PROBLEM 9: BOTTLE TONE

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Team Switzerland
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**THE PROBLEM**

**Bottle tone**

Take an empty bottle and blow air across its mouth to produce a sound. Now fill the bottle with some water and study how the sound changes.
DEMONSTRATION
\[ f = \frac{v}{4L} \quad \rightarrow \quad f = 504\text{Hz} \]

Actual frequency = 236Hz
THEORY: HOW THE SOUND IS CREATED
THEORY: HELMHOLTZ RESONANCE

\[ f = \frac{v}{2\pi} \sqrt{\frac{A}{V \cdot L_{eq}}} \]

\[ L_{eq} = L_n + 0.3D \]

**Sound parameters**
- \( f \): frequency of the resonance [Hz]
- \( v \): propagation speed of sound [m/s]

**Bottle parameters**
- \( L_{eq} \): equivalent length of neck [m]
- \( L_n \): length of neck [m]
- \( D \): diameter of neck [m]
- \( A \): cross-sectional area of neck [m\(^2\)]
- \( V \): static volume of cavity [m\(^3\)]

Theory suggests: frequency and volume vary inversely
Model of Frequency

\[ f = \frac{v}{2\pi} \sqrt{\frac{A}{V_{Leq}}} = \frac{343 \text{ m/s}}{2\pi} \sqrt{\frac{0.003142 \text{ m}^2}{V \cdot 0.046 \text{ m}}} \]
EXPERIMENTS
Set up
Glass bottles
Scale
Plastic bottle
Measuring cup
THE BOTTLES
Frequency of Empty Bottle with Volume 343 cm³
Comparing Experiments to the Theory

\[ L_n = 0.04 \text{[m]} \]
\[ D = 0.02 \text{[m]} \]
\[ A = 0.3142 \times 10^{-3} \text{[m}^2\text{]} \]
\[ V = 0.343 \times 10^{-3} \text{[m}^3\text{]} \]
Comparing Experiments to the Theory

\[ L_n = 0.057 \text{[m]} \]
\[ D = 0.009 \text{[m]} \]
\[ A = 0.64 \times 10^{-6} \text{[m}^2\text{]} \]
\[ V = 0.37 \times 10^{-6} \text{[m}^3\text{]} \]
Comparing Experiments to the Theory

\[ L_n = 0.11 \text{[m]} \]
\[ D = 0.019 \text{[m]} \]
\[ A = 0.28 \times 10^{-3} \text{[m}^2\text{]} \]
\[ V = 0.36 \times 10^{-3} \text{[m}^3\text{]} \]
Comparing Experiments to the Theory

$L_n = 0.09\,[\text{m}]$

$D = 0.0019\,[\text{m}]$

$A = 0.28 \times 10^{-3}\,[\text{m}^2]$

$V = 0.48 \times 10^{-3}\,[\text{m}^3]$
Comparing Experiments to the Theory

$L_n = 0.09\text{[m]}$
$D = 0.02\text{[m]}$
$A = 0.31 \times 10^{-3}\text{[m}^2]\$
$V = 0.75 \times 10^{-3}\text{[m}^3]\$
Comparing Experiments to the Theory

- $L_n = 0.102\, [m]$
- $D = 0.02\, [m]$
- $A = 0.31 \times 10^{-3}\, [m^2]$
- $V = 1.5 \times 10^{-3}\, [m^3]$
WHAT DOES THIS MEAN?

Helmholtz resonance is valid

Volume comparable neck volume → Edge case (assumptions break down)
EXPERIMENT WITH “MY HEART WILL GO ON”

\[ f \sim \frac{1}{\sqrt{V}} \implies V \sim \frac{1}{f^2} \]

low D  F#  G  A

low D  B  C  high D
TILT OF THE BOTTLE
CONCLUSION

Notes with bottles and water:

\[ f(V) = \frac{\text{constant}}{\sqrt{V}} \]

Same bottle:

- the shape of cavity does not influence the frequency
THANK YOU FOR LISTENING
Special thanks to Marc Bitterli and Ella Blakely


The Use of Helmholtz Resonance for Measuring the Volume of Liquids and Solids, E.S. Webster, C.E. Davies, Journal of Engineering and Advanced Technology, New Zealand, 2010

http://hyperphysics.phy-astr.gsu.edu/hbase/shm.html

https://hyperphysics.phy-astr.gsu.edu/hbase/Waves/cavity.html

https://newt.phys.unsw.edu.au/jw/Helmholtz.html

https://pages.mtu.edu/~suits/notefreqs.html

www.youtube.com/channel/UCp96ZKaODsGQwWYCR5T3DeA
EXPERIMENT WITH SPECIFIC NOTES

\[ V = \frac{S \cdot v^2}{4\pi^2 \cdot L_{eq} \cdot f^2} \]

<table>
<thead>
<tr>
<th>Musical note</th>
<th>low D</th>
<th>F#</th>
<th>G</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>high D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency in Hz (theory)</td>
<td>293.66</td>
<td>369.99</td>
<td>392.00</td>
<td>440.00</td>
<td>493.88</td>
<td>523.25</td>
<td>587.33</td>
</tr>
<tr>
<td>Volume in cm³ (model)</td>
<td>236</td>
<td>148</td>
<td>132</td>
<td>105</td>
<td>83</td>
<td>74</td>
<td>59</td>
</tr>
<tr>
<td>Frequency in Hz (experiment)</td>
<td>293</td>
<td>373</td>
<td>408</td>
<td>456</td>
<td>496</td>
<td>530</td>
<td>594</td>
</tr>
<tr>
<td>Frequency error</td>
<td>0.2%</td>
<td>0.8%</td>
<td>4.1%</td>
<td>3.6%</td>
<td>0.4%</td>
<td>1.3%</td>
<td>1.1%</td>
</tr>
</tbody>
</table>
SPEED OF SOUND IN FLUIDS

\[ v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma p}{\rho}} \]

\( \gamma \): Adiabatic index  
\( R \): Universal gas constant  
\( T \): Temperature  
\( M \): Molecular mass  
\( v \): Propagation speed of sound  
\( p \): Pressure  
\( \rho \): Density
HELMHOLTZ DERIVATION

\[ a = \frac{F}{m} = \frac{\Delta p A}{\rho A L_{eq}} \]

\[ = -\frac{\gamma p A x}{\rho V L_{eq}} = -\frac{v^2 A}{V L_{eq}} x \]

\[ x(t) = B \sin(\omega t + \varphi) \]

\[ \omega = 2\pi f = v \sqrt{\frac{A}{V L_{eq}}} \]
DIMENSIONS OF THE BOTTLE

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