



- inverse period of the oscillation 1/(T/s) as a function of U→ all balls show a linear dependence on U] less inelastic reflection → more evidence of the dependence table tennis ball).

Special Arrangement



In this case the ball is deflected at every bounce and moves outward until the outward motion which is caused at every reflection is compensated by the inward motion due to the torque build up.
..\IYPT 2004 Turnier\Electric Pendulum\Objekte

Conclusions

- Energy loss ~ v^2 for all balls
- 1/U dependence for the period of oscillation for all balls
- 1/U dependence increases elasticity
- for low voltages the period of oscillation is limited by the free oscillation period of the ball

HANDOUT "ELECTRIC PENDULUM"

$$F_{Nwtn} = c_D A\rho/2 v^2 \tag{1}$$

$$m\frac{dv}{dt} = \alpha\frac{dU^2}{D} - \beta d^2 v^2 \tag{2}$$

$$\frac{dv}{dt} = 0 \quad v = U\sqrt{\frac{\alpha}{\beta D d}} \tag{3}$$

$$T_\sigma = 2\frac{(D-d)}{v} \equiv \frac{2(D-d)}{U}\sqrt{\left(\frac{\beta D d}{\alpha}\right)} \tag{4}$$

$$m\frac{dv}{dt} = QE \quad D = \frac{\alpha d U^2}{2mD}T_\sigma^2 \quad T_\sigma = \frac{D}{U}\sqrt{\frac{2m}{\alpha d}} \tag{5}$$

2. Problems for the IYPT from the 7th to the 16th IYPT

2.1. Problems for the 7th IYPT

NETHERLANDS, 1994

Think up a problem yourself
(problems 1, 2, 3) Invent yourself and solve a problem on the given theme.

1. Optics
Think up and solve a problem connected with employing a thin lens of a large focal length.

2. Compass
"In sledge trips we use liquid compasses, the most exact of the small ones. But you understand of course that due to proximity to the magnetic pole the arrow usually points downwards. To make it horizontal, its opposite end is balanced with a weight". (From the letter of Cherry-Garrad, member of the last expedition of R. Scott.) Use the context of this quotation to formulate a problem.

3. Magnetization
A cylindrical permanent magnet falling inside a copper tube is found to move at an almost constant velocity, the slower the thicker and the walls of the tube. Use this fact to formulate a problem (see also 14).

Gravitation machine
(problems 4, 5, 6). A horizontal plate (a vibrator) oscillates harmonically up and down. A steel ball put on the surface of the plate starts jumping higher or lower. For the experimental device one may successfully use a ferrite core in a coil connected to an alternating current generator (a sound generator). The butt-end of the ferrite core will play the part of the vibrating horizontal plane. Steel balls of diameter 1 or 2 mm are suitable for the experiment. The glass tube approximately 1m long can also be very helpful.

4. Upper boundary
Measure experimentally the maximum height to which the ball rises to and explain the result.

5. Distribution function
Determine experimentally what part of a sufficiency large time interval the ball is in the range of heights H, H+ dH and explain the result.

6. Acceleration
The mechanical energy of the ball changes after every impact. The mean mechanical energy (averaged overall successive impacts) increases at the beginning of the process and then tends to a constant value. Try to obtain experimentally the time dependence of the mean mechanical energy of the ball.

7. Aspen leaf
Even in windless weather aspen leaves tremble slightly. Why does an aspen leaf tremble?

8. Superball
A highly elastic ball (a superball) falls on a horizontal surface from a small height (4cm or less) and recoils several times. What is the number of impacts of the superball against a table?

9. Meteorite
A meteorite of mass 1000 tons files directly to the Sun. Can modern instruments register the fact of its fall on the Sun?

10. Water dome
A vertical water jet falls on the butt-end of a cylindrical bar and creates a bell-like water dome. Explain this phenomenon and evaluate the parameters of the dome.

11. Siphon
A rubber tube is used as a siphon to flow water from one vessel into another. The vessels are separated by a high partition and the levels of water in them are different. If one withdraws the tube from one vessel, lets the pole of air enter it and then puts the tube into the water again, the action of the siphon may be resumed or not. Investigate this phenomenon.

12. Boiling
Put a metallic ball heated to the temperature 150^0 C – 200 ^{0}C into hot water at the temperature close to 100^0 C and observe the process of intensive evaporation of the water. Explain the observed phenomenon.

13. Spirits
A closed vessel (a bottle) contains spirits – pure or substantially diluted by water. Suggest a method of estimation of the concentration of spirits without opening the vessel.

14. Magnetic friction
To investigate the phenomenon described in the problem 3 we suggest to create the device containing the following elements:
a) a copper plate (or a set of plates) 0. 3 to 15 mm thick. The length and the width of the plate may be chosen according to one`s convenience, but they should be large enough to avoid the effect of the boundaries;

b) a cylindrical electromagnet with a flat butt-end;
c) a device providing free motion of the flat butt-end of the electric magnet over the horizontal surface of the copper plate. It is very important that the gap between the magnet and the plate is small as possible and constant everywhere;
d) the push providing the uniform motion of the magnet at a given velocity over the plate surface.
Introduce the following notation: T – the push (and the force of magnetic friction), v – the velocity of the magnet, h – the thickness of the plate. Investigate and determine experimentally the dependence of T on h at v = constant for several values of v.

15. Transmission of energy
Transmit without wires to a distance of 3 meters the largest possible part of the energy stored in a capacitor having capacity of C= 10μF charged to voltage U= 100 V. Measure this energy.
Your device should not contain energy sources. Naturally the capacitor itself must not be transported.

16. The Moon and the Sun
“ If you are asked what is more important, The Sun or the Moon, you should answer the Moon. For the Sun shines in daytime when there is enough light without it”, says a joke. When is it possible to see the Sun and the Moon at the same time? Calculate the schedule of the events for the European countries during 1994.

17. Straw
The Russian proverb says “Had I known the place where I fell, I would have laid some straw there”. How much straw should be laid to guarantee a safe fall?

2.2. Problems for the 8th IYPT

SPALA, POLAND, 1995

1. Think up a problem yourself (paradox)
Try to puzzle your rivals by a paradoxical physical experiment.

2. Boiling water
Some people say it is important to put a lid on the pot when you want to boil water for tea to save energy and time. Investigate this phenomenon and determine the energy and time saving.

3. Drop
A drop of salted water drying on a smooth surface creates a system of rings. Investigate and explain this phenomenon.

4. Gravitational spacecraft
A spacecraft (having a shape of a dumb-bell of variable length) can shift from the Earth orbit (300 km above the Earth surface) to the Moon orbit without the use of jets. Calculate the time taken by such a manoeuver.

5. Sound
Transfer the electric energy stored in a capacitor of 0.1 mF charged to the voltage of 30 V into the energy of the sound, with the highest efficiency possible. No external energy sources are allowed. Determine the fraction of energy converted into sound in the discharge.

6. Curtain
A light curtain (light scatters on dust particles) is used in some theatres. Suggest the design of a light curtain, which allows its effective action with the minimum power supplied for one meter of stage width?

7. Three discs
Investigate collisions of three homogeneous, rigid discs which can move in a plane. At first two discs are at rest. The third disc:
a) collides at exactly the same time with two other discs,
b) collides at first with one of the discs.

8. Carpet
When a carpet is rolled into a cylinder it sometimes unrolls by itself or with the help of a gentle push. Determine the factors on which the speed of the rolling carpet depends.

9. Ice cream
Obtain super-cooled water in an experimental setup. By how many degrees below 0 C did you manage to super-cool it? What can be the record in this experiment? Determine the freezing point of water.

10. Cathode –ray tube
While a well-known physicist A. First watched a football match by TV, another well-known physicist B.Second made a hole of diameter 0.001 mm in the cathode-ray tube. Did A. First manage to see the football match up to the end?

11. Moon light
It is possible to set paper on fire using a lens and solar radiation. Could it be possible using lunar instead of solar light? If yes – invent an optimal optical system for such a purpose. If not – what should the Moon be like, for being this possible?

12. Tinder box
When someone strikes two pieces of flint rock, sparks are created. Investigate and explain this phenomenon.

13. Air lens
Lenses are usually made of solids and sometimes made of liquids. Construct an optical lens made of air in such a way that light can travel through the lens without crossing any material but air. Determine on which factors the focal length of an air lens depends.

14. Frozen lake
The water surface of a lake is in winter exposed to cold air at a fixed temperature below zero. There is no wind. Determine the thickness of the ice layer as a function of time.

15. Bottle
A plastic bottle of a capacity between 1 and 2 litres completely filled with water is "accidentally" dropped on the floor from the height H = 1 m. What maximum height can the spray reach and why? Determine the minimal height from which the bottle should be dropped to burst?

16. Oscillation of plates
Water has been poured on a horizontal glass plate and a second glass plate placed on it. If the lower plate is oscillating in a horizontal plane, at certain amplitudes and frequencies, the upper plate begins to oscillate in vertical direction. Investigate and describe this phenomenon. Is there any difference when you use another liquid ?

17. Epic Hero

An epic Russian hero Ilya Muromets had once thrown his mace weighing forty poods (1 pood = 16 kg) and in forty days this mace fell at the same place. Estimate the parameters of the throw of the hero.

2.3. Problems for the 9th IYPT

KUTAISSI, GEORGIA, 1996

1. Invent yourself.
Invent and solve yourself a problem concerning the ozone holes.

2. Paper clot.
Crumple arbitrarily a sheet of paper A4 in your hand. This clot can be approximated by a sphere. Making many of this clots and measuring their average diameters a histogram of distribution of diameters can be plotted. Try to explain the result obtained. Make more comprehensive investigation of the dependence of the average diameter of a clot on the parameters which you consider important.

3. Cycle racing
According to the forecast of specialists two very strong and "absolutely identical" sportsmen had to show equal time in a highway race for 100 km. But, alas, one sportsman lagged behind. Later it was found out that some malefactor adjusted a nut of mass 5 g to the rim of the rear wheel of his bicycle. For what time is the victim?

4. Self-formation of a pile.
A horizontal rigid plate vibrates vertically at a frequency of the order of 100 Hz. A cone-shaped pile of fine dispersed powder (e.g. Licopodium or talc) which is heaped up on the plate remains stable at small amplitudes of the vibration. If the amplitude is increased the cone decays. Further increase of the amplitude yields a distribution confined by a sharp border and at still higher amplitudes a pile appears again .Investigate and explain this phenomenon.

5. Auto oscillations.
Produce and investigate auto oscillating system containing thermistor as a single non-linear element.

6. Water generator.
If some volume of water is frozen from one side, a potential difference appears across the ice-water frontier. Measure this potential difference and explain the phenomenon.

7. Sun.
In the centre of the Sun suddenly an extra quantity of energy is produced which is equal to the energy emitted by the Sun per year. How will the parameters of the Sun observed on the Earth change during one year?

8. Surface information.
Develop a method for transferring information by the waves on the surface of water. Investigate the angular characteristics of the emitter and the receiver (the antennas) which you constructed.

9. Floor-polisher.
A device stands on two identical disks lying flat on a horizontal surface. The disks can rotate in opposite directions at a given velocity. Investigate how the value of a force providing a uniform motion this device along a horizontal plane depends on the velocity of this motion and the velocity of rotation of these disks.

10. Soap bubbles.
Dip the ring of a children's toy for blowing out soap bubbles into a soap solution and blow on the film formed in the ring. At what velocity of the air flux blown into the ring will the bubbles form ? How must the velocity of the air flux be adjusted to produce the bubble of maximum size ?

11. Candle
Some candles twinkle before dying out. Investigate and explain this phenomenon.

12. Motor car.
A car driven at constant power moves onto a wet section of a straight road. How will its speed change when the thickness of the water layer increases slightly and linearly with the distance ?

13. Grey light.
Construct a source of light which would seem to be grey.

14. Coherer.
It is known that a glass tube with two electrodes and metallic filings between them (coherer) has different resistance in d.c. and a.c. circuits. Investigate the frequency dependence of the coherer's resistance.

15. Salt water oscillator.
A cup with a small hole in its bottom containing salt water is partially immersed in a big vessel with fresh water and fixed. Explain the mechanism of the observed periodical process and investigate the dependence of its period on different parameters. To visualize the process, the water in the cup should be coloured.

16. Hail
Explain the mechanism of hail formation and propose your own method to prevent the hailing.

17. Gloves
Some people refuse to wear gloves in winter because they suppose to feel colder than without gloves. Others prefer to wear mittens instead. What is your opinion?

2.4. Problems for the 10th IYPT

CHEB, CZECH REPUBLIC, 1997

1. Invent yourself
Construct and demonstrate a device which moves in a definite direction under chaotic influence.

2. Coin
From what height must a coin with heads up be dropped, so that the probability of landing with heads or tails up is equal?

3. Paper
How does the tensile strength of paper depend on its humidity?

4. Electron Beam
An electron beam is cast upon a planparallell plate of known homogenous material. Some of the electrons get through it, some do not. Try to simulate processes taking place, e.g. using Monte Carlo method and compare your results with the ones described in literature.

5. Blue Blood
Human blood is known to be red, but the veins seem to be blue. Explain this phenomenon and illustrate it by a model.

6. Magic Tube
A compressor blows air into Ranque-Hilsch T-shaped tube at a pressure of 0,5 Mpa or higher so that the air begins to circulate. In such a case hot air is coming out from one end of the tube and cold air from the opposite one. Find out which end of the tube is the "hot" one and explain the difference of the temperatures obtained. Investigate the parameters this difference depends on.

7. Water Jet
A water jet streaming vertically downwards from a tube is divided into drops at some distance from the tube. Choose the conditions under which the length of the unseparated jet is largest. What maximum length did you obtain?

8. Floatation
A piece of chocolate, which is dropped into a glass of soda water, periodically sinks and goes back to the surface. Investigate the dependence of the period of these oscillations on various parameters.

9. Jet-spread
A water jet falling onto a horizontal plane spreads out radially. At some distance from the center the thickness of the layer increases dramatically. Explain the phenomenon.

10. Cooling the Earth
How would the temperature of the Earth change with time, if the Sun suddenly stopped radiating?

11. Candle Generator
Construct a device for charging an electric capacitor (1000 μ F/100 V) using the energy of a candle burning for a period of one minute.

12. Static Friction
A force of motion friction is known to independent on the rubbing surface area of a body. How does the static friction depend on the rubbing surface area?

13. Tea Cup
If one fills a cup with hot tea (60° – 80° C), a thin layer of steam emerges above the surface. One can see that some parts of the steam layer disappear suddenly and reappear after a few seconds. Investigate and explain this phenomenon

14. Rain
On a long-time exposure photograph of night rain taken in the light of a projector, the tracks of drops appear interrupted. Explain this phenomenon.

15. Cell and Accumulator
How does the voltage-current characteristics of a cell and of an accumulator change during discharging?

16. Roghe Spiral
The Roghe Spiral is a device where a source of current is connected to a vertically suspended spring, the lower end of which dipped mercury. Mercury is a highly dangerous chemical substance and thus the experiments with it are not permitted. Substitute the mercury with a less dangerous substance and investigate the functioning of this device.

17. Leap
To make a leap it is necessary to squat. How does the height of a leap depend on the depth of the s

2.5. Problems for 11th IYPT

DONAUESCHINGEN, GERMANY, 1997

1. Invent yourself
Construct an aeroplane from a sheet of paper (A4, 80 g/mI). Make it fly as far and/or as long as possible. Explain why it was impossible to reach a greater distance or a longer time.

2. Popping body
A body is submerged in water. After release it will pop out of the water. How does the height of the pop above the water surface depend on the initial conditions (depth and other parameters)?

3. Spinning disc
Investigate and explain the phenomenon of a spinning annular disc as they progress down a straight, cylindrical rod. If the rod is moved upwards at a defined velocity, the disc spins at constant height. Investigate the mechanism.

4. Water streams
A can with three holes in the side-wall at the same height slightly above the bottom is filled with water. The water will escape in three separate streams. By gently touching the streams with a finger they may unite. Investigate the conditions for this to happen.

5. Water jet
If a vertical water jet falls down onto a horizontal plate, standing waves will develop on the surface of the jet. Investigate the dependence of this phenomenon on different parameters.

6. Mount Everest
Can you see Mount Everest from Darjeeling?

7. Air bubble
An air bubble rises in a water-filled, vertical tube with inner diameter 3 to 5 mm. How does the velocity of the rising bubble depend on its shape and size?

8. Trick
It is known that a glass filled with water and covered with a sheet of paper may be turned upside down without any loss of water. Find the minimum amount of water to perform the trick successfully.

9. Woven textiles
Look at a point-like light source through different woven textiles. Describe what you see. What is the explanation of the phenomenon?

10. Repeated freezing

While a vessel filled with an aqueous solution of a volatile fluid, e.g., ammonia, ethanol or acetone, is being cooled, repeated freezing and melting may be observed near the surface. Describe and explain the phenomenon.

11. Current system

In a Petri dish (shallow bowl), small metal balls, e.g., 2 mm in diameter, are immersed in a layer of castor oil. The inner rim of the dish contains an earthed metal ring. Above the centre of the dish there is a metal needle which does not touch the oil surface. Investigate what happens when the voltage between needle and earth is about 20 kV.

Warning: The high voltage should be obtained by means of a safe generator, e.g., an electrostatic generator!

12. Powder conductivity

Measure and explain the conductivity of a mixture of metallic and dielectric powders wit various proportions of the two components.

13. Rope

How is it possible that a very long and strong rope can be produced from short fibers? Prepare a rope from fibers and investigate its tensile strength.

14. Water rise

Immerse the end of a textile strip in water. How fast does the water rise in the strip and what height does it reach? In which way do these results depend on the properties of the textile?

15. Luminescent sugar

Investigate and explain the light produced when sugar crystals are pulverized. Are there other substances with the same property?

16. Strange motion

Make a mixture of ammonium nitrate and water, proportion 5 to 1. When the mixture is heated to about 100 °C it melts. When it cools,it crystallizes and you may observe a strange motion below the surface. Investigate and explain the phenomenon.

Safety rules: Do not heat the ammonium nitrate without water, preferably use a water bath! Use protection glasses during the experiment!

17. Icicles

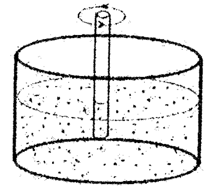
Investigate and explain the formation of icicles.

2.6. Problems for the 12th IYPT

VIENNA, AUSTRIA, 1999

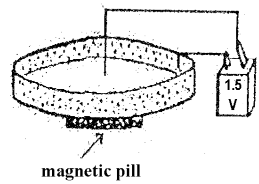
1. Rotation

A long rod, partially and vertically immersed in a liquid, rotates about its axis. For some liquids this will cause an upward motion of the liquid on the rod and for other liquids a downward motion



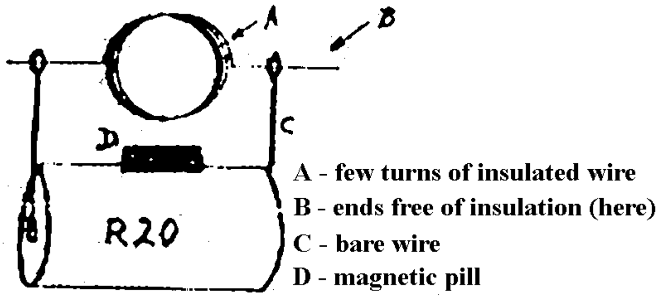
2. Ionic Motor

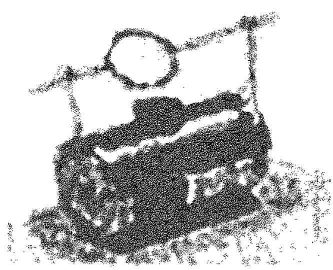
An electrolyte (an aqueous solution of $CuSO_4$, NaCl, ...) in a shallow tray is made to rotate in the field of a permanent magnet (a small "pill" placed under the tray). The electric field is applied from a 1.5 V battery in such a way that one electrode is in the form of a conducting ring immersed in the electrolyte --- the other is a tip of wire placed vertically in the centre of the ring. Study the phenomenon and find possible relations between the variables.



3. Magic Motor

Construct a DC motor without a commutator, using a battery, permanent magnet and a coil. Explain how it functions





4. Soap Film

Explain the appearance and development of colours in a soap film, arranged in different geometries.

5. Dropped Paper

If a rectangular piece of paper is dropped from a height of a couple of meters, it will rotate around its long axis whilst sliding down at a certain angle. How does this angle depend on various parameters?

6. Singing Glass
When rubbing the rim of a glass containing a liquid a tone can be heard. The same happens if the glass is immersed in a liquid. How does the pitch of the tone depend on different parameters.

7. Heated Needle
A needle hangs on a thin wire. When approached by a magnet the needle will be attracted. When heated the needle will return to its original position. After a while the needle will be attracted again. Investigate this phenomenon, describe the characteristics and determine relevant parameters.

8. Energy Converter
A body of mass 1 kg falls from a height of 1 m. Convert as much as possible of the released potential energy into electric energy and use that to charge a capacitor of 100 mikroF.

9. Air Dryer
During 4 minutes collect as much water as possible from the air in the room. The mass of the equipment must not exceed 1 kg. The water should be collected in a glass test tube, provided by the jury.

10. Charged Balloon
An air-filled balloon rubbed with wool or dry paper may stick to the ceiling and stay there. Investigate this phenomenon and measure the charge distribution on the surface of the balloon.

11. Billiard
Before a snooker game starts, 15 balls form an equilateral triangle on the table. Under what conditions will the impact of the white ball (16th ball) produce the largest disorder of the balls.

12. Flour Craters
If you drop a small object in flour, the impact will produce a surface structure which looks like moon crater. What information about the object can be deduced from the crater?

13. Gas Flow
Measure the speed distribution of the gas flow in and around the flame of a candle. What conclusions can be drawn from the measurements?

14. Wheat Waves
The wind blowing through a wheat field creates waves. Describe the mechanism of wave formation and discuss the parameters which determine the wavelength.

15. Bright Spots
Bright spots can be seen on dew drops if you look at them from different angles. Discuss this phenomenon in terms of the number of spots, their location and angle of observation.

16. Liquid Diode
Make an electrochemical diode and investigate its properties, in particular the frequency dependence.

17. Sound from Water
When you heat water in a kettle you hear a sound from the kettle before the water starts to boil. Investigate and explain this phenomenon.

2.7. Problems for the 13th IYPT

BUDAPEST, HUNGARY, 2000

1. Invent for yourself
Suggest a contact-free method for the measurement of the surface tension coefficient of water. Make an estimate of the accuracy of the method.

2. Tuning fork
A tuning fork with resonant frequency of about 100 Hz is struck and held horizontally, so that its prongs oscillate up and down. A drop of water is placed on the surface of the upper prong. During the oscillation of the tuning fork standing waves appear on the surface of the drop and change with time. Explain the observed phenomena.

3. Plasma
Investigate the electrical conductivity of the flame of a candle. Examine the influence of relevant parameters, in particular, the shape and polarity of the electrodes. The experiments should be carried out with a voltage not exceeding 150V.

4. Splash of water
Measure the height reached by splashes of water when a spherical body is dropped into water. Find a relationship between the height of the splashes, the height from which the body is dropped, and other relevant parameters.

5. Sparkling water
Bubbles in a glass of sparkling water adhere to the walls of the glass at different heights. Find a relationship between the average size of the bubbles and their height on the side of the glass.

6. Transmission of signals
Using a bulb, construct the optimum transmitter of signals without any modulation of the light beam between transmitter and receiver. Investigate the parameters of your device. The quality of the device is defined by the product of the information rate (bits/sec) and the distance between transmitter and receiver.

7. Merry-go-round
A small, light, ball is kept at the bottom of a glass filled with an aqueous solution and then set free. Select the properties of the solution, so that a moving up time of several seconds is achieved. How will this time change if you put your glass on the surface of a rotating disk?

8. Freezing drop
Drops of melted lead or tin fall from some height into a deep vessel filled with water. Describe and explain the shape of the frozen drops as a function of height of fall.

9. Radioactivity
Use efficient methods to collect as much radioactive material as you can in a room. Measure the half-life of the material you have collect

10. Liquid fingers
When a layer of hot salt solution lies above a layer of cold water, the interface between the two layers becomes unstable and a structure resembling fingers develops in the fluid. Investigate and explain this phenomenon.

11. Throwing stone
A student wants to throw a stone so that it reaches the greatest distance possible. Find the optimum mass of the stone that should be used.

12. Tearing paper
Tear a sheet of paper and investigate the path along which the paper tears

13. Rolling can
A can partially filled with water rolls down an inclined plane. Investigate its motion.

14. Illumination
Two bulbs, 100 and 40 watts, respectively, illuminate a table tennis ball placed between them. Find the position of the ball, when both sides of the ball appear to be equally lit. Explain the result.

15. Cooling water
Two identical open glasses, filled with hot and warm water, respectively, begin to cool under normal room conditions. Is it possible that the glass filled with hot water will ever reach a lower temperature than the glass filled with warm water? Make an experiment to investigate this and explain the result.

16. Coloured sand
Allow a mixture of differently coloured, granular materials to trickle into a transparent, narrow container. The materials build up in distinct bands. Investigate and explain this phenomenon.

17. A strange sound
Pour hot water into a cup containing some cappuccino or chocolate powder. Stir slightly. If you then knock the bottom of the cup with a teaspoon you will hear a sound of low pitch. Study how the pitch changes when you continue knocking. Explain the phenomenon.

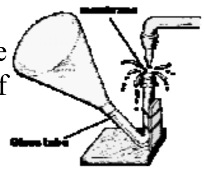
2.8. Problems for the 14th IYPT

ESPOO, FINLAND, 2001

1. Electrostatic motor
Is it possible to create a motor which works by means of an electrostatic field? If yes, suggest how it may be constructed and estimate its parameters.

2. Singing saw
Some people can play music on a handsaw. How do they get different pitches? Give a quantitative description of the phenomenon.

3. Tuning dropper
Make the music resonator shown in the picture. Investigate the conditions that affect the pitch. Can you observe amplification of external sounds? If yes, how can you explain this?



4. Dancing sand clock
Investigate the trickling of sand when a sand clock (egg-timer) is placed on a vibrating base.

5. Rubber heat machine
Investigate the conversion of energy in the process of deformation of rubber. Construct a heat machine, which uses rubber as the working element and demonstrate how it works.

6. Fractal diffraction
Produce, demonstrate and analyse diffraction pictures of fractal structures of different orders.

7. Cracks
When drying a starch solution, you will see cracks forming. Investigate and explain this phenomenon.

8. Speedometer
Two electrodes of different metal are immersed in an electrolyte solution. Investigate the dependence of the measured potential difference on the relative motion of electrodes and their shapes.

9. Pouring out
Investigate how to empty a bottle filled with a liquid as fast as possible, without external technical devices.

10. Water stream pump
Construct and demonstrate a water stream vacuum pump. What is your record value for the minimum pressure?

11. Rolling balls
Place two equal balls in a horizontal, V-shaped channel, with the walls at 90 degrees to each other, and let the balls roll towards each other. Investigate and explain the motion of the balls after the collision. Make experiments with several different kinds of ball pairs and explain the results.

12. Reaction
Make an aqueous solution of gelatine (10g gelatine in 90ml of water), heat it to 80 degrees C in a water bath and mix it with a solution of potassium iodide. Pour the solution in a test tube and cool it. Pour a solution of copper sulphate on the surface of the gel. Find a physical explanation to the observed phenomena.

13. Membrane electrolyser
In an electrolyser, containing a membrane which completely divides the space between two inert electrodes, the pH-value of the diluted salt solution will change substantially after electrolysis. Investigate how this difference depends on the pore size of the membrane.

14. Thread dropper
One end of a thread is immersed in a vessel filled with water. The other end hangs down outside without contact with the outer wall of the vessel. Under certain conditions, one can observe drops on that end of the thread. What are those conditions? Determine how the time of appearance of the first drop depends on relevant parameters.

15. Bubbles in magnetie field
Observe the influence of an alternating magnetic field (50 or 60 Hz) on the kinetics of gas bubbles in a vessel filled with water. The bubbles can be generated by blowing air into the water.

16. Adhesive tape
Investigate and explain the light produced, when adhesive tape is ripped from a smooth surface.

17. Seiches
Seiching is a phenomenon known for long, narrow and deep lakes. For reasons of changes in the atmospheric pressure, the water of the lake can start moving in such a way that the water level at both ends of the lake makes periodic motions, which are identical, but out of phase. Make a model that predicts the period of seiching depending on the appropriate parameters and test its validity.

2.9. Problems for the 15th IYPT

ODESSA, UKRAINE, 2002

Heat engine
A tall glass cylinder is half-filled with hot water and topped up with cold water. A small ampoule, containing a few drops of ether or alcohol (and closed off by a rubber pipette cap), is then put in. Describe the phenomena occurring in the system. How does the motion of the ampoule change with time?

2. Spider's web
A spider's thread looks like a string of pearls. What is the reason for this? Make experiments to investigate the relevant parameters.

3. Flying colours
Why do flags flutter in the wind? Investigate experimentally the airflow pattern around a flag. Describe this behaviour.

4. Hazy
The colour of a distant forest appears not green, but hazy blue. What is the minimum distance at which this phenomenon is observed? How do weather conditions affect this? Is it possible that a forest can appear grey?

5. Pond skater
It is known that unwettable small bodies can float on water due to the surface tension force. Construct a floating raft based on this principle and determine its static and dynamic parameters.

6. Stop and start
Sometimes a flow of traffic can experience sudden stops and starts for no apparent reason. Build a physical model to explain why this occurs.

7. Ohm's Law for a liquid
It is said that electric current “flows”. Is this the only analogy between electric current and the flow of a liquid? Investigate theoretically and experimentally other analogies between these two.

8. Charged sand
Fine, well-dried quartz sand is poured out of a short thin tube into a conical metallic vessel connected to an electrometer. Investigate the behaviour of the sand stream as the vessel fills up. What changes if the stream is lit by a UV-lamp?

9. Chromatography
Put a drop of coloured liquid on a piece of absorbant paper. Describe quantitatively the observed phenomena.

10. Sound cart
Construct and demonstrate a device that can be propelled solely by sound. Investigate its properties.

11. Equilibrium
Fill a glass with water up to the point where a convex meniscus is formed. Place a table tennis ball on the surface of the water. Investigate and explain the stability of its equilibrium. Repeat your experiment with other liquids.

12. Electroconductivity
How can you measure the electroconductivity of salt solutions without using direct contact electrodes? Analyse the problem and demonstrate your device.

13. Spinning ball
A steel ball of diameter 2-3 cm is put on a horizontal plate. Invent and construct a device, which allows you to spin the ball at high angular velocity around a vertical axis. The device should have no mechanical contact with the ball.

14. Torn sail
Determine the dependence of the efficiency of a sail on its degree of perforation. What would be the effect of using a fishing net as a sail?

15. Pulsating air bubble
Trap an air bubble of radius 1-2 cm under an inverted watch glass beneath a water surface. Introduce alcohol into the bubble through a thin tube, controlling and adjusting the rate of flow until the bubble pulsates rhythmically. Study the phenomenon and explain your observations.

16. Elastic pendulum
Study and describe the behaviour of a pendulum where the bob is connected to a spring or an elastic cord rather than to a stiff rod.

17. Bottle battle
Take two opened glass bottles of cola and knock one against the other. After a short while, the cola spurts out of one of the bottles. Investigate and explain the phenomenon.

2.10. Problems for the 16th IYPT

UPPSALA, SWEDEN, 2003

1. Motion of a kite
On windy days one can see kites flying in the wind. Often, one-string kites move on a stable track, which looks like a number 8. Why does a kite move in such a way? Are there other stable tracks?

2.Water drops
Investigate and explain the movement of raindrops on a window pane.

3. Transparent film
If you cover printed text with a piece of transparent polyethylene film you can still easily read it. As you gradually lift up the film, the text becomes increasingly blurred and may even disappear. Study the properties of the film. On what parameters of the film is the phenomenon based?

4. Bright spots
Blow a soap bubble and allow it to rest on a liquid surface or a glas plate. When illuminated by sunlight, bright spots can be observed on the bubble. Investigate and explain the phenomenon.

5. Bubbles at an interface
Certain liquids can be layered one above the other with a sharp interface between them. If the surface tensions of the liquids are different, then an interesting phenomenon can be observed. Blow bubbles of different sizes into the lower liquid and observe their behaviour near the interface. Investigate and explain the phenomenon.

6. Freezing soft drinks
On opening a container of cold soft (carbonated) drink the liquid inside sometimes freezes. Study the relevant parameters and explain the phenomenon.

7. Oscillating box
Take a box and divide it into a number of small cells with low walls. Distribute some small steel balls between the cells. When the box is made to oscillate vertically, the balls occasionally jump from one cell to another. Depending on the frequency and the amplitude of the oscillation, the distribution of the balls can become stable or unstable. Study this effect and use a model to explain it.

8. Heat engine
Construct a heat engine from a U-tube partially filled with water (or another liquid), where one arm of the tube is connected to a heated gas reservoir by a

length of tubing, and the other arm is left open. Subsequently bringing the liquid out of equilibrium may cause it to oscillate. On what does the frequency of the oscillation depend? Determine the pV diagram of the working gas.

9. Falling chimney
When a tall chimney falls it sometimes breaks into two parts before it hits the ground. Investigate and explain this.

10. Tungsten lamp
The resistance of the tungsten filament in a light bulb shows a strong temperature dependence. Build and demonstrate a device based on this characteristic.

11. Light scattering
Construct an optical device for measuring the concentration of non-soluble material in the 'viscous' properties of hens' eggs that have been boiled to different extents.

12. Boiled egg
Construct a torsion viscometer. Use it to investigate and explain the difference in the 'viscous' properties of 'hens' eggs that have been boiled to different extends.

13. Electro-osmosis
Develop a device that will drain wet sand, whit the aid of an electrical voltage but without significant heating.

14. Rotating disk
Find the optimum way of throwing a 'frisbee' as far as possible. Explain your findings.

15. Vortices
Make a box that has a hole in its front wall and a membrane as its back wall. Hitting the membrane creates a vortex that propagates out from the hole. Investigate the phenomenon and explain what happens when two vortices interact.

16. Pot and ice
It is sometimes argued that to cool a pot effectively one should put ice above it. Estimate to what extend this is more effective than if the ice is put under the pot.

17. Prometheus problem
Describe and demonstrate the physical mechanism, based on friction, which allowed our ancestors to make fire. Estimate the time needed to make fire in this way.

V. Problems for the 19th IYPT

BRATISLAVA, SLOVAKIA, 2006

1. Froth
Investigate the nature of the decay in height of the “froth” or “foam” on a liquid. Under what conditions does the froth remain for the longest time?

2. Shades
If small non-transparent objects are illuminated with light, patterns in the shadows are observed. What information can be obtained about these objects using these patterns?

3. Duck`s cone
If one looks at the wave pattern produced by a duck paddling across a pond, this reminds one of Mach`s cone. On what parameters does the pattern depend?

4. Whispering Gallery
The Whispering Gallery at St Paul`s Cathedral in London, for example, is famous for the fact that the construction of the circular gallery makes a whisper against its walls on the side of the gallery audible on the opposite side of the gallery. Investigate this phenomenon.

5. Probability
A coin is held above a horizontal surface. What initial conditions will ensure equal probability of heads and tails when the coin is dropped and has come to rest?

6. Wet cleaning
A wet rag is hard to drag when it is spread out and pulled across the floor. What does the resistive force depends on?

7. Airglider
A paper sheet is on the table. If one blows along the table the sheet begins to glide over it. Determine the flight characteristics of the paper.

8. Electrostatics
Propose and make a device for measuring the charge density on a plastic ruler after it has been rubbed with a cloth.

9. Sound and foam
Investigate the propagation of sound in foam.

10. Inverted pendulum
It is possible to stabilize an inverted pendulum It is even possible to stabilize an inverted multiple pendulum (one pendulum the top of the other). Demonstrate the stabilization and determine on which parameters this depends.

11. Singing tube
A tube open at both ends is mounted vertically. Use a flame to generate sound from tube. Investigate the phenomenon.

12. Rolling magnets
Investigate the motion of a magnet as it rolls down an inclined plane.

13. Sound
Measure the speed of sound in liquids using light.

14. Cellular materials
Investigate the behaviour of a stream of liquid when it strikes the surface of a sponge-like material.

15. Heat and temperature
A tube passes steam from a container of boiling water into a saturated aqueous salt solution. Can it be heated by the steam to a temperature greater than 100°C? Investigate the phenomenon.

16. Hardness
A steel ball falls onto a horizontal surface. If one places a sheet of paper onto the surface with a sheet of carbon paper on top of it, a round trace will be produced after the impact. Propose a hardness scale based on this method.

17. Magnetohydrodynamics
A shallow vessel contains a liquid. When an electric and magnetic fields are applied, the liquid can start moving. Investigate this phenomenon and suggest a practical application.