

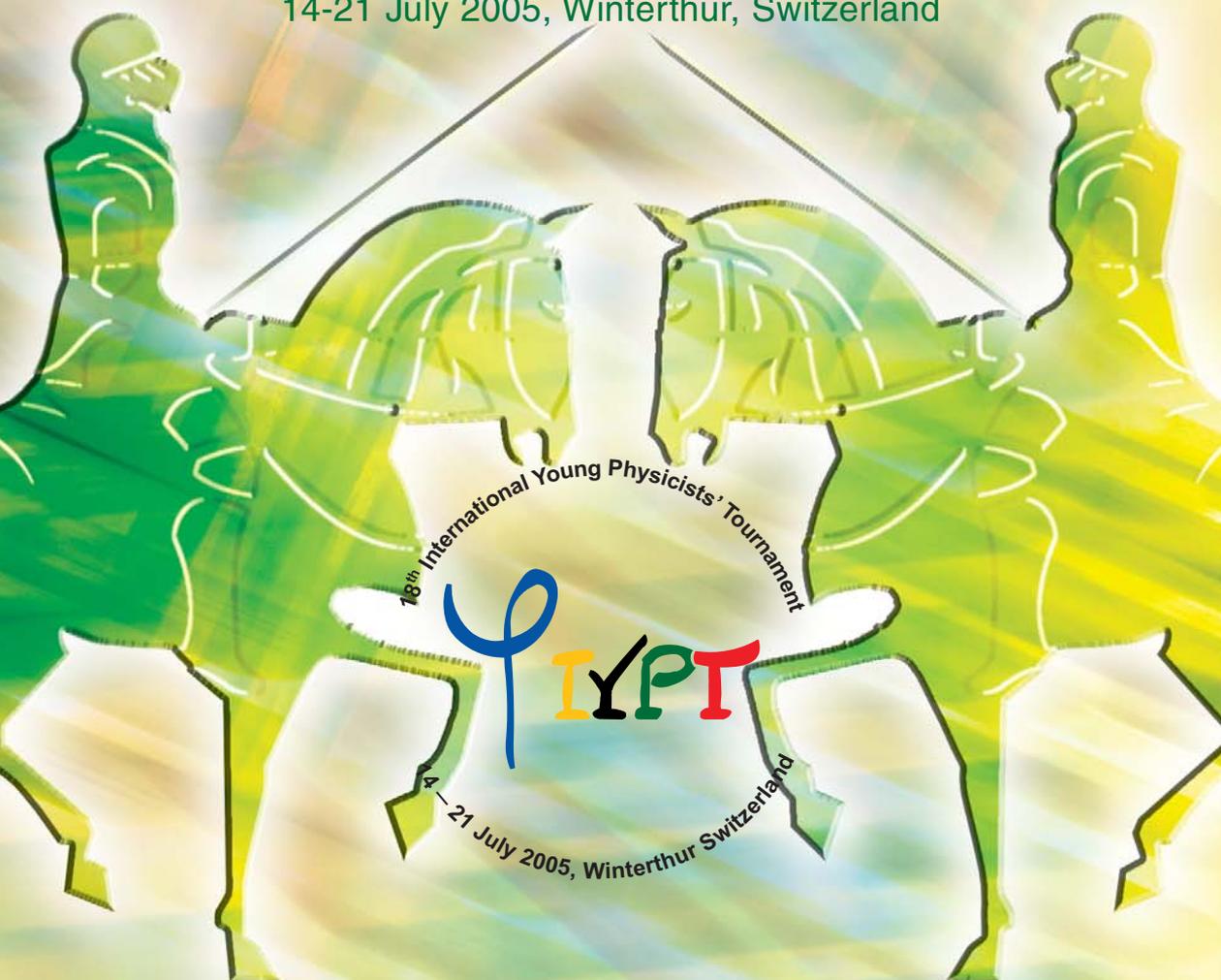
# IYPT

## The International Young Physicists' Tournament

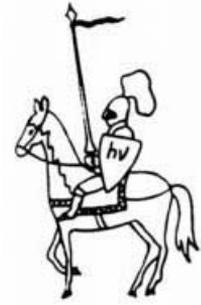
The Physics World Cup

Proceedings of the 18<sup>th</sup> IYPT 2005

14-21 July 2005, Winterthur, Switzerland





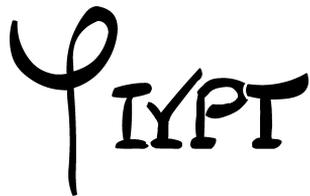


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**IYPT: The International Young Physicists' Tournament. The Physics World Cup.  
Proceedings of the 18th IYPT 2005**

*Silvina Simeonova (ed.)*

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# I. Introduction

This edition is an attempt to collect and present solutions of the problems for the IYPT, prepared and most of them presented at the 18<sup>th</sup> IYPT in Winterthur, Switzerland.

The main aim of the edition is to reveal and summarize the results of the investigations concerning the problems of the IYPT, done by the participants in the IYPT from different countries.

Another aim is the popularization of the tournament and estimating its educational effectiveness it to be used for the raising of the level of physics education.

## 1. AIMS AND TASKS OF THE IYPT

The IYPT is a competition between teams of secondary school students in their ability to solve complicated problems, to present solutions to these problems in a convincing form and defend them in scientific discussion, called Physics Fight (PF) [*Regulation of the IYPT*, [www.iypt.org](http://www.iypt.org)].

The IYPT is an organization of participants, which share its aims, mainly to provoke and enhance of the interests of the students from the secondary schools to physics and another branches of knowledge, to contribute for the realization of a connection between schools and research scientific institutions.

The concrete tasks of the tournament are:

- To develop the scientific thinking of the students and their investigation skills, communicative skills and skills to work in a team. By the problems of the tournament the students to come across with real scientific problems, giving them the possibility to become well acquainted and to make sense of the essence and stages of each investigation process, to enrich their knowledge;

- To quicken the interest of the young people and their desire to realize themselves in the area of engineering and scientific specialities and especially in the area of hi-tech industry, which is very important at the present in a global aspect;

- To create new kind of relationships between researchers, teachers and students, thanks to the specific content of the problems and the need of the consultations with scientists and specialists during the preparation and participation in the tournament;

- To popularize the tournament widely among schools and universities, seeking for new forms for the collaboration between them.

## 2. HISTORY

The tournament was established in 1979 as a Tournament of Young Physicists (YPT) and was developed at the Physics Faculty of Moscow State University for students from Moscow and its vicinity.

The important role for its foundation was played by Academician E.P.Velichov, Academician G.T,Zacepin, profesor S. M. Gudinov and Dr. Yanusov (2).

The reason of the establishment of the tournament was, by original and different from the traditional physics textbook tasks, to attract attention of the young people not only to physics, but as well as to the science and nature.

During the first tenth years the YPT was organized as a tournament of Moscow secondary school students. Since 1985 till 1988 the secondary schools students from the former Soviet Union could participate in the tournament The tenth annual tournament was then organized as a Soviet Tournament and simultaneously as an International Young Physicists' Tournament.

The first sixth IYPT took place in Russia and after that in other countries, participated in the tournament (2,4).

Now in the tournament take part teams from many European countries, from Australia, Brazil, Indonesia, Kenya, Korea, New Zealand, USA. The number of the participating countries in the 18<sup>th</sup> IYPT was 23 (see Section "IV. Participating Countries in the 18<sup>th</sup> IYPT").

### **3. ORGANIZATIONAL STRUCTURE OF THE IYPT**

The IYPT administration consists of the following units:

- a) The IOC (International Organization Committee)
- b) The LOC (Local Organization Committee)
- c) The Executive Committee

The President of the IYPT is Prof. Gunnar Tibell (Department of Radiation Sciences, Upsala University, Sweden).

The General Secretary is Dr. Andrzej Nadolny (Institute of Physics, Polish Academy of Science, Poland).

### **4. REGULATIONS OF THE IYPT**

The IYPT is a competition between teams of secondary school students.

Each team, participating in the IYPT consists of five students and two team leaders.

From each country participates one team.

The participants of the IYPT can be national teams, teams of regions, towns, clubs and colleges. The decision about participation of the latter may be taken by the LOC.

The working language is English.

The IYPT is a competition, which examines and compares the abilities of the students to present, opponent and review convincingly in a scientific discussion the solutions of the preliminary put 17<sup>th</sup> physics problems. On these problems students have to do approximately during one school year the needed experimental activity and to give the respective theoretical explanation. The 17<sup>th</sup> problems are prepared of the representatives of the participating countries in the International Organizing Committee (IOC).

The competition includes **selective discussions and a final**.

The basic structure unit of the competition is the Discussion Group, less formally called a Physics Fight (PF). All teams participate in Selective Physics Fight (SPF).

In one PF three teams (or four depending on entire number of teams) compete in three (or four) stages. In each stage the teams perform one of the three (or four) roles:

*Reporter, Opponent, Reviewer (Observer).*

In the next stages the teams change their roles, according to definite scheme. On the base of the received results from the SPF three teams are in the final.

More details about the regulations can be seen on the website of the IYPT – [www.iypt.org](http://www.iypt.org).

## **5. THE PROBLEMS**

The problems of the tournament possess the characteristic features of the interesting and original problems, presented to the students by R.Feynman, P.L.Kapitza and Nobel Prize Winner (open-ended problems).

They are complex problems with scientific and technology content, from real life and nature, from different areas of physics and science and their boundary fields.

The problems reflect the role and influence of physics on many areas of human activity such as engineering, building, art, sports, etc.

The solution of the problems requires doing the necessary activities for every scientific research-experimental investigation and theoretical analysis.

## **6. EDUCATIONAL EFFECTIVENESS**

### **6.1 .Development of the cognitive skills of the students**

The untraditional problems and discussion character of the tournament is new and unknown challenge for the students to enter into the real world of the science. It allows many of the aims of the education of physics to be realized.

One of them is the development of the cognition of students, their adoption of the contemporary style of scientific thinking. The latter are in direct connection with the formation of a system of scientific terms and knowledge, with mastery of the common methodological knowledge through concrete educational content.

### Adoption of the methodological terms

Independently of a great variety of the problems of the IYPT their solutions are based on such common scientific terms as a system, structure, state of the system, model and hypothesis.

### Making sense of key ideas and principles

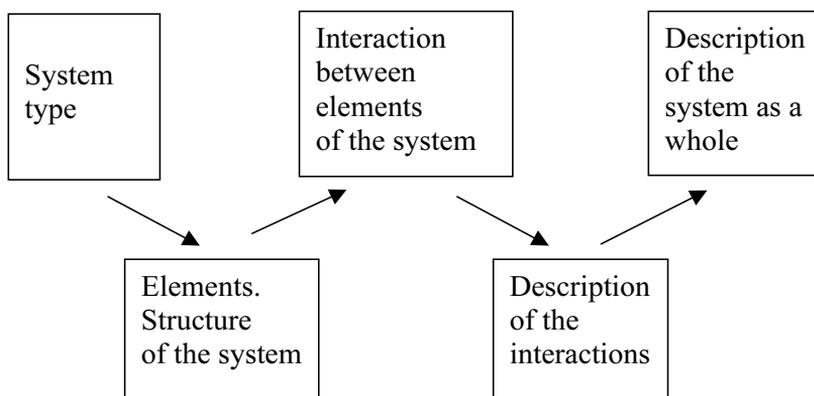
The solutions of the problems are also based on key ideas and principles (wave-particle duality, correspondence principle – “Optical Tunneling”, conservation of energy, momentum conservation, angular momentum conservation – “Hydraulic Jump”, “Einstein–de Haas Experiment”), which as whole defines the attitude of the students towards science, systematizes and make sense of their knowledge, outlines their vision about nature, universe and human life.

### Adoption of scientific approaches and methods

Ones of the most widely used methods for the solution of the problems are the modeling, analogy and system-structure approach.

The term with the greatest weight here is the term a physics system.

The content of the problems gives the possibility to understand and to repeat the stages of the studying of a given system in the sense of the system-structure approach, namely:

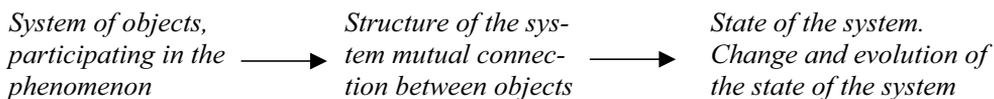


The analysis of the presented solutions in the edition shows that the above scheme is the base to do comparison and **analogies between classical and quantum systems in physics**, for example the problems “Optical Tunneling” (Brazil, Croatia, Poland, Ukraine), “Einstein–de Haas Experiment” (Brazil, Bulgaria, Czech, Poland).

Analogical scheme to the above is used to do modeling of the granular system, which is far complicated than non-granular (“Obstacle in a Funnel” – Hungary). In the solution there is analogy between **different kind of systems-physics** (granular system) **and social system** (so called pedestrian escape panic).

Using the analogy **between different physics systems** (the glass is interpreted as a spring with a weight), the complex phenomenon in the problem “The Sound in the Glass” is investigated (New Zealand, Ukraine).

The common scheme of the study of the complex system and complex phenomenon, used in the presented solutions allows creating of different didactical schemes for the study of the physics phenomena, for example:



## **6.2. Extension of knowledge of students, acquaintance with new ideas and contemporary problems of the science**

Another very important feature of the problems is that they give the possibility to convey in very easy and attractive way the study of the system with predictable behaviour (the study basic for the secondary school) to the study of the system with unpredictable behaviour and connected to them new terms and problems (chaotic motion, catastrophe theory, fold catastrophe system, etc.). By this way the students would learn more about new area of researches and acquire broad vision for the complexity of the phenomena in the nature.

## **6.3. Creating communicative and organization skills**

The necessity to do serious preliminary theoretical and experimental activity to the problems and the character of their presentation in the PF is a condition to create skills with students to work in team, to distribute tasks, to take responsibilities, to organize and plan their activity and preparation.

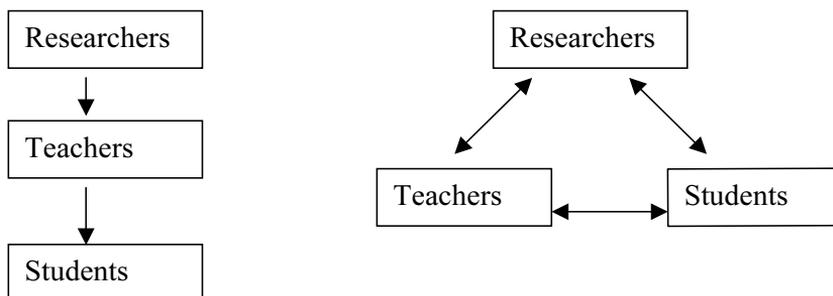
## **6.4. Influence over the relationships between the participants in the educational system**

The practice of the preparation of the problems of the tournament and the realization of the latter, lead to the inference that thanks of the essence of its problems and its regulations, also thanks to the broad using of the information technologies, we can observe interesting direction of delivering of new knowledge, interesting and new relationships between students, teachers and researchers as participants in the educational system in principle.

We can say that now the relationships teacher-student doesn't build on the presumption that the teacher is this who knows, but student is this who doesn't know. They have to in position as partners and the teacher is this who will help student to accumulate himself the information, to classify it, to check and give it a meaning.

About the delivering of new knowledge the practice of the tournament suggests the following parallel. The first figure/fig.1/ presents the being vision for the delivering of the new knowledge, subordinated to a defined hierarchy from top to the bottom.

The second one presents the relationships between respective participants in educational system and possible obtaining of new knowledge, thanks to the open-ended character of the problems and character of the tournament. It is realized by a continuous feedback between participants in education, typical and needed for the effective function of every complicated system, including and the educational system.



Such parallel can be a basis for didactic analyses, regarding the change of learning, teaching and education, as result of new positions and functions, which gives in concrete case the participation in the tournament.

## 7. CONCLUSION

The IYPT is a school competition with great educational effectiveness.

As a domino wave the main aims of the tournament entail the realization of many other aims, concerning the upbringing and growth of the young people- to be well educated with wide spirit and mental horizons.

Maybe the most beautiful feature of the problems of the tournament is this one that they bring up with students the skills to observe, investigate and understand nature, to enjoy its beauty, to provoke and evaluate their curiosity to the things around us and seeking for answer to the questions concern simple at first glance things, such as why the color of the sky is blue, why the sunrises and sunsets are red, the young people to take the path of discovering of the deep secrets of the nature.

## ACKNOWLEDGMENTS

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Especially, I would like to thank Dr. Zdenek Kluiber (IOC member of the IYPT from the Czech Republic) and Dr. Szuszana Rajkovits (IOC member of the IYPT from Hungary) for the contribution to the edition with their experience, ideas and opinions, concerning the IYPT.

## REFERENCES

1. Kluiber, Z., The Development of Talents in Physics, The Union of Czech Mathematicians and Physicists, Prometheus, Prague 1995.
2. Kluiber, Z., 10<sup>th</sup> International Young Physicists' Tournament, Hradec Kralove 1998.
3. Kluiber, Z., Proceeding of the 10<sup>th</sup> International Conference of Young Scientist, Christian Doppler Grammar School, Prague 2003.
4. Lobysev, V., Junosov E., Tournament of Young Physicists, Kvant, 1996, №.1
5. Rajkovits, Z., Physics Competitions, Vol.1, №1, 1999.
6. Simeonova, S., The Information technologies and Creation of New Relationships between Students, Teachers and Researchers, The XXXI National Conference of Physics, Bulgaria 2003.

*Silvina Simeonova*

IOC member of the IYPT 2003, 2004, 2005  
Bulgaria



## **II. Problems for the 18<sup>th</sup> IYPT**

### **1. Dragonfly**

Propose a model of how a dragonfly flies, investigate the major parameters and validate your model.

### **2. The two balls problem**

Two balls placed in contact on a tilted groove sometimes do not roll down. Explain the phenomenon and find the conditions, under which it occurs.

### **3. Avalanche**

Under what conditions may an avalanche occur? Investigate the phenomenon experimentally.

### **4. Hydraulic jump**

When a smooth column of water hits a horizontal plane, it flows out radially. At some radius, its height suddenly rises. Investigate the nature of the phenomenon. What happens, if a liquid more viscous than water is used?

### **5. Mirage**

Create a mirage like a road or desert mirage in a laboratory and study its parameters.

### **6. Noise**

When a droplet of water or other liquid falls on a hot surface, it produces a sound. On what parameters does the sound depend?

### **7. Bouncing plug**

A bathtub or sink is filled with water. Remove the plug and place a plastic ball over the plughole. As the water drains the ball starts to oscillate. Investigate the phenomenon.

### **8. Wind car**

Construct a car, which is propelled solely by wind energy. The car should be able to drive straight into the wind. Determine the efficiency of your car.

### **9. Sound in the glass**

Fill a glass with water. Put a tea-spoon of salt into the water and stir it. Explain the change of the sound produced by the clicking of the glass with the tea-spoon during the dissolving process.

## **10. Flow rate**

Combine powdered iron/ iron filings/ with a vegetable oil. Connect two containers with plastic tubing and allow the mixture to drain through the tube. Develop an external mechanism to control the flow rate of the mixture.

## **11. Water droplets**

If a stream of water droplets is directed at a small angle to the surface of water in a container, droplets may bounce off the surface and roll across it before merging with the body of water. In some cases the droplets rest on the surface for a significant length of time. They can even sink before merging. Investigate these phenomena.

## **12. Ball spin**

Spin can be used to alter the flight path of balls in sport. Investigate the motion of a spinning ball, for example a table-tennis or tennis ball, in order to determine the effect of the relevant parameters.

## **13. Hard starch**

A mixture of starch (e.g. cornflour or cornstarch/ and a little water has some interesting properties. Investigate how its viscosity changes when stirred and account for this effect. Do any other common substances demonstrate this?

## **14. Einstein-de-Haas Experiment**

When you apply a vertical magnetic field to a metallic cylinder suspended by a string it begins to rotate. Study this phenomenon.

## **15. Optical tunneling**

Take two glass prisms separated by a small gap. Investigate under what conditions light incident at angles greater than the critical angle is not totally internally reflected.

## **16. Obstacle in a funnel**

Granular material is flowing out vessel through a funnel. Investigate if it is possible to increase the outflow by putting an obstacle above the outlet pipe.

## **17. Ocean “Solaris”**

A transparent vessel is half-filled with saturated salt water solution and then fresh water is added with caution. A distinct boundary between these liquids is formed. Investigate its behaviour if the lower liquid is heated

# III. Solution of the problems for the 18<sup>th</sup> IYPT

## 1. PROBLEM № 2: THE TWO BALLS PROBLEM

SOLUTION OF BRAZIL

### Problem № 2: The Two Balls Problem

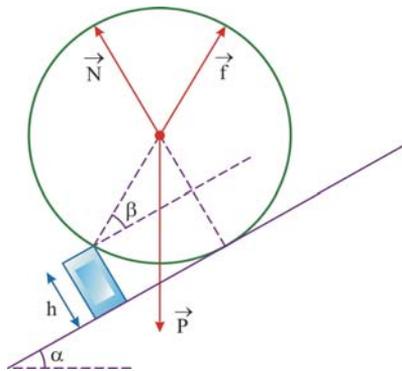
Marcelo Puppo Bigarella, Brazil

#### The problem

Two balls placed in contact on a tilted groove sometimes do not roll down. Explain the phenomenon and find the conditions, under which it occurs.

#### Physic Insight:

First, we will treat about the case where we have just one sphere in a plan. If we want the sphere does not roll down, we need to fix an obstacle (as showed in the picture) in front of it.



Picture 1: A ball in a ramp with an obstacle

In this case we have  $\vec{f}$  force: contact force that obstacle exert in the sphere (with  $R$  radius),  $\vec{P}$ : sphere's weight,  $\vec{N}$ : Normal force that the titled ramp exert in the sphere;  $h$  is the obstacle height fixed in the ramp,  $\alpha$  is the ramp's inclination and  $\beta$  is the angle formed between the ramp and  $\vec{f}$  direction.

In the equilibrium, the external force sum at the sphere must be equal to zero (null). Thus,

$$\sum_i F_{ix} = \sum_i F_{iy} = 0 \quad (1)$$

where  $F_{ix}$  and  $F_{iy}$  are the  $x$  and  $y$  component of the  $i$ -esimal force, respectively. In this way:

- X axis:  $f \cdot \cos(\alpha + \beta) = N \cdot \sin \alpha$
- Y axis:  $P = f \cdot \sin(\alpha + \beta) + N \cdot \cos \alpha$

Solving these two equations (in function of  $P$ ) we have:

$$f = \frac{\sin \alpha}{\cos \beta} \cdot P \quad (2) \qquad N = \frac{\cos(\alpha + \beta)}{\cos \beta} \cdot P \quad (3)$$

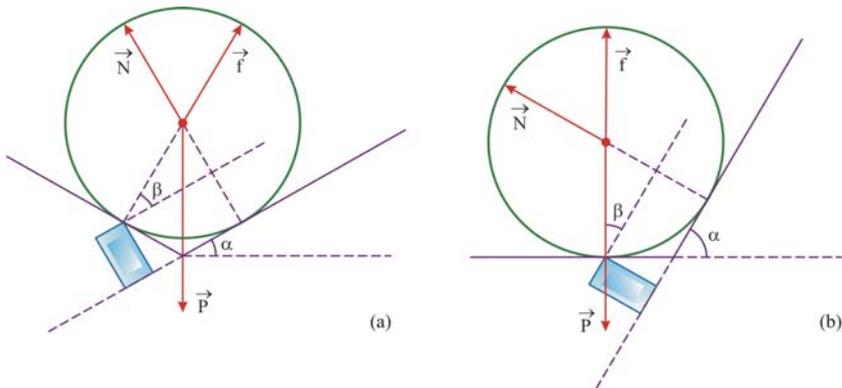
This solution above just have a physical sense, when  $N \geq 0$ . Thus, from equation 3 we obtain:

$$\alpha + \beta \leq \frac{\pi}{2} \quad (4)$$

With this, we can conclude that the contact point between the sphere and the obstacle must stay at the left of  $\vec{P}$  vector; otherwise the sphere will roll over the obstacle.

We could thing in other way to understand the equilibrium: let's imagine that the ball is placed in a groove with V shape (picture 2a). Therefore, we can consider that the ball is in under a tilted ramp with an obstacle as the spotted lines in the drawing shows.

The equilibrium configuration happens when both walls of the groove are inclined upwards (stability). It is way, in picture 2b, the ball is under a permanent equilibrium configuration (instability), which means that the sphere can roll to left in one moment. (It is possible because the left groove wall is in a horizontal position). Geometrically, this condition happens when  $\alpha + \beta = \pi/2$ . In this way we also arrive in the equilibrium condition (equation 4).



**Picture 2: Equilibrium configurations**

With assistance of this theory, we can substitute the obstacle for a small sphere, placed in the same local: at the left of the superior ball. The two balls would not roll down because the friction force torque over them must be

compensates by the torque of the contact force between them. Therefore, we can expect that the torques cancel themselves and do not occur any rolling movement.

We can notice that if the left sphere is bigger than the right sphere, it will be impossible maintaining the equilibrium. It would be equivalent to having the obstacle at the right side of the ball. So, we would not deal with this situation.

### The Two Balls Problem:

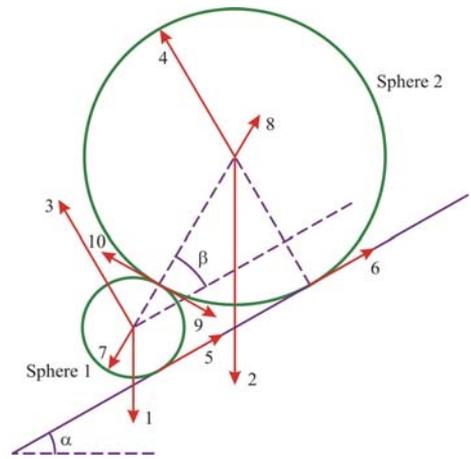
We need to analyze the condition in under which two balls in touch in a tilted groove do not roll down.

Picture 3 shows us the two balls in a tilted ramp and the external forces that are applied over each ball.

The ball at the left is *sphere 1* and the ball at the right is *sphere 2*.

These forces are:

1. Sphere 1 weight force:  $\vec{P}_1$ ;
2. Sphere 2 weight force:  $\vec{P}_2$ ;
3. Sphere 1 Normal:  $\vec{N}_1$ ;
4. Sphere 2 Normal:  $\vec{N}_2$ ;
5. Friction force in sphere 1:  $\vec{f}_1$ ;
6. Friction force in sphere 2:  $\vec{f}_2$ ;
7. Sphere 2 contact force in sphere 1, which tends to push down the sphere e 1:  $\vec{f}_3$
8. Reaction to force  $\vec{f}_3$ :  $-\vec{f}_3$ ;
9. Sphere 2 contact force in sphere 1, which tends to rotate sphere 1 in a clockwise movement:  $\vec{f}_4$ ;
10. Reaction to force  $\vec{f}_4$ :  $-\vec{f}_4$



**Picture 3: The two balls problem**

the angle  $\alpha$  represents the ramp inclination and  $\beta$  represents the angle formed between the tilted ramp and the forces pair  $\vec{f}_3$  and  $\vec{f}_4$  actuation line

As we want that the system continues in equilibrium, the sum of the forces over each spheres have to be zero.

$$\sum_i F_{ix} = \sum_i F_{iy} = 0$$

So, such as the external forces, as well the external torques which acts in the two balls must annul themselves. First, the force equilibrium equations for each ball are:

#### Sphere 1:

- X axis:  $f_1 \cos \alpha + f_4 \sin(\alpha + \beta) = f_3 \cos(\alpha + \beta) + N_1 \sin \alpha$  ;
- Y axis:  $\vec{P}_1 + f_3 \sin(\alpha + \beta) + f_4 \cos(\alpha + \beta) = f_1 \sin \alpha + N_1 \cos \alpha$  ;

- Rotation:  $f_1 = f_4$

Sphere 2:

- X axis:  $f_2 \cos \alpha + f_3 \cos(\alpha + \beta) = f_4 \sin(\alpha + \beta) + N_2 \sin \alpha$  ;
- Y axis:  $\vec{P}_2 = f_2 \sin \alpha + f_3 \sin(\alpha + \beta) + f_4 \cos(\alpha + \beta) + N_2 \cos \alpha$  ;
- Rotation:  $f_2 = f_4$

The system solution is showed below:

$$N_1 = P_1 \cos \alpha + \frac{\sin \alpha}{2 \cos \beta} [P_1 + P_2 + (P_2 - P_1) \sin \beta],$$

$$N_2 = P_2 \cos \alpha - \frac{\sin \alpha}{2 \cos \beta} [P_1 + P_2 + (P_2 - P_1) \sin \beta],$$

$$f_3 = \frac{\sin \alpha}{2 \cos \beta} [P_2 - P_1 + (P_2 + P_1) \sin \beta],$$

$$f_1 = f_2 = f_4 = \frac{1}{2} \sin \alpha (P_1 + P_2).$$

In order not roll any ball, it is necessary that

$$f_1 \leq \mu_1 N_1 \quad (6), \quad f_2 \leq \mu_2 N_2 \quad (7), \quad f_4 \leq \mu_3 f_3 \quad (8)$$

where  $\mu_1, \mu_2, \mu_3$  are the static friction coefficient between the sphere 1 and the ramp, between the sphere 2 and the ramp and between sphere 1 and sphere 2, respectively. The system will be in movement imminence as soon as one of these equations becomes saturated.

Now, we must study the module of  $\alpha, \beta, P_1, P_2, \mu_1, \mu_2, \mu_3$  that saturate the equilibrium's conditions (equation 6, 7, 8) with purpose of finding out the imminence conditions in which the spheres roll down.

Firstly, let's analyze the restriction 8. So,

$$\cos \beta - \mu_3 \sin \beta \leq \mu_3 \frac{P_2 + P_1}{P_2 - P_1} \quad (9)$$

which solution is:

$$\cos \beta \geq \frac{\mu_3 p + \sqrt{\mu_3^2 + (1 - p^2) \mu_3^4}}{1 - \mu_3^2},$$

where  $p = \frac{P_2 - P_1}{P_2 + P_1}$ . We must to emphasize that is not the unique solution. There are negative solutions which refers to case in where the smallest ball is at the right side.

As we know that:

$$\beta_{\min} \leq \beta \leq \frac{\pi}{2} - \alpha \quad (10)$$

we have:

$$\beta_{\min} = \arccos \frac{\mu_3 p + \sqrt{\mu_3^2 + (1 - p^2) \mu_3^4}}{1 - \mu_3^2}$$

In an analog form, to the equation 7:

$$\cot \alpha \geq \frac{P_1 + P_2}{2P_2} \left( \frac{1}{\mu_2} + \frac{1 + p \sin \beta}{\cos \beta} \right) \quad (11)$$

In the same form, to equation 6:

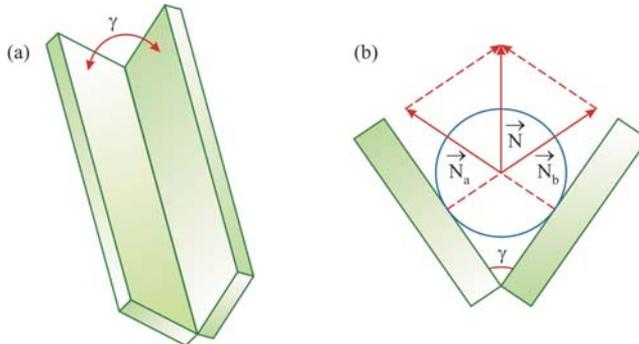
$$\cot \alpha \geq \frac{P_1 + P_2}{2P_2} \left( \frac{1}{\mu_1} - \frac{1 + p \sin \beta}{\cos \beta} \right) \quad (12)$$

The restriction ensemble (10), (11) e (12) defines  $\alpha$  and  $\beta$  modules, in function of the others parameters  $P_1, P_2, \mu_1, \mu_2, \mu_3$

### **Groove:**

The first point we want to discuss is the tilted ramp (opening groove wall's angle =  $180^\circ$ ). Two balls in a tilted ramp always roll down because the bi-dimensional configuration is unstable. Let's remember that we are working with the hypothesis that the spheres are confined to move along the line which defines the tilted plan (conform picture 3).

The experiment in a tilted plan would just be successful if we worked with cylinders, instead of balls. As we want to respect the problem, using spheres, we must have to use a gutter with a V format (the groove), as it is showed in picture 4.



**Picture 4: Groove's characteristic**

The difference between this model and the model utilized in the previous section is the  $\beta$  angle's determination. We can notice that the equilibrium force

equations remain unaltered. For example, the Normal force vector which acts over a sphere in this groove (see picture 4b) remains in the rolling plan. The transversal component to the vectors  $\vec{N}_a$  and  $\vec{N}_b$  rolling plan cancel themselves (see picture 4b) and yet remains in the rolling plan.  $\beta$  angle's determination in the assembly of picture 3 is extremely simple:  $\sin \beta = (R - r)/(R + r)$ . In the groove, the  $\beta$  angle's determination should depend on  $\gamma$ . As illustrated in picture 4b, the distance between the groove basis (intersection point between the two walls) and the sphere centre (with radius equal to  $R$ ) is:

$$D = \frac{R}{\sin \frac{\gamma}{2}}$$

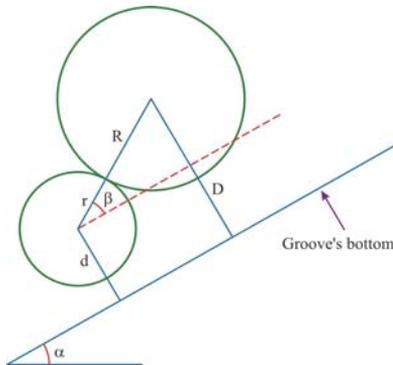
In this way,  $\beta$  angle have to be determined by (see picture 5),

$$\sin \beta = \frac{R - r}{(R + r) \sin \frac{\gamma}{2}} \quad (16)$$

There is a minimum  $\gamma$ . It is necessary to remember condition  $\alpha + \beta \leq \frac{\pi}{2}$ , therefore:

$$\frac{R - r}{(R + r) \sin \frac{\gamma}{2}} \leq \cos \alpha, \Rightarrow \sin \frac{\gamma}{2} \geq \frac{R - r}{(R + r) \cos \alpha}$$

To finish, the experiment to be done still have seven free parameters:  $\alpha, \beta, P_1, P_2, \mu_1, \mu_2, \mu_3$ . In the true, we can simplify it to just six parameters because in the restriction inequations (6), (7), (8)  $P_1$  and  $P_2$  appears in the two sides, so, the interest quantity is the relative mass between the spheres, and not the absolute mass of each one.



**Picture 5: A lateral view of the groove**

In the attached [Excel plan two\\_balls](#) we manipulated all the formulas, where we can place the variables (theoretical values or experimental data) to obtain the maximum angle for a specific situation. The angle formed between the two groove walls is fixed (in our case  $90^\circ$ ). We fixed this angle in order to a better experimental performance. However, as we fixed this angle, we can not apply it into a future theoretical angle's maximum model.

As we are working with an opening groove's angle equal to  $90^\circ$ , equation 13 becomes:

$$\sin \beta = \frac{R - r}{(R + r) \sin \frac{\pi}{4}}$$

We are going to use its plan to compare the experiment with theory and see the difference between these two parts of the problem's resolution

**Not rolling conditions:**

In this section we will determine the conditions and the inclination angle  $\alpha$  which must satisfy the equations presented, in order to occur the phenomenon. It means that  $\alpha$  must satisfy:

$$\alpha_{\min} \leq \alpha \leq \alpha_{\max}$$

where  $\alpha_{\min}$  and  $\alpha_{\max}$  depends on  $\beta, P_1, P_2, \mu_1, \mu_2, \mu_3$ .

$\alpha_{\min}$  determination is simple: in the case the groove is in the horizontal position, the system will stay in rest, hence,  $\alpha_{\min} = 0$  for any of the others values.

$\alpha_{\max}$  determination is more complex: it demands that all restrictions (4), (6), (7) and (8) is satisfied. From condition 4:

$$\alpha_1 = \frac{\pi}{2} - \beta \tag{13}$$

where  $\alpha_1$  is a possible  $\alpha_{\max}$  value, and, by inequation 10,  $\beta \geq \beta_{\min}$ .

If  $\beta < \beta_{\min}$ ,  $\alpha_{\max} = 0$ .

It is important to notice that condition if the sphere at the left is smaller than the sphere at the right; otherwise  $\beta$  would be smaller or equal to zero.

The others possible  $\alpha_{\max}$  values come from inequation (11) and (12) and they are, respectively:

$$\alpha_2 = \arctan \frac{2P_2\mu_2 \cos \beta}{(P_1 + P_2)(\cos \beta + \mu_2 + \mu_2 p \sin \beta)} \tag{14}$$

$$\alpha_3 = \arctan \frac{2P_1\mu_1 \cos \beta}{(P_1 + P_2)(\cos \beta - \mu_1 - \mu_1 p \sin \beta)} \tag{15}$$

Therefore,

$$\alpha_{\max} = \min\{\alpha_1, \alpha_2, \alpha_3\}$$

For determining the maximum inclination that the groove can has, without the spheres rolling down, we must calculate angles  $\alpha_1, \alpha_2, \alpha_3, \beta_{\min}$ . If  $\beta \geq \beta_{\min}$ , then the maximum groove inclination will be given by the smaller value among  $\alpha_1, \alpha_2, \alpha_3$ . Otherwise, If  $\beta < \beta_{\min}$ , the phenomenon (problem) will not occur.

This conclusion ends up the study about phenomenon, but they do not give a good indication how to vary  $\beta, P_1, P_2, \mu_1, \mu_2, \mu_3$  parameters in the way that it maximizes the groove inclination. It happens because the equation which determines  $\alpha_1, \alpha_2, \alpha_3$  are completely non-linear equation.

First, we need to understand more about the opening wall's angle (which is always, in our case,  $90^\circ$ )

In the next topics we will explain the manipulation possibilities of these equations in one more simplified context.

### **Simplified Situation:**

In this section we will treat about the realization possibilities of the described phenomenon in situations more simplified. Our simplification choice will be specified in each case

- Same Size Spheres

Considering two spheres with same size (same radius), is it possible that they do not roll down?

In this situation,  $\beta$  is equal to zero (see equation 13). In this way:

$$f_3 = \frac{\sin \alpha}{2} (P_2 - P_1) \qquad f_4 = \frac{\sin \alpha}{2} (P_1 + P_2)$$

$$\text{As } f_4 \leq \mu_3 f_3, \qquad \mu_3 \geq \frac{P_2 + P_1}{P_2 - P_1} > 1$$

Hardly we will find in the nature material which have static friction coefficient bigger than 1. So we conclude that the experiment success in this situation is remote (if it is not impossible).

We confirm our hypothesis supported in the excel plan *two\_balls*.

- Sphere and groove made of the same material

Now we will analyze the case in which we have both spheres and groove made of the same material (e. g. wood). This imply in  $\mu_1 = \mu_2 = \mu_3 = \mu$ . Defining  $\Delta$  as the relation between the spheres weight ( $P_2 = \Delta P_1$ ), the minimum  $\beta$  is given by:

$$\beta_{\min} = \arccos \frac{\mu(1 - \Delta) + \mu\sqrt{(1 + \Delta)^2 + 4\Delta\mu^2}}{(1 + \Delta)(1 + \mu^2)}$$

We will test this equation experimentally and see the difference between the theoretical value and the experimental one.

**Experiment:**

Our experiment consists in reproducing the problem according to the explanation giving. After it, we intend to place the experimental data in the theory equation (previous studied) and see how much approximate is our experience.

We made a groove and fixed an angle's measure equipment at its base. Photos of the apparatus are attached. Our variables, in this experiments, were:

The surface material: cardboard paper, polystyrene paper, paper, wood or plastic.

The balls were made of: polystyrene, plastic, leather, steel, glass.

In our experiment, we choose two balls and one surface. We measure the maximum inclination angle ( $\alpha$ ). After this, we compared with the maximum  $\alpha$ 's angle given by equations 13, 14 or 15 (we can use this equation because we know the relation between the two ball radius, and, consequently,  $\beta$ ). The theoretical values were taken from the Excel plan.

We repeat the experience changing the ball combinations and marking the respective angles.

As we said before, in our experiment the angle between the walls of the groove have not been changed (as we used two fixed wood board as our groove, the angle was 90° angle)

The first step in the experiment was measuring the characteristic of the ball (for substituting in the equations), like the diameter, and consequently, the radius, the mass, and, consequently, the weight

For measuring the mass, we made use of a balance (with maximum precision as a hundredth of the gram) and for measuring the diameter we used special ruler (with maximum precision as a tenth of the millimeter).

The balance's error is 0,01g (it is written in the technical characteristic protocol)

The measure instrument error is the half of the smallest measure step. So, the error in the diameter was 0,005mm

The experimental measure data as well the correspondent error is attached in the Excel plan two\_balls\_measures.

It is important to emphasis that we did not work just with full solid balls: we worked also with spherical husks (ball with air inside).

Completely solid (6):	Golf, iron 1 and 2, glass 1 and 2, plastic ball
Spherical husk (12):	All polystyrene balls, tennis balls, blue, orange and violet balls, ping-pong balls

The unique difference between this two kind of ball (hollow or full) is the great difference between the weight force in balls with almost the same size.

As we need to place the static friction coefficient to calculate theoretically the maximum angle, we need to know its value for each material combination in the experiment.

A complete static friction coefficient table is attached in a Word document *two\_balls\_friction*

Those coefficients we did not managed to find in any table, we calculate using the equation below (where  $\theta$  is the movement imminence angle).

$$\mu_{static} = \tan \theta$$

Just in four material's combination we did not have a theory value we made this experience. The results are showed in the table below

	$\theta$	$\tan \theta = \mu$ (static)
Cardboard and polystyrene	27°	0,50
Plastic and polystyrene	24,5°	0,45
Paper and polystyrene	20,5°	0,37
Leather and plastic	32°	0,62

## DEDUCTIONS

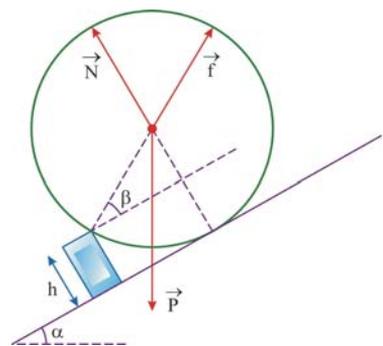
### DEDUCTION 1

- X axis:  $f \cdot \cos(\alpha + \beta) = N \cdot \sin \alpha$  (I)
- Y axis:  $W = f \cdot \sin(\alpha + \beta) + N \cdot \cos \alpha$  (II)

$$\text{As, } \sum_i F_{ix} = \sum_i F_{iy} = 0$$

$$(I) N = \frac{f \cdot \cos(\alpha + \beta)}{\sin \alpha}$$

$$(II) N = \frac{W - f \cdot \sin(\alpha + \beta)}{\cos \alpha}$$



**Picture 1: A ball in a ramp with an obstacle**

$$N = N \rightarrow \frac{f \cdot \cos(\alpha + \beta)}{\sin \alpha} = \frac{W - f \cdot \sin(\alpha + \beta)}{\cos \alpha}$$

$$\square f \cdot \cos(\alpha + \beta) \cdot \cos \alpha = [W - f \cdot \sin(\alpha + \beta)] \cdot \sin \alpha$$

$$\square f \cdot \cos(\alpha + \beta) \cdot \cos \alpha + f \cdot \sin(\alpha + \beta) \cdot \sin \alpha = W \cdot \sin \alpha$$

$$\square f \cdot [\cos(\alpha + \beta) \cdot \cos \alpha + \sin(\alpha + \beta) \cdot \sin \alpha] = W \cdot \sin \alpha$$

$$\square f \cdot [\cos(\alpha + \beta - \alpha)] = W \cdot \sin \alpha$$

$$\square f \cdot [\cos(\alpha + \beta - \alpha)] = W \cdot \sin \alpha$$

$$\square f \cdot (\cos \beta) = W \cdot \sin \alpha \quad \rightarrow$$

$$f = \frac{\sin \alpha}{\cos \beta} \cdot W$$

Placing  $f$  result in equation (I), we have:

$$N \cdot \sin \alpha = f \cdot \cos(\alpha + \beta) \rightarrow N \cdot \sin \alpha = \frac{\sin \alpha}{\cos \beta} W \cdot \cos(\alpha + \beta) \rightarrow N = \frac{\cos(\alpha + \beta)}{\cos \beta} \cdot W$$

## Deduction of $\text{Alfa}_1$

$$N = \frac{\cos(\alpha + \beta)}{\cos \beta} \cdot W$$

$$N \geq 0$$

$$\alpha + \beta \leq \frac{\pi}{2}$$

$$\alpha_1 = \frac{\pi}{2} - \beta$$

## Deduction of $\text{Alfa}_2$

For finding  $\beta$  minimum, we need to pick condition

$$f_1 \leq \mu_1 N_1$$

We need to substitute  $f_1$ ,  $N_1$  and  $\mu_1$  (static friction coefficient between the two ball's material). Therefore,

$$f_1 = \frac{1}{2} \sin \alpha (W_1 + W_2)$$

$$N_1 = W_1 \cos \alpha + \frac{\sin \alpha}{2 \cos \beta} [W_1 + W_2 + (W_2 - W_1) \sin \beta]$$

$$\square f_1 \leq \mu_1 N_1$$

$$\square \frac{1}{2} \sin \alpha (W_1 + W_2) \leq \mu_1 \left[ W_1 \cos \alpha + \frac{\sin \alpha}{2 \cos \beta} [W_1 + W_2 + (W_2 - W_1) \sin \beta] \right]$$

$$\square \frac{\frac{1}{2} \sin \alpha (W_1 + W_2) \leq \mu_1 \left[ W_1 \cos \alpha + \frac{\sin \alpha}{2 \cos \beta} [W_1 + W_2 + (W_2 - W_1) \sin \beta] \right]}{\frac{\sin \alpha}{2}}$$

$$\square (W_1 + W_2) \leq \mu_1 \left[ \frac{2W_1 \cos \alpha}{\sin \alpha} + \frac{[(W_1 + W_2) + (W_2 - W_1) \sin \beta]}{\cos \beta} \right] \text{ and}$$

$$\frac{\cos \alpha}{\sin \alpha} = \cot \alpha \rightarrow$$

$$\square (W_1 + W_2) \leq \mu_1 \cdot 2W_1 \cot \alpha + \frac{\mu_1 (W_1 + W_2)}{\cos \beta} + \frac{\mu_1 (W_2 - W_1) \sin \beta}{\cos \beta}$$

$$\text{and } w = \frac{W_2 - W_1}{W_2 + W_1} \rightarrow w \cdot (W_2 + W_1) = W_2 - W_1$$

$$\square (W_1 + W_2) - \frac{\mu_1 (W_1 + W_2)}{\cos \beta} - \frac{\mu_1 (w \cdot (W_2 + W_1)) \sin \beta}{\cos \beta} \leq \mu_1 \cdot 2W_1 \cot \alpha \rightarrow$$

$$\square \frac{(W_1 + W_2) \cos \beta}{\mu_1 \cos \beta} - \frac{\mu_1 (W_1 + W_2)}{\mu_1 \cos \beta} - \frac{\mu_1 w \cdot (W_2 + W_1) \sin \beta}{\mu_1 \cos \beta} \leq 2W_1 \cot \alpha \rightarrow$$

$$\square (W_1 + W_2) \left( \frac{\cos \beta}{\mu_1 \cos \beta} - \frac{\mu_1}{\mu_1 \cos \beta} - \frac{\mu_1 w \cdot \sin \beta}{\mu_1 \cos \beta} \right) \leq 2W_1 \cot \alpha$$

$$\square \frac{(W_1 + W_2)}{2W_1} \left( \frac{1}{\mu_1} - \frac{1}{\cos \beta} - \frac{w \cdot \sin \beta}{\cos \beta} \right) \leq \cot \alpha \rightarrow$$

$$\frac{(W_1 + W_2)}{2W_1} \left( \frac{1}{\mu_1} - \frac{1 + w \cdot \sin \beta}{\cos \beta} \right) \leq \cot \alpha$$

$$\square \boxed{\cot \alpha \geq \frac{W_1 + W_2}{2W_1} \left( \frac{1}{\mu_1} - \frac{1 + w \sin \beta}{\cos \beta} \right)}$$

$$\cot \alpha = \frac{1}{\tan \alpha} \rightarrow \frac{1}{\tan \alpha} \geq \frac{W_1 + W_2}{2W_1} \left( \frac{1}{\mu_1} - \frac{1 + w \sin \beta}{\cos \beta} \right)$$

$$\square \frac{1}{\frac{W_1 + W_2}{2W_1} \left( \frac{1}{\mu_1} - \frac{1 + w \sin \beta}{\cos \beta} \right)} \geq \tan \alpha \rightarrow$$

$$\frac{2W_1}{(W_1 + W_2) \left( \frac{\cos \beta - \mu_1 - \mu_1 w \sin \beta}{\mu_1 \cos \beta} \right)} \geq \tan \alpha$$

$$\square \tan \alpha \leq \frac{2W_1 \mu_1 \cos \beta}{(W_1 + W_2)(\cos \beta - \mu_1 - \mu_1 w \sin \beta)}$$

$$\square \alpha_3 = \arctan \frac{2W_1 \mu_1 \cos \beta}{(W_1 + W_2)(\cos \beta - \mu_1 - \mu_1 w \sin \beta)}$$

## Deduction of $\alpha_3$

For finding  $\beta$  minimum, we need to pick condition

$$f_2 \leq \mu_2 N_2$$

We need to substitute  $f_2$ ,  $N_2$  and  $\mu_2$  (static friction coefficient between the two ball's material). Therefore,

$$f_2 = \frac{1}{2} \sin \alpha (W_1 + W_2) \quad N_2 = W_2 \cos \alpha - \frac{\sin \alpha}{2 \cos \beta} [W_1 + W_2 + (W_2 - W_1) \sin \beta]$$

$$\square f_2 \leq \mu_2 N_2$$

$$\square \frac{1}{2} \sin \alpha (W_1 + W_2) \leq \mu_2 \left[ W_2 \cos \alpha - \frac{\sin \alpha}{2 \cos \beta} [W_1 + W_2 + (W_2 - W_1) \sin \beta] \right]$$

$$\square \frac{1}{2} \sin \alpha (W_1 + W_2) \leq \mu_2 \left[ W_2 \cos \alpha - \frac{\sin \alpha}{2 \cos \beta} [W_1 + W_2 + (W_2 - W_1) \sin \beta] \right]$$

$$\square \frac{\sin \alpha}{2}$$

$$\square (W_1 + W_2) \leq \mu_2 \left[ \frac{2W_2 \cos \alpha}{\sin \alpha} - \frac{[(W_1 + W_2) + (W_2 - W_1) \sin \beta]}{\cos \beta} \right] \quad \text{and} \quad \frac{\cos \alpha}{\sin \alpha} = \cot \alpha$$

$\rightarrow$

$$\square (W_1 + W_2) \leq \mu_2 \cdot 2W_2 \cot \alpha - \frac{\mu_2 (W_1 + W_2)}{\cos \beta} - \frac{\mu_2 (W_2 - W_1) \sin \beta}{\cos \beta}$$

and  $w = \frac{W_2 - W_1}{W_2 + W_1} \rightarrow w \cdot (W_2 + W_1) = W_2 - W_1$

$$\square (W_1 + W_2) + \frac{\mu_2(W_1 + W_2)}{\cos \beta} + \frac{\mu_2(w \cdot (W_2 + W_1)) \sin \beta}{\cos \beta} \leq \mu_2 \cdot 2W_2 \cot \alpha \rightarrow$$

$$\square \frac{(W_1 + W_2) \cos \beta}{\mu_2 \cos \beta} + \frac{\mu_2(W_1 + W_2)}{\mu_2 \cos \beta} + \frac{\mu_2 w \cdot (W_2 + W_1) \sin \beta}{\mu_2 \cos \beta} \leq 2W_2 \cot \alpha \rightarrow$$

$$\square (P_1 + P_2) \left( \frac{\cos \beta}{\mu_2 \cos \beta} + \frac{\mu_2}{\mu_2 \cos \beta} + \frac{\mu_2 p \cdot \sin \beta}{\mu_2 \cos \beta} \right) \leq 2P_2 \cot \alpha$$

$$\square \frac{(W_1 + W_2)}{2W_2} \left( \frac{1}{\mu_2} + \frac{1}{\cos \beta} + \frac{w \cdot \sin \beta}{\cos \beta} \right) \leq \cot \alpha \rightarrow$$

$$\frac{(W_1 + W_2)}{2W_2} \left( \frac{1}{\mu_2} + \frac{1 + w \cdot \sin \beta}{\cos \beta} \right) \leq \cot \alpha$$

$$\square \boxed{\cot \alpha \geq \frac{W_1 + W_2}{2W_2} \left( \frac{1}{\mu_2} + \frac{1 + w \sin \beta}{\cos \beta} \right)}$$

$$\cot \alpha = \frac{1}{\tan \alpha} \rightarrow \frac{1}{\tan \alpha} \geq \frac{W_1 + W_2}{2W_2} \left( \frac{1}{\mu_2} + \frac{1 + w \sin \beta}{\cos \beta} \right)$$

$$\square \frac{1}{\frac{W_1 + W_2}{2W_2} \left( \frac{1}{\mu_2} + \frac{1 + w \sin \beta}{\cos \beta} \right)} \geq \tan \alpha \rightarrow$$

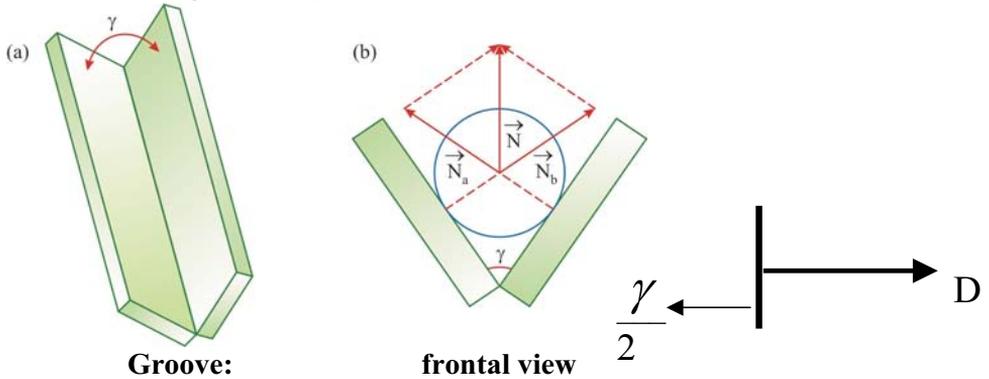
$$\frac{2W_2}{(W_1 + W_2) \left( \frac{\cos \beta + W_2 + \mu_2 w \sin \beta}{\mu_2 \cos \beta} \right)} \geq \tan \alpha$$

$$\square \tan \alpha \leq \frac{2W_2 \mu_2 \cos \beta}{(W_1 + W_2)(\cos \beta + \mu_2 + \mu_2 w \sin \beta)}$$

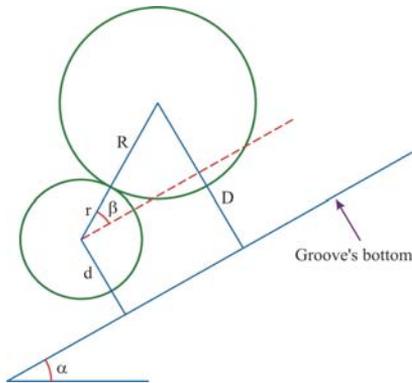
$$\square \boxed{\alpha_2 = \arctan \frac{2W_2 \mu_2 \cos \beta}{(W_1 + W_2)(\cos \beta + \mu_2 + \mu_2 w \sin \beta)}}$$

## Deduction of Beta

- In relation to gamma ( $\gamma$ ) and to the ball's size -



From the rectangle triangle, we have:  $D = \frac{R}{\sin \frac{\gamma}{2}}$  and  $d = \frac{r}{\sin \frac{\gamma}{2}}$



$$\square \sin \beta = \frac{D - d}{R + r} \rightarrow \sin \beta = \frac{\frac{R}{\sin \frac{\gamma}{2}} - \frac{r}{\sin \frac{\gamma}{2}}}{R + r} \rightarrow$$

$$\sin \beta = \frac{(R - r)}{R + r} \left( \frac{1}{\sin \frac{\gamma}{2}} \right) \rightarrow \sin \beta = \frac{R - r}{(R + r) \sin \frac{\gamma}{2}}$$

$$\square \beta = \arcsin \frac{R - r}{(R + r) \sin \frac{\gamma}{2}}$$

**Groove: lateral view**

## Deduction of $\beta_{\min}$

For finding  $\beta$  minimum, we need to pick condition

$$f_4 \leq \mu_3 f_3$$

We need to substitute  $f_4$ ,  $f_3$  and  $\mu_3$  (static friction coefficient between the two ball's material). Therefore,

$$f_4 = \frac{1}{2} \sin \alpha (W_1 + W_2) \quad f_3 = \frac{\sin \alpha}{2 \cos \beta} [W_2 - W_1 + (W_2 + W_1) \sin \beta] \quad f_4 \leq \mu_3 f_3$$

$$\square \frac{1}{2} \sin \alpha (W_1 + W_2) \leq \mu_3 \left[ \frac{\sin \alpha}{2 \cos \beta} [W_2 - W_1 + (W_2 + W_1) \sin \beta] \right]$$

$$\square (W_1 + W_2) \leq \mu_3 \left[ \frac{1}{\cos \beta} [W_2 - W_1 + (W_2 + W_1) \sin \beta] \right]$$

$$\square \cos \beta \leq \mu_3 \left[ \frac{1}{(W_1 + W_2)} [(W_2 - W_1) + (W_2 + W_1) \sin \beta] \right]$$

$$\square \cos \beta \leq \mu_3 \left[ \frac{(W_2 - W_1)}{(W_1 + W_2)} + \frac{(W_2 - W_1) \sin \beta}{(W_1 + W_2)} \right] \text{ and } w = \frac{W_2 - W_1}{W_2 + W_1}$$

$$\square \cos \beta \leq \mu_3 [w + \sin \beta] \rightarrow \cos \beta \leq \mu_3 w + \mu_3 \sin \beta$$

$$\square \cos \beta - \mu_3 w \leq \mu_3 \sin \beta \text{ and } \sin \beta = \sqrt{1 - \cos^2 \beta}$$

$$\square \cos \beta - \mu_3 w \leq \mu_3 \sqrt{1 - \cos^2 \beta} \rightarrow (\cos \beta - \mu_3 w)^2 \leq (\mu_3 \sqrt{1 - \cos^2 \beta})^2$$

$$\square \cos^2 \beta - 2\mu_3 w \cos \beta + \mu_3^2 w^2 \leq \mu_3^2 (1 - \cos^2 \beta)$$

$$\square \cos^2 \beta (1 + \mu_3^2) - \cos \beta \cdot 2\mu_3 w + \mu_3^2 (w^2 - 1) \leq 0 \text{ (second class equation)}$$

$$a = (1 + \mu_3^2) / b = -2\mu_3 w / c = \mu_3^2 (w^2 - 1)$$

$$\square \text{ If } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow \cos \beta \leq \frac{-(-2w\mu_3) \pm \sqrt{(-2w\mu_3)^2 - 4(w^2 - 1)(1 + \mu_3^2)\mu_3^2}}{2a}$$

$$\square \cos \beta \leq \frac{2w\mu_3 + \sqrt{4w^2\mu_3^2 - 4(w^2 - 1)(1 + \mu_3^2)\mu_3^2}}{2(1 + \mu_3^2)} \rightarrow$$

$$\cos \beta \leq \frac{\mu_3 w + \sqrt{w^2\mu_3^2 - (w^2 - 1)(\mu_3^2 + \mu_3^4)}}{1 + \mu_3^2} \rightarrow$$

$$\cos \beta \leq \frac{\mu_3 w + \sqrt{[w^2\mu_3^2 - (w^2 - 1)\mu_3^2] + (1 - w^2)\mu_3^4}}{1 + \mu_3^2} \rightarrow$$

$$\cos \beta \geq \frac{\mu_3 w + \sqrt{\mu_3^2 + (1 - w^2)\mu_3^4}}{1 - \mu_3^2}$$

As we know that:

$$\beta_{\min} \leq \beta \leq \frac{\pi}{2} - \alpha$$

we have

$$\beta_{\min} = \arccos \frac{\mu_3 w + \sqrt{\mu_3^2 + (1 - w^2)\mu_3^4}}{1 - \mu_3^2}$$

## 2. PROBLEM № 4: HYDRAULIC JUMP

SOLUTION OF KOREA

### **Problem № 4: Hydraulic Jump by Overlapping of Gravitational Wave with Viscous Fluid**

*Yoon JongMin<sup>1)</sup>, Yang Il<sup>2)</sup>, Noh JiHo<sup>2)</sup>, Ro YongHyun<sup>3)</sup>, Han MinSung<sup>1)</sup>, and  
Kwon MyoungHoi<sup>4)</sup>.*

<sup>1)</sup>Korea Science Academy, Backyangkwanmunro, Busanjin, Busan, 614-103, Korea

<sup>2)</sup>Korea Minjok Leadership Academy, Sosari, Anheungmyun, Hoengseonggun,  
Gangwondo, 225-823, Korea

<sup>3)</sup>Incheon Science High School, Unseodong, Junggu, Incheon, 403-300, Korea

<sup>4)</sup>Physics Department, Incheon University, Dohwadong, Namgu, Incheon, 402-739,  
Korea

### **The problem**

*When a smooth column of water hits a horizontal plane, it flows out radially. At some radius, its height suddenly rises. Investigate the nature of the phenomenon. What happens, if a liquid more viscous than water is used?*

We investigated hydraulic jump of a radially spreading film of water originated by column-like jet that falls onto a horizontal plate. The reason of formation was suggested in terms of Froude number and overlapping of gravitational waves upstream and downstream. With volume-flux and momentum-flux constancy, some equations were made which describe the jump. The role of viscosity of fluid was explained by laminar boundary layer flow. In experimental parts, the water depth before and after the jump, and the radius of the jump were measured with the variation of water column radius, spreading speed, and viscosity. The depth could be measured with steel probe which was attached to micrometer and connected to ammeter, using the fact that when the probe touches the water surface the voltage changes suddenly. In jump radius, it increases when the efflux radius and speed increase and kinematic viscosity decreases and well matched with equations made.

### **Introduction**

In normal kitchen sink, we can see very interesting phenomenon called ‘Hydraulic jump’. (See Fig. 1.) This phenomenon has been issued for almost one century [1][2][3], and this can be easily identified in everyday life. However the reason of formation and characteristics of the jump have not been explained fully, and still many studies are conducted on it.

In this paper, the hydraulic jump was explained in terms of overlapping of gravitational wave and especially roll of viscosity to the jump was investigated.

## Theoretical background

In theoretical part, we approached to the jump in terms of gravitational wave first, and made equations with volume-flux and momentum-flux constancy.

### The reason of formation ; Overlapping of wave

Waves in water can be divided into mainly two waves; gravitational wave, and surface wave. For our fluid, of which depth is shorter than the half of the wavelength of wave made, the wave has the characteristic of gravitational wave. That is, the speed of wave is determined only by the depth of the water. When  $h$  is the depth of water,



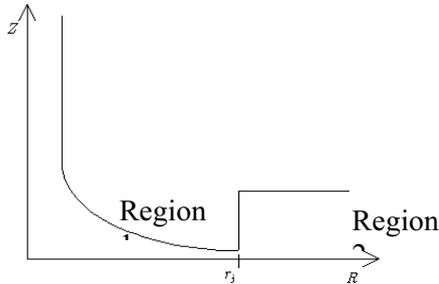
$$v_{wave} = \sqrt{gh}$$

Froude number  $Fr$  is the significant number for fluid, which is the ratio between wave speed and fluid speed.[4] Especially for gravitational wave,

**Fig. 1.** Sample hydraulic jump in kitchen sink.

$$Fr = \frac{v_{water}}{v_{wave}} = \frac{v}{\sqrt{gh}}$$

From now on,  $v$  is the speed of water. The fluid can be divided into 2 regions in terms of the  $Fr$ .(See Fig. 2.)



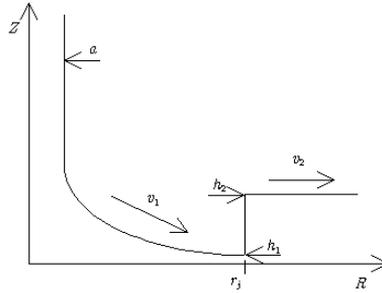
**Fig. 2.** A sketch of simplified hydraulic jump. Region 1 is supercritical and region 2 is subcritical.  $r_j$  means the radius of jump.

Before the jump, in the region 1,  $Fr$  is less than 1, and the region 1 is called subcritical. In this region, because the water flows faster than the wave, the wave can go only downstream, but upstream. After the jump, in the region 2,  $Fr$  is bigger than 1, and the region 2 is called supercritical. Because the speed of water is slower than that of wave, the wave can go upstream and downstream both now and can be overlapped. At the point of jump,  $r_j$ ,  $Fr$  is equal to 1, which is critical region, and it can be said that the jump position is the point where the wave

upstream and downstream can start overlapping of wave. Therefore, the jump was made by the overlapping of gravitational wave.

### Hydrodynamic approach

**Modeling** To make some equations to describe the jump quantitatively, the simplified model of the jump is necessary. In order to make some conditions simple, we made a model like Fig. 3. To make the  $v_1$  constant, we assumed that the depth of water in region 1 decreases until the point of jump. Also, although the jump occurs with some thickness, it was ignored.



**Fig. 3.** The modeled hydraulic jump.  $r_j$  is the jump radius,  $a$  is the radius of the vertical water column.  $v_1$  ( $v_2$ ) is the speed of the water before the jump (after the jump) and  $h_1$  ( $h_2$ ) is the depth before the jump (after the jump).

**Volume constancy** The volume of the water will be preserved because water is incompressible fluid.

$$Q = \pi a^2 v_1 = 2\pi r_j h_1 v_1 = 2\pi r_j h_2 v_2 \quad (1) \quad [5]$$

Then,

$$h_1 v_1 = h_2 v_2 \quad (2)$$

**Momentum constancy** Also, the momentum of the stream should be constant. Make a momentum constancy equation by finding the force of the stream in two different ways.

First, consider a cylindrical shell element of fluid from radius  $r_\alpha$  to  $r_\beta$ . The total

force to deform the element is  $F_{tot} = F_\beta - F_\alpha$   $F_\alpha = \frac{d(mv)}{dt} = m \frac{dv}{dt} + v \frac{dm}{dt}$

$$\frac{dm}{dt} = \rho Q \quad \text{and} \quad \frac{dv}{dt} = 0 \quad \text{at steady state}$$

$$F_{tot} = (\rho Q v)_\beta - (\rho Q v)_\alpha \quad (3)$$

Second, find the deforming force due to the pressure.

$$F = \int P dA = 2\pi r \int_0^h \rho g (h - z) dz = \pi r \rho g h^2 \quad (4)$$

Equating (3) and (4), we have

$$(\rho Qv)_\beta - (\rho Qv)_\alpha = \pi r \rho g h_\alpha^2 - \pi r \rho g h_\beta^2$$

Using (1)  $Q = 2\pi r h v$ ,

$$2\pi r \rho v_\beta^2 h_\beta - 2\pi r \rho v_\alpha^2 h_\alpha = \pi r \rho g h_\alpha^2 - \pi r \rho g h_\beta^2$$

$$v_\alpha^2 h_\alpha + \frac{1}{2} g h_\alpha^2 = v_\beta^2 h_\beta + \frac{1}{2} g h_\beta^2,$$

That is,

$$v_1^2 h_1 + \frac{1}{2} g h_1^2 = v_2^2 h_2 + \frac{1}{2} g h_2^2 \quad (5)$$

**Depth relationship** With two constancy equations, the relationship between  $h_1$  and  $h_2$  can be known easily, and later we will check this relationship is valid with the experimental results.

Use equation (1) to change  $v_1$  and  $v_2$  term in equation (5).

$$v_1 = \frac{Q}{2\pi r_j h_1}, \quad v_2 = \frac{Q}{2\pi r_j h_2}$$

$$\frac{Q^2}{4\pi^2 r_j^2 h_1} + \frac{1}{2} g h_1^2 = \frac{Q^2}{4\pi^2 r_j^2 h_2} + \frac{1}{2} g h_2^2 \quad \frac{Q^2}{4\pi^2 r_j^2} \left( \frac{h_2 - h_1}{h_1 h_2} \right) = \frac{1}{2} g (h_2^2 - h_1^2)$$

Since  $h_1 \neq h_2$ , and  $q \equiv \frac{Q}{2\pi r_j}$  as the volume flux per unit width,

$$\frac{q^2}{h_1 h_2} = \frac{1}{2} g (h_1 + h_2)$$

Multiply  $h_2$  to both side and change the equation in terms of  $h_2$ .

$$\frac{1}{2} g h_1^2 + \frac{1}{2} g h_1 h_2 - \frac{q^2}{h_1} = 0 \quad h_2 = -\frac{h_1}{2} \pm \frac{h_1}{2} \sqrt{1 + \frac{8q^2}{g h_1^3}}$$

Since  $h_2 > 0$ ,

$$h_2 = h_1 \left( -\frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{8q^2}{g h_1^3}} \right) = \frac{h_1}{2} \left( -1 + \sqrt{1 + \frac{8q^2}{g h_1^3}} \right)$$

$$h_2 = \frac{h_1}{2} \left( \sqrt{1 + \frac{8v_1^2}{g h_1}} - 1 \right) \quad (6) \quad \text{since } q = \frac{Q}{2\pi r_j} = \frac{2\pi r_j h_1 v_1}{2\pi r_j} = h_1 v_1$$

We now found the relationship between  $h_1$  and  $h_2$ . However, let's try to simplify more. When  $Fr_1 = \frac{v_1}{\sqrt{g h_1}}$ ,  $\frac{h_2}{h_1} = \frac{1}{2} \left( \sqrt{1 + \frac{8v_1^2}{g h_1}} - 1 \right) = \frac{1}{2} \left( \sqrt{1 + Fr_1^2} - 1 \right) \quad (7)$

Now we can check that the our explanation about the hydraulic jump in terms of the Froude number. In the equation (7), when  $Fr_1 > 1$ ,  $\frac{h_2}{h_1} > 1$ , which means  $h_2$  s bigger than  $h_1$  and when  $Fr_1 \leq 1$ ,  $\frac{h_2}{h_1} \leq 1$ , which means the jump is not created.

To find out the radius of the jump, use equation (1).

$$Q = \pi a^2 v_1 = 2\pi r_j h_1 v_1 = 2\pi r_j h_2 v_2 - (1)$$

$$h_1 = \frac{a^2}{2r_j} - (8) \quad r_j = \frac{a^2}{2h_1} - (9)$$

In the equation (5), because  $v_1^2 h_1 \gg v_2^2 h_2$  and  $\frac{1}{2} g h_2^2 \gg \frac{1}{2} g h_1^2$ , then

$$v_1^2 h_1 = \frac{1}{2} g h_2^2$$

and by applying equation (9),

$$r_j = \frac{v_1^2 a^2}{g h_2^2} - (10)$$

And here, we can make one more  $h_1 - h_2$  relationship equation with equation (9) and (10).

$$h_2 = \sqrt{\frac{2h_1}{g}} v_1 - (11)$$

Actually, This is the same results with equation (6) because when  $\frac{8v_1^2}{gh_1}$  is big enough to ignore the 1,

$$\begin{aligned} h_2 &= \frac{h_1}{2} \left( \sqrt{1 + \frac{8v_1^2}{gh_1}} - 1 \right) \approx \frac{h_1}{2} \left( \sqrt{\frac{8v_1^2}{gh_1}} - 1 \right) \\ &\approx \frac{h_1}{2} \left( \sqrt{\frac{8v_1^2}{gh_1}} \right) = \sqrt{\frac{2h_1}{g}} v_1 \end{aligned}$$

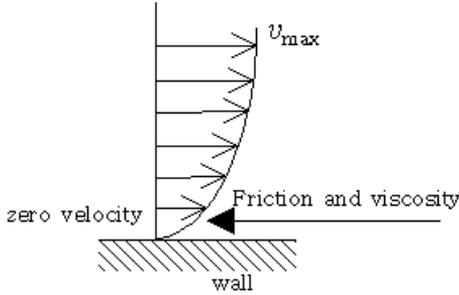
and in our experimental condition,  $v_1 = 1.2m/s$  and  $h_1 = 0.4mm$ ,  $\frac{8v_1^2}{gh_1}$  was about 3000, which is big enough.

**Roll of viscosity** In the problem, the roll of the viscosity of the liquid is asked. The theoretical explanation above does not have any concern of viscosity. It is for the inviscid liquid. For the roll of the viscosity, the boundary layer can be concerned.[6][7][8] The boundary layer means the layer of the water stream which is influenced by the friction with bottom surface and kinematic viscosity of the liquid(See Fig. 4.). Near the bottom, the liquid does not have same speed with the surface; in fact, the speed of the stream is much slower at the bottom. Because there is sudden decrease of the stream at the point of the hydraulic jump, the thickness of the boundary layer and the whole stream becomes same at the point of the jump. The thickness of the viscous laminar boundary layer is

$$\Delta = k \sqrt{\frac{\nu r}{\nu}} \quad - (12)$$

where  $k$  can be experimentally acquired.

For  $h \gg \Delta$ , the deviation from inviscid flow is negligible. However, as  $h \rightarrow \Delta$ , the no-slip boundary condition becomes important and eventually dominate the whole flow behavior.



**Fig. 4.** Side shape of water flow considering laminar boundary layer. At the surface, the speed of water is the biggest.

In our experiment, the Reynolds number is smaller than the transition between laminar and turbulent flow, so we can use the laminar boundary layer.

Consider  $h_1 = \Delta$  in the equation (9).

$$r_j = \frac{a^2}{2} \frac{1}{k} \sqrt{\frac{\nu_1}{\nu r_j}}$$

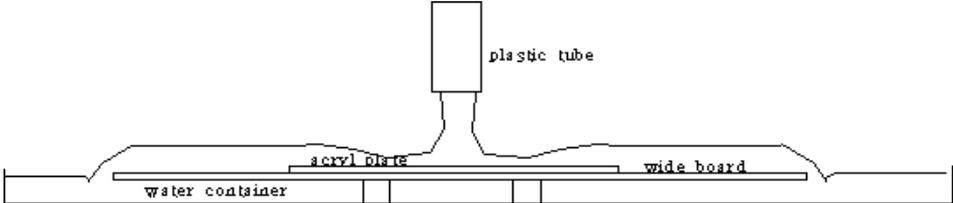
$$r_j = \frac{a^{4/3}}{(2k)^{2/3}} \left(\frac{\nu_1}{\nu}\right)^{1/3} = 0.63 \frac{a^{4/3}}{k^{2/3}} \left(\frac{\nu_1}{\nu}\right)^{1/3} \quad - (13)$$

And equation (13) can be very useful because we can know the radius of the jump without the  $h_1$  or  $h_2$  which should be measured to be known. On following experiments, it will be the main equation to compare the experimental data with theory.

## Materials & Methods

With the wide water container, the flat board was placed upon the surface of the water and the acryl plate was put on the board.(See Fig. 5.) Between the board and plate, there was plotting paper which made it easier to measure the radius of jump. Next to the water container, there was water reservoir and by the small pump water was sprinkled onto the plate along the plastic tube. The amount of flowing water was controlled by the clamp attached to the end-point of plastic tube. The height of end-point of tube and water reservoir was able to be changed and it changed the efflux speed of water.

Near the place at which the jump was made, the sawn micrometer was set with steel stick tightly fixed by steel stand. At the end point of micrometer, steel pin was attached and it was connected to the ammeter which showed the voltage difference between the pin and the water in water container by putting the other end of ammeter into it. At normal condition the ammeter shows 0V, but by rotating micrometer when the endpoint of pin touches the water surface, the voltage changes drastically because water is not perfect insulator. The surface level could be known by measuring the scale of micrometer when the voltage changes, and the bottom level could be known by rotating the micrometer continuously until the endpoint of pin touches the bottom plate. By this method, it was able to measure the depth of water flow in 10 unit of length.



**Fig. 5.** Side shape of simplified experimental setup. Plastic tube was connected to water reservoir and pump. Between acryl plate and board, the plotting paper was put.

With three variables, efflux speed, efflux radius, and kinematic viscosity, the experiments were conducted. Kinematic viscosity was controlled by mixing the glycerin into the water. The table of dynamic viscosity of water-glycerin solution[9] was used and by measuring the density of solution for each concentration, the kinematic viscosity was obtained. With same conditions, same experiment was conducted for 5 times and the average value was used in analysis.

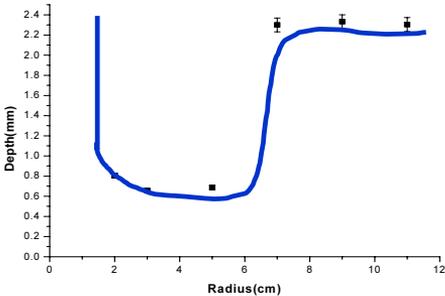
**Results and Discussion**

**Sample hydraulic jump**

One sample hydraulic jump was made, and the water depth along the radius was measured.(See Fig. 6.) Near the radius of 6cm, the jump occurred. And with this experiment, the value of  $k$  could be known; 0.51.

**Changes in efflux speed**

Fig. 7. is the graph of jump radius as the efflux speed changes. With the equation (13),  $k$  value 0.51 and the



**Fig. 6.** Sample hydraulic jump

condition  $a = 2\text{mm}$ ,  $\nu = 1 \times 10^{-6} \text{m}^2/\text{s}$ , theoretically expected red line was made. First five dots are well matched with the line. However, when the speed becomes bigger than critical speed, about  $1.6\text{m}/\text{s}^2$ , the radius becomes much bigger than expected. This can be explained by the edge effect of the board. The all equations were made with the assumption that the jump was made on the infinitely wide plate. However, when the board is finite, in our experiment  $50\text{cm} \times 50\text{cm}$ , the edge effect occurs. When water falls down to the water container, it drags other water of the plate by the cohesion of water. Therefore, the  $h_2$  becomes lower and  $r_j$  becomes bigger than expected.

In Fig. 8.( $h_1$ ) and 9.( $h_2$ ) the expected trend of  $h_1$  and  $h_2$  was identified; as efflux speed increases,  $h_1$  decreases and  $h_2$  increases. The last three dots are also by edge-effect.

### Changes in efflux radius

Fig. 10. is the graph of jump radius as the efflux radius changes. With the equation (13),  $k$  value 0.51 and the condition  $v = 1.4\text{m}/\text{s}$ ,  $\nu = 1 \times 10^{-6} \text{m}^2/\text{s}$ , theoretically expected red line was made. We can see the red line is very well matched with the experimental data.

In Fig. 11.( $h_1$ ) the expected trend was identified. And in Fig. 12.( $h_2$ ), it was identified that over the critical efflux radius, the  $h_2$  remains same value. This can be also explained by the edge effect.

### Changes in kinematic viscosity

Fig. 13. is the graph of jump radius as the kinematic viscosity changes with the condition  $v = 1.4\text{m}/\text{s}$ , and  $a = 1.328\text{mm}$ . The blue line is experimentally made. With the theoretically expected line, the power of kinematic viscosity was rather different. However, the trend that the jump radius decreases as the kinematic viscosity increases.

Also in Fig. 14.( $h_1$ ) and in Fig. 15.( $h_2$ ), expected trend was identified; as kinematic viscosity increase,  $h_1$  and  $h_2$  increases both. In Fig. 16., the different shape of hydraulic jump can be identified with the eye between water and water-glycerin.

### Conclusion

First, we described what the hydraulic jump is. The overlapping of wave at the critical point was suggested as the reason of the formation of jump. For quantitative investigation, some equations were made with two useful constancy; volume and momentum. With the laminar boundary layer flow, the viscosity influences the jump. At the experimental parts, all the trend of the jump radius and the depth before and after the jump were identified. Especially for jump radius with the variation of efflux speed and radius, the theoretically expected equation was almost perfectly matched with experimental data.

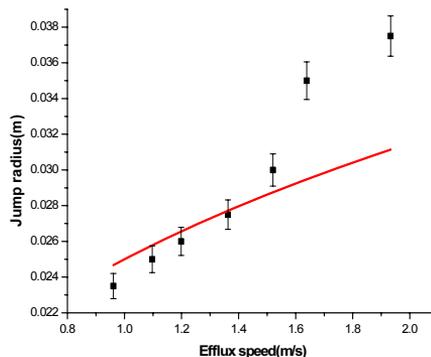
## Acknowledgement

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## References

- [1] Lord Rayleigh, Proc. Roy. Soc. A, 90, 324 (1914).
- [2] Watson, E.J. "The radial spread of a liquid jet over a horizontal plane" *J. Fluid Mech.* **20**, 1964 : 481 – 499.
- [3] Higuera, F.J. "The hydraulic jump in a viscous laminar flow" *J. Fluid Mech* **274**, 1994 : 69 – 92.
- [4] Jearl Walker, *The Flying Circus of Physics With Answers*, p. 93 (John Wiley & Sons, 1977).
- [5] C. Bloom. "The Circular Hydraulic Jump; pursuit of analytic predictions" 25 April 1997.
- [6] Godwin, R.P. "The hydraulic jump (shocks and viscous flow in the kitchen sink)". *Am. J. Phys.* **61** (9), 1993 : 829 – 832.
- [7] Blackford, B.L. "The hydraulic jump in radially spreading flow". *Am. J. Phys.* **64** (2), 1996 : 164 – 169.
- [8] Tani, I. "Water jump in the boundary layer" *J. Phys. Soc. Japan* **4**, 1949 : 212 – 215.
- [9] "Properties of Ordinary Water-Substance." N.E. Dorsey, p. 184. New York (1940).

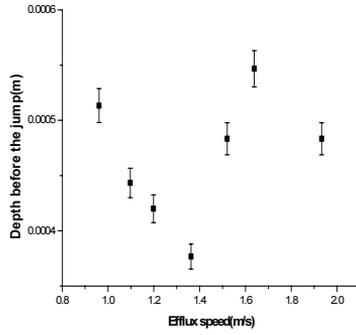
## Captions 7-16



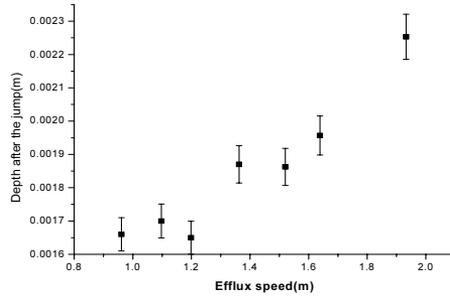
**Fig. 7.** Jump radius along efflux speed

The conditions are

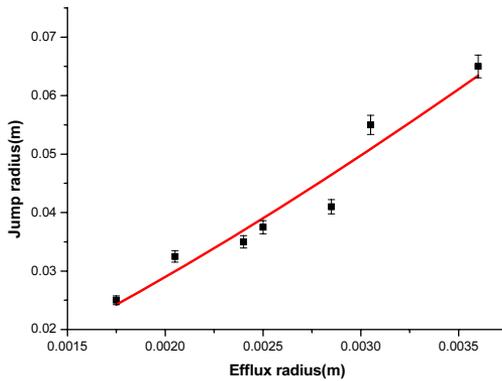
$$a = 2\text{mm}, \nu = 1 \times 10^{-6} \text{ m}^2 / \text{s}$$



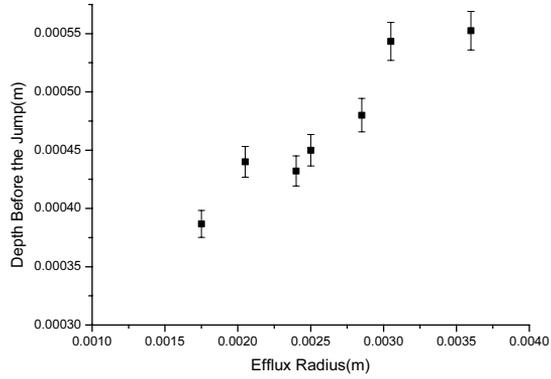
**Fig. 8.** Changes of  $h_1$  along the efflux speed. The conditions are  $a = 2mm$ , and  $\nu = 1 \times 10^{-6} m^2 / s$ .



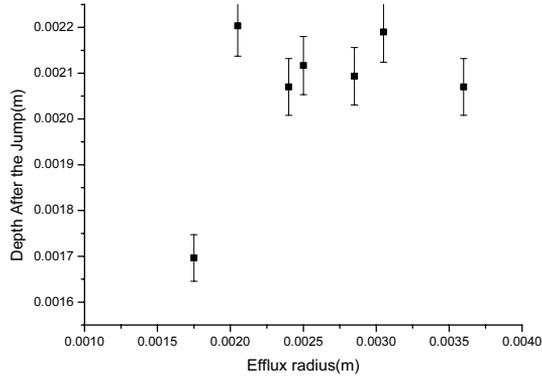
**Fig. 9.** Changes of  $h_2$  along the efflux speed. The conditions are  $a = 2mm$ , and  $\nu = 1 \times 10^{-6} m^2 / s$ .



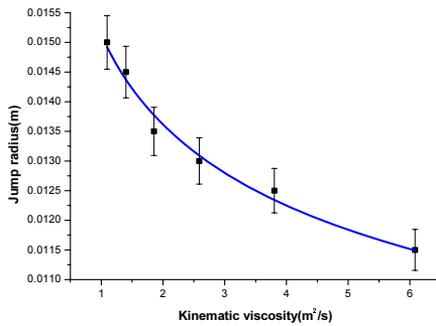
**Fig. 10.** Jump radius along efflux radius. The conditions are  $\nu = 1.4m / s$ , and  $\nu = 1 \times 10^{-6} m^2 / s$ .



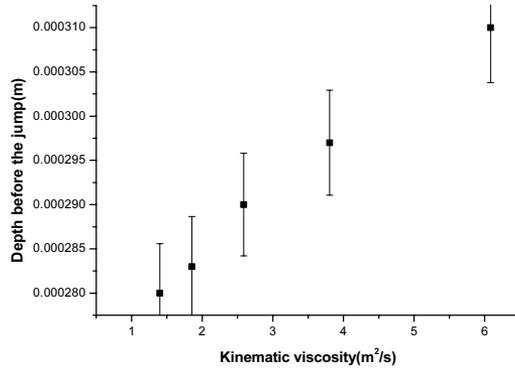
**Fig. 11.** Changes of  $h_1$  along the efflux radius. The conditions are  $v = 1.4m/s$ , and  $\nu = 1 \times 10^{-6} m^2/s$ .



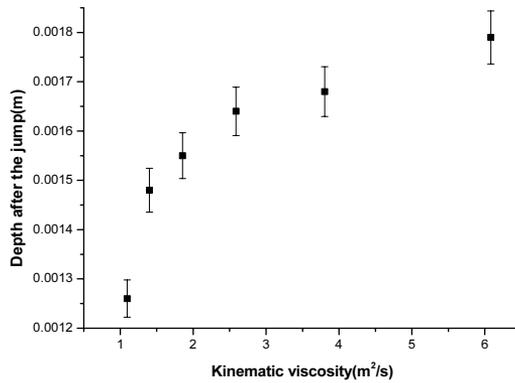
**Fig. 12.** Changes of  $h_2$  along the efflux radius. The conditions are  $v = 1.4m/s$ , and  $\nu = 1 \times 10^{-6} m^2/s$ .



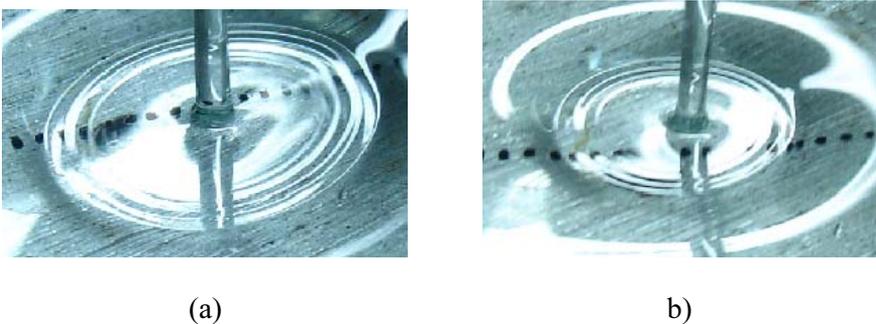
**Fig. 13.** Jump radius along the kinematic viscosity. The conditions are  $v = 1.4m/s$ , and  $a = 1.328mm$ .



**Fig. 14.** Changes of  $h_1$  along the kinematic viscosity. The conditions are  $v = 1.4 \text{ m/s}$ , and  $a = 1.328 \text{ mm}$ .



**Fig. 15.** Changes of  $h_2$  along the kinematic viscosity. The conditions are  $v = 1.4 \text{ m/s}$ , and  $a = 1.328 \text{ mm}$ .



**Fig. 16.** Hydraulic jump of (a) water, (b) water-glycerin solution

### 3. PROBLEM № 8: WIND CAR

SOLUTION OF BRAZIL

#### Problem № 8: WINDCAR

Marcelo Puppo Bigarella, Brazil

#### The problem

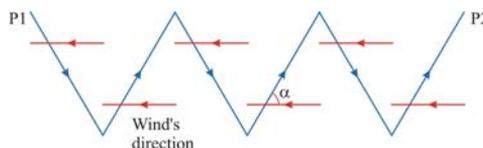
*Construct a car which is propelled solely by wind energy. The car should be able to drive straight into the wind. Determine the efficiency of your car.*

---

**1. Objective:** In this problem we are supposed to construct a car, which can be able to drive straight into the wind. The car has to be propelled just by wind energy, which means that we can not use others energy sources to move the car. The problem also asks us to determine the car's efficiency. In this part of the resolution (calculating efficiency) we have to establish the car's performance. The experimental section of this problem is extremely necessary (indispensable). We have to construct a prototype that respect the problem's limitation, in order to make the necessary measurements to explain the car functionality and to determine its efficiency.

**2. Theoretical Background:** In this part of the resolution we will focus on each variable that could interfere with the final result, such as studies about momentum transmission as well studies about different kind of speed: scalar speed and angular speed. We will also exemplify possible models and the energy loss in each one, as well how to calculate efficiency. At the beginning, we thought in three kinds of cars that could drive straight into the wind:

1- A sail's car (moved by sail): In this car we would utilize the force of the wind (blowing directly in the sail) to make the car walk. But as the car has to walk straight into the wind, this car would not respect the problem's limitation because it would have to drive in a zigzag trajectory to use the wind buoyancy.



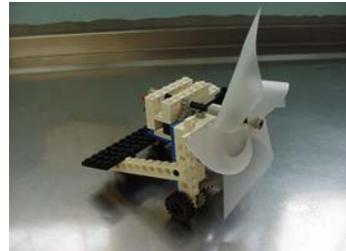
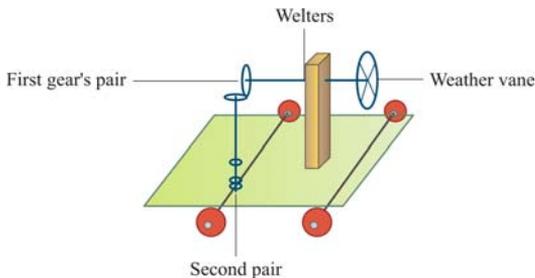
2- An electric car: The principle of the electric car is to transform the kinetic energy into electric energy (using a dynamo moved by a weather vane). Then, it would transform, with an electric engine, this electric energy into mechanical energy that would make the car walk.

- Loss in this car: air viscous force in the weather vane as well a considerable loss in the energy transformations (mechanic-electric-mechanic): energy dissipation.

3- A mechanic car: Using a weather vane (twirled by the wind) and pairs of gears, we would make the mechanic car move by energy transmission.

Loss in this car: friction in the axes (energy dissipation), gears heating (another example of energy dissipation) and tendency to slide in the wind direction (as the wind is blowing against the car (opposite to the problem's direction).

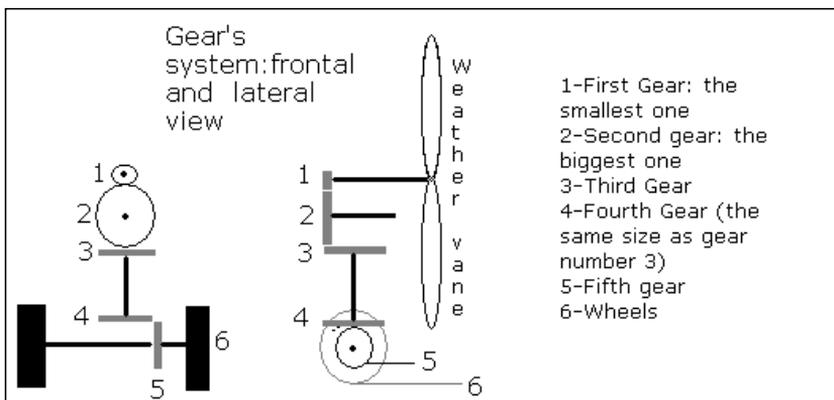
Considering this three models, and the energy loss of each one, we chose the third one (mechanic car) to develop the prototype. Below, a initial idea of the mechanic car



2.1 Mechanic car: Our mechanic car consists in a base (with four wheels) that support all the car structure, the gear's structure and its welters

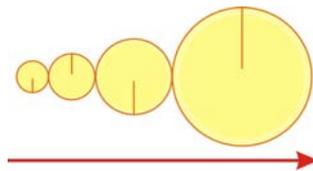
The car inner working is simple: basically the wind will twirl the weather vane, which, by gear's transmission, will roll a pair of wheels, making the car move. The transmission of momentum (wind blowing and twirling the weather vane) will be made by gears and axes.

Below, a drawing of the prototype (graphic project) and its photo:

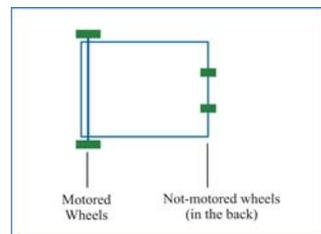


Important things for considering:

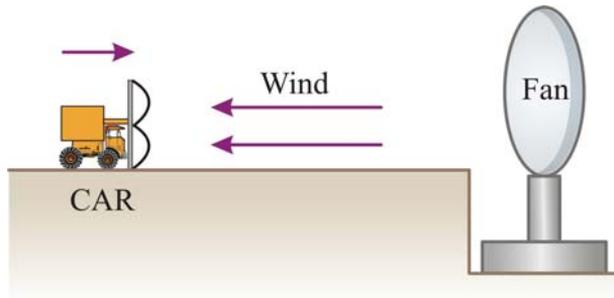
- Frontal Area: It is important to have the smallest frontal area (not considering the weather vane area). Air resistance force depends on the frontal area of the car. Smaller frontal area have smaller resistance (wind buoyancy), which is capable to reduces the car final speed and, consequently, the car efficiency.
- The weather vane size: The total torque on the weather vane determines the driving speed of the car. A bigger vane will have a larger torque on it, and, consequently, the car will drive faster into the wind. However, a larger weather vane has more contact surface area, thus it will have more resistance force, which tends to push back the car. So, we need to find a optimum size that delivers a maximum torque and a minimum resistance force.
- The heating process in the gear's tooth: Since we are working with gears to transmit momentum, we need to consider the heating process in the gear's tooth. The gear is heated by friction on its teeth. This) heating process can not be put aside because to twirl another gear, one gear has to hit its teeth in the other gear teeth. This process one form of energy dissipation and represents a reduction in the final performance.
- Gear's position: For a better performance, we have to put the smaller gear in the same axis of the weather vane and then, we need to increase the gear's size, until the wheels axis. (When we have the gears in the same axis, the gears can have the same size, because there are no force raise in the same axis). In this way we "give" a bigger torque to the wheel (responsible for moving the car).



- Car's weight: The car can not be too heavy, because the friction force increases directly proportional to the weight, given by the equation:  $\vec{F}_{at} = \vec{N} \cdot \mu$ ; where  $\vec{N}$  is the compression (Normal) force between the surface and the car, which is also the weight force reaction pair;  $\mu$  is the static friction coefficient between the wheel material and the surface material. With a heavier car, we will have a bigger Normal force, thus more attrition. However, the car weight is limited by the fact that too little friction increases the chance of wheels skidding (not a good "interaction" between the plan and the wheels).
- Wheels: There are four wheels in our mechanic car: Two are moved, indirectly, by the wind (motored wheels) and two wheels (not motored) have the function of equilibrating the car (this wheels are located at the back of the car). Below we show a drawing of the wheel's arrangement.

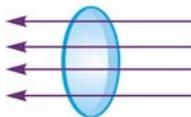


- Wind Source: In our experiment, we will use a fan as the wind source. Our fan has three different powers: it means that there are three possibilities of wind power (three different wind escape intensities). For a more complete resolution, we will utilize all powers to calculate the efficiency.
- Wind direction: The efficiency depends on the direction that the wind blows at the car frontal area. For a better performance, we used a step to put the fan, so we had a better wind utilization (picture)

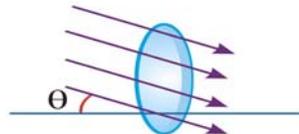


If we had used the wind in a transversal form (forming an angle ( $\theta$ ) with the horizontal), the air flux would change, as we show in the pictures below (because just the horizontal speed component will be utilized).

$$\text{flux} = V_n \cdot \text{Area} \quad (1)$$



$$F = |\vec{V}| \cdot A$$

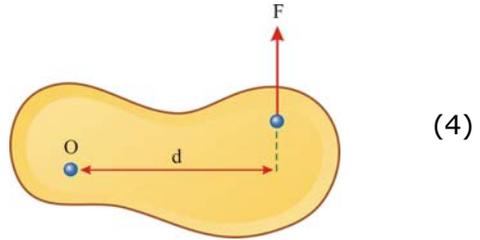


$$F = (\vec{V} \cdot \vec{n}) \cdot A \quad (2)(3)$$

In our case we will just utilize the equation 2 because in our setup the wind will always be perpendicular to the weather vane area.

2.2 Force Momentum (torque): In mechanics, generally, we work with particles. But in our experimental setup it is necessary to work with objects (spatial corpus) and not just particles. Considering objects which do not deform itself when an external force is applied, we define momentum ( $M$ ), or torque of a force  $\vec{F}$  (acting in this body in relation to a axis which pass in  $O$ ), by the relation below (where  $d$  is the distance between the  $O$  and the perpendicular projection of the force):

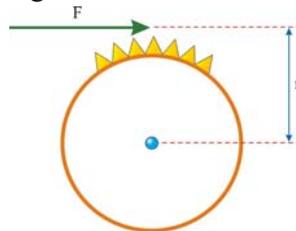
$$M = F \cdot d$$



(4)

In our case, we will use momentum as one form to calculate the efficiency and also to relate the transmission among the gears and the weather vane. In the gear, momentum will be calculated by the force applied in the gear's tooth (extremity of the gear) times  $r$ , which will represent the gear's radius.

$$M = F \cdot r$$



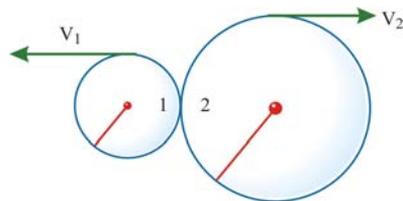
(5)

2.1.1 – Gears: In this car, the gear will be used to transmit linear and angular momentum. The gears also will help us in: (1) Changing the rotation direction, (2) Increasing or decreasing the rotation speed, (3) Changing the rotation axis and (4) synchronizing the rotation direction.

Gear's size: The relation between two gears in touch determines the rotation speed of each one. A reduction of the relative radius between to gears reduces the angular speed of the bigger gear (It happens because all the points in the extremity of the both gear have the same speed).

- Gears that are contact have the same scalar speed. Scalar speed is a speed defined by the equation below, where  $C$  is the length of the circle (perimeter),  $r$  is the radius (in our problem we just consider the radius as the maximum possible radius) and  $T$  is the period of gear revolution:

$$\vec{v} = \frac{\Delta S}{\Delta T} = \frac{C}{T} = \frac{2 \cdot \pi \cdot r}{T}$$

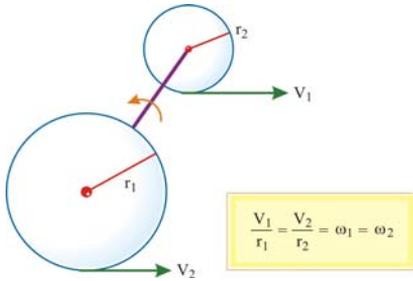


(6)

$$v_1 = v_2$$

- Gears that are in the same axis have the same angular speed. Angular speed is a speed defined by the equation below, where  $2\pi$  is the angle's variation (in our case, a complete lap),  $r$  is the radius (in our problem we just consider the

radius as the maximum possible radius) and  $T$  is the time that the gear takes to complete one revolution. Two gears in the same axis have the same angular speed and their scalar speed is proportional to their radius.



$$\omega = \frac{\Delta\theta}{\Delta T} = \frac{2 \cdot \pi}{T}$$

## 2.2 Efficiency:

Classically, in general problems, we calculate efficiency as the ratio between the effective energy used to our purpose and the total energy available. Ratio equal to 1, means the system has a 100% performance.

Firstly, we thought about efficiency like being the result of the division below:

$$\eta = \frac{\overline{V}_{car}}{\overline{V}_{wind}} \quad (9)$$

But, in the wind car problem, more specifically in our experiments, this division is ambiguous. The wind speed is measured in our reference frame? Or in the car's frame? Clearly the energy available will be different. Thus we must state if we are referring to wind relative speed or to the wind absolute speed.

Therefore, let's deduce the efficiency model.

- Speed and Kinetic energy: The kinetic energy modules are related to the speed modules. The kinetic energy also depends on the body mass and it is defined by the equation below (where  $m$  is the body mass and  $V$  is its speed):

$$K = \frac{m \cdot V^2}{2} \quad (10)$$

- The done work by the car: Physically speaking, the car does not do work. The work is done by the forces which propels the car. But, we will deal with the phrase that the work is done by the car to make things easier.

The work done by the car is defined by the kinetic energy theorem, which says that the work is the difference between the final kinetic energy and the initial kinetic energy:

$$W = \Delta K = \frac{m \cdot V_F^2}{2} - \frac{m \cdot V_I^2}{2} \quad (11)$$

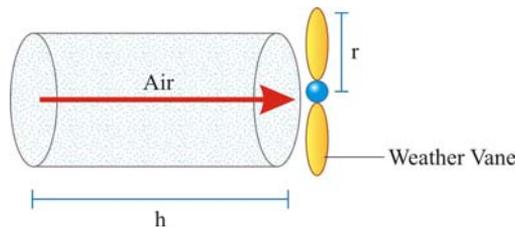
We can obtain the work modules because we have the average speed of the car (assuming that the car has a constant acceleration). With the average speed (and with constant acceleration) we can find the final speed, using the equation below (where  $\bar{V}$  is the average speed and  $V_F$  is the final speed):

$$V_F = 2 \cdot \bar{V} \quad (12)$$

Using this result and the fact that the car starts from rest, we conclude that the total car's kinetic energy is:

$$W = \frac{m \cdot (2 \cdot \bar{V})^2}{2} = \frac{m \cdot 4 \cdot \bar{V}^2}{2} = 2 \cdot m \cdot \bar{V}^2 \quad (13)$$

- The done work by the wind: Physically saying, wind does not do work, the work is done by the resistive force applied by the wind. But, we will deal with the phrase that the work is done by the wind to make things easier. Considering a frontal air cylinder, (in front of the weather vane), like in the picture, with a fixed volume ( $V$ ) and a the average wind speed ( $\bar{V}$ ), we can say:



- a) The cylinder volume is the relation between mass and density (air density):

$$\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho} \quad (14)$$

- b) The cylinder volume is the same as the cylinder height multiplied by the cylinder's base (the weather vane area). So,

$$V = A \cdot h \quad \text{and} \quad V = \frac{m}{\rho} \quad (15) \quad A \cdot h = \frac{m}{\rho} \Rightarrow m = A \cdot h \cdot \rho \quad (16)$$

$$A = \pi \cdot r^2 \quad (17) \quad m = h \cdot \pi \cdot r^2 \cdot \rho \quad (18)$$

$$h = \frac{m}{\pi \cdot r^2 \cdot \rho} \quad (19)$$

where  $h$  is the cylinder height (length),  $r$  is the weather vane radius and  $\rho$  the air density.

c) The air cylinder speed is calculated by:

$$\bar{V} = \frac{h}{\Delta T} \quad \text{So,} \quad \bar{V} \cdot \Delta T = h \quad (20)$$

Comparing equation (19 and 20) we can conclude that:

$$\bar{V} \cdot \Delta T = \frac{m}{\pi \cdot r^2 \cdot \rho} \quad (21)$$

$$m = \bar{V} \cdot \Delta T \cdot \pi \cdot r^2 \cdot \rho \quad (22)$$

Using the kinetic equation (10) we substitute the mass (22) and discover that:

$$K = \frac{m \cdot \bar{V}^2}{2} \Rightarrow K = \frac{\Delta T \cdot \pi \cdot r^2 \cdot \rho \cdot \bar{V}^3}{2} \quad (23)$$

As we know the kinetic energy (work), the efficiency (performance) is defined as the division of the car work (13) by the wind work (23):

$$\eta = \frac{W_{car}}{W_{wind}} = \frac{2 \cdot (\bar{V}_{car})^2 \cdot m_{car}}{\Delta T \cdot \pi \cdot r^2 \cdot \rho \cdot (\bar{V}_{wind})^3} = \frac{4 \cdot (\bar{V}_{car})^2 \cdot m_{car}}{\Delta T \cdot \pi \cdot r^2 \cdot \rho \cdot \bar{V}_{wind}^3} \quad (24)$$

Where:

$\bar{V}_{car}$	is the average car speed
$\bar{V}_{wind}$	is the average Wind speed
$m_{car}$	is the car mass
$r$	is the weather vane radius
$\rho$	is the average air density
$\Delta T$	is the total time that the car took to cross over the fixed distance (in our case, 05, meters)

**Efficiency**  $\rightarrow$  
$$\eta = \frac{4 \cdot (\bar{V}_{car})^2 \cdot m_{car}}{\Delta T \cdot \pi \cdot r^2 \cdot \rho \cdot (\bar{V}_{wind})^3} \quad (25)$$

### 3. Experimental Setups:

1. Prototype: characteristics

#### Measuring:

2. Wind speeds (three different intensities wind sources)
3. Car speeds (three different intensities)
4. Car efficiency (performance)

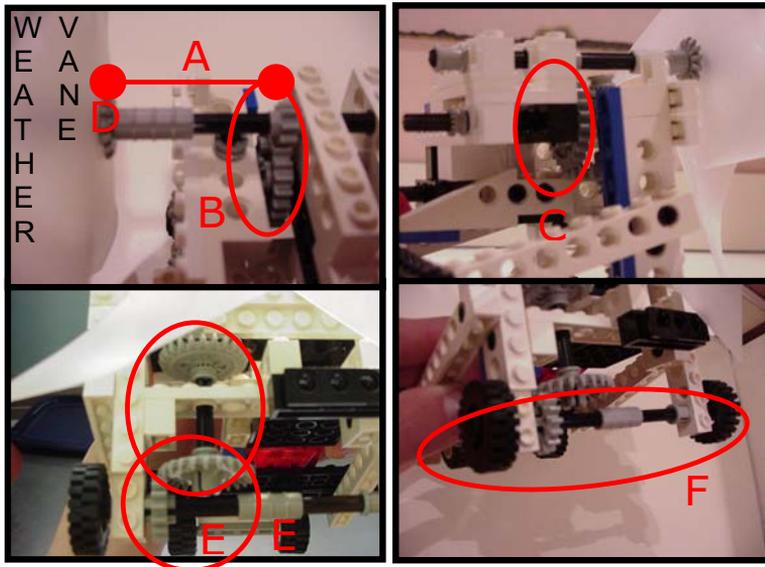
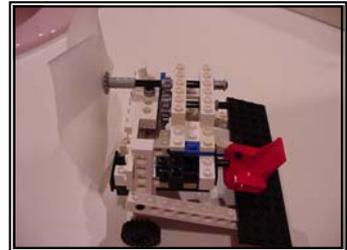
#### Additional Information:

5. Acceleration

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#### 1. Prototype: characteristics

Our car has five gears, four wheels (two pairs) and one weather vane. Below, we show photos of the car and also a photo of each pair of gears (in the same axis and in contact also). It was made with pieces of LEGO™ and with a plastic weather vane.



- A- First pair: gear and weather vane (same axis: equal angular speed)
- B- Second pair of gear (equal scalar speed)
- C- Third pair of gear (equal scalar speed)
- D- Fourth pair of gear (same axis: equal angular speed)
- E- Fifth pair of gear (equal scalar speed)
- F- Sixth pair: gear and wheel axis (same axis: equal angular speed)

## Gear's Characteristics:

Sizes: with a help of a measure instrument we measure the gears and wheels diameter and then, dividing per two, we found the average radius:

	Diameter (cm)	Radius (cm)
Weather Vane	$15,49 \pm 0,05$	$7,745 \pm 0,025$
First gear (the smallest)	$0,96 \pm 0,05$	$0,480 \pm 0,025$
Second gears (the biggest)	$2,57 \pm 0,05$	$1,285 \pm 0,025$
Third gear	$2,56 \pm 0,05$	$1,280 \pm 0,025$
Fourth gear	$2,56 \pm 0,05$	$1,280 \pm 0,025$
Fifth gear	$1,77 \pm 0,05$	$0,885 \pm 0,025$
Motored wheels	$2,44 \pm 0,05$	$1,220 \pm 0,025$
Not-motored wheels	$2,10 \pm 0,05$	$1,050 \pm 0,025$

Car Mass: It is important to know the car mass because we need it for knowing the kinetic energy (that involves speed and mass)

Car mass	0,150 kg
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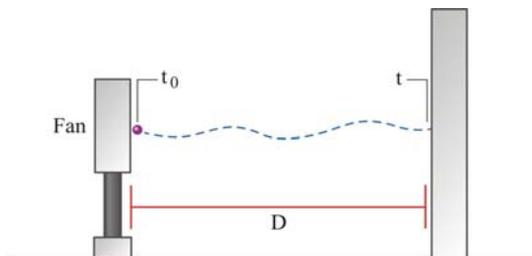
## **Measuring:**

### 2. Wind speeds (three different intensities of the same wind sources)

Wind Speed: For calculating the car efficiency, it is necessary to know the wind speed. There are a lot of methods to measure the wind speed: one method, homemade, consists in throwing small polystyrene balls in front of the fan and the, measuring the average time that the balls take to reach a fixed distance.

For measuring it, we had to do an experiment. The results for 2 meter runs are shown in the next topics, in all wind intensities. We then calculated the average time over all runs and the average wind speed for all three different wind intensities (fan's power).

First, a draw of the wind speed calculation experience:



$$\bar{V}_{wind} = \frac{\Delta S}{\Delta T} = \frac{D}{t - t_0} \quad (26)$$

- Wind speed in the fan's third power: The time that the particle (polystyrene ball) took to cross over two meters applying the “wind third” power (the fastest/strongest one). We measured the time in 20 runs and average over them:

T(s)

0,39	0,40	0,44	0,42	0,44	0,47	0,36	0,35	0,39	0,32
0,39	0,48	0,44	0,39	0,45	0,35	0,47	0,47	0,28	0,49

$$\bar{M} = \frac{\sum t_n}{n}$$

Thus:

$$\boxed{\bar{M} = 0,4095 = 0,41s} \quad \text{Wind speed: } \rightarrow \bar{V} = \frac{2,0m}{T_{medium}} \quad \bar{V} = \frac{2,0m}{0,41s}$$

(equation 26)

- Average wind speed (in the third power):

$$\boxed{\bar{V}_3 \cong 4,9m / s}$$

- Wind speed in the fan's second power: The time that the particle (polystyrene ball) took to cross over two meters applying the “wind second” power (middle one). We measured the time in 20 runs and average over them:

T(s)

0,57	0,56	0,55	0,53	0,59	0,53	0,46	0,51	0,51	0,49
0,53	0,59	0,52	0,52	0,51	0,48	0,51	0,54	0,53	0,56

$$\bar{M} = \frac{\sum t_n}{n}$$

Thus,

$$\boxed{\bar{M} = 0,5295 = 0,53s} \quad \text{Wind speed: } \rightarrow \bar{V} = \frac{2,0m}{T_{medium}} \quad \bar{V} = \frac{2,0m}{0,53s}$$

(equation 26)

- Average wind speed (in the second power):

$$\boxed{\bar{V}_3 \cong 3,8m / s}$$

- Wind speed in the fan's first power: The time that the particle (polystyrene ball) took to cross over two meters applying the "wind first" power (the smallest one). We measured the time in 20 runs and average over them:

T(s)

0,57	0,66	0,64	0,61	0,55	0,62	0,65	0,69	0,72	0,54
0,74	0,70	0,66	0,62	0,59	0,59	0,64	0,68	0,71	0,59

$$\bar{M} = \frac{\sum t_n}{n}$$

In our case:

$$\bar{M} = 0,6385 = 0,64s$$

Wind speed: (equation 26)  $\bar{V} = \frac{2,0m}{T_{medium}} \quad \bar{V} = \frac{2,0m}{0,64s}$

- Average wind speed (in the third power):

$$\bar{V}_3 \cong 3,1m/s$$

### 3. Car speeds (three different intensities)

For measuring the car speed, important for calculating the car efficiency, we fixed a distance (in our case 0,5 meter) and then, divided for the average time that car spent to cross over this distance.

In the tables below, we show the times that the car took, in each fan's power, to cross 0,5 meters. After this, we did, for each fan's power, the calculus of the average time, to express correctly the car speed.

- Car speed in the fan's first power:

In the first fan power (wind intensity), we obtained these values:

T(s)

2,00	2,18	1,95	1,92	1,97	1,84	2,12	2,06	1,99	2,18
2,08	2,20	1,87	2,03	2,09	2,11	2,16	2,14	1,96	1,87
2,06	2,09	2,23	2,00	1,90	2,29	2,04	2,28	2,12	2,24
2,25	2,15	2,20	2,23	2,08	2,17	2,17	1,95	2,00	2,18

$$\bar{M} = \frac{t_1 + \dots + t_n}{n}$$

In our case:

$$\bar{M} = \frac{t_1 + \dots + t_{40}}{40} = \frac{2,00 + \dots + 2,18}{40} = \frac{83,35}{40} = 2,0837 \approx 2,1s$$

Car speed:  $\rightarrow \bar{V} = \frac{0,5m}{T_{medium}} \rightarrow \bar{V} = \frac{0,5m}{2,1s}$

- Final car speed (in the first power):

$$\boxed{V_1 \cong 0,24m / s}$$

➤ Car speed in the fan's second power:

In the second fan power we obtained these values:

T(s)

1,78	1,67	1,59	1,75	1,59	1,69	1,94	1,52	1,65	1,75
1,56	1,60	1,72	1,62	2,00	1,71	1,51	1,44	1,81	1,94
1,50	1,55	1,70	1,65	1,57	1,53	1,48	1,78	1,47	1,59
1,72	1,47	1,66	1,54	1,51	1,61	2,00	1,68	1,72	1,79

$$\bar{M} = \frac{t_1 + \dots + t_n}{n}$$

In our case:

$$\bar{M} = \frac{t_1 + \dots + t_{40}}{40} = \frac{1,78 + \dots + 1,79}{40} = \frac{66,36}{40} = 1,659 = 1,7s$$

Car speed:  $\rightarrow \bar{V} = \frac{0,5m}{T_{medium}} \rightarrow \bar{V} = \frac{0,5m}{1,7s}$

- Final car speed (in the second power):

$$\boxed{V_1 \cong 0,30m / s}$$

➤ Car speed in the fan's third power:

In the third fan power, we obtained these values:

T(s)

1,47	1,44	1,46	1,65	1,53	1,50	1,63	1,53	1,36	1,38
1,33	1,46	1,50	1,46	1,43	1,44	1,44	1,53	1,53	1,49
1,50	1,39	1,53	1,42	1,43	1,56	1,47	1,50	1,43	1,34
1,51	1,40	1,50	1,43	1,39	1,38	1,53	1,37	1,69	1,42

$$\bar{M} = \frac{t_1 + \dots + t_n}{n}$$

In our case:

$$\bar{M} = \frac{t_1 + \dots + t_{40}}{40} = \frac{1,47 + \dots + 1,42}{40} = \frac{58,75}{40} = 1,4687 = 1,5s$$

Car speed:  $\rightarrow \bar{V} = \frac{0,5m}{T_{medium}} \rightarrow \bar{V} = \frac{0,5m}{1,5s}$

- Final car speed (in the third power):

$$\boxed{V_3 \cong 0,33m/s}$$

#### 4. Car efficiency:

To determine the car efficiency we use equation (25). We will just substitute, in the three cases (different wind intensity) the average car speed, as also the average wind speed, the air density, the weather vane radius and the time variation ( $\Delta T$ ) and the car mass.

Efficiency

Equation:

$$\eta = \frac{4 \cdot (\bar{V}_{car})^2 \cdot m_{car}}{\Delta T \cdot \pi \cdot r^2 \cdot \rho \cdot (\bar{V}_{wind})^3}$$

Information that we need to use:

DATA	VALUE (International Units System)
Wind speed in fan's first power	3,1 m/s
Wind speed in fan's second power	3,8 m/s
Wind speed in fan's third power	4,9 m/s
Car speed in fan's first power	0,24 m/s
Car speed in fan's second power	0,30 m/s
Car speed in fan's third power	0,33 m/s
Average time that the car took to cross over 0,5 meter in the fan's first power	2,1 s
Average time that the car took to cross over 0,5 meter in the fan's second power	1,7 s
Average time that the car took to cross over 0,5 meter in the fan's third power	1,5 s
Car mass	0,150 kg
Weather Vane radius	0.07745 m
Air density	1,21 kg/m <sup>3</sup>

Substituting the wind average speed the car average speed the air density, the weather vane radius, the car mass and the time that the car took to cross over 0,5 meter into equation (25), we calculate an efficiency of

➤ First Power: \_\_\_\_\_ Second Power: \_\_\_\_\_ Third Power: \_\_\_\_\_

$$\eta = 2,4\%$$

$$\eta = 2,5\%$$

$$\eta = 1,6\%$$

Presenting the car efficiency:

	<b>Efficiency (%)</b>	<b>Error (%)</b>
First Fan's Power	2,40	±0,04
Second Fan's Power	2,50	±0,02
Third Fan's Power	1,60	±0,06

### 3. Car acceleration in the three different wind intensities

We suppose that the car has a constant acceleration. In reality this is not the case, as when the car comes closer to the fan the torque on the weather vane can increase, and, consequently, the final car speed increases also. However, the wind speed does not vary considerably along five meters. Thus, supposing the torque and acceleration constant is a good approximation.

To measure the car acceleration we need to consider as if it is constant. We determined the car acceleration in the three fan's power using the equation below

$$V_F^2 = V_0^2 + 2 \cdot a \cdot \Delta S \quad (27)$$

Substituting  $V_F$  of the car according to equation (12),  $\Delta S$  equal to 0,5 (meter), we find that the acceleration is:

$$a = 4 \cdot \bar{V}_{car}^2 \quad (28)$$

Information we will need to calculate the acceleration:

Average Car speed in fan's first power	0,24 m/s
Average Car speed in fan's second power	0,30 m/s
Average Car speed in fan's third power	0,33 m/s

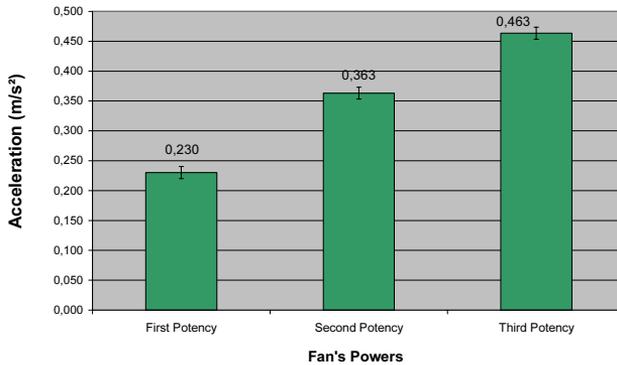
Substituting the average car speed and placing them in the equation (28): we have, in this fan's power, an acceleration of:

➤ First Power: \_\_\_\_\_ Second Power: \_\_\_\_\_ Third Power: \_\_\_\_\_

$$a = 0,23\text{m/s}^2$$

$$a = 0,36\text{m/s}^2$$

$$a = 0,44\text{m/s}^2$$



### **3.1 Materials:**

In our prototype, we used pieces of LEGO™ and a square plastic to make the weather vane, and as the wind source, we used a domestic fan. For doing the measure and calculating efficiency, we use a ruler, chronometers, a measure tape, stickers and polystyrene small balls.

### **3.2 Possible errors sources:**

We did a lot of approximation, like considering the wind speed, as well the car acceleration, constant. There is also friction between the wheels and its wetlers that could perturb the car.

-Optimization for a next experiment: Maybe if we could work with ideal condition the experimental analyzes as well the mathematical and physical deductions, would be more precise.

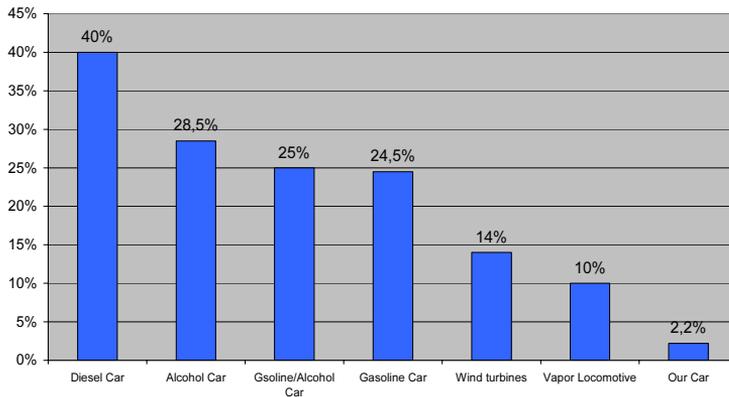
#### Importance:

- Future energy lapse
- Recent researches about new clean energy
- Environment preservation (pollution)

### **4. Conclusion:**

In this problem, we develop a prototype and also calculated the efficiency of the prototype for different wind intensities (we discover that in the middle power the car has a better performance). The car's efficiency is high if we compare with other systems that also have wind as theirs energy sources.

Gasoline cars have an efficiency of around 25%, diesel cars have efficiency of around 40% (the highest one) and alcohol cars have efficiency of around 28%. Thermo Machines (e.g. vapor locomotive) have efficiency of 10%. Wind turbines (for energy) have an efficiency of 17%. Therefore, our car (average efficiency of 2,2%) is good if we compare it with others professionals models. Below, a graphs of others systems efficiency.



## Bibliography

- HALLIDAY, David; **Physic Basis**, EDIUSP, First Volume, São Paulo, SP, 2001
- HALLIDAY, David; **Physic Basis**, EDIUSP, Second Volume, São Paulo, SP, 2001
- TIPLER, Paul A.; **Physic 1**, Guanabara (Comp: UFRJ), Rio de Janeiro, RJ, 1998
- TIPLER, Paul A.; **Physic 2**, Guanabara (Comp: UFRJ), Rio de Janeiro, RJ, 1998
- MAXIMO, Antônio and ALVARENGA, Beatriz, **Physic Course**, Second Volume, SP- 2003
- **Larrouse Cultural Encyclopedia**, 23rd Volume (pg 1345-1347)

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- Prof. Ronaldo Fogo
- Luiz Tizei
- Alberto Helito

## Deduction of efficiency model

- Speed and Kinetic energy:  $K = \frac{m \cdot V^2}{2}$

- 

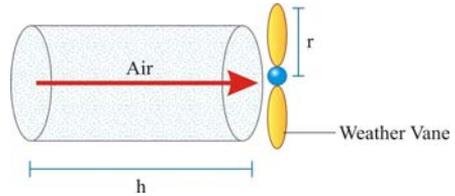
- The done work by the car:

- 

- $W = \Delta K = \frac{m \cdot V_F^2}{2} - \frac{m \cdot V_I^2}{2} \quad V_F = 2 \cdot \bar{V}$

$$W = \frac{m \cdot (2 \cdot \bar{V})^2}{2} = \frac{m \cdot 4 \cdot \bar{V}^2}{2} = 2 \cdot m \cdot \bar{V}^2$$

- The done work by the wind:



a)  $\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho}$

b)  $V = A \cdot h \quad \text{and} \quad V = \frac{m}{\rho}$

$$A \cdot h = \frac{m}{\rho} \Rightarrow m = A \cdot h \cdot \rho \quad \rightarrow \quad A = \pi \cdot r^2 \quad \rightarrow \quad m = h \cdot \pi \cdot r^2 \cdot \rho \quad \rightarrow$$

$$h = \frac{m}{\pi \cdot r^2 \cdot \rho}$$

c)  $\bar{V} = \frac{h}{\Delta T} \quad \text{So,} \quad \bar{V} \cdot \Delta T = h$

d)  $\bar{V} \cdot \Delta T = \frac{m}{\pi \cdot r^2 \cdot \rho} \Rightarrow m = \bar{V} \cdot \Delta T \cdot \pi \cdot r^2 \cdot \rho$

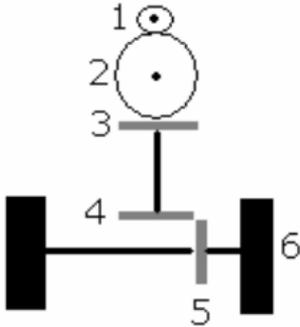
e)  $K = \frac{m \cdot \bar{V}^2}{2} \Rightarrow K = \frac{\Delta T \cdot \pi \cdot r^2 \cdot \rho \cdot \bar{V}^3}{2}$

f)  $\eta = \frac{W_{car}}{W_{wind}} = \frac{2 \cdot (\bar{V}_{car})^2 \cdot m_{car}}{\Delta T \cdot \pi \cdot r^2 \cdot \rho \cdot (\bar{V}_{wind})^3} \Rightarrow \eta = \frac{4 \cdot (\bar{V}_{car})^2 \cdot m_{car}}{\Delta T \cdot \pi \cdot r^2 \cdot \rho \cdot (\bar{V}_{wind})^3}$

## Wheels, Gears and Weather Vane Revolution - Model -

We will make a model capable to predict how many revolutions does a gear (or also the weather vane) make with  $n$  revolutions of the wheel. After, we also can

substitute the real values to see the interdependence of the number of revolutions. Our first referential will be the “motored” wheel (number 6, in the drawing). The number of each gear can be identified in the drawing below:



Gear NUMBER 5: As it is in the same axis than the wheel, we have:

$$n_5 = n_{wheels}$$

Gear NUMBER 4: As it is in contact with gear number 5, we have:

$$n_4 = n_5 \cdot \frac{r_5}{r_4} \rightarrow n_4 = n_{Wheels} \cdot \frac{r_5}{r_4}$$

Gear NUMBER 3: As it is in the same axis of gear number 4, we have:

$$n_3 = n_4 \rightarrow n_3 = n_{Wheels} \cdot \frac{r_5}{r_4}$$

Gear NUMBER 2: As it is contact with gear number 3 and  $r_3 = r_4$ , we have:

$$n_2 = n_3 \cdot \frac{r_3}{r_2} \rightarrow n_2 = n_{Wheels} \cdot \frac{r_5}{r_4} \cdot \frac{r_3}{r_2} \rightarrow n_2 = n_{Wheels} \cdot \frac{r_5}{r_2}$$

Gear NUMBER 1: As it is in contact with gear number 2, we have:

$$n_1 = n_2 \cdot \frac{r_2}{r_1} \rightarrow n_1 = n_{Wheels} \cdot \frac{r_5}{r_2} \cdot \frac{r_2}{r_1} \rightarrow n_1 = n_{Wheels} \cdot \frac{r_5}{r_1}$$

Weather Vane: As it is in the same axis of gear number 1, we have:

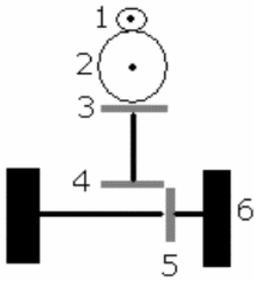
$$n_{Vane} = n_{Wheels} \cdot \frac{r_5}{r_1}$$

## Wheels, Gears and Weather Vane Revolution - Results -

We will substitute the radius of each gear in this part of the resolution to see how many revolution does each gear (or also the weather vane) make with one complete revolution of the “motored” wheel (number 6, in the drawing).

The number of each gear can be identified in the drawing below:

For each wheel complete revolution...



Gear NUMBER 5: makes:  
 $n = 1$  revolution

Gear NUMBER 4: makes:  
 $n \approx 0,7$  revolution

Gear NUMBER 3: makes:  
 makes:  
 $n \approx 0,7$  revolution

Gear NUMBER 2:  
 $n \approx 0,7$  revolution

Gear NUMBER 1: makes:  
 $n \approx 1,8$  revolutions

Weather Vane: makes:  
 $n \approx 1,8$  revolutions

## Scalar Speed

As we know the relation between angular speed of the weather vane and the wheel, and we also know the radius of both, we will now calculate the scalar speed (of the extremity of the wheel and of the extremity of the weather vane, respectively)

→ According to the relation of angular speed and scalar speed, we find that the scalar speed of the wheel and the weather vane is, in each fan's power:

$$V = \frac{2 \cdot \pi \cdot r}{T}$$

### Scalar speed

#### Wheels

	Radius (m)	Time (T)	Scalar Speed (m/s) (Error: 0,05m/s)
First Power	0,024	2,1 s	$0,023 \pi$
Second Power	0,024	1,7 s	$0,028 \pi$
Third Power	0,024	1,5 s	$0,032 \pi$

#### Weather Vane

	Radius (m)	Time (T)	Scalar Speed (m/s) (Error: 0,03m/s)
First Power	0,077	2,1 s	$0,067 \pi$
Second Power	0,077	1,7 s	$0,091 \pi$
Third Power	0,077	1,5 s	$0,10 \pi$

## Angular Speed

As we know the relation between the number of revolutions in the Wheel and in the weather vane, we will calculate now the angular speed.

For this part, we need to know how many times does the length of the wheel fit the fixed 0,5 meter distance:

$$x = \frac{0,5}{C} = \frac{0,5}{d \cdot \pi} = \frac{0,5}{0,021 \cdot \pi} \approx 7,6$$

7,6 means the number of revolution that the wheel makes in that distance. It is, approximately,  $15\pi$  Rad. Using the *models for revolution*, we find that the weather vane, make, in the same space:

13,7 revolutions (approximately,  $27\pi$  Rad)

→ According to the formula of angular speed, we find that the angular speed of the wheel and the weather vane is, in each fan's power:

$$\omega = \frac{\Delta\theta}{T}$$

### Angular Speed:

#### Wheels

	Angle's variation ( $\Delta\theta$ ) (Error: $\pm 0,6 \pi$ Rad)	Time (T)	Angular Speed (Rad/s) (Error: $\pm 0,3 \pi$ Rad)
First Power	$15\pi$ Rad	2,1 s	$7,14 \pi$
Second Power	$15\pi$ Rad	1,7 s	$8,82 \pi$
Third Power	$15\pi$ Rad	1,5 s	$10,00 \pi$

#### Weather Vane

	Angle's variation ( $\Delta\theta$ ) (Error: $\pm 0,4 \pi$ Rad)	Time (T)	Angular Speed (Rad/s) (Error: $\pm 0,5 \pi$ Rad)
First Power	$27\pi$ Rad	2,1 s	$12,86 \pi$
Second Power	$27\pi$ Rad	1,7 s	$15,88 \pi$
Third Power	$27\pi$ Rad	1,5 s	$18,00 \pi$

### Statistic treatment

Average:  $\bar{x} = \frac{1}{N} \sum x_i$

Error:  $\sigma = \sqrt{\frac{1}{N-1} \sum (\bar{x} - x_i)^2}$

Error of the average:  $\sigma_M = \frac{\sigma}{\sqrt{N}}$

## 4. PROBLEM № 9: SOUND IN THE GLASS

### 4.1. SOLUTION OF NEW ZEALAND

#### Problem № 9: Sound in the Glass

*/.Power point Presentation/*

#### The problem

● *Fill a glass with water. Put a tea-spoon of salt into the water and stir it. Explain the change of the sound produced by the clicking of the glass with the tea-spoon during the dissolving process.*

#### Key definitions

##### ● *Glass*

– Approx cylindrical container of rigid glass

##### ● *Tea-spoon of salt*

≈ 10 g of standard table salt. Can vary

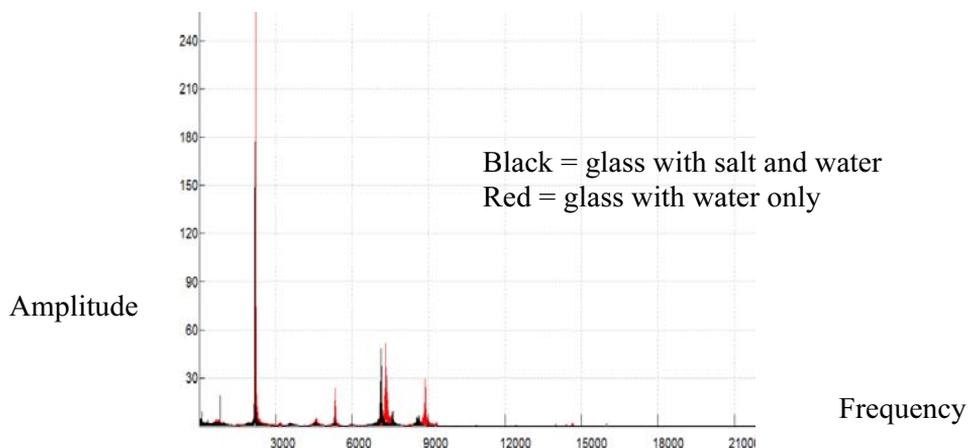
##### ● *Change of sound*

– *Any variation observed in average frequency, amplitude and overall timbre of the clicking*

#### *The Change in sound*

Please listen!!

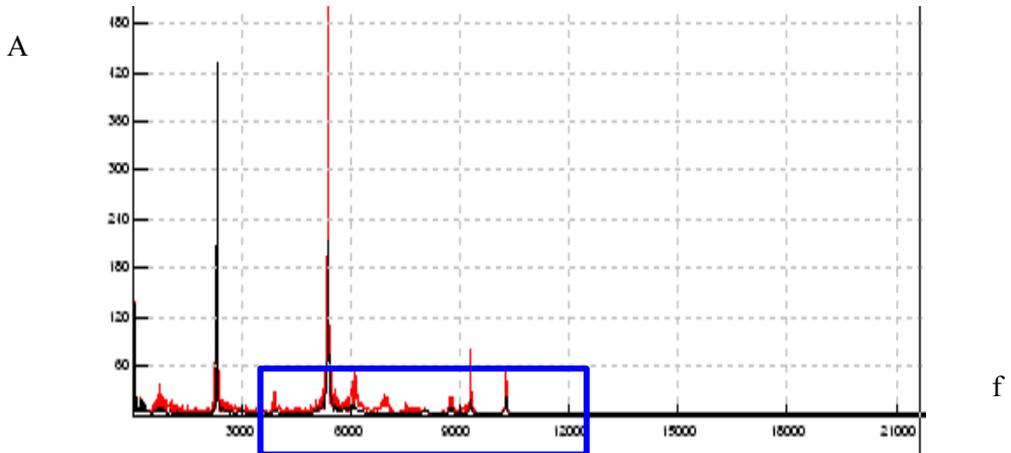
–1. Water in glass only 2. Same glass during dissolution of 1 teaspoon of salt



Not very clear, is it??

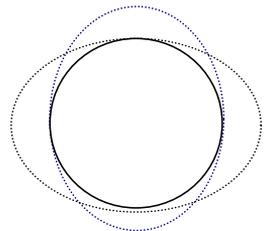
- “Sound” from glass
- à Mixture of different sound components
- Cup and water vibrating
- Vibrations of spoon. **Not all components change!!**

Compare...metal vs rubber



Sounds produced in “clicking”

- On impact...
- Glass wall flexes
- Produce vibrations
- Produce pressure variations in surrounding media
- à SOUND



**Clicking Mechanism.** Clicking inside = Clicking outside !  
 .Clicking outside is easier to regulate

**Overview of problem** ● Dissolving the salt...

- STEP A : Air bubbles released from powder
- STEP B: Salt disperses into suspension
- STEP C: Salt gradually dissolves (MAIN EFFECT)

- Changes...
  - Density
  - Bulk Modulus (small change)
  - Attenuation

**Bulk Modulus** ??? Reciprocal of *compressibility* ( $\kappa$ )

- *Waves = rarefactions/compressions in medium!*
- *Travel of sound waves in medium will be affected*

## PRELIMINARY THEORY

What defines a sound

- Chief sound characteristics governed by:
  - Change in one parameter = change in others
- STRING VIBRATION ANALOGY
- Wavelength determined by *physical constraints of system*
- *Frequency determined by medium properties*
- Therefore, wavelength doesn't change

$$c^2 = \frac{\beta_T}{\rho_0}$$

Speed of sound 'c' in liquids

- c = speed of sound in liquids (m/s)
- $\beta_T$  = Bulk modulus (Pa)
- $\rho_0$  = density of medium (kg/m<sup>3</sup>)

### STEP A

**Air bubbles released from powder:**

**Air bubbles...**? Salt crystals not perfectly even – Air between gaps in particles

- Air escapes as bubbles
- Negligible solubility (N<sub>2</sub>, O<sub>2</sub>, CO<sub>2</sub> etc.)
- During bubble transition
- Density/bulk mod of medium changes

## Equation for sound speed change

$$c_{new} \approx \sqrt{\frac{1}{(1-\varepsilon)\varepsilon\rho_w\kappa_a}}$$

- $\rho_w$  = in/of water
- $\kappa$  = adiabatic compressibility ( $\text{Pa}^{-1}$ )
- $\varepsilon$  = fraction of air in water

## Consequences

Assume, for example, 1% vol of air in water?

- $\kappa$  for air =  $7.04 \times 10^{-6} \text{ Pa}^{-1}$
- Speed of sound in cup of water
- Approx 120 m/s, less than in air!!!
- Unreasonable...1% is very liberal
- $c$  has decreased  $\Rightarrow$   $f$  has decreased

## Problems with practical investigation

- VERY short time span
- Effect of aeration...use aerator??
- Any aerator will introduce NOISE
- Air concentration will be very different
- Doesn't always occur
- Depends on “click” during air-escape process

## STEP B

### Salt dispersal & attenuation

#### Salt suspension

- Before dissolution
- Salt “disperses” into suspension
- Observable during process
- Suspension
- Attenuates sound wave within water medium
- Slight timbre change, amplitude change
- More salt = more attenuation



# Attenuation due to solid suspension

$$Att \propto freq^2$$

• Higher attenuation at higher frequencies

• Combined effect of

– Scattering

– Absorption

**Practical investigation 1** Use *large* quantities of salt – Clearer demonstration of effect

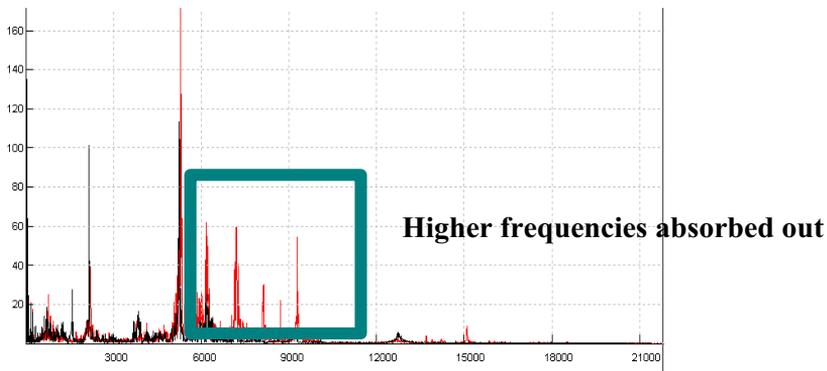
– Record clicking spectrum before and after suspension is achieved

## Comparison of spectrum

General disturbance of spectrum – “scattering”

• Red line – clicking sound before agitation

• Black line – clicking sound after suspension forms



## Other things to note

• As salt dissolves, attenuation dissipates

• Effect on  $c$ ??

– Minimal

– Bulk mod/density remain approx constant

• Very small fraction of solid in suspension

• Quantitative measurements

– Unreliable with available equipment and such a small scale

## STEP C

Density change

Bulk modulus variation with salinity

- Increase in salinity = increase in bulk mod.
- Non-linear variation
- Increments become less with greater salinity
- Linear extrapolation can be made

## Theory: c change in water column

.Density increase  
 Speed decreases – DOMINANT EFFECT  
 Bulk modulus increase  
 Speed increases

$$c^2 = \frac{\beta_T}{\rho_0}$$

## Overall effect?

Recall

- If c decreases
  - Wavelength remains constant
  - Frequency decreases
- Negative frequency shift observed in spectrum
- More salt = greater shift

## Analogy

- Water not only acts as resonating column
  - Vibrates itself, with vessel providing restorative force
  - Approximated to a spring
  - Natural frequency relationships determined by

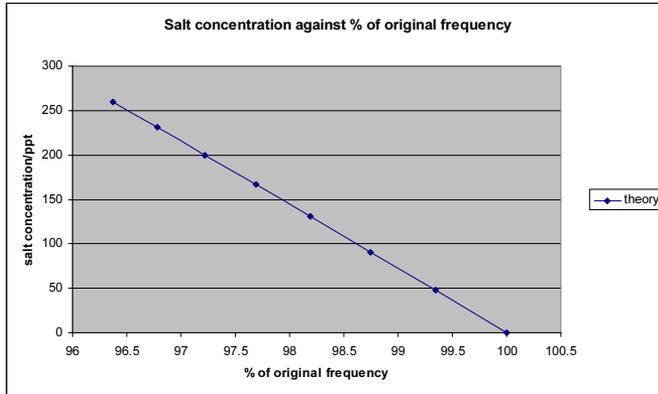
$$f \propto \sqrt{\frac{k}{m}}$$

- As salt is added, mass increases
- Therefore, negative frequency shift
- N.B. k & B, m &  $\rho$

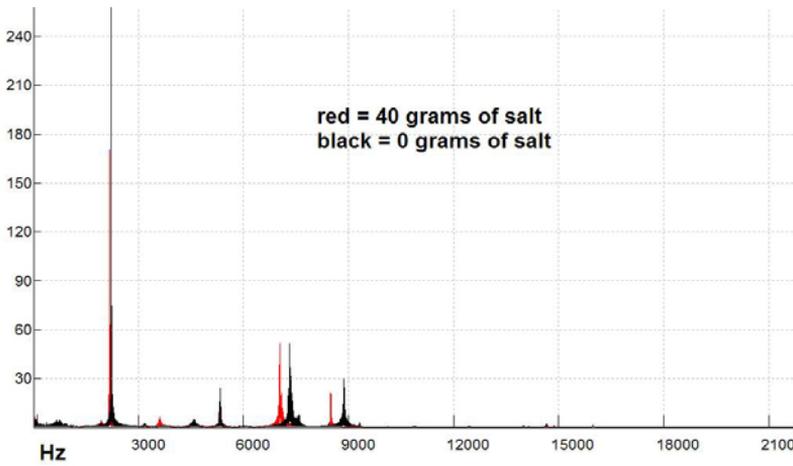
## Practical investigation 2

- Dissolve salt in water
- Record clicking spectrum
- Repeat for different amounts

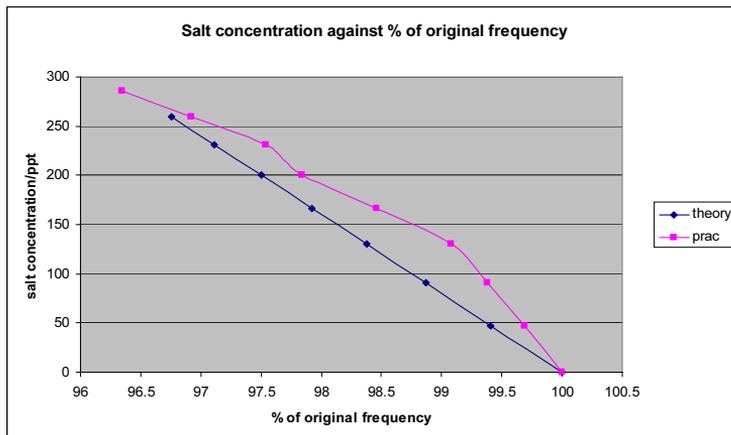
# Predicted Spectrum shift



## No salt vs. 40 grams of salt



## Spectrum shift



# Reason for variation in results

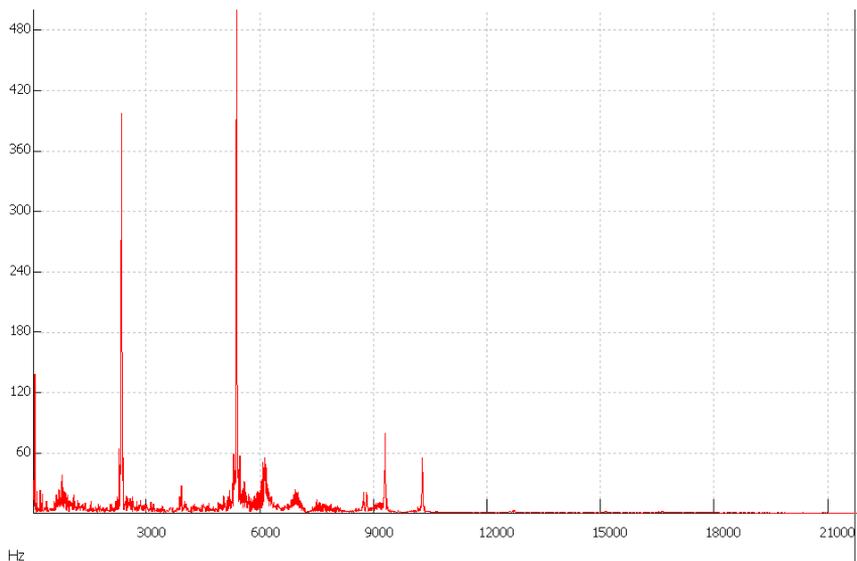
Linear extrapolation of bulk modulus used in theory

## Conclusion

Question asks: “CHANGE in sound”... “DURING dissolving process”

- Amplitude drops
  - Due to attenuation, *while salt in suspension*
- Frequency drops
  - Due to change in  $c$ , change in density (and bulk modulus)
  - Lower frequencies more noticeable – due to attenuation
- (Possible large frequency drop
  - Due to release of air from solid, very short time span)

## Example – Empty cup & spoon



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## PROBLEM № 9: SOUND IN THE GLASS

### 4.2. .SOLUTION OF UKRAINE

#### Problem № 9: Sound in the Glass

*Oleg Matveichuk,, Lena Filatova, Grygoriy,Fuchedzhy, Alyeksyey Kunitskiy, Valentin Munitsa*

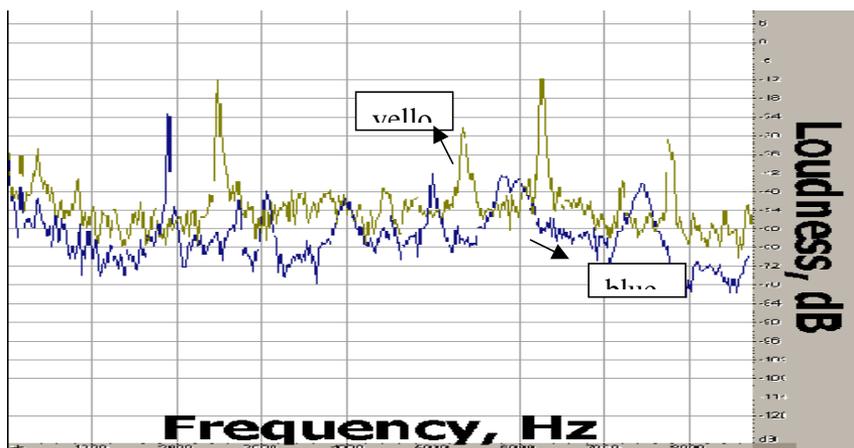
*Richelieu lycium, Shepkina Str 5, Odessa*

#### The problem:

*Fill the glass with water. Put a tea-spoon of salt into the water and stir it. Explain the change of the sound produced by the clicking of the glass with the tea-spoon during the dissolving process.*

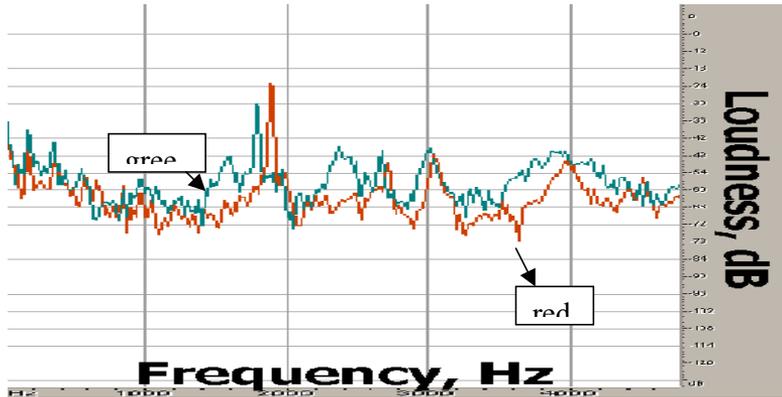
To solve the problem the aim of the work is always needed. We will look for the frequency change of clicking, which appears after stirring in the glass, experimentally and theoretically.

We wanted to observe the phenomenon so took a glass of water and recorded sounds of clicking for the empty glass, for the glass filled with water and for the glass with water and dissolved coffee in it. Using computer programmes frequency analysis was done:



Here you can see **blue line** that represents frequency analysis for the glass with water and **yellow line** for the empty glass. It is obvious that water makes the sound duller, its frequency lower. We can use a mechanical similarity: the glass will be interpreted as a spring with a weight and it oscillates with some frequency; adding of the water corresponds to increasing of the weight.

On the next graph **red line** represents frequency analysis for the glass with water and **green line** for water with coffee.



What changes with adding coffee? When we put a tea-spoon of coffee or other soluble in water material into the glass and stir it, the dissolving starts. It causes the changing of the gas dissolubility (as you know some gas always exists in water). So the gas evolves in the form of bubbles. Because of this compressibility and density of the liquid changes consequently the speed of the sound also changes. This works for small bubbles, which don't influence on the sound path, which we observe in the glass. Digressing into the mechanical similarity we can say that dissolving of coffee means changing rigidity for the spring and mass for the weight.

As mechanism of the phenomenon is understood, we can start mathematical investigation of the problem. In our model such assumptions were made:

1. Wave that travels in the water is longitudinal;
2. Bubbles that form in the water are small  $R_b \ll \lambda_{oscill}$ ;
3. While sound travels in the glass compression of water and gas occurs adiabatically.

To find the frequency change we need new compression modulus and density of the water with bubbles.

So long as compression of water and gas occurs adiabatically:

$$pV_{gas}^\gamma = const \Rightarrow dpV_{gas} + \gamma p dV_{gas} = 0 \quad (1)$$

$$\Delta p = -k \Delta V_{water} = -\frac{\gamma p}{V_{gas}} \Delta V_{gas} \quad (2)$$

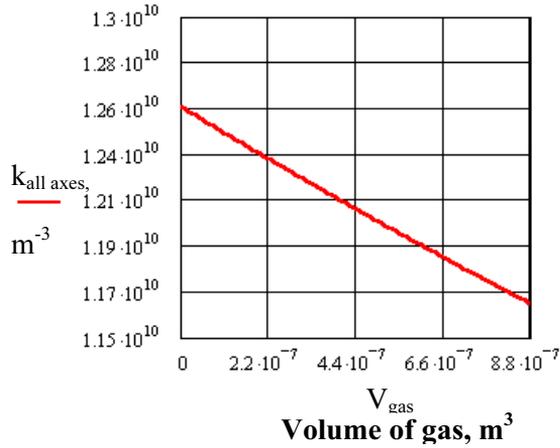
$$k = \frac{1}{V_{water} K_{water}} \quad (3)$$

$$\left( \frac{\partial p}{\partial V_{all}} \right)_S \approx \left( \frac{\Delta p}{\Delta V_{gas} + \Delta V_{water}} \right) = \left( \frac{-\frac{\gamma p}{V_{gas}}}{1 + \frac{\gamma p}{k V_{gas}}} \right) = -\frac{k}{1 + \frac{k V_{gas}}{\gamma p}} \quad (4)$$

$$k_{all\ axes} = \frac{k}{1 + \frac{k V_{gas}}{\gamma p}} \quad (5)$$

## Compression modulus

$d$  is the characteristic geometrical size of the glass with water.



$$\rho_{new} = \frac{\rho_{water} V_{water}}{V_{gas} + V_{water}} \Rightarrow \frac{\rho_{old}}{\rho_{new}} = 1 + \frac{V_{gas}}{V_{water}} \quad (6)$$

$$C_{sound} = \sqrt{\frac{E}{\rho}} \Rightarrow \frac{C_{new}}{C_{old}} = \sqrt{\frac{\rho_{old}}{\rho_{new} \left(1 + \frac{kV_{gas}}{\gamma P}\right)}} \quad (7)$$

$$v_0 = \frac{C_{old}}{d} \quad (8)$$

$$v_1 = \frac{C_{new}}{d} \quad (9)$$

$$\Delta v = v_1 - v_0 = \frac{C_{old}}{d} \left( \frac{C_{new}}{C_{old}} - 1 \right) = v_0 \left( \sqrt{\frac{1 + \frac{V_{gas}}{V_{water}}}{1 + \frac{kV_{gas}}{\gamma P}}} - 1 \right)$$

In our experiments we saw:

Frequency before dissolving, Hz	Frequency after dissolving, Hz	$\Delta v$ , Hz	$\frac{\Delta v}{v}$
1880	1800	80	0.04
3036	2910	126	0.04

The only problem for theoretical solution is to find the volume of the gas that evolved. At the same time definite volume of gas corresponds to the definite frequency change. So the experiment gives us information about the frequency

$$v_0 = \frac{C_{old}}{d} \quad (8) \qquad v_1 = \frac{C_{new}}{d} \quad (9)$$

$$\alpha = \frac{1}{V_{water}} \qquad \beta = \frac{k}{\gamma p}$$

and

we can find the volume of gas theoretically. Then we'll try to estimate this value in the other way. But our last formula is too complicated and can be simplified.

Substitution of these magnitudes gives:

$$K = 4.5 \cdot 10^{-10} \text{ Pa}^{-1} \qquad V_{water} = 0.177 \text{ m}^3$$

$$p = 10^5 \text{ Pa} \qquad \gamma = \frac{C_p}{C_v} = 1.4$$

$$\frac{C_{new}}{C_{old}} = \sqrt{\frac{1 + \frac{V_{gas}}{V_{water}}}{1 + \frac{kV_{water}}{\gamma p}}}$$

$$\boxed{\Delta V_{gas} = 8.8 \cdot 10^{-7} \text{ m}^3} \quad \longrightarrow \quad dV = v_0 \left[ -\frac{1}{2}(\beta - \alpha) \right] dV_{gas}$$

$$\boxed{\Delta V_{gas} = \frac{-2}{(\beta - \alpha)} \frac{\Delta v}{v_0} = -\frac{2\Delta v}{v_0} \cdot \frac{K_{water} \gamma p}{1 - K_{water} \gamma p} V_{water} \quad (10)}$$

We heard that after some time sound of the clicking becomes the same as before appearance of any bubbles. It means all of them have risen to the surface. By these considerations we can estimate the average size of the bubble and consequently their volume.

We equalize buoyant and resistant forces, thinking that velocity becomes constant very quickly.

$$\begin{array}{l}
 F_{res} + mg = F_b \\
 F_{res} = 6\pi\eta RV, \quad V = \frac{L}{t} \\
 F_b = \frac{4}{3}\pi R^3 \rho g \\
 \eta = 10^{-3} \text{ Pa} \cdot \text{s} \\
 t = 15\text{s}
 \end{array}
 \left. \vphantom{\begin{array}{l} F_{res} + mg = F_b \\ F_{res} = 6\pi\eta RV, \quad V = \frac{L}{t} \\ F_b = \frac{4}{3}\pi R^3 \rho g \\ \eta = 10^{-3} \text{ Pa} \cdot \text{s} \\ t = 15\text{s} \end{array}} \right\} \begin{array}{l}
 mg \ll F_{res}, \quad mg \ll F_b \\
 \longrightarrow \\
 F_{res} = F_b \\
 L \approx 10^{-1} \text{ m} \\
 n \approx 10^3
 \end{array}
 \quad \rho = 10^3 \frac{\text{kg}}{\text{m}^3}$$

$$R = 3 \cdot \sqrt{\frac{\eta L}{2\rho g t}} \longrightarrow \Delta V_{gas} = \frac{4}{3}\pi R^3 n$$

$$\Delta V_{gas} \approx 6.7 \cdot 10^{-7} \text{ m}^3$$

As you see both results for the volume of the evolved gas are comparable with each other so frequencies got in the experiment and predicted by our theory are in quite good agreement.

In conclusion I would like to add that such bubbles lead to one more effect which wasn't considered here: high frequencies are dampened in the water with such bubbles. But nevertheless considerations of the effect of the frequency shift because compressibility and density of the liquid change gives good results for explanation and prediction of the phenomenon.

Special thanks to:

Oleg Matveichuk, main author of the idea.

## 5. PROBLEM № 11: WATER DROPLETS

SOLUTION OF NEW ZEALAND

### Problem № 11: Water Droplets

/Power Point Presentation/

#### The problem

➤ *If a stream of water droplets is directed at a small angle to the surface of water in a container, droplets may bounce off the surface and roll across it before merging with the body of water. In some cases the droplets rest on the surface for a significant length of time. They can even sink before merging. Investigate these phenomena.*

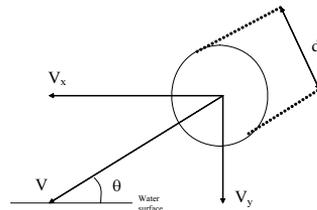
#### Definition of question

- **Droplets**; spherical balls of water
- **Bounce**; the droplets, after impact with the water surface must rebound off.
- **Roll**; the droplets don't coalesce with the water surface as they float on the surface while rotating.
- **Merging**; when the droplet coalesces with the water in the container.
- **Investigate**; Explore the nature as to why these phenomena happen and what will affect these phenomena.

#### Parameters of the problem

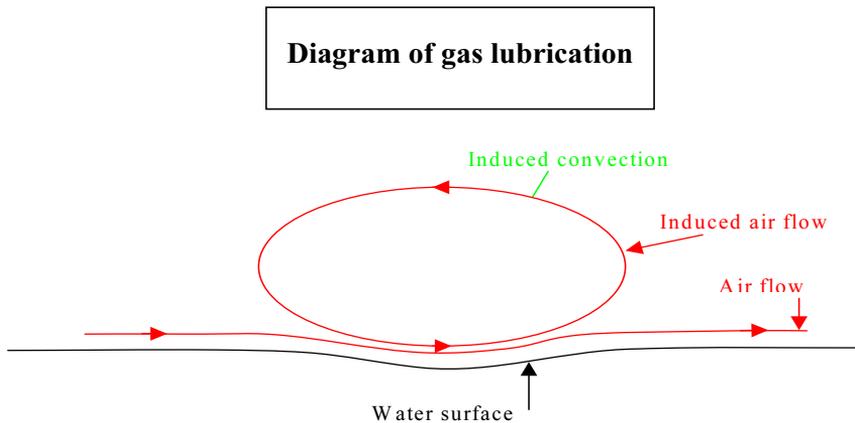
##### Diagram of a droplet

- Droplet diameter ( $d$ )
- Impact velocity ( $V$ )
- Vertical component of  $V$  ( $V_y$ )
- Horizontal component of  $V$  ( $V_x$ )
- Bounce height ( $b$ )
- Incident angle ( $\theta$ )



## Theory (why the droplet doesn't mix with the water surface)

- The droplet doesn't coalesce with the water surface.
- The reason for this non-coalescence is because of gas lubrication.
- This is when a very thin layer of air is trapped between the interface.



## Theory (bouncing)

- The effect of gas lubrication has to be sufficient to maintain separation.
- The downwards momentum makes droplet spread out, creating a dimple in the water surface.
- They recoil to their original states, if this is forceful enough, the droplet will rebound of the water surface.

## Sequence of droplet bounce



## Theory (rolling)

- If  $V_x$  is large enough to maintain sufficient gas lubrication, the droplet won't coalesce.
- But if  $V_y$  is too small the droplet doesn't exert a large enough force on the water surface.
- So it will instead roll across the surface until  $V_x$  becomes too small.
- Then it will coalesce.

## Theory of sinking droplets

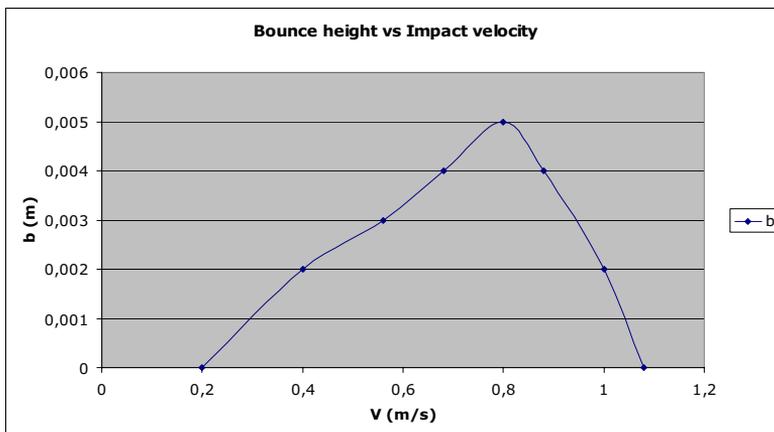
- Heat capacity of a droplet is very small.
- This causes the density to drop.
- It also increases the surface tension of the droplet.
- The droplet must break the surface.
- This can be done by weight ( $>4\text{mm}$  d) or downwards momentum.

## Theory of sinking droplets

- For this to occur the droplet cannot be allowed to coalesce.
- This non-coalescence occurs again because of gas lubrication.
- Initial air flow under the droplet causes the droplet to rotate.
- A layer of air will fully enclose the droplet and circulate around it maintaining separation.

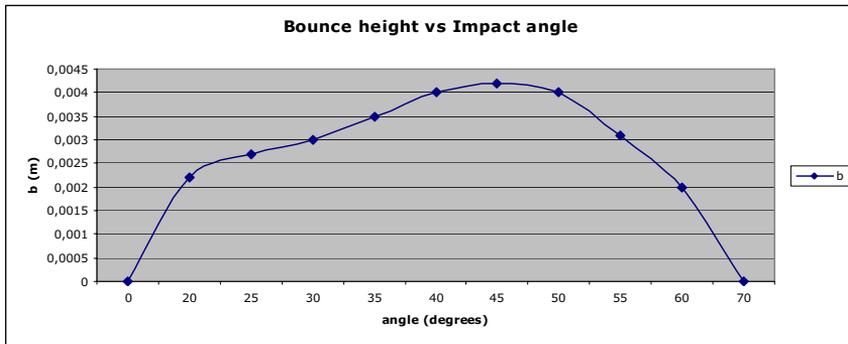
## Theory (V)

- The more kinetic energy it has the more kinetic energy can be converted into potential energy.
- Therefore the higher it can bounce.
- The greater V the longer the collision time.
- The more energy loss.
- The lower the bounce height.
- There will be optimum V for any given  $\theta$ .



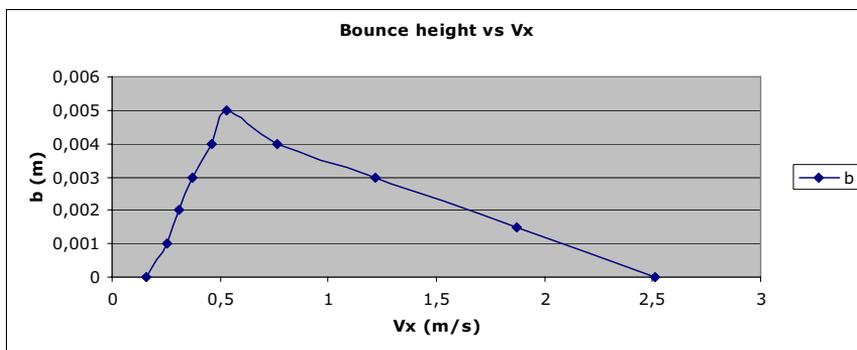
## Theory ( $\theta$ )

- The less  $\theta$  is the greater  $V_x$  is in relation to  $V_y$ .
- The greater  $\theta$  is the more force the droplet can apply to the surface
- So there is also an optimum  $\theta$  for a given  $V$ .
- At a lower  $V$  a greater  $\theta$  is better and at a higher  $V$  a smaller  $\theta$  is better.



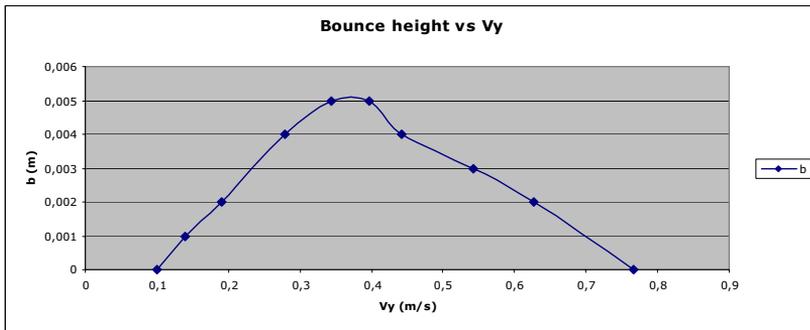
## Theory ( $V_x$ )

- Greater  $V_x$  means better gas lubrication.
- Hence more downwards force it can overcome.
- Hence the higher it can bounce.
- Greater  $V_x$  also means greater  $V$  hence a longer the collision time.
- Therefore the more energy loss there is.



## Theory ( $V_y$ )

- Greater  $V_y$  means more downwards force exerted by the droplet.
- The effect of gas lubrication has to also increase to cope with the extra downwards force.
- Greater  $V$  also means a longer collision time.



## Theory (d)

- Smaller  $d$  means a higher ratio of surface tension to volume.
- Therefore the more robust the droplet is.
- Therefore the higher  $V$  the droplet can withstand and not break up.
- Also the more efficient the collisions are.

## Video of small/large droplets

## Conclusion

- The smaller the droplet diameter the higher it bounces
- There will be an optimum impact velocity ( $V$ ) for any given angle ( $\theta$ ).
- To gain the highest bounce an increase in  $V_y$  must have an increase in  $V_x$ .
- For the longest roll time a large  $V_x$  is needed with a small  $V_y$  and if possible a forwards spin.
- For sinking the faster the spin and the larger the droplet (assuming it stays spherical) the better.

Presentation is supported by the Royal Society of New Zealand

## 6. PROBLEM № 13: HARD STARCH

SOLUTION OF CZECH REPUBLIC

### Problem № 13: Hard Starch

Klára Roženková

Mendelovo gymnázium, Opava

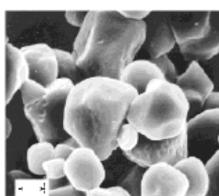
#### The problem

*A mixture of starch (e.g. corn flour or cornstarch) and a little water has some interesting properties. Investigate how its “viscosity” changes when stirred and account for this effect.*

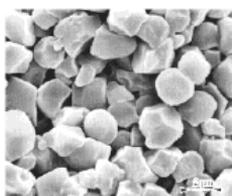
*Do any other common substances demonstrate this effect?*

Starch has been used for production of papyrus and glue since 3500 B.C. In 1525 it was used for solidification of shirt's collars. There are many sources of starch in nature –bulbs and roots (potatoes, manioca), seeds (grain), fruits (chestnut, pulses). Content of starch is very different, e.g. rice 70-75 %, potatoes 12-20%.

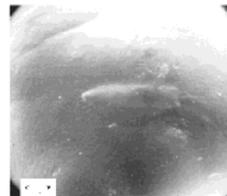
The mixture of starch and a little water is made of small insoluble particles in liquid, which means that it is a suspension.



corn starch, 2400x



rice starch, 3000x



potato starch, 2400x

corn starch, 2400x

rice starch, 3000x

potato starch, 2400x

The interesting property of the mixture of starch and a little water is the fact, that during stirring (force application) the liquid mixture starts to behave as a solid material, but just for the duration of the force application..

After ending the force application the solid becomes liquid again and we can observe how the “solid” is melting back to liquid.

Another remarkable property to be mentioned; immerse your finger into the glass with the mixture very quickly and you will find out that it's simply impossible, because you



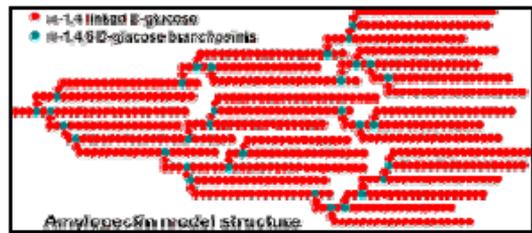
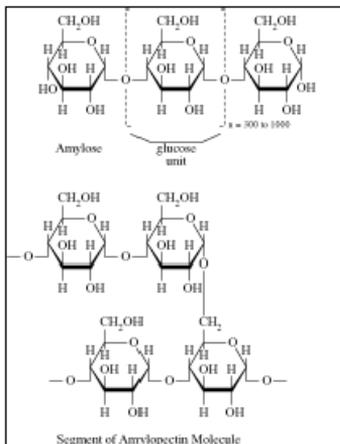
bump into the hard upper level of the mixture. On the other hand, if you immerse your finger slowly, you can easily reach the bottom of the glass.

## Account for this effect?

Water penetrates into the molecules of starch and creates hydrogen bonds with free hydroxides. Although this suspension has a high viscosity, it is possible to immerse objects with high density into it. But how is it possible that the mixture seems to be solid for a while? We can say that the starch corns in the mixture of starch and water are freely floating surrounded by water. If we apply mechanical force, the water is crushed out, the starch corns join and create an impression of a solid material. But if the mechanical force is small, the corns can freely move and the water acts as a lubricant.

It is easier to explain this effect on the microscopic structure of starch. Starch consists of two different polymeric polysaccharides – amylose (30%) and amylopectin (70%).

While amylose is linear and is made of few thousands of monomers, the structure of amylopectin is branched and can be made of millions of monomers. Molecules of water which are between the chains of amylose and amylopectin are during the force application crushed out and the chains wedge. The hydrogen links are forming and the structure of amylopectin is misshaped. It results in growth of the viscosity – this happens just for the duration of the force application. After ending the force application, the solid becomes liquid again. The bigger force we apply, the bigger viscosity and more solid properties we get



*structure of amylose and amylopectin*

*amylopectin structure in space*

This property is called **rheopexicity**. The suspension owing to the movement (stirring, crumpling, shaking) becomes solid but at ease it becomes liquid again.

The better known property is **thixotrophy** – which is the opposite effect to rheopexicity. All of us know ketchup – at ease it's solid, you can't get it out of the bottle but after shaking it becomes liquid.

This means that the viscosity of starch during stirring increases so much that the mixture seems to behave as a solid material.

What is the definition of viscosity and rheopecticity? **Viscosity** is a measure of the resistance of a fluid to deformation under shear stress. It is commonly perceived as "thickness", or resistance to pouring. Viscosity describes a fluid's internal resistance to flow and may be thought of as a measure of fluid friction.

**Rheoplectic fluids** are a type of non-Newtonian fluids. Rheopecticity shows a time dependant change in viscosity; the longer the fluid undergoes shear, the higher its viscosity. Rheoplectic fluids are a rare type of fluids, in which shaking for time causes solidification.

How to measure viscosity using different force and intensity of the mechanical force in order to prove the increase of the viscosity?

The best how to do it is to use Stokes's figure for resistance force acting on the ball during drawing through the liquid.

$$F = 6 \pi \eta R v$$

We measured viscosity using drawing an iron ball in a mixture of starch and a little water with a constant force application, which we kept with a help of dynamometer.

Because we drew the ball in a volumetric cylinder, instead of  $v$  into the figure we had to institute

$$v_l = v_m (1 + 2,4 R/RT)$$

where  $v_l$  is the velocity of drawing after Landerburg's correction and  $v_m$  is the measured velocity.



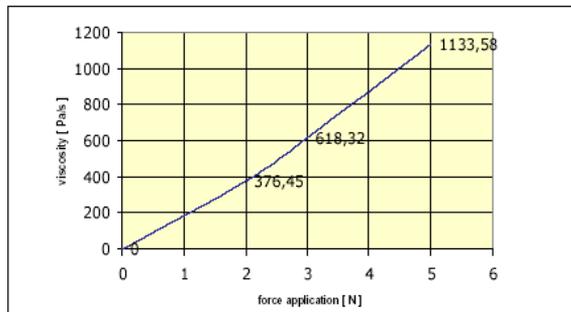
In our case::

At first we measured the velocity of starch during the force application of 2N. Average time of this measurement was  $t = 9,13$  s, we used this data for a calculation of the viscosity.

After calculating  $v_l$  and institution into the figure for viscosity we got a viscosity 376,45Pa/s.

During the force application of 3N and  $t = 10$ s calculated viscosity was 618,32 Pa/s.

During the force application of 5N and  $t = 11$ s calculated viscosity was 1133,58 Pa/s.



viscosity dependence on the force application

This experiment proved that during stirring (=force application) the viscosity of starch is increased. The bigger force we apply, the bigger viscosity we get.

Other values we used for the calculation:

$RT = 0,045 \text{ m}$

$s = 0,13 \text{ m}$

$r = 0,012 \text{ m}$

For our experiment we used a suspension of water and potato starch in weigh ratio 1: 1.

Other interesting property is that this effect occurs when the proportion of the amount of water and starch is 1: 1. If using more water for the mixture, this surplus water separates from the mixture on the bottom of the glass after some time. The arisen mixture has perfect rheopectic properties.



The same effect doesn't occur only during stirring but it also appears while shaking or crumpling. Other common substances that demonstrate this effect are all rheopectic substances, for example asphalt, gum arabic, mud or gypsum.

### References:

<http://www.pf.jcu.cz/toCP1250/stru/katedry/fyz/veletrh/holubova.doc>

<http://en.wikipedia.org/wiki/Starch>

<http://www.lsbu.ac.uk/water/hysta.html>

<http://www.portal.cz/scripts/detail.asp?id=3829&tmplid=27>

<http://www.andweighing.com/and.nsf/html/Measuring+Viscosity>

Pavol Hrnčiar, DrSc., ORGANICKÁ CHÉMIA, Slovenské pedagogické nakladateľstvo v Bratislave, 1977, vydání první, 67-002-77

Otakar Červinka, Václav Dědek, Miloslav Ferles, ORGANICKÁ CHEMIE, Nakladatelství technické literatury v Praze, 1970, vydání první, 04-608-70

Bohumil Škárka, Miroslav Ferenčík, Katarína Horáková, BIOCHÉMIA, ALFA v Bratislavě, 1973, vydání první, 63-085-73

František Petruž, Karel Andrlík, ORGANICKÁ CHEMIE, Státní nakladatelství technické literatury v Praze, 1957, vydání druhé, L 16-C2-4-II

Michaels L, Shah NS, DANGERS OF CORN STARCH POWDER, *Br Med J* 1973; 2: 714.

## 7. PROBLEM № 14: EINSTEIN–DE HAAS EXPERIMENT

### 7.1. SOLUTION OF BRAZIL

#### Problem № 14: Einstein–de Haas Experiment

*Emanuelle Roberta da Silva*

*Colégio Objetivo – São Paulo – SP*

#### The problem

*When you apply a vertical magnetic field to a metallic cylinder suspended by a string it begins to rotate. Study this phenomenon.*

#### 1. Objectives:

The original experiment, did in 1915, had the objective to determine the change in angular momentum which accompanies a known change in magnetic moment, or in other words, to determine the *gyromagnetic ratio*. Doing some experiments, we intend to prove our hypothesis that the cause of the cylinder movement is microscopic and interior, because there is no external torque acting on the cylinder. With experiments and theory, we will show that the movement is related with the angular momentum and magnetic moment of the electron and with the cylinder magnetization.

#### 2. Theoretical Introduction:

##### Torque and angular momentum:

In order to make an object rotate, it is necessary that we apply a force. The torque is the vector product of the position vector and the force vector:

$$\vec{\tau} = \vec{r} \times \vec{F} \quad (1)$$

The angular momentum is defined as the vector product of the position vector and the linear momentum vector:

$$\vec{L} = \vec{r} \times \vec{p} \quad (2)$$

By the Newton's Second Law, we can relate the torque and the angular momentum:

$$\sum \vec{\tau}_{ext} = \frac{d\vec{L}}{dt} \quad (3)$$

When the resultant of the external torques is null, the angular momentum is constant, and we can enunciate the Conservation of the Angular Momentum Law:

If the resultant of the torques that are acting on a system is zero, the total angular momentum is conserved.

**Magnetic Moment:**

The magnetic moment is defined as the product of the electrical current and the area of the circuit:

$$\mu = iA \tag{4}$$

We can arrive at the classic relation between magnetic moment and angular momentum:

$$\begin{aligned} \mu = iA &= q \left( \frac{v}{2\pi r} \right) (\pi r^2) = \frac{1}{2} qvr = \frac{1}{2} q \left( \frac{L}{m} \right) \\ \bar{\mu} &= \frac{q}{2m} \bar{L} \end{aligned} \tag{5}$$

The magnetic momentum of the electron, associated with its orbit is given by this equation:

$$\bar{\mu}_l = -g_l \mu_B \frac{\bar{L}}{\hbar} \tag{6}$$

where  $\mu_B$  is the Bohr magneto:

$$\mu_B = \frac{e\hbar}{2m_e} = 9,274.10^{-24} A.m^2 \tag{7}$$

Classically, the prediction for the *gyromagnetic factor* ( $g$ ) is 1, but experimentally, the researches have found values near 2, and it happens because the existence of the *spin* (the electron has an intrinsic angular momentum called spin).

So, we can calculate the magnetic moment of the electron due to the spin:

$$\bar{\mu}_s = -\frac{g_s \mu_B}{\hbar} \bar{S} \tag{8}$$

Where the *gyromagnetic fator* ( $g$ ) is closer to 2.

**Magnetization:**

Magnetization is defined as the sum of the magnetic moments of the electrons that are in a solid.

$$\bar{M} = \sum_{i=1}^N \bar{\mu}_i \tag{9}$$

It is proportional to the magnetic field applied to the body.

$$\vec{M} = \chi_m \frac{\vec{B}}{\mu_0} \quad (10)$$

where  $\chi_m$  is the magnetic susceptibility of the material and  $\mu_0$  is the magnetic permeability. For the vacuum:

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{T \cdot m}{A}$$

**Development:**

Developing the equation that shows the relation between magnetic moment and angular momentum:

$$\vec{\mu}_l = -g \frac{e}{2m_e} \vec{L} \quad (5)$$

$$\Delta \vec{\mu} = -\frac{ge}{2m_e} \Delta \vec{L}$$

$$\sum \Delta \vec{\mu} = -\frac{ge}{2m_e} \sum \Delta \vec{L} \qquad \sum \Delta \vec{\mu} = \frac{ge}{2m_e} \Delta \vec{L}_{mac}$$

$$\Delta \vec{M} = \frac{ge}{2m_e} \Delta \vec{L}_{mac}$$

$$\chi_m \frac{\Delta \vec{B}}{\mu_0} = \frac{ge}{2m_e} \Delta \vec{L}_{mac} \quad (11)$$

Analyzing this last equation, we can conclude that a variation in the magnetic field will provoke a variation in the macroscopic angular momentum.

**Magnetic field:**

Inside a coil, the magnetic field is proportional to the electrical current:

$$B = ki \quad (12)$$

**3. Experiment:**

**Materials:**

- |  |  |  |                                       |
|--|--|--|---------------------------------------|
| <input type="checkbox"/> 2 Coils           | <input type="checkbox"/> Iron cylinder       | <input type="checkbox"/> Thread            | <input type="checkbox"/> Oscilloscope |
| <input type="checkbox"/> Source of current | <input type="checkbox"/> Steel cylinder      | <input type="checkbox"/> Banana connectors |                                       |
| <input type="checkbox"/> Amplificator      | <input type="checkbox"/> Functions generator | <input type="checkbox"/> Mirror            |                                       |
| <input type="checkbox"/> Holder            | <input type="checkbox"/> Multimeter          | <input type="checkbox"/> Laser             | <input type="checkbox"/> Rampart      |

**EXPERIMENT 1:**

We plugged the source of current in the coil with the banana connectors. We moored the thread in the holder and in the cylinder, and we put the cylinder inside the coil. We switched on the source of current and start to increase the current. We could see the movement of the cylinder, when we reach 3.12 ampères. (*pictures a and b*)

**Coil:**

□ Number of turns: 760

**Iron cylinder:**

□ Length:  $(3.50 \pm 0.05)$ cm

□ Diameter:  $(1.30 \pm 0.05)$ cm

□ Mass:  $(26.60 \pm 0.05)$ g

**EXPERIMENT 2:**

We repeated the proceeding of the experiment 1, but with a source of current that gives us it in a sinoidal function. At this time we glued a small mirror on the cylinder. We used a laser that we put between the cylinder and a rampart. When the laser incise on the mirror, we could measure the amplitude, and looking to a oscilloscope, that we also connected in our source of current, we could measured the period. We changed the frequency in our functions generator and we obtained the following data: (*pictures c and d*)

<b>Time (s) *</b>	<b>2.Amplitude(cm)</b>
$(18.0 \pm 0.2)$	$(29 \pm 1)$
$(16.1 \pm 0.2)$	$(39 \pm 1)$
$(13.8 \pm 0.2)$	$(72 \pm 3)$
$(11.1 \pm 0.1)$	$(40 \pm 2)$
$(9.9 \pm 0.2)$	$(18 \pm 1)$

\* time for 5 oscillations

$$\text{Distance (cylinder – rampart)} = (70.0 \pm 0.5)\text{cm}$$

$$R = 9.5 \Omega$$

$$U = 1.3 \text{ V}$$

**Coil**

□ Number of turns: 1000

**Steel cylinder**

□ Length:  $(3.70 \pm 0.05)$ cm

□ Diameter:  $(0.15 \pm 0.05)\text{cm}$

□ Mass:  $(1.14 \pm 0.05)\text{g}$

**Calculating the gyromagnetic ratio:**

Developing the equation (11) we can arrive in an equation to calculate the gyromagnetic ratio:

$$\frac{\chi_m}{\mu_0} \frac{dB}{dt} = \frac{ge}{2m_e} \frac{dL_{mac}}{dt} \qquad \frac{\chi_m}{\mu_0} k \frac{di}{dt} = \frac{ge}{2m_e} \tau$$

$$\frac{\chi_m}{\mu_0} k i_{\max} \cos(\Omega t) = \frac{ge}{2m_e} \tau$$

$$\tau = \frac{\chi_m 2m_e k i_{\max}}{\mu_0 ge} \cos(\Omega t) \tag{13}$$

$$\tau = A \cos(\Omega t) \qquad \sum \tau = I\theta''(t) \tag{14}$$

$$A \cos(\Omega t) - k\theta(t) - P\theta'(t) = I\theta''(t)$$

$$\theta''(t) + \frac{P}{I} \theta'(t) + \omega^2 \theta(t) = \frac{A}{I} \cos(\Omega t)$$

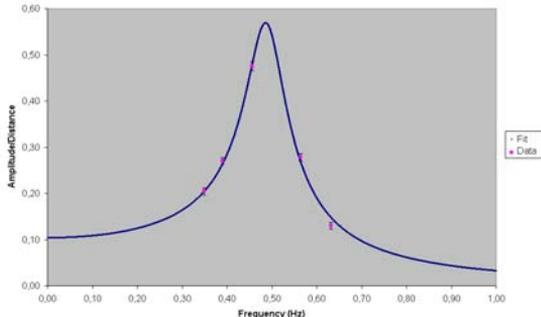
Solving this equation we have:

$$\theta(t) = \frac{A \cos(\Omega t + \Phi)}{I \sqrt{(\omega^2 - \Omega^2)^2 + P^2 \Omega^2 / I^2}} + \text{transient}$$

The frequency and the angle we could determine experimentally:

$$\Omega = \frac{2\pi}{T} \quad \text{and} \quad \tan \theta = \frac{\text{ampl}}{\text{dist}}$$

To determine the others parameters, we used a computer program that fit the parameters, so the distance of the curve to our points is the minimum possible. We obtained the following graphic:



After the software determined the parameters, we could arrive in a value for the gyromagnetic factor:

$$\frac{A}{I} = a$$

from the equation (13) we have:

$$Ia = \frac{\chi_m 2m_e k i_{\max}}{\mu_0 g e} \quad g = \frac{\chi_m 2m_e k i_{\max}}{\mu_0 e I a} \quad (15)$$

Finally, we used our data:

- $a = 3.5$
- $I = 5.88 \cdot 10^{-9} \text{ Kg.m}^2$
- $e = 1.60 \cdot 10^{-19} \text{ C}$
- $m_e = 9.11 \cdot 10^{-31} \text{ Kg}$
- $i_{\max} = 0.137 \text{ A}$
- $\chi_m = 49$
- $k = 0.006 \text{ T/A}$
- $\mu_0 = 1.26 \cdot 10^{-6} \text{ T.m/A}$

$g = 17.7$

### Error Sources:

In our experiment, we can not affirm that the cylinder was in the middle of the coil, so the magnetic field was not constant.

Another possible error source are the perturbations on the system, because our system was not isolated of external perturbations, for example, air motion.

Finally, we have the influence of the Earth's magnetic field, that we could not minimize in our experiment.

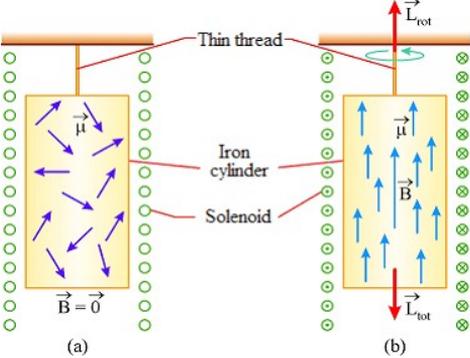
### 4. Analysis and Conclusions:

Firstly, we need to understand, why the cylinder begins to rotate when we apply a magnetic field on it.

Initially, the magnetic field is zero in the region where the cylinder is located, and the atomic magnetic moments are randomly oriented. The angular momentum have the same direction as the magnetic moment but pointed to the other side, due to this reason, there are also randomly oriented.

When we apply a vertical magnetic field on the cylinder, the atomic magnetic moments align on the direction of the magnetic field and consequently, the angular momentum align to the opposite side. So, the cylinder will have an angular momentum different of zero.

As there is no external torque acting on the cylinder, the angular momentum of it must be constant during the time. Because of the Conservation of the Angular Momentum, the cylinder begins to rotate to produce an angular momentum in order to keep the total angular momentum constant.

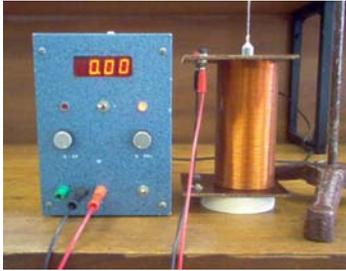


The rotation of the cylinder shows that the magnetic moment and the angular momentum are connected. Classically, the value predicted for the *gyromagnetic ratio* is 1. But doing the experiment, many researches found the numbers near to 2, and we also find a result different of 1. This results prove the theory about the existence of the *spin*.

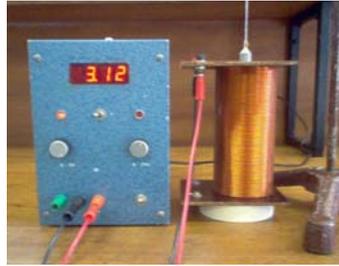
To conclude, we have a table showing the values for the *gyromagnetic ratio* calculated for different investigators, after extremely precise experiments:

Investigator	Year	g
BARNETT	1944	$1.938 \pm 0.006$
MEYER	1951	$1.936 \pm 0.008$
SCOTT	1951	$1.927 \pm 0.004$
BARNET & KENNY	1952	$1.929 \pm 0.006$
SCOTT	1955	$1.919 \pm 0.006$
MEYER & BROWN	1957	$1.932 \pm 0.008$
Scott (cylinder)	1960	$1.917 \pm 0.002$
Scott (ellipsoid)	1960	$1.919 \pm 0.002$

## 5. Pictures



*Picture a = experiment 1*



*Picture b = experiment 1*



*Picture c = experiment 2*



*Picture d = experiment 2*

## 6. Bibliography

- <http://hyperphysics.phy-astr.gsu.edu/hbase/tables/magprop.html>
- SCOTT, G.G.  
Review of Gyromagnetic Ratio Experiments
- ATKINS, Peter e Julio de Paula.  
Físico-química 7a edição, volume 2. LTC editora, 2004.
- FERRARO, Nicolau Gilberto; Caio S.V. Calçada; e Ronaldo Fogo.  
Coleção Objetivo: 6- Eletricidade. Editora Sol, 2005.
- GALISON, P.  
How experiments end. The University of Chicago Press, Chicago, USA
- HALLIDAY, David; Robert Resnick; e Jearl Walker. Fundamentos de Física 6a edição, volumes 2 e 4. LTC editora, 2003.
- ROBOTELLA, José Luis de Campos; OLIVEIRA, Edson Ferreira de; ALVES FILHO, Avelino.  
Física- Eletromagnetismo. Editora Ática, 1982
- TIPLER, Paul.  
Física 4a edição, volumes 1, 2 e 3. LTC editora, 2000.

## PROBLEM № 14: EINSTEIN–DE HAAS EXPERIMENT

### 7.2. SOLUTION OF BULGARIA (AMERICAN COLLEGE OF SOFIA)

#### Problem № 14: Einstein–de Haas Experiment

(Power Point Presentation of the National Team of Bulgaria)

Team Leaders: Krasimira Chakarova, Vanya Angelova

Team Members: Miloslava Evtimova, Yavor Kostov, Kalin Dimitrov, Velin Djidjev, Jordan Ivanchev, American College of Sofia, Bulgaria

#### The problem:

When you apply a vertical magnetic field to a metallic cylinder suspended by a string it begins to rotate. Study this phenomenon.

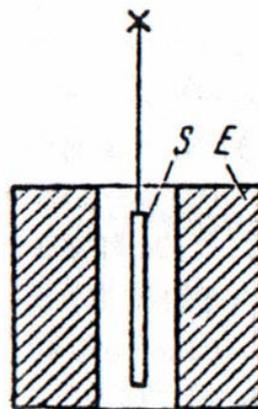
#### Historical Background

- Experimenteller Nachweis der Ampereschen Molekularströme. Naturwissenschaft, 1915, 3, 237-238
- Experimenteller Nachweis der Ampereschen Molekularströme. (Mit W.J. de Haas) Verhndl. Dtsch. Phys. Ges., 1915, 17, 152-170  
(oral report 19.02.1915 article submitted 10.04.1915)
- Berichtigung zu meiner gemeinsam mit Herrn W.J. de Haas Veröffentlichen Arbeit “Experimenteller Nachweis der Ampereschen Molekularströme”. Verhndl. Dtsch. Phys. Ges., 1915, 17, 203
- Notiz zu unserer Arbeit “Experimenteller Nachweis der Ampereschen Molekularströme” (Mit W.J. de Haas) Verhndl. Dtsch. Phys. Ges., 1915, 17, 420
- Ein einfaches Experiment zum Nachweis der Ampereschen Molekularströme. Verhndl. Dtsch. Phys. Ges., 1916, 18, 173-177. (oral report 25.02.1915 article submitted)

The quoted works of Einstein and de Haas, all published or reported in Germany reveal the idea and the results of an experiment to prove the existence of the so called molecular currents of Ampere, which interpret the magnetic properties of materials.

#### Nature of magnetism of the elements

- Atoms of the elements contain electrons
- Electrons move in orbits and “spin” around their axes
- They are the currents in the atoms (molecular Ampere’s currents) => reason for magnetic properties of atoms



- Electrons possess :
  - Orbital magnetic moment and spin magnetic moment
  - Orbital angular momentum and electron spin mechanical momentum

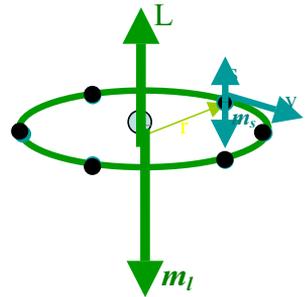
At the time Einstein performed his experiment only the orbital motion and the relevant magnetic moment were known. The modern concept of the atom involves orbital, spin magnetic moments and their interactions.

## Moments and Momenta

### Orbital moments

- orbital angular momentum  $\mathbf{L} = l \cdot \hbar$
- orbital magnetic moment  $\boldsymbol{\mu}_l = -(e/2m) \cdot \mathbf{L} = \gamma_l \cdot \mathbf{L}$

$$\boldsymbol{\mu}_l = \gamma_l \cdot \mathbf{L}, \text{ where } \gamma_l = g_l e/2m$$



### Spin moments

Spin mechanical moment

$$s = \frac{1}{2} \cdot \hbar$$

Spin magnetic moment

$$\boldsymbol{\mu}_s = -(e/2m) \cdot \hbar \cdot \gamma_s \cdot s$$

$$\boldsymbol{\mu}_s = \gamma_s \cdot s, \text{ where } \gamma_s = g_s \cdot e/2m$$

*Gyro magnetic ratio*

$$\gamma_l = \mu_l/L = -(e/2m) = 0,88.1011C/kg$$

$$\gamma_s = \mu_s/s = -(e/m) = 1,76.1011C/kg$$

### Mixed magnetism

$$\mathbf{j} = (\mathbf{l} + \mathbf{m}_s) \cdot \hbar$$

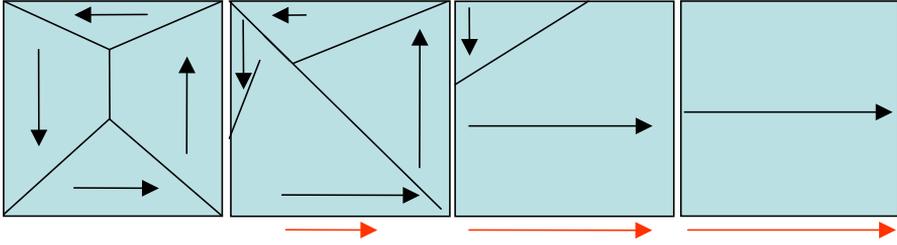
$$\boldsymbol{\mu}_j = \gamma_j \cdot \mathbf{j}, \text{ where } \gamma_j = g_j \cdot e/2m$$

$$g_j = 1 + \frac{\mathbf{j}(\mathbf{j}+1) + \mathbf{m}_s(\mathbf{m}_s + 1) - \mathbf{l}(\mathbf{l}+1)}{2\mathbf{j}(\mathbf{j}+1)}$$

The theory of atomic magnetism defines the mechanical momenta of the electron (orbital and spin) and its magnetic orbital and spin momenta. The ratio between the magnetic to the mechanical moment is known as the gyromagnetic ratio, g. In the case of spin-orbital interaction this ratio is known as the Landé factor  $g_j$

### Domains

- Magnetic domains are regions in a crystal with different directions of the magnetizations
- Ferromagnets



## Magneto-mechanical phenomena

- The sum of the magnetic moment vectors of the domains can be said to be the vector of the magnetic moment of the substance.
- Let  $I$  be the vector of magnetization,  $V$  is the volume of the body

$$I \cdot V = \sum \mu_d$$

- Let  $Q$  be the total mechanical momentum of the domains

$$Q = \sum L_d = \chi \cdot I \cdot V = \chi \cdot \sum \mu_d$$

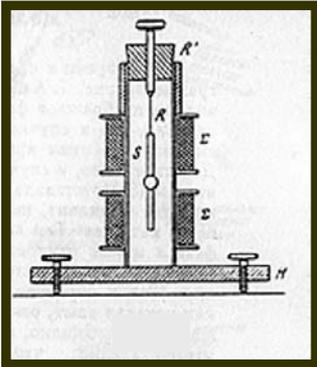
## Magneto-mechanical phenomena

- When the body is not magnetized  $I=0 \Rightarrow Q=0$
- When the body is magnetized  $I$  is no more  $0$
- Then according to the formula  $Q=\chi \cdot I \cdot V$ ,  $Q$  also changes
- According to the **law of conservation of mechanical momentum**
- $Q_{tot} = Q_D + Q_B$
- In the beginning the sum of the mechanical momenta of the domains is  $0$  and the body is not moving  $\Rightarrow$  **the total momentum is  $0$**
- $Q_D \neq 0 \Rightarrow {}_B B \neq 0$
- **So we must observe the spinning of the body**

In fact in Einstein-de Haas experiment the orientation and reorientation of domains, which are effects on a larger than molecular scale have been detected. The change of the magnetic moment of the domains causes a macro magneto-mechanical effect, which results in the rotation of the body.

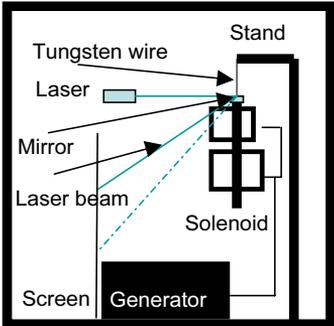
# Experiment Comparison

## Einstein's experiment

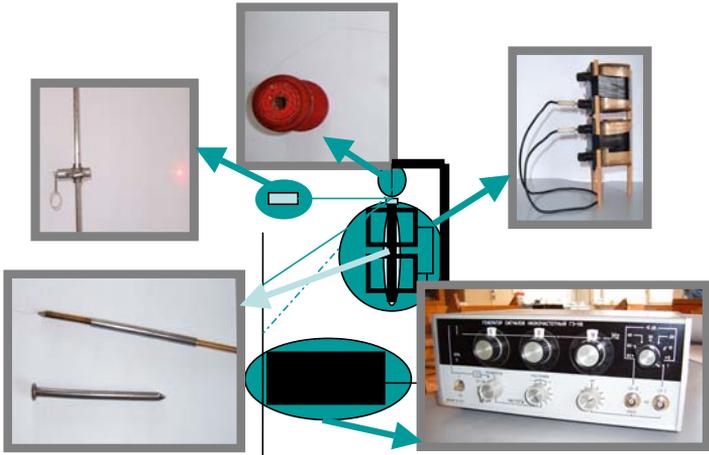


Picture from: Albert Einstein-selected scientific works, "Nauka", Moscow 1966

## Our experiment

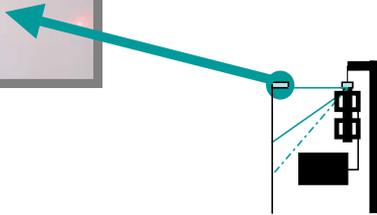
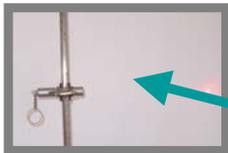


## Experiment Setting

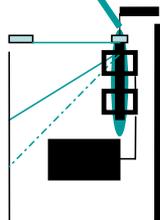
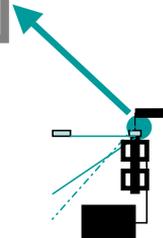


## Laser

Semiconductor laser  
1.5 euro from the market



- Wire
  - Wire made of Tungsten with thickness of 15  $\mu$ m
  - Connected to a reel
- Frequency – measured
- Torsion balance



### Sample

- The cylinder
- Nail (Fe)
- Two parts of brass (Cu and Zn) up and down
- Weight 7,052 g
- Length 7,44 cm
- Diameter 4mm
- Mirror – aluminum foil

The brass parts above and below the iron nail possess non-ferromagnetic properties. In our experiment they serve to damp the parasitic libration motion, which occurs at magnetization of the iron part.

### Solenoids

- Two solenoids connected in series
- 12000 turns each
- Magnetic field of about 20 G is created



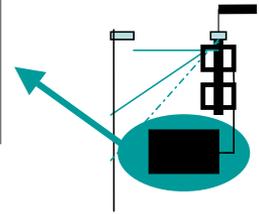
- Alternative current of **6 mA**

## Generator

Generator of sinusoidal vibrations with changing frequency

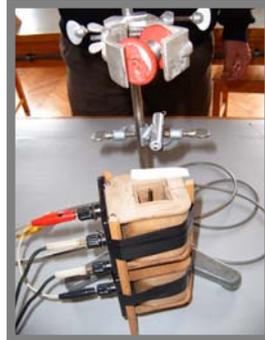
Creates resonance to strengthen the effect

Beating effect, when slightly different frequency is set



## Experiment

$$\gamma = \frac{\omega \cdot J}{I \cdot V}$$



## Video

To run the videos that show the experiment performance, the observed oscillations and the beating effect, see [www.acs.bg](http://www.acs.bg) . Look in the link “student life”.

$$\gamma = \frac{I \cdot V \cdot T_e}{\varphi \cdot J_s \cdot \Lambda}$$

The major factor calculated is the gyromagnetic ratio of the suspended body in our experiment.

## Parameters

**I** – vector of magnetization, its table value is  
**I=1,59155.106 A/m**

**V** – volume of the sample (only ferromagnetic part)

$$V = \pi \cdot r^2 \cdot h = 0,9349 \cdot 10^{-6} \text{ m}^3$$

$$\gamma = \frac{I \cdot V \cdot T_e}{\varphi \cdot J_s \cdot \Lambda}$$

- r – radius of the cylinder r=0.002 m
- h – length of the cylinder h=0.0744 m

**φ** – angle of declination, experimentally found:

$$\varphi \approx 180^\circ = 3,14 \text{ rad}$$

- **T<sub>e</sub>** – experimental time for measuring the angle of declination

$$T_e = \frac{1}{f} = 1,43 \text{ s}$$

$$\gamma = \frac{I \cdot V \cdot T_e}{\varphi \cdot J_s \cdot \Lambda}$$

- f – frequency of the
- torsion balance f=0.07Hz

**J<sub>s</sub>** – moment of inertia of the sample

- for cylinder:

$$J_s = \frac{1}{2} \cdot m_s \cdot r_s^2$$

for our sample:

$$J_s = \frac{1}{2} \cdot m_b \cdot r_b^2 + \frac{1}{2} \cdot m_f \cdot r_f^2 = 1,7883 \cdot 10^{-8} \text{ kg} \cdot \text{m}^2$$

$$\gamma = \frac{I \cdot V \cdot T_e}{\varphi \cdot J_s \cdot \Lambda}$$

m<sub>b</sub> and r<sub>b</sub> – mass and radius of the brass parts  
m<sub>b</sub>=0,003359 kg and r<sub>b</sub>=0,0015 m  
m<sub>f</sub> and r<sub>f</sub> – mass and radius of the ferromagnetic part  
m<sub>f</sub>=0,007052 kg and r<sub>f</sub>=0,002 m

**Λ** – decrement of decrease

$$\Lambda = \ln \frac{\varphi_n}{\varphi_{n+1}} = \ln \frac{180}{179,5} = 0,0028$$

$$\gamma = \frac{I \cdot V \cdot T_e}{\varphi \cdot J_s \cdot \Lambda}$$

$\varphi_n$  and  $\varphi_{n+1}$  are two consecutive angles of declination. The difference is about 0.5 degrees

## Final Calculation

$$\gamma = \frac{I \cdot V \cdot T_e}{\varphi \cdot J_s \cdot \Lambda} = \frac{1,59155 \cdot 10^6 \cdot 0,9349 \cdot 10^{-6} \cdot 1,43}{3,14 \cdot 1,7883 \cdot 10^{-8} \cdot 0,0028} = 0,1353298 \cdot 10^{11}$$

This will be true if :

$$g \approx 0,15$$

Modern experiments performed with precision much higher than that of Einstein and de Haas give a value of about 2 for the g factor. Our result differs substantially from it.

## Sources of error

- The torsion balance is not centered
- Brass part – Barnett's effect
- Earth's magnetic field
- Noises and vibrations
- Domains that do not get remagnetized
- Eddie currents

## Conclusions

- Einstein – de Haas experiment was successfully demonstrated
- The observed magneto-mechanical effect was predicted and explained using quantum mechanics concepts

## References

- Albert Einstein – Selected Scientific Works, "Nauka", Moscow 1966
- Experimenteller Nachweis der Amperschen Molekularströme,
- Naturwissenschaft, 1915,3,237-238
- Electricity, S.G.Kalashnikov, "Nauka", Moscow, 1977

## PROBLEM № 14: EINSTEIN–DE HAAS EXPERIMENT

### 7.3. SOLUTION OF BULGARIA (SHUMEN)

#### Problem № 14: Einstein–de Haas Experiment

*SVilen Rusev, Todor Bogdanov, Ivan Ivanov – Maths&Science School, Shumen  
Dimitar Ribarov – University of Shumen, Bulgaria*

#### The problem

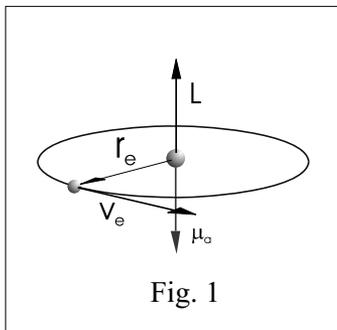
*When you apply a vertical magnetic field to a metallic cylinder suspended by a string, it begins to rotate. Study this phenomenon.*

#### Theoretical part

The experiment, done by Einstein and de Haas is connected with determination of the ratio between the magnetic moment of the electron and its angular momentum during its motion around the nucleus.

According to the known by that time the theory, the magnetization of a given magnetic material is a result of the orientation of the electrons' orbit and the summarizing of all atomic magnetic moments in the external magnetic field.

Fig.1 presents the classical understanding for the motion of the electrons around the nucleus in circular orbit.



For the ratio

$$\gamma_c = \frac{\mu_a}{L},$$

where  $\mu_e$  is the magnetic moment of electrons and  $L$  is the angular momentum we can obtain:

$$\mu_a = \frac{e \cdot \omega \cdot S}{2\pi} \quad \text{and} \quad L = m_e v_e r_e$$

$$\gamma_c = \frac{1}{2} \frac{e}{m_e}.$$

Einstein and de Haas realize experiment in which ferromagnetic cylinder is suspended to an elastic thread into homogeneous magnetic field in parallel to its magnetic field lines.

When the magnetic field is applied, the atoms would be orientated in the field and according to the law of the conservation of the angular momentum, the cylinder to rotate, twisting the thread.

Magnetization of the cylinder

$$J_m = \frac{P_m}{\Delta V} = \frac{N\mu_a}{\Delta V} = \chi H$$

where  $\chi_m$  is the magnetic susceptibility of the cylinder,  $\Delta V$  - the value of the cylinder, and  $H$  - the intensity of the magnetic field in which it is put.

The angular momentum is determined by the torsion moment of the thread  $D$ , the inertial moment of the cylinder  $J$  and the angle  $H$ , to which the thread is twisted after the applying of the field

$$L = \varphi \sqrt{J.D} .$$

Because of the small angle of twisting of the thread in the experiment of Einstein and de Haas, the mechanical resonance has been used and the obtained result has been two times more than expected one. Consequently the magnetization is due not only to the orientation of the electrons orbits but also and to the own magnetic moments of the electrons.

$$\gamma_q = \frac{e}{m_e}$$

The ratio: 
$$g = \frac{\gamma_q}{\gamma_c} = 2$$

is called gyro-magnetic ratio and in ideal case is equal to 2.

## Experiment

In order to obtain the value 2 for the gyro-magnetic ratio it is needed all atoms of the ferromagnetic cylinder to be orientated into the magnetic field. Even to the intensities of the saturation of the materials, by which the cylinder is made only a part of the atoms would be orientated (interaction between domains, heat fluctuations, etc.)



Fig.2

In our experiment we have used the steel with low content of carbon, which have relative magnetic permeability

$$\mu = 0.988 \cdot 10^4 \text{ T. m/A.}$$

In order we to be in the area of the saturation of the ferromagnetic material we have used solenoid, creating along its axis the intensity of the magnetic field:

$$H = 3880 \frac{A}{m}$$

In order to be realized the condition for the homogeneity of the field, in which the cylinder is hung, the length of the solenoid is  $l = 510 \text{ mm}$  and its diameter is  $D = 65 \text{ mm}$  (Fig.2).

The experiment is divided in two parts:

**The first part** – qualitative part of the experiment. Establishment of the rotation of the cylinder in the homogeneous magnetic field and observing the effect of Einstein-de Haas.

**The second part** – quantitative measurement of the gyro-magnetic ratio.

## Carrying out of the first part of the experiment

For the implementation of this part of the experiment we have used a cylinder, made from the same ferromagnetic material, with a diameter  $D = 14\text{mm}$  and a height  $h = 9\text{mm}$ , fixed on the bottom of hollow semi-sphere, leaved freely to float in a little vessel filled with water- a float (Fig. 3 and Fig. 3a).



Fig. 3

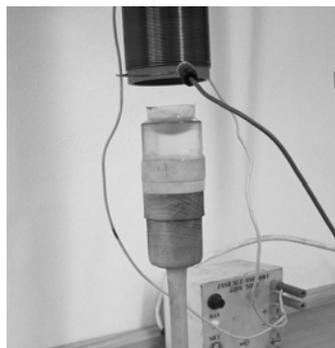


Fig. 3a

Wetting surface of the semi-sphere and not wetting material of the vessel gives a possibility of the obtaining of stable equilibrium of the semi-sphere in the middle of the vessel with the water. Because of the extremely little resistance of the liquid, the appearance of the moment of the force even with very little value would rotate the float.

When the magnetic field is applied, when the vessel with the float is inside of the solenoid, the orientation of the magnetic moments starts. In consequence of the low of the conservation of the angular momentum the ferromagnetic cylinder together with the semisphere rotate.

After the ending of the process of the orientation of the atomic moments the force of internal friction stop rotation of the cylinder. The existence of the section of the free stopping of the rotation is a proof of that the effect of Einstein-de Haas is observed. If such section missed it would mean that the rotation is provoked by magnetic forces, creating by the non-homogeneity of the field.

In order to specify the existence of such section two dependences are compared:

$\omega_L = f(t)$  - for the free stopping float and

$\omega_E = g(t)$  - for the float in magnetic field.

With a digital camera the float have been photographed when it stops freely and in the magnetic field of the solenoid (Fig. 4).

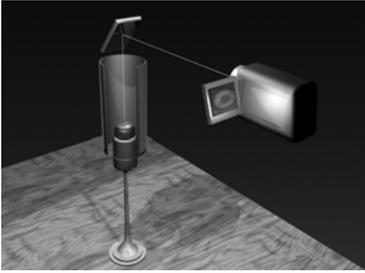


Fig.4

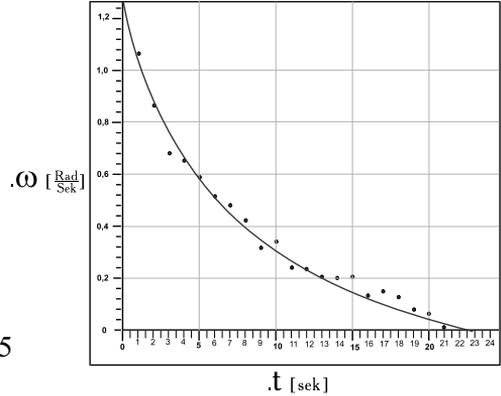


Fig.5

By the analysis of the slides the angle velocities have been determined at different time moments. The data are graphically presented on Fig. 5 for the freely stopped float.

On Fig.6 are presented the data for the float in the magnetic field.

On the right part above is shown a slide of the photographed float inside the solenoid. Along the diameter of the float is put a marker for the determination of the angle of rotation.

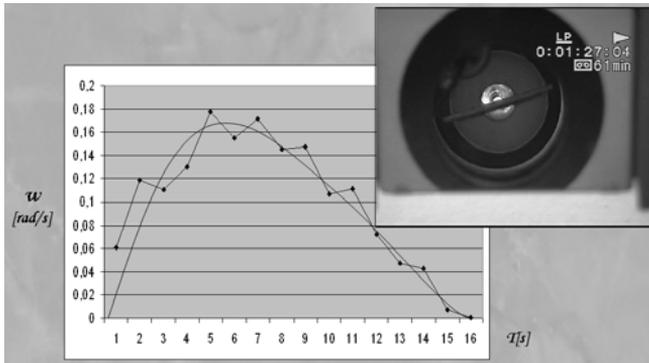
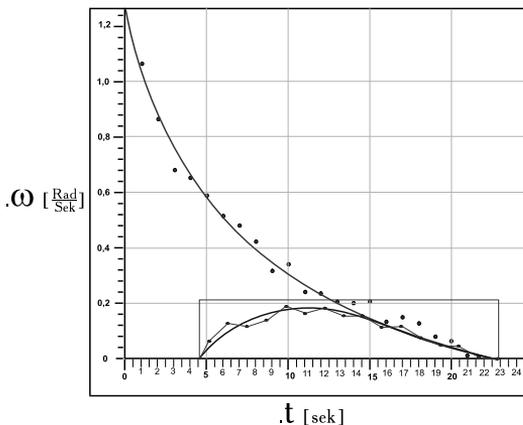


Fig6



On Fig7.the two graphs are united in order to be seen the coincidence of the two curves in the passive section (15s-22s) of the stopping of the float. It proves the effect of Einstein- de Haas.

Fig. 7

## Realization of the second part of the experiment

For the quantitative determination of the gyro- magnetic ratio we have carried out the experiment of Einstein- de Haas. By this reason we have used the sample of ferromagnetic material with fixed to it a mirror (Fig. 8).

- 1- a sample of ferromagnetic material.
- 2- holder of the mirror by diamagnetic.
- 3- mirror.

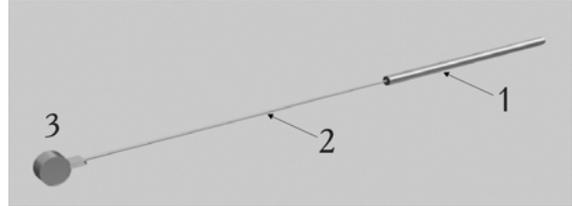


Fig.8

The sample is hung into the solenoid (3), in such way that the mirror (2) is outside it – Fig.9. The beam of the laser (1), reflected by the mirror (2) is directed to the screen (6).

The angle of rotation can be determined by direct measurement, but not by the resonance. It decreases the mistake of the final result.

The sample is hung to the thin bronze thread. In order to decrease the horizontal vibrations is used a thread (4), in the lower part of the sample, which is constantly strained (5).

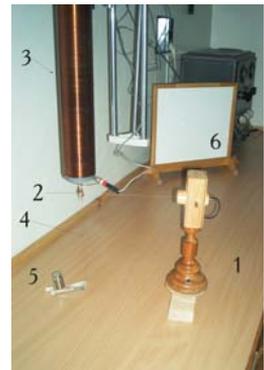


Fig. 9

## Data from the experiment

### 1. Parameters of the setting and the sample

Sample- iron  $d = 5 \text{ mm}, L = 113 \text{ mm}$

Holder of the mirror –copper  $d = 1.2 \text{ mm}, L = 220 \text{ mm}$

Mirror- glass  $d = 10 \text{ mm}, L = 6 \text{ mm}$

Current through the solenoid –  $5A$

Intensity along the axis of the solenoid –  $H = 3880 \frac{A}{m}$

Magnetic permeability of the sample –  $\mu = 0.998 \cdot 10^4$

### 2. Data from measurement of the angle of deviation

The distance between the screen and the mirror-  $l = 547 \text{ cm}$

Measurement	1	2	3	4	5	6
Deviation of the light mark [ cm]	39.5	37	38	38	37.5	38.5
Angle of displacement of the light beam $\varphi_L$ [rad]	0,0722	0,0676	0,0694	0.0694	0,0685	0,0703

Average value of the displacement of the light beam-  $\varphi_L = 69.6 \cdot 10^{-3} \text{ [rad]}$

Angle of rotation of the sample-  $\varphi = \frac{\varphi_L}{2} = 34.78 \cdot 10^{-3} \text{ [rad]}$

## Result

Angular momentum of the sample -  $L = 2.3 \cdot 10^{-9} \frac{\text{kg} \cdot \text{m}^2}{\text{s}}$

Magnetic moment of the sample –  $P_m = 85.8 \text{ A} \cdot \text{m}^2$

$$g = \frac{e/m}{P_m / L} = 4,7$$

## References:

Савельев.И. ; Курс общей физики II часть, Мир, 1975 г.

Персел.Е. ; Берклиевский курс физики, Наука, 1972г.

Giancoli.D.; General Physics, Prentice-Hill, Inc, 1984.

## PROBLEM № 14: EINSTEIN–DE HAAS EXPERIMENT

### 7.4. SOLUTION OF CZECH REPUBLIC

#### Problem № 14: Einstein–de Haas experiment

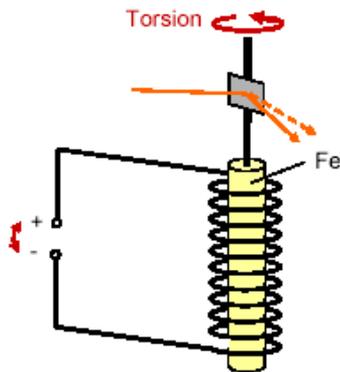
Janečka, A. – Kudela, J. – Lejnar, O. – Roženková, K. – Solný, P. – Kluiber, Z.

#### The problem

*When you apply a vertical magnetic field to a metallic cylinder suspended by a string, it begins to rotate. Study this phenomenon.*

Einstein–de Haas experiment was performed in 1915 with an iron cylinder and should clarify the cause of magnetism in ferromagnetic materials, results of their experiment were more than surprising, as the result was two times bigger than they'd expected. This 'inaccuracy' have been clarified several decades later.

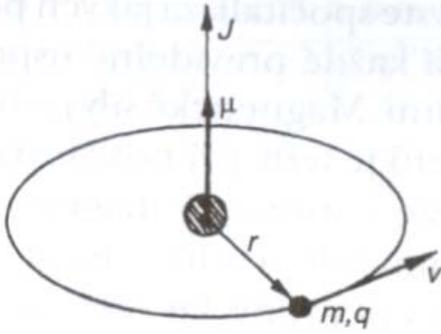
As the ferromagnetic material they used an iron cylinder hung on a thin thread. Around the cylinder is wound a coil. At the beginning, there are no moments of outer forces acting on the cylinder. Once a electric current goes through the coil, a magnetic field  $\mathbf{B}$  is formed inside the coil. Due to this magnetic field, angular momentum of every single atom inside changes its direction so the total angular momentum is now not equal to zero. To preserve the total angular momentum (and to equal to the primal angular momentum, which was equal to zero), the cylinder starts rotating around its axis. If no thread was used, the cylinder would rotate as long as the magnetic field would be applied. Due to the torsion of the thread, a torsional momentum is formed (depending on the torsion modulus of the thread). The torsional momentum makes the cylinder stops rotating and makes it start rotating in a opposite direction (and thread gets straight). The thread will twist and straighten as the cylinder rotates with harmonic motion around its equilibrium like a torsional pendulum.



A mechanic analogy also exists: one stands on a rotatable pad and holds a horizontally revolving wheel (balance-wheel). When he turns the balance-wheel upside down, he starts revolving himself on the pad to preserve the total angular momentum.

First statement of classical mechanics says that if an electron moves along a circle (e.g. circulates around core due to centripetal force), its magnetic momentum and angular momentum are proportional.

Angular momentum  $\mathbf{J}$  is defined as  $\mathbf{J} = \mathbf{m}\mathbf{r} \times \mathbf{v}$ , where  $m$  is mass of electron;  $r$  is the distance from electron to the core and  $\mathbf{v}$  is the velocity of



electron. Direction of angular momentum of the electron is perpendicular on the plane of trajectory. Radius  $r$  is perpendicular on the vector of velocity  $\mathbf{v}$ , so the equation  $\mathbf{J} = \mathbf{m}r \times \mathbf{v}$  can be written as:

$$J = mvr$$

Magnetic momentum  $\mu$  of the electron circulating around the core is equal to  $\boldsymbol{\mu} = I \cdot \mathbf{A}$ , where  $\mathbf{I}$  is the electric current and  $\mathbf{A}$  the area inside the trajectory of the electron. Electric current is defined as charge which goes in time through any place at the trajectory, i.e. charge  $q$  times frequency of circular motion  $I = q \cdot f$ . Frequency is velocity divided by the length of trajectory  $f = \frac{v}{2\pi \cdot r}$ , so:

$$I = \frac{qv}{2\pi \cdot r}$$

The area is  $A = \pi \cdot r^2$ , magnetic momentum is then:

$$\mu = \frac{qvr}{2}$$

Magnetic momentum is perpendicular on the plane of trajectory, just like the angular momentum, their direction is the same, then:

$$\frac{\mu}{J} = \frac{\frac{qvr}{2}}{mvr} = \frac{q}{2m} \Rightarrow \boldsymbol{\mu} = \frac{q}{2m} \mathbf{J} \quad (\text{Orbital movement})$$

Ratio of magnetic momentum and angular momentum is called the gyromagnetic ratio. This ratio depends neither on velocity nor on the radius of trajectory. Magnetic momentum  $\mu$  of every particle on the circular track is  $(q/2m)$ -multiple of angular momentum  $J$ . The electron's charge is negative (we'll call it  $-q_e$ ):

$$\boldsymbol{\mu} = -\frac{q_e}{2m} \mathbf{J} \quad (\text{Orbital movement of the electron})$$

This equation is valid also in the quantum mechanics. But we know that the orbital movement isn't the only causer of magnetism. Electron has also a spin (it's

like Earth and its rotation around its own axis) and like a consequence, there is also a spin angular momentum and related magnetic momentum. From the quantum mechanics, we know that, the spin angular momentum of an electron is equal to:

$$J_{Spin} = \frac{n \cdot h}{4 \cdot \pi}$$

And spin angular momentum is:

$$\mu_{Spin} = \frac{n \cdot e \cdot h}{4 \cdot \pi \cdot m}$$

After substitution into the gyromagnetic ration:

$$\frac{\mu_{Spin}}{J_{Spin}} = \frac{4 \cdot \pi \cdot n \cdot e \cdot h}{4 \cdot \pi \cdot n \cdot m \cdot h} = \frac{e}{m}$$

Then:

$$\mu = -\frac{q_e}{m} J \text{ (electron spin)}$$

Generally, in every atom exist many of electrons and by compounding their spin and orbital movements the total angular momentum and the total magnetic momentum is formed. In spite of lack of classical mechanics explanation, in quantum mechanics is generally valid a statement that the direction of the magnetic momentum of an isolated atom is exactly opposite than the direction of angular momentum. Their ration don't have to be  $-q_e/m$  or  $-q_e/2m$ , though, but can be somewhere in between these values because the magnetic momentum is compound of orbital and spin portions. We can write their ration as:

$$\mu = -g \left( \frac{q_e}{2m} \right) \mathbf{J}$$

Where g, so-called Landé Faktor, is a factor characterizing the state of atom. It's either equal to one for solely orbital movement, to two for solely spin movement and to any other number between one and two for a complicated system like an atom is. Landé factor is a non-dimensional constant. From the

equation  $\mu = -g \left( \frac{q_e}{2m} \right) \mathbf{J}$  results that the magnetic momentum is parallel with the angular momentum, the size can be different though, depending on the Landé factor.

Magnetic momentum of an electron is in equation with the magnetization of the cylinder:

$$\mu = M \cdot V / N$$

Where M is magnetization, V is volume of the cylinder and N is the number of particles inside the cylinder.

Total angular momentum of the cylinder is:

$$J_{celk} = \sum_{i=1}^n m_i v_i r_i$$

$$J_{celk} = N \cdot J$$

After substitution  $\mu$  and J:

$$M \cdot V = -g \left( \frac{q_e}{2m} \right) J_{celk}$$

In Einstein – de Haas experiment, magnetization angular momentum change within time because the system oscillates. Then, the time change of these quantities is observed:

$$\dot{M} \cdot V = -g \left( \frac{q_e}{2m} \right) D \wedge D = \dot{J}_{celk}$$

$$\Leftrightarrow \underline{\underline{g = -2 \frac{m}{e} \cdot V \cdot \frac{\dot{M}(t)}{D(t)}}}$$

Experiment of Einstein and de Haas showed g-factor 2. So ferromagnetism is based on spin of the electron and not on the orbital angular momentum.

In Einstein – de Haas experiment, the cylinder behaves like a torsional pendulum, whose period of oscillation is:

$$T = \sqrt{\frac{2 \cdot \pi \cdot l_v \cdot m_T \cdot r_T^2}{r_v^4 \cdot \mu_v}}$$

Where  $l_v$  is the length of thread,  $m_T$  is the weight of cylinder,  $r_T$  is the radius of cylinder,  $r_v$  is the radius of thread and  $\mu_v$  is torsion modulus of thread.

From the equation is obvious that period doesn't depend on current, voltage or on the used coil. If coil creates a bigger magnetic field, cylinder will revolve faster, on the other hand, it will deflect more, so the period won't change at all.

## PROBLEM № 14: EINSTEIN–DE HAAS EXPERIMENT

### 7.5. SOLUTION OF POLAND

#### Problem № 14: Einstein–de Haas Experiment

*Team members: Tomasz Bobiński, Pawel Debski (Captain), Maciej Lisicki, Krzysztof Wojtowicz, Maciej Zielenkiewicz*

*Team leaders: MSc. Stanislaw Lipinski, Karolina Kocko, Michal Oszmaniec  
XIV Stanislaw Staszic Secondary School, Warsaw,*

#### The problem

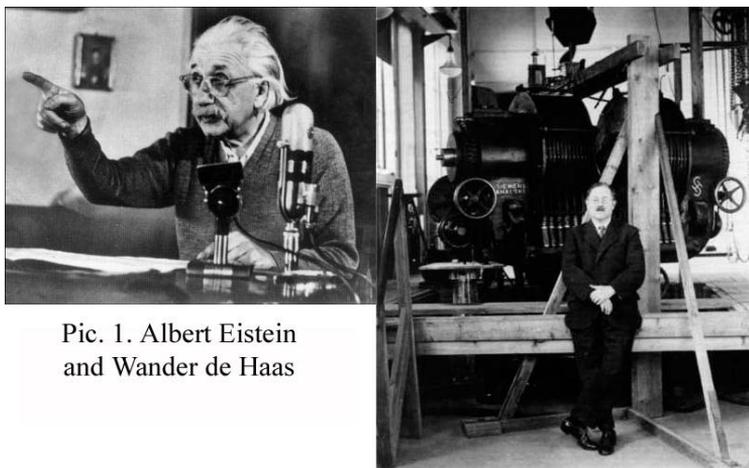
***When you apply a vertical magnetic field to a metallic cylinder suspended by a string, it begins to rotate. Study this phenomenon.***

The described phenomenon was firstly investigated in 1915, when Albert Einstein, searching for a proof of existence of Ampere molecular currents, conducted along with Wander de Haas, an experiment, which would confirm his theory.

Through the late years of the XIX cent. the scientific world was searching for the reason of magnetic properties of certain materials. In 1908 Richardson was the first one to mention that orbital motion of electrons is responsible for such behaviour of magnetic materials in presence of external magnetic field.

The described phenomenon was firstly investigated in 1915, when Albert Einstein (pic.1) , searching for a proof of existence of Ampere molecular currents, conducted along with Wander de Haas, an experiment, which would confirm this theory. The results of the experiment confirmed the theoretical predictions very well. Einstein published their work and did not put more attention on the analysis of the effect.

However, in later years many experimentators (including W.J. de Haas) tried to repeat this experiment, but their results were completely different. This uncertainty of the ultimate, correct result caused a discussion on Einstein's work and in further research,



Pic. 1. Albert Eistein and Wander de Haas

lead to discovery of the electron spin as partially responsible for magnetic behaviour of materials.

## Theoretical explanation

At first, let's try to analyze a simple mechanical situation, in which we shall observe the basic effect which is in fact responsible also for Einstein – de Haas effect.

Let's consider a man sitting on a revolving chair, which is initially not rotating. A man holds a rotating bicycle wheel is such way, that its angular momentum vector is directed vertically (pic. 2). What happens, when we change the direction of the angular momentum vector?



Pic. 2

Obviously, as a consequence of the angular momentum conservation principle, the angular momentum vector of the whole system is constant, so the revolving chair starts to rotate if we change the angular momentum vector direction (pic. 3).



Pic. 3

We shall now consider situation, in which we can observe the Einstein – de Haas effect. At first, let's analyse orbital motion of an electron (pic. 4)

Orbiting electron can be considered as a small current in a loop. Such a current has a magnetic moment:

$$\mu_{orb} = IS = ve \cdot \pi r^2 = \frac{ev\pi r^2}{2\pi r} = \frac{evr}{2}$$

Magnetic moment is a vector, and its direction can be noted as:

$$\vec{\mu}_{orb} = \frac{e(\vec{v} \times \vec{r})}{2}$$

An orbiting electron has also an angular momentum vector, which can be noted as:

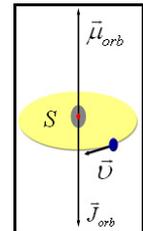
$$\vec{J}_{orb} = m(\vec{r} \times \vec{v})$$

So the magnetic moment vector, which of course changes its direction in presence of external magnetic field, is strictly connected with the angular momentum vector of the atom. The relation is as follows:

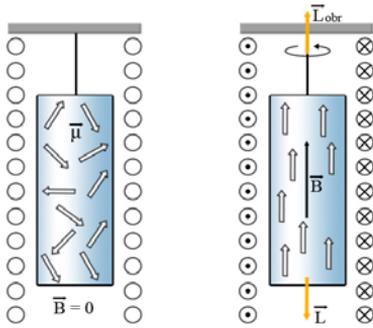
$$\vec{\mu}_{orb} = -\frac{e}{2m} \vec{J}_{orb}$$

Basing on this relation, we can qualitatively explain the occurring effect. Let's describe it using a drawing (pic.5 and 6).

Initially, because of thermal motion and absence of external magnetic fields,



Pic. 4 An orbiting electron (according to Bohr model)



Pic. 5.6. Magnetic moment vectors in a probe.

the magnetic moment vectors of the atoms are randomly directed in the whole probe, so the resultant magnetic moment, as well as the resultant angular momentum, equals 0. When we apply a vertical magnetic field, the magnetic moment vectors tend to line up with the magnetic field lines, so they change their direction, so the resultant magnetic moment vector is directed vertically. We shall note, that this change is also connected with changes in angular momentum vectors,

which then change their direction. The angular momentum conservation principle demands the resultant angular momentum vector to be 0, so the probe gains angular momentum directed oppositely to the resultant angular momentum vector of the atoms. This angular momentum causes the rotation of the probe.

We shall remember, that this is only a qualitative explanation. After an electron spin has been discovered, an important lacks in the experimental results could be explained. It occurred, that an electron has its own, internal angular momentum and magnetic moment, but the relation between those vectors is as follows:

$$\vec{\mu}_s = -\frac{e}{m} \vec{J}_s$$

We shall note that the proportionality factor between those vectors is 2 times bigger than when considering an orbiting electron.

This discovery revealed new facts connected with theoretical explanation of the behaviour of magnetic materials. It caused a discussion about the exact reason of the magnetic behaviour. Caused questions, like: what is more important in this phenomenon – orbital motion or electron spin? To solve this problem, precise quantitative experiments had to be conducted.

The idea was to measure the proportionality constant, called the g-factor or Lande factor:

$$\vec{\mu}_{metal} = -g \left( \frac{e}{2m} \right) \vec{J}_{metal}$$

If the g-factor value was closer to 1, it would confirm the hypothesis, that crucial for this effect are the orbiting electrons, if it was closer to 2, it would prove, that electron spin has the key role for this phenomenon.

## Experimental Approach

We wanted to use the same method, which Einstein and de Haas described in their work in 1915. Their idea was to apply an external field and measure the deflection of an iron cylinder. Their setup consisted of coils, generating the magnetic field and a cylinder suspended on a glass fibre, with a mirror attached to it (pic. 6 and 7).

The cylinder can be treated as a torsional pendulum – using a resonance phenomenon, we can amplify the oscillations. Therefore, the frequency of current in the coils, generating the magnetic field equals the natural frequency of oscillations of the cylinder, the oscillations will be amplified and easier to observe.

To measure the deflection we have used a laser and a screen.

To calculate the value of the g-factor, we have tried to use Einstein’s method, but to make the measurements easier, we have modified it, as did the Berlin Technical University students. The formula for calculating g, given with no derivation, is as follows:

$$g = \frac{m_e}{q} \cdot \frac{U_{ind} \cdot V}{N_2 \cdot \mu_0 \cdot A \cdot X \cdot \omega \cdot \beta \cdot \alpha_{max}}$$

$m_e$  – electron mass

$q$  – electron charge

$U_{ind}$  – inducted voltage

$\mu_0$  – magnetic permeability of vacuum

$\omega$  – resonant angular frequency

$X$  – moment of inertia

$\beta$  – damping constant

So we had to measure various parameters of the whole system.

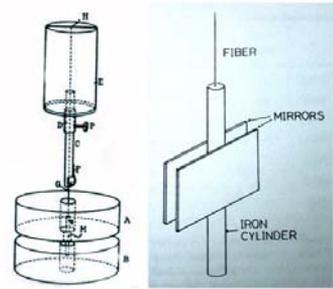
We have constructed our first experimental setup (pic. 11), hoping to measure necessary parameters. As it occurred, it was very imprecise and allowed us only to estimate g as about 0.002.

These wooden constructions, which can be seen in this photography are Helmholtz coils, used to compensate the nonvertical components of earth magnetic field.

We have decided to build much more precise experimental setup, which would allow us to conduct measurements (pic.12).

This time the setup was very stable, symmetric, with special devices used to center the cylinder inside the coil and regulate the tension of the suspension (pic.13).

Our cylinder was long (150 mm) and very thin (2.4 mm), to avoid any nonuniformities of the magnetic field inside the coil. It was made of ARMCO



Pic. 7.8. Einstein's original scheme of experimental setup

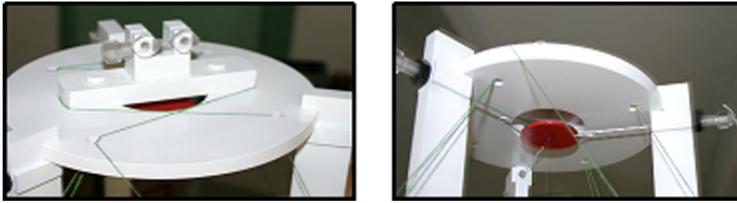


Pic. 11 First experimental setup.

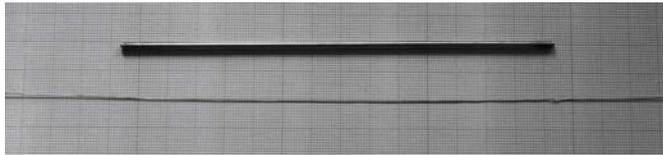


Pic. 12 Second experimental setup.

(contained 99% of iron) We have suspended it on a CuNi wire (with diameter of 0.2 mm) (pic. 14).

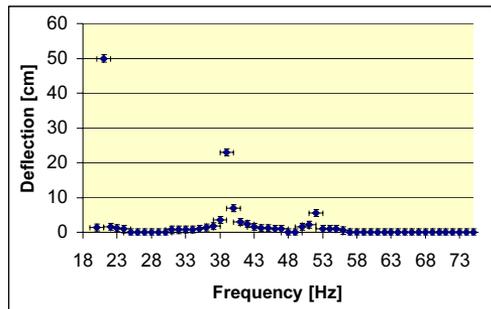
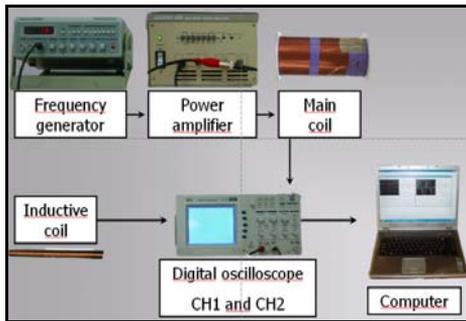


Pic. 13 String tension controller and centering device



Pic. 14. CuNi wire and iron cylinder

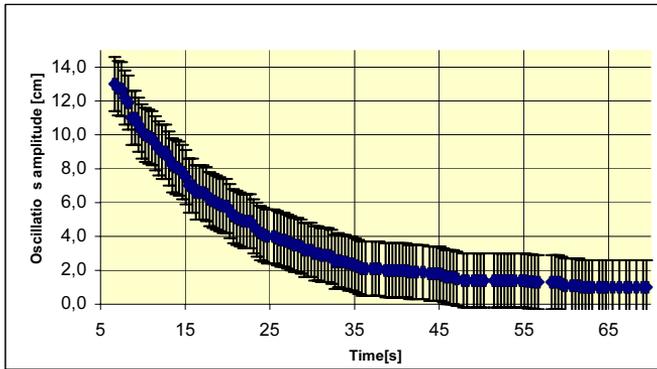
To generate the external magnetic field, we have used a frequency generator, connected to a power amplifier and a digital oscilloscope. The signals generated by presence of an iron cylinder inside the coil were gathered by an inductive coil, and this signal was also analyzed using a computer.



First of our measurements was the estimation of the resonant frequency of the cylinder. Obtained results are presented in the above graph:

We have measured the deflection on the screen while changing the external magnetic field change frequency (our generator). Three harmonics can be clearly visible. We therefore assume the lowest and strongest one, at about 19 Hz, to be the resonant frequency of the cylinder.

Another important parameter was the damping constant. We could measure it by turning off the generator at the resonant frequency and measuring the deflection decrease in time. Then, using GNUPlot, we have fitted a curve, showing the expected dependence, as it is shown below:



Pic. 15 Oscilloscope signals

The expected dependence was:

$$f(t) = Ae^{-\beta t}$$

After estimating curve-fitting parameters, we could estimate the damping constant as:

$$\beta = 0.060 \pm 0.005$$

The induced voltage was measured using the digital oscilloscope, as it can be seen in the illustration (pic. 15).

The two signals in this screen are signals from main coil (sinusoidal) and from the inductive coil (in this signals, peaks mark the demagnetisation of the cylinder, as the current in main coil changes its direction).

$$\begin{aligned} m &= 9.1 \cdot 10^{-31} \text{ kg} \\ q &= 1.6 \cdot 10^{-19} \text{ C} \\ \mu_0 &= 1.257 \cdot 10^{-6} \text{ T}^2 \text{ m}^3 \text{ J}^{-1} \\ V &= 6.79 \cdot 10^{-7} \text{ m}^3 \\ U_{ind} &= 2.2 \text{ V} \\ N_2 &= 600 \\ A &= 4.52 \cdot 10^{-6} \text{ m}^2 \\ X &= 3.744 \cdot 10^{-9} \text{ kg} \cdot \text{m}^2 \\ \omega &= 131.95 \text{ Hz} \\ \beta &= 0.06 \\ \alpha_{max} &= 5.22 \cdot 10^{-2} \text{ rad} \end{aligned}$$

$$\begin{aligned} \frac{\Delta V}{V} &\approx 16.5\% \\ \frac{\Delta X}{X} &\approx 16\% \\ \frac{\Delta \omega}{\omega} = \frac{\Delta f}{f} &\approx 5\% \\ \frac{\Delta \alpha_{max}}{\alpha_{max}} &\approx 14\% \\ \frac{\Delta \beta}{\beta} &\approx 8.4\% \\ \frac{\Delta U}{U} &\approx 4.5\% \end{aligned}$$

After conducting necessary measurements, we were ready to estimate the value of  $g$  and analyse possible sources of error in our measurements.

The total error of our measurements was therefore:

$$\sqrt{\left(\frac{\Delta V}{V}\right)^2 + \left(\frac{\Delta X}{X}\right)^2 + \left(\frac{\Delta \omega}{\omega}\right)^2 + \left(\frac{\Delta \alpha_{max}}{\alpha_{max}}\right)^2 + \left(\frac{\Delta \beta}{\beta}\right)^2 + \left(\frac{\Delta U}{U}\right)^2} = 0.296$$

The calculated value of g:

$$g = \frac{m_e V U_{ind}}{q N \mu_0 A X \alpha_{max} \omega \beta}$$

$$g = 1.61 \pm 0.48$$

### **Therefore we can draw some conclusions on this effect:**

1. Einstein – de Haas effect is connected with atoms and electron having angular momentum and magnetic moment. In fact, professional laboratory measurements estimate g as about 1.8, so it proves that electron spin and magnetic moments have bigger influence on the behaviour of magnetic materials in presence of external field.

2. Change in direction of magnetic moments vectors causes change in angular momentum of the probe; rotation is the effect of the angular momentum conservation principle.

3. This effect has a wide historical background and had many implications on the concept of magnetism of matter (from molecular currents to spin of the electron).

4. Although quantitative analysis is very hard, but it is possible to do it in school conditions. In our case, we've obtained value, which is completely sufficient, even surprisingly close to professionally measured value.

### **Acknowledgement:**

We'd PhD. Piotr Kossacki from Warsaw University for valuable discussion and theoretical help. We'd like to thank Mr Krzysztof Bobiński from Warsaw Univeristy of Technology, for help in experimental setup construction and PhD. Jan Grabski from Warsaw Univeristy of Technology, for advice, theoretical support and providing us with electronic equipment. We'd also like to thank MSc. Anna Mazurkiewicz from Jozef Poniatowski Secondary School for help in search for theoretical materials.

## 8. PROBLEM № 15: OPTICAL TUNNELING

### 8.1. SOLUTION OF BRAZIL

#### Problem № 15: Optical Tunneling

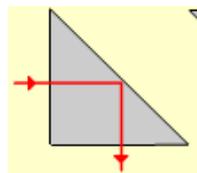
*Daniel Nogueira Meirelles de Souza, São Carlos – SP  
Escola Educativa – Instituto de Educação e Cultura*

#### The problem

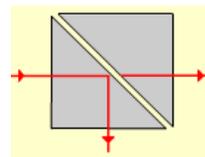
**"Take two glass prisms separated by a small gap. Investigate under what conditions light incident at angles greater than the critical angle is not totally internally reflected."**

#### Introduction

Total internal reflection is a well known optical phenomenon. It occurs when light propagating in a medium of index of refraction  $n$  reaches a separation boundary between this medium and one of index of refraction smaller than  $n$  at an angle of incidence greater than a critical angle  $\theta_c$ . All light is reflected back into the first medium (of greater index of refraction). Total internal reflection can be well visualized if we have a triangular  $90^\circ$  prism (of, for example, glass) and a light source (for example, a laser pen). If we simply make the light enter the prism and reach the separation boundary with air on its hypotenuse at an angle greater than the critical angle, we will easily see total internal reflection. View top picture.



If a second prism or piece of glass is approached to the hypotenuse of the one in which total reflection is taking place, making the two prisms separated by a small gap, an unexpected phenomenon might occur (bottom picture). A normal total internal reflection would be expected, since air is still surrounding the prism. However, if the gap is sufficiently small, part of the light that would suffer total internal reflection is unexpectedly transmitted into the second prism, leaving a smaller amount of light to suffer reflection:

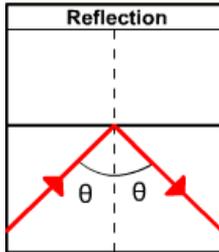


This phenomenon is given the name of Frustrated Total Internal Reflection (which we shall now call FTIR). It was first observed by Isaac Newton, about 300 years ago, and reported in his famous book *Optics*. Newton brought a convex lens close to the region into contact with the reflecting surface of the prism and realized some light started to travel through the lens. Although reported, the phenomenon could not be successfully explained by Newton. In fact, the

phenomenon is not predicted by geometric optics. In this problem, we shall investigate FTIR and the conditions in which it occurs.

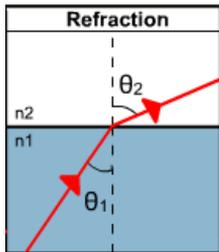
## Theory

**Geometric Optics:** it is important for the understanding of FTIR knowledge of basic concepts in geometric optics.



### Reflection:

Light suffers reflection when it reaches the boundary of a reflecting surface. The angle between the incident ray of light and the direction normal to the reflecting surface is equal to the angle of the reflected ray with this normal. This is the law of reflection.



### Refraction:

Is what happens to a wave when it changes its medium of propagation and consequently its propagation speed. In optics, light suffers refraction when it changes its propagation medium into one of different index of refraction. A medium's index of refraction,  $n$ , is given by:

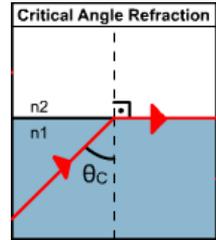
$$n = \frac{c}{v}$$

In which  $c$  is the speed of light in vacuum (aprox.  $3 \cdot 10^8$  m/s) and  $v$  is the speed of propagation in the considered medium. The index of refraction is also referred as optical density. When a light ray suffers refraction, its speed and direction changes. The law of Snell – Descartes relates the sine of the incident and refracted angles and the index of refraction of both mediums:

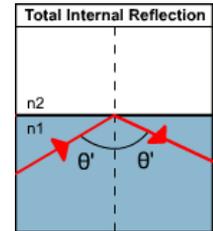
$$\sin \theta_1 \cdot n_1 = \sin \theta_2 \cdot n_2$$

We can conclude from the equation that if light is traveling initially in a medium of smaller index of refraction and refracts into a medium of greater index, the angle of the refracted ray will be smaller than the angle of the incident ray. If light is traveling initially in a medium of greater index of refraction and refracts into a medium of smaller index, the angle of the refracted ray will be bigger. It is important to add that not all incident light is refracted: part of it can be reflected, returning back into the first medium. If we take the case of the initial medium of propagation be of bigger index, we will find that at a certain angle the refracted ray will be perpendicular to the normal direction and therefore parallel to the surface. The incident angle in which this occurs is the *critical angle*. From Snell – Descartes, we find that the sine of the critical angle  $\theta_c$  is given by:

$$\sin \theta_c = \frac{n_2}{n_1}$$



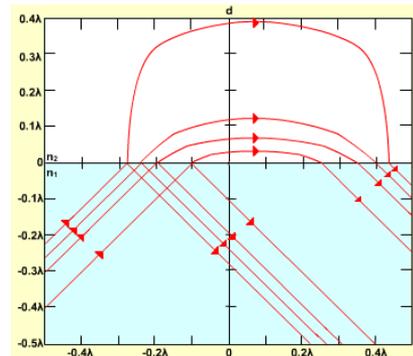
If, in these conditions, the incident angle becomes greater than  $\theta_c$ , total internal reflection will occur, in which all light is reflected and none is refracted:



### Evanescent wave in Total Internal Reflection:

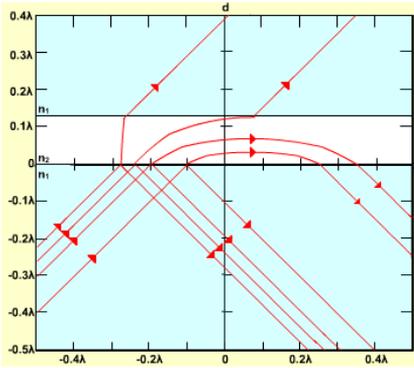
Frustrated Total Internal Reflection could only be properly explained with 19th century Maxwell's electromagnetic theory. A deeper look into total internal reflection was possible. In total internal reflection, we have the penetration of the electromagnetic wave in the region beyond the totally reflecting interface, into the second medium. This penetrating wave is called the **evanescent wave** (see pic.).

This picture was taken from the third reference. It shows total internal reflection of light incident at  $45^\circ$  on the interface, in which  $n_1 = 1,5$  and  $n_2 = 1,0$ . The flow lines are represented, showing that when light is incident in an angle greater than the critical angle, part of it is reflected back into the first medium and part, surprisingly, actually penetrates into the less optically dense medium, creating an existence of electromagnetic energy in the region beyond the interface, which travels according to the flow lines represented.



This is a strange behaviour. The wave that penetrates into the second medium runs along the direction parallel to the interface, and, after a distance of the order of the wavelength  $\lambda$ , returns to the first medium, parallel to the reflected rays. This is what actually happens in total internal reflection. The picture also shows that, increasing the distance  $d$  from the interface, we have a smaller concentration of flow lines, which means the amplitude of the evanescent field drops if the distance from the interface is increased, so that, at some distance, the amplitude would be too small to be considered.

If we were to approach a second piece of glass to the first, at a distance in which the amplitude of the evanescent wave is appreciable, we would have that



some of the electromagnetic energy would enter the second glass in the form of a light wave. This would result in a smaller amount of light returning to the first medium:

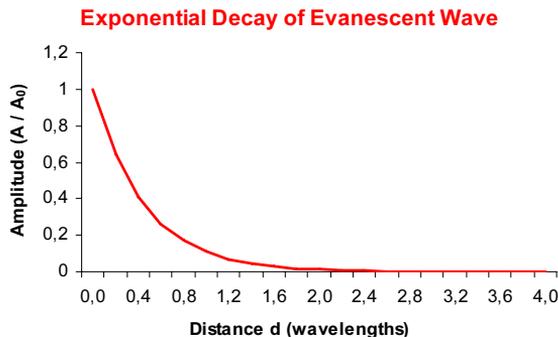
This partially explains frustrated total internal reflection. If the second prism is approached at a distance small enough, it will be able to capture the electromagnetic energy in the evanescent wave. Due to conservation of energy, less light returns to the first prism. If the distance between the prisms is too great,

the amplitude of the evanescent wave will be practically zero and no light would be frustrated. The drop of the amplitude of the evanescent wave with the distance from the interface in the direction normal to this interface is exponential:

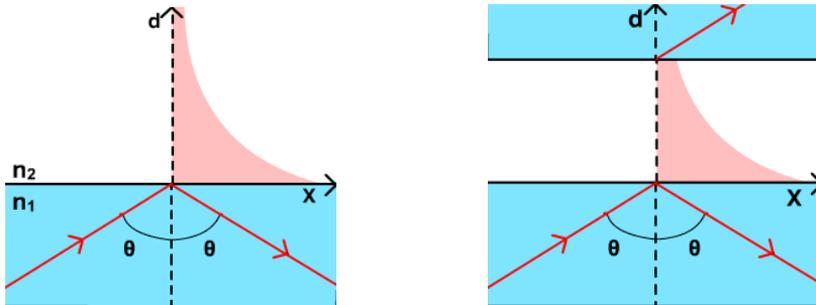
$$A = A_0 e^{-d\alpha}$$

$$\alpha = \frac{2\pi}{\lambda} \sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2 \theta_1 - 1}$$

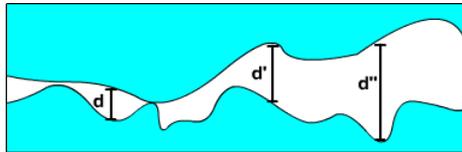
Where  $A_0$  is the amplitude at distance 0,  $d$  is the distance in the direction normal to the interface,  $\lambda$  is the wavelength of the electromagnetic wave and  $\theta_1$  the incidence angle. The deduction of this equation may be viewed by the curious reader in the **appendix**. From the exponential equation, we can observe that the amplitude of the evanescent wave is considerable at distances smaller to or of the order of  $\lambda$ . The graphic below shows how the amplitude of the evanescent wave (in relation to  $A_0$ ) varies with distance from the interface (in units of wavelength) when the incidence angle is  $45^\circ$  and  $n_1$  and  $n_2$  are, respectively, 1,5 and 1:



Below we have another representation for the evanescent wave and FTIR, in which the exponential decay is represented by the reddish curve:



So, in order to obtain the phenomenon, we must place our prism at a distance no much greater than 2 wavelengths. The wavelength of visible light varies from approximately 400 to 800 nanometers. We must then place our prisms at a distance of the order of  $10^{-7}$  m from each other. The best way to try to do this is pressing one prism against the other, or at least putting them into maximum contact possible. This is because most prisms are still irregular in microscopic terms.



So we have great differences in distance between the prisms along their surfaces, if compared to the wavelength of visible light. This makes it impossible to obtain FTIR with prisms which don't have a very good surface quality.

## Experiment

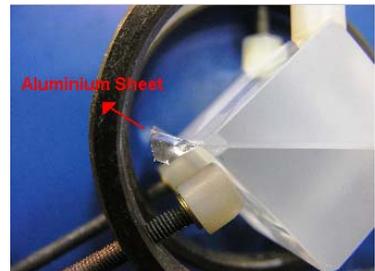
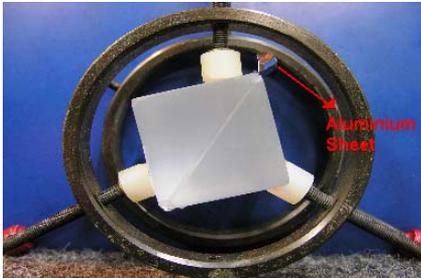
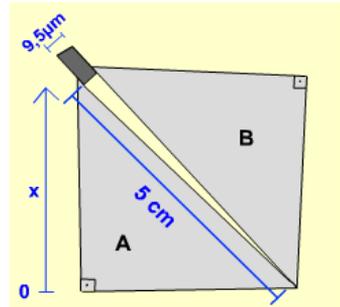
The Objective of the experiment was to verify and measure the dependence of the transmitted light (in frustrated total internal reflection) on the separation air gap  $d$  between the prisms.

### Utilized Materials:

- 2 BK7 45° Prisms ( $n = 1,515$  for 635nm)
- Red laser ( $\lambda = 635$  nm)
- Aluminium Sheet ( $9,5 \pm 0,5$   $\mu\text{m}$  width).
- Photocell
- Voltmeter
- 2 lens
- Micrometer
- 2 Fasteners
- Voltmeter

### Experimental Methods:

With the use of the very thin aluminium sheet, the prisms were arranged so that the air gap (or distance) between them changed uniformly. The prisms used had a very good surface regularity:



2 Fasteners were used to press the prisms against each other in order to assure they were really in contact at one end. The prisms used had a very high optical quality. Along a distance of 5 cm, the separation gap between the prisms changed continuously from 0 to 10µm. It is important to know the linear relationship between the separation gap  $d$  and the position in the direction  $x$  (shown in figures above). The relationship was obtained using basic trigonometry. The major error source is the error of the width of the aluminium sheet:

$$\begin{aligned}d &= \text{Width of air gap} \\d &= x (0,27 \pm 0,01) \cdot 10^{-3} \\d(\mu\text{m}) &= x(\text{mm}) (0,27 \pm 0,01)\end{aligned}$$

### Setup:



**1. Laser ( $\lambda = 635 \text{ nm}$ ):** The laser was placed so that light entered the first prism at an angle of incidence of  $0^\circ$ , without changing direction of propagation. Consequently, the light encountered the hypotenuse of the first prism at a  $45^\circ$  angle of incidence, which is slightly greater than the critical angle between the used glass and air ( $41,3^\circ$ ).

**2. Lens:** One lens was placed in front of the laser to focalize the light, in order to make the beam thin enough to be considered punctual. Another lens was placed after the prisms for the same reason, avoiding the possibility of the beam becoming too large to be detected by the photocell.

**3. Micrometer:** All experimental setup was maintained still, except for the prisms. The prisms were placed on top of the micrometer, which made it possible to move them in the direction of  $x$ , causing light to reach the hypotenuse of the first prism at points of different position in  $x$ , and to measure this movement. Using the relationship between  $x$  and  $d$ , it is possible to calculate the width of the air gap at the point in which light is incident on the hypotenuse of the first prism.

**4. Prisms:** were put on top of the micrometer. The ones used (BK7) had a high optical quality.

**5. Photocell:** Used to capture the transmitted light.

**6. Voltmeter:** Attached to the photocell, this was used to measure the intensity of the transmitted light.

The prisms were moved in the  $x$  direction, and the intensity of the transmitted light beam was measured once each half millimetre moved. The experiment was realized in the dark, so that no external light influenced the measurements.

## Results:

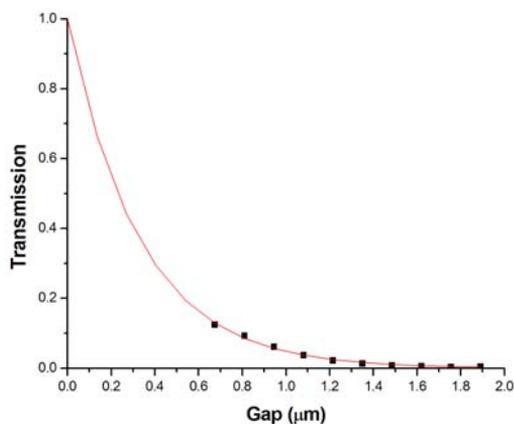
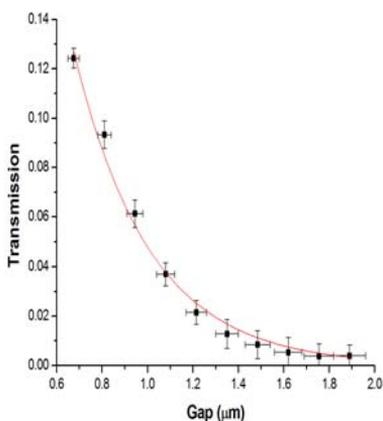
The table below shows the values obtained for each position in  $x$ :

Position in X (mm)	Measure 1 (mV)	Measure 2 (mV)	Measure 3 (mV)	Average (mV)	Error (mV)
2,5	26,3	26,1	27,7	26,7	0,9
3,0	19,6	19,1	21,4	20,0	1,2
3,5	13,0	12,1	14,4	13,2	1,2
4,0	7,8	7,0	9,0	7,9	1,0
4,5	4,0	4,0	5,8	4,6	1,0
5,0	2,0	2,0	4,2	2,7	1,3
5,5	1,1	1,1	3,2	1,8	1,2
6,0	0,6	0,2	2,6	1,1	1,3
6,5	0,4	0,0	2,0	0,8	1,1
7,0	0,3	0,3	1,9	0,8	0,9

Measurements before 2,5 mm were not included due to their uncertainty, since the light incident at these regions was greatly scattered, so the intensity of transmission could not be measured in positions before 2,5 mm. With the data above, it was possible, using the program Origin 6.0, to obtain the equation which best describes the relationship of the voltage accused (I) as a function of the position in x. It is the equation which best adjusts to our experimental data and the error:

$$I = (210 \pm 30)e^{-x/(1,24 \pm 0,07)}$$

where x is given in mm and I in mV. So, according to the equation, the voltage accused at x = 0 mm would be  $210 \pm 30$  mV. Knowing this, the intensity of the transmitted beam was normalized so that this maximum intensity (at x = 0) would be equal to 1. It was possible to plot a graphic of the transmission (which can go from 0 to 1) versus the size of the gap:



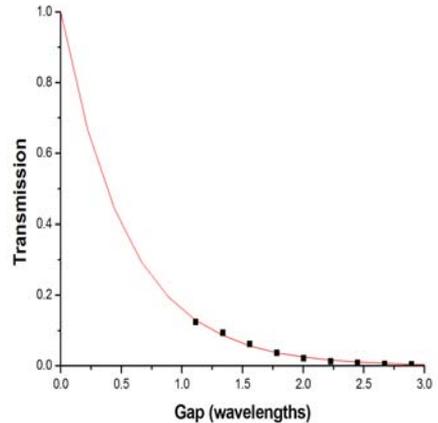
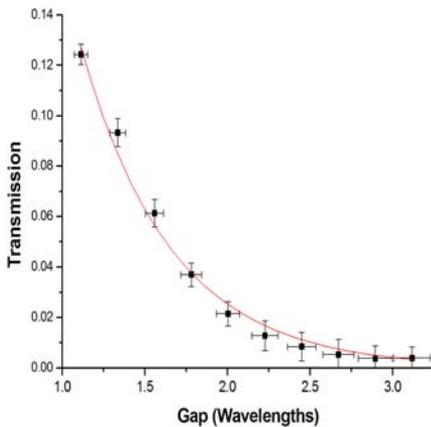
The dots represent the experimental values, and the line represents the exponential equation which best fits these results. The equation that best fits the experimental results is:

$$T = T_0 e^{-d/(0,33 \pm 0,02)}$$

where T stands for transmission and d is the gap in μm. The value  $(0,33 \pm 0,02) \mu\text{m}$  would be our experimental value for  $1/\alpha$  (of the equation at the end of page 5), so the experimental value for  $\alpha$  would be  $(3,0 \pm 0,2) \mu\text{m}^{-1}$ . The expected value for  $\alpha$  under the conditions of the experiment would be  $3,79 \mu\text{m}^{-1}$ , and  $1/\alpha$  would be  $0,26 \mu\text{m}$ . A possible explanation for the small difference between experimental and theoretical  $\alpha$  would be that the width of the aluminium sheet would get smaller because it is being compressed by the two prisms. This is a non-quantifiable error source in the

experiment. In fact, if we consider the width changed to 8  $\mu\text{m}$ , we would have  $1/\alpha$  equal to  $(0,27 \pm 0,2)$   $\mu\text{m}$  accused by Origin.

Knowing the wavelength of the incident light used it was possible to build graphics with the gap distance in units of wavelength:



The last graphic proves the fading of the evanescent wave within a few wavelengths, which is predicted in theory. We can see that at a distance of about or greater than  $2,5\lambda$ , the transmitted light is practically null.

## Conclusions

- The phenomenon will occur if the distance between the prisms is of the order of the wavelength  $\lambda$ .
- Very well polished prisms are needed in order to perform the experiment with visible light.
- An application of the experiment would be the determination of a medium's index of refraction, because we can determine  $\alpha$ .

## References

- Castro, J.C. (1974) Optical Barrier Penetration: A Simple Experimental Arrangement. *American Journal of Physics*, **43**, 107-108.
- Coon, D.D. (1965) Counting Photons in the Optical Barrier Penetration Experiment, *American Journal of Physics*, **34**, 240-243.
- Mahan, A.I. and Bitterli, C.V. (1978) Total Internal Reflection: A deeper Look. *Optical Society of America*, **17**, 509-519.
- Hecht, E and Zajac, A. *Optics*. Addison-Wesley.
- Eisberg, R. *Mecânica Quântica*, Editora Campus.

- Newton, I. Óptica, EdUSP.
- OSA Unicamp *IYPT booklet*.

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## APPENDIX

### 1. Resolution of electromagnetic wave equation.

The following equation comes from solving Maxwell's equations of electromagnetism. It describes the wave function of an electromagnetic radiation of frequency  $f$  propagating in a medium of index of refraction  $n$  in one dimension ( $x$ ):

$$\frac{d^2\psi(x)}{dx^2} + \left(\frac{2\pi f}{c} n\right)^2 \psi(x) = 0$$

It can be rewritten as:

$$\frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0 \quad k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} n$$

In which  $k$  is called the *wave number*. Two possible solutions for this equation are:

$$\psi(x) = \psi(0)e^{\pm ikx}$$

It can be proved that these two solutions are possible. First, the proof for the positive exponential:

$$\begin{aligned} \frac{d^2\psi(x)}{dx^2} &= \psi(0)i^2k^2e^{ikx} = -\psi(0)k^2e^{ikx} \\ k^2\psi(x) &= \psi(0)k^2e^{ikx} \end{aligned}$$

Proceeding in the same manner for the negative exponential:

$$\frac{d^2\psi(x)}{dx^2} = \omega_{(0)} i^2 k^2 e^{-ikx} = -\omega_{(0)} k^2 e^{-ikx}$$

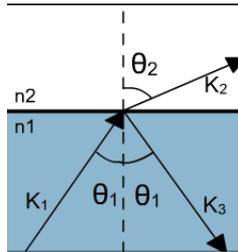
$$k^2 \psi(x) = \psi_{(0)} k^2 e^{-ikx}$$

A more general solution, for three dimensions, can be used to represent the spatial variation of an electromagnetic wave:

$$\psi = \psi_{(0)} e^{\pm i(\mathbf{k} \cdot \mathbf{r})}$$

where  $\mathbf{k}$  is the wave vector or propagation vector, and  $\mathbf{r}$  is the vectorial position.

## 2. Proof of Evanescent wave in total internal reflection.



Here we have an example of wave vectors  $\mathbf{k}$  in refraction and reflection.

The equation describing the spatial variation of the electromagnetic wave is:

$$\psi = \psi_{(0)} e^{\pm i(\mathbf{k} \cdot \mathbf{r})}$$

The law of Snell can be applied whenever light encounters a boundary between two mediums:

$$\sin \theta_1 \cdot n_1 = \sin \theta_2 \cdot n_2$$

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1 \quad \longrightarrow \quad \sin \theta_2 = \frac{\sin \theta_1}{\sin \theta_c}$$

$$\sin^2 \theta_2 = \frac{\sin^2 \theta_1}{\sin^2 \theta_c} \quad \longrightarrow \quad 1 - \sin^2 \theta_2 = 1 - \frac{\sin^2 \theta_1}{\sin^2 \theta_c}$$

$$\cos^2 \theta_2 = 1 - \frac{\sin^2 \theta_1}{\sin^2 \theta_c} \quad \longrightarrow \quad \cos \theta_2 = \sqrt{1 - \frac{\sin^2 \theta_1}{\sin^2 \theta_c}}$$

Total internal reflection occurs when:

$$\theta_c < \theta_1 < 90^\circ \quad \frac{\sin^2 \theta_1}{\sin^2 \theta_c} > 1$$

$$\cos \theta_2 = \sqrt{-1 \left( \frac{\sin^2 \theta_1}{\sin^2 \theta_c} - 1 \right)} \quad \longrightarrow \quad \cos \theta_2 = i \sqrt{\left( \frac{\sin^2 \theta_1}{\sin^2 \theta_c} - 1 \right)}$$

$$\cos \theta_2 = i \sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2 \theta_1 - 1}$$

Having determined the cosine of  $\theta_2$  in total internal reflection, we may apply the wave equation to describe the wave that penetrates into the second medium in total internal reflection. To determine the behaviour of this wave in the direction  $d$  (normal to the surface) we must include in the wave equation a scalar product between  $\mathbf{k}$  and the position in  $\mathbf{d}$ :

$$\Psi = \Psi_{(0)} e^{\pm i k d (\cos \theta_2)}$$

$$\Psi = \Psi_{(0)} e^{\pm i (\mathbf{k} \cdot \mathbf{d})}$$

$$i k d (\cos \theta_2) = d i^2 \frac{2\pi}{\lambda} \sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2 \theta_1 - 1} = -d\alpha$$

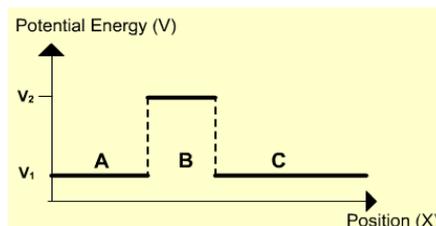
$$\alpha = \frac{2\pi}{\lambda} \sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2 \theta_1 - 1}$$

We have then 2 solutions, a positive and a negative exponential:

$$\Psi = \Psi_{(0)} e^{\pm d\alpha}$$

The positive exponential is discarded because it is physically impossible due to conservation of energy. The evanescent wave, therefore, decays exponentially.

**3. Analogy to Quantum Physics:** Every optical phenomenon has an analogue in quantum physics. Frustrated Total Internal Reflection is an optical analogue to particle potential barrier penetration (Quantum Tunnelling Effect). If a quantum particle (ex: an electron) encounters a potential barrier ahead of it during its movement, it will have a probability of being reflected from this barrier and a probability of passing through the barrier and therefore be encountered in the region beyond it.



Supposing our electron is moving in the sense of a growing position, in region A, and reaches a region of greater potential, B. We say it has encountered a

potential barrier. It will have a probability either of returning back into region A and a probability of tunnelling into region C. This behaviour is different from macroscopic physics, in which we could predict if a body is or not to trespass a potential barrier. In quantum physics however, we must work with probabilities. The higher the potential barrier in quantum physics, the greater the probability of the particle being reflected, and the smaller the probability of tunnelling to occur. The length of the barrier also influences: the “longer” the potential barrier, the greater probability of reflection and the smaller the probability of tunnelling. In quantum mechanical optics analogies, the particles are the so – called photons that make up light. A beam of light can be well interpreted as a stream of photons. The quantum potential of a medium analogue to light will depend on the medium’s index of refraction  $n$ .

$$\frac{d^2\psi(x)}{dx^2} + \left(\frac{2\pi f}{c} n\right)^2 \psi(x) = 0$$

This is the equation that describes electromagnetic radiation of frequency  $f$  propagating in a medium of index of refraction  $n$ . An equation applied in electromagnetism. Here  $\psi$  is the electric or magnetic field,  $x$  is position and  $c$  is the speed of light in vacuum.

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2} (E - V(x)) \psi(x) = 0$$

This is Schrödinger’s non time-dependent equation. It comes from quantum mechanics. Here  $\psi$  is the wave function,  $x$  is position,  $m$  is the mass of the photon (which is  $h \cdot f / c^2$ , in which  $h$  is the Planck constant =  $6,626 \cdot 10^{-34}$  J.S),  $\hbar$  is  $h / 2\pi$ .  $E$  is the energy of the photon ( $h \cdot f$ ) and  $V$  is the quantum potential in the position  $x$ . The two equations express the same mathematical relation, so we can make an analogy between them. Comparing the equations, we obtain:

$$\left(\frac{2\pi f}{c} n\right)^2 = \frac{2m}{\hbar^2} (E - V(n))$$

So we have the relationship between a medium’s quantum potential analogue to light ( $V$ ) as a function of it’s index of refraction  $n$ :

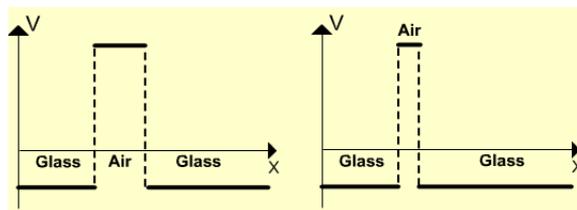
$$V(n) = E - \frac{\left(\frac{fnh}{c}\right)^2}{2m}$$

By looking at the equation we can see that the bigger the medium's index of refraction  $n$ , the smaller the potential this medium offers to photons. Here we have some quantum potentials for red light ( $f = 4,48 \cdot 10^{14}$  Hz):

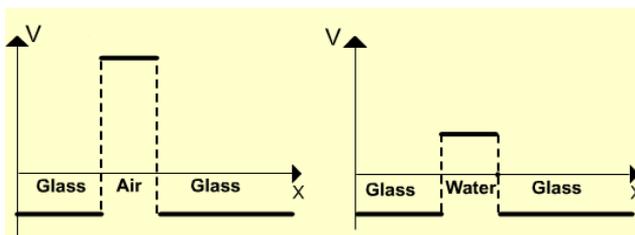
$$\begin{aligned} \text{Air } (n = 1) &= 1,45 \cdot 10^{-19} \text{ J} \\ \text{Water } (n=1,3) &= 3,21 \cdot 10^{-20} \text{ J} \\ \text{Glass } (n = 1,5) &= -3,61 \cdot 10^{-20} \text{ J} \end{aligned}$$

These values depend on the frequency of the light considered. However, the **difference** between the potential offered by two mediums is the same for all frequencies.

The greatest potential is that of air. So we can say that the photon, when reaching the separation surface between glass and air in total internal reflection, encounters a potential barrier. If a second prism is placed at a distance of the order of  $\lambda$ , the length of the potential barrier will be small enough for quantum tunnelling of the photons to occur. The smaller the distance between the two prisms, the greater the probability of tunnelling of a single photon. So, the smaller the gap, the greater the intensity of light frustrated, because more photons tend to tunnel:



If the distance between the prisms is maintained, but the medium between them changes, the probability of tunnelling will also change because the height of the potential barrier will change. If we spread a fluid of greater index of refraction than air's, for example, water, on the surface of the prisms and put them macroscopically in contact, the distance between them will be the same as if there were air between them. However, the photons would encounter a potential barrier far smaller. Tunnelling would be made easier, because a same distance between prisms would offer a greater probability:



## PROBLEM № 15: OPTICAL TUNNELING

### 8.2. SOLUTION OF CROATIA

#### Problem № 15: Optical Tunneling

M. Lukas<sup>1</sup>, M. Marković<sup>2</sup>, D. Pelc<sup>1</sup>, M. Popović<sup>1</sup>, I. Sudić<sup>1</sup>

<sup>1</sup> V. gimnazija, Klaićeva 1, 10000 Zagreb, Croatia

<sup>2</sup> XV. gimnazija, Jordanovac 8, 10000 Zagreb, Croatia

#### The problem

*Take two glass prisms separated by a small gap. Investigate under what conditions light incident at angles greater than the critical angle is not totally internally reflected.*

#### Introduction

If light is entering a prism at an angle greater than the critical, the ray will be totally reflected. However, if we place another prism close to the reflection plane of the first prism, some light will make it across the gap between the prisms. This effect, which is in fact the optical analogy of the quantum – mechanical potential barrier tunnelling of particles, is called optical tunnelling. As we shall see, it can be explained both classically with the aid of the Maxwell equations and quantum – mechanically, both explanations giving the same results; but as the classical picture is somewhat closer to us we will use the Maxwell explanation.

In the beginning, before we make any quantitative analyses, we will mention some properties of the tunnelling process observed experimentally; the conclusions we shall draw from the observations will be of great aid in constructing the mathematical model:

- The intensity of the tunnelled light falls off exponentially when the gap is increased linearly
- The passing light can be observed as long as the gap is a few wavelengths wide; if the gap is further increased, the light is too faint to detect, which means that the intensity falls off very rapidly
- The intensity of the tunnelled light depends on the refractive index of the medium between the prisms
- Naturally, the effect cannot be seen if the refractive index of the medium is larger than the refractive index of the prisms because total reflection doesn't occur either

The explanation of all these effects is in fact very simple: when the wave is reflected off the prism surface, the electromagnetic field can't be discontinuous at the boundary between the prism and the medium beyond, it has to extend a little further into the medium, decaying rapidly. That field can indeed be detected, and

in the classical picture it presents the smooth transition between the field in the prism and the "no – field" in the medium beyond the reflection plane. The most understandable explanation of the occurrence of this transition uses Huygens' principle; it is known that a light wave in a crystal (here the prism) or medium is the result of the interference of all waves scattered on the atoms of the medium. During total reflection there is only one principal interference maximum, the reflected ray. However, in the vicinity of the reflective surface "tails" of the light that didn't manage to interfere completely are formed, decaying fast. These tails are referred to as the evanescent wave. If the second prism is placed in that region a new wave will be formed in that prism because of the "tails" shaking its atoms generating secondary emissions. Only in that case does energy leave the first prism; the evanescent wave itself carries no energy whatsoever because the electric and magnetic fields are in counterphase. The more beautiful quantum – mechanical explanation uses the Indeterminacy Principle: a photon cannot be localized with 100% accuracy, so there is always a finite probability that some of them are beyond the reflection surface, in the "forbidden" region. The classical, as well as the quantum theory, gives very reliable and simple mathematical results, although the interpretations are somewhat different. As we already mentioned, we will follow the classical theory, using the boundary conditions for the light waves on the reflection surface<sup>1,2</sup>. In the first part of the article we will present a short quantitative description of this theory and the determination of the dependence of the tunnelled light intensity on the main parameter – the gap width – in order to proceed to the experimental results and their evaluation. The quantum – mechanical theory won't be examined in detail because it gives the same numerical results as the classical<sup>3,4</sup>.

## Theory

To gain an exact relation between the tunnelled light intensity, the gap width and the refractive index of the medium between the prisms, we have to solve the Maxwell equations implementing the boundary conditions for reflection. As the prism material and the medium between them are dielectrics, we will work with Maxwell equations for a dielectric medium:

$$\begin{aligned}\nabla\mathbf{E} &= -\frac{1}{\varepsilon_0}\nabla\mathbf{P} \\ \nabla\times\mathbf{E} &= -\frac{\partial\mathbf{B}}{\partial t} \\ \nabla\mathbf{B} &= 0 \\ c^2\nabla\times\mathbf{B} &= \frac{1}{\varepsilon_0}\frac{\partial\mathbf{P}}{\partial t} + \frac{\partial\mathbf{E}}{\partial t}\end{aligned}$$

where  $\mathbf{E}$  is the electric field,  $\mathbf{P}$  the polarization,  $\mathbf{B}$  the magnetic field induction,  $\varepsilon_0$  the permeability of vacuum and  $c$  the speed of light *in vacuum*. We know that

plane sinusoidal waves of the electric and magnetic fields are particular solutions of these equations; the electric field is

$$\mathbf{E} = \mathbf{E}_0 e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})}$$

with  $\mathbf{E}_0$  the amplitude,  $\omega$  the circular frequency,  $t$  time,  $\mathbf{k}$  the wavenumber vector (which points in the direction of wave propagation and has a length equal to  $\frac{\omega n}{c}$ , with  $n$  being the refractive index of the medium through which the wave is propagated) and  $\mathbf{r}$  a radius vector. For the magnetic field one readily obtains

$$\mathbf{B} = \frac{1}{\omega} \mathbf{k} \times \mathbf{E}$$

The magnetic field is thus perpendicular to the electric field, a well – known result. What concerns us now are the amplitudes of the fields in the three regions: the first prism, the gap, the second prism. To obtain them we must find the boundary conditions at the region boundaries. First we must define the geometry and polarization of the incident light, because the relation between the direction of the electric field vector and the plane of propagation is quite important. The coordinate system will be such that light propagates in the  $xy$  – plane, and as a special case we will take the light to be linearly polarized in the  $z$  – direction. The reflecting plane of the prism lies in the  $yz$  – plane, and due to the ray being practically infinitely thin we assume the face to be of infinite size. The face of the

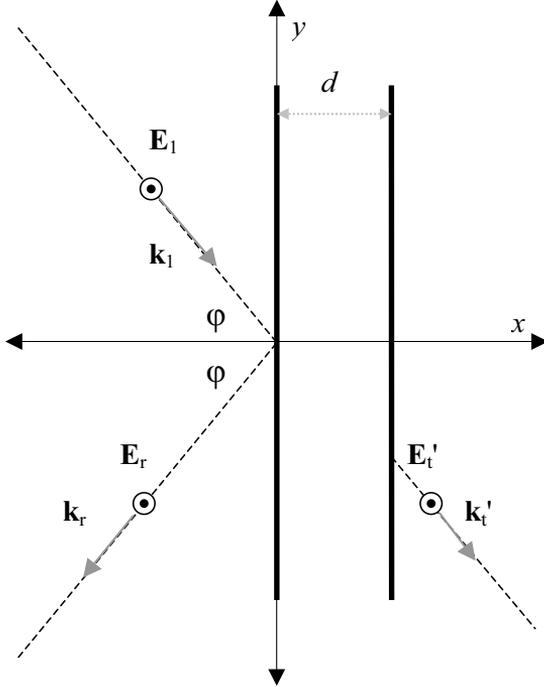


Fig. 1. Geometry of the problem

second prism is parallel to the first, the distance between them being  $d$ . The wave vector of the incident light we have denoted by  $\mathbf{k}_1$ , the vector of the reflected light is  $\mathbf{k}_r$  and the vector of the transmitted light  $\mathbf{k}_t$ . However, as we shall see, this vector is complex in the space between the prisms, becoming real in the second prism; that real vector is denoted by  $\mathbf{k}_t'$ . The notation is analogous for the electric and magnetic fields. The angle of incidence or reflection (it is clear that those two angles are equal) is  $\varphi$  (Fig. 1). Now we can write the waves in the first prism in the exponential form;

$$E_1 = E_{10} e^{i(\omega t - k_{1x}x - k_{1y}y)}$$

is the incident electric field, and

$$E_r = E_{r0} e^{i(\omega t - k_{rx}x - k_{ry}y)}$$

the reflected field. In the space between the prisms the wave will be

$$E_t = E_{t0} e^{i(\omega t - k_{tx}x - k_{ty}y)}$$

Now we want to find the relations between the wave vector components of the incident, reflected and transmitted waves. Due to energy conservation we can readily conclude that

$$E_i + E_r = E_t$$

for all times and coordinates; that induces:

- The frequencies of the incident, transmitted and reflected waves are the same
- For the amplitudes we get

$$E_{t0} + E_{r0} = E_{i0}$$

and

$$k_{1x}E_{i0} + k_{rx}E_{r0} = k_{tx}E_{t0}$$

because the derivatives have to be continuous at the boundary (or the wave equation would break down at the edges which is physically impossible). Knowing that  $k_{1x} = -k_{rx}$  (the incident angle is equal to the reflected) we obtain

$$E_{r0} = \frac{k_{1x} - k_{tx}}{k_{1x} + k_{tx}} E_{i0}$$

$$E_{t0} = \frac{2k_{1x}}{k_{1x} + k_{tx}} E_{i0}$$

Of course, if the wave vector of the transmitted wave is complex, the amplitude is equal to the modulus of the complex amplitude obtained with the above relation.

- Due to light speed constancy, one obtains for the wavenumbers of the incident and transmitted waves

$$k_{ty} = k_{1y}$$

$$k_{tx} = \left( \frac{n_1}{n_0} \right)^2 k_1^2 - k_{1y}^2$$

where  $n_0$  is the refractive index of the prisms and  $n_1$  the refractive index of the medium between them.

Now we are in possession of all ingredients necessary for finding the equation of the wave in the gap. For total reflection, the angle  $\varphi$  must be greater than the critical angle defined by

$$\frac{n_0}{n_1} \sin \varphi_c = 1$$

where  $\varphi_c$  is the critical angle. That relation is of course just a special case of Snell's law. As  $k_{1y} = k_1 \sin \varphi$ , the  $x$  – component of the wave vector of the transmitted wave becomes

$$k_{x'}^2 = k_1^2 \left[ \left( \frac{n_1}{n_0} \right)^2 - \sin^2 \varphi \right]$$

If the angle of incidence is larger than the critical angle,  $\sin^2 \varphi > \left( \frac{n_1}{n_0} \right)^2$  and the  $x$  – component of the wave vector is a pure imaginary; inserting it into the wave equation one gets the predicted exponential drop of amplitude (the positive solution, corresponding to exponential growth, makes no physical sense):

$$E_t = E_{t0} e^{-\frac{n_1}{n_0} k_{1x} \sqrt{\left( \frac{n_0}{n_1} \right)^2 \sin^2 \varphi - 1}} e^{i(\omega t - k_{1y} y)}$$

If there is only a medium of refraction index  $n_1$  the field in this medium will decay very fast and the reflected wave will show no energy losses; energy only oscillates to and fro at the boundary but doesn't leak into the medium. That can be easily shown considering the Poynting vector; the cross – product (and accordingly the vector itself, which represents the energy carried by the wave) of the electric and magnetic fields is zero due to their counterphase oscillations. However, if we put a second prism near to the first, the evanescent wave will shake the electrons in the second prism and cause another emission, with the fields in phase this time, and carrying energy. The energy is taken from the reflected wave, causing it to faint as the prisms are drawn closer, vanishing when they touch. Following these arguments (or referring to the expressions for the amplitudes found above) we arrive at the formula for the wave intensity in the second prism:

$$E_t' = E_{t0} e^{-2\pi n_1 \frac{d}{\lambda} \sqrt{\left( \frac{n_0}{n_1} \right)^2 \sin^2 \varphi - 1}}$$

with  $\lambda$  the wavelength *in vacuum*. Due to simplicity we introduce a parameter  $\Theta$ :

$$\Theta = 2\pi n_1 \sqrt{\left( \frac{n_0}{n_1} \right)^2 \sin^2 \varphi - 1}$$

which will be measured and compared to theoretical values. To sum it all up, the presented Maxwell theory arrived at an easily investigable relation between the intensity of the tunnelled wave and the relevant parameters (gap width, refraction indices) considering only boundary conditions at the interfaces. As we shall see, the obtained expression also quite well agrees with the experimental data we produced.

# Experiment

In order to obtain a larger precision and reliability, the measurements were conducted in two different frequency ranges: microwaves and visible light. With the microwaves the investigation of the phenomenon posed no problem because the gap can be up to a few centimetres wide; however, the procedure is more complicated in the optical range due to the very small gaps necessary for the effect to be measurable. The prisms must be very smooth and clean, and the measurement of the gap size is rather difficult<sup>5</sup>. That's why we conducted the measurements in time – the intensities of the tunnelled light were logged at fixed time intervals while the prisms were uniformly moving towards each other pushed by a slow motor. That method gives quite good results but the fast logging necessary for such small distances (a few microns!) can be a problem. But using a very slow motor (1/2 rpm) and large sampling we succeeded in measuring the effect.

The setups for the optical and the microwaves experiments look rather similar (Fig. 2.). The prisms for the microwaves were made of paraffin, with dimensions 150x150x100 mm. The gap between the prisms was varied with a simple wooden translator moved by a long screw. The source of the necessary radiation was a

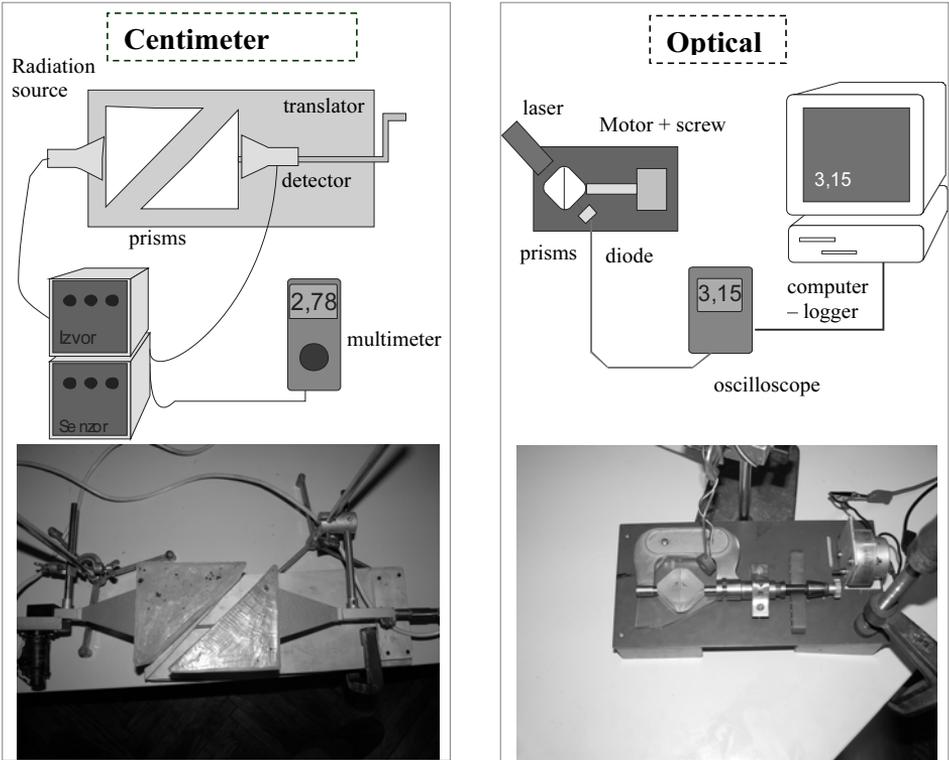


Fig. 2. Experimental setups. The setup for microwaves is on the left and the optical setup on the right

magnetron pentode with an amplifying horn which was placed close to the prism during measurement. The detector for the radiation which was used in tunnelled wave measurement consisted of an amplifying horn with a resonant space containing a 100mH coil which was placed at a node of the wave in the tube. Thus the voltage read from the coil is directly proportional to the field, not the intensity (like when working with photodiodes). The wavelength of the waves was 3.0 cm, making little scratches and defects on the prisms unnoticeable. The optical setup was quite similar, only that the prisms were made of glass, mounted in the screw driven by the motor. The velocity of the prisms was 0.025 cm/min (0.6 seconds for a micron), and the intensity was logged every fiftieth of a second. The data had later to be smoothed due to imperfections (and dust) on the prisms and noise, resulting in a 10 measurements per second sampling. A 780 nm laser diode with polarizer was used as the light source. The transmitted light was detected by a photodiode which gave signal proportional to the intensity (the *square* of the electric field) of the light.

The results of the measurements are in relatively good agreement with the theoretically predicted curves, the agreement being better for the microwaves due to much greater precision and a larger number of points (Fig. 3.). The slight deviations from the theoretical curve at low intensities are probably caused by the unfocusedness of the source or voltage variations on it; anyway, the deviations were completely random throughout the measurements so they are probably some indeterminate noise. In the optical range (Fig. 4.) the disturbances were very large, making high – sampling measurements impossible, and the discrepancies are larger due to smaller precision and defects (for example dust particles, the size of about a micron, can influence the results badly).

A complete comparison between experiment and theory wasn't possible because we didn't know the relation between the measured detector voltage and the real field, but as the functions look the same it was enough to compare the decaying factors in the exponents. In the optical case we have to perform a little transformation due to the time – measurements; the gap width,  $d$ , becomes  $d_0 - vt$ , with  $d_0$  being some starting value

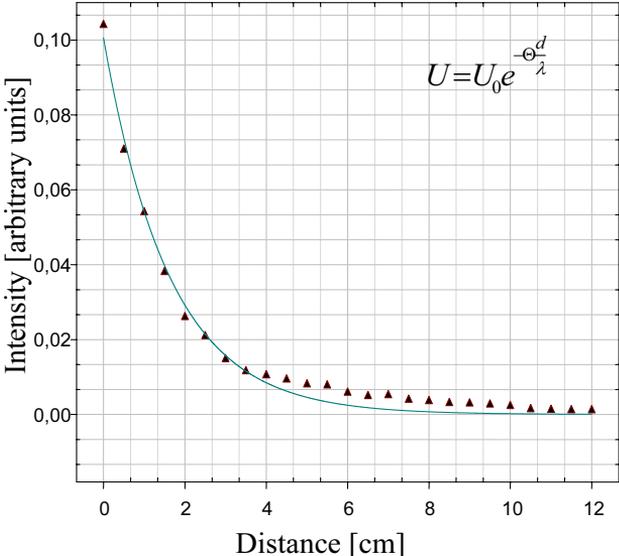


Fig. 3. The experimental curve and fit for the microwave experiment

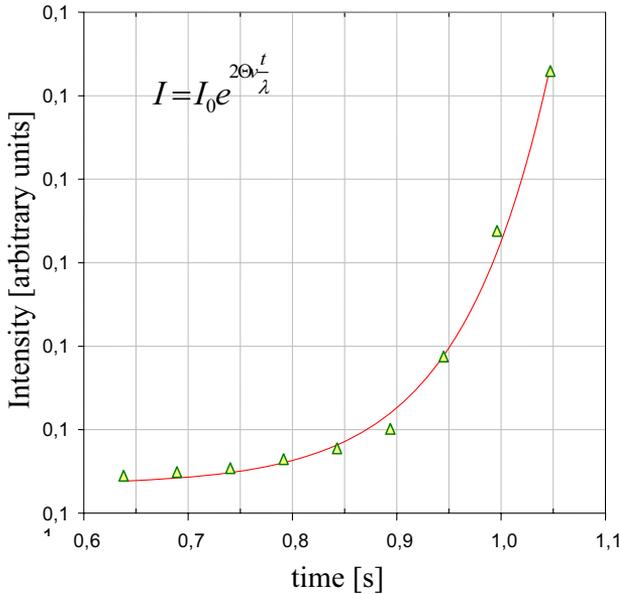


Fig. 4. The experimental curve and fit for the optical experiment in time mode.

(not necessary for the comparison),  $v$  the prism velocity and  $t$  time, the measured variable, and of course we had to account for the fact that the measured output voltage of the photodiode was proportional to the square of the field. The refractive index of the glass prisms was 1.48 (measured) and the paraffin had 1.50 for our wavelength. The refractive index of the air between the prisms was taken to be 1.00 and the angle of incidence was  $45^\circ$ . That leads to the following results for the decay factor  $\Theta$  (all the data have been given again for sum – up):

Prism refractive index, $n_0$	1.48
Wavelength	780 nm
Angle of incidence	$45^\circ$
Theoretical $\Theta$	1.9
Experimental $\Theta$	$1.1 \pm 0.1$

Prism refractive index, $n_0$	1.50
Wavelength	3.0 cm
Angle of incidence	$45^\circ$
Theoretical $\Theta$	2.2
Experimental $\Theta$	$2.5 \pm 0.1$

We see that the agreement is rather good, especially in the centimetre range; the measurement was, as mentioned, rather more precise there; the agreement to a factor of 1.7 in the optical range is quite satisfactory considered all errors.

## Conclusion

To sum things up, we can conclude that we have approached the problem of optical tunnelling and solved it to a certain depth, appropriate to our resources. We have obtained the fundamental relations between the intensity of the tunnelled light and the gap width using elementary classical theory and checked the theoretical results in experiment, with two different wavelengths in two different ranges of the spectrum. The microwave measurements enabled us to perform high – precision tunnelling measurements and obtain valuable quantitative data thanks to the macroscopical size of the gap, while the optical experiment served a more

demonstrational purpose due to the difficulty of obtaining reliable measurements, though we tried to do that too. The parameters entering our intensity formula are the gap width as the most obvious and the prism and medium between prisms refractive indices. In our investigations we mainly focused on the gap, somewhat neglecting the refraction indices. However, we are of the opinion that the gap is indeed much more important and maybe more fundamental to the effect itself than the refraction indices – their variation in fact only induces a change of the critical incidence angle which is not as important. On the other hand, measurements with varying refractive indices could provide an even more firm confirmation of the theory. Also, our formulas were obtained using classical theory; a more complete investigation of optical tunnelling might include a more detailed treatment of the quantum, photonic picture, linking it to the classical formalism. We have chosen the classical theory in our work due to its simplicity, in spite of the beauty of the quantum model, not having the space or time to make more detailed theoretical investigations.

With the suggested adds, for which we didn't have the time or equipment, included, we could say that the problem of optical tunnelling would be rather completely solved. And in the end we may briefly answer the question posed by the problem itself: light incident at angles greater than the critical angle is not totally internally reflected if the second prism is as close to the first as a few wavelengths of the light used.

## Acknowledgements

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## References

- <sup>1</sup> D. B. Sivuhin, *Opći kurs fizike*
- <sup>2</sup> R. Feynman, *Lectures on Physics*
- <sup>3</sup> E. Mertzbacher, *Quantum Mechanics*, J. Wiley & sons, New York, 1961.
- <sup>4</sup> G. Joos, I. M. Freeman, *Theoretical Physics*, Dover, New York, 1950.
- <sup>5</sup> D. D. Coon, *Counting Photons in the Optical Barrier Penetration Experiment*, Am. J. Phys. **4**, 240, 1965.

## PROBLEM № 15: OPTICAL TUNNELING

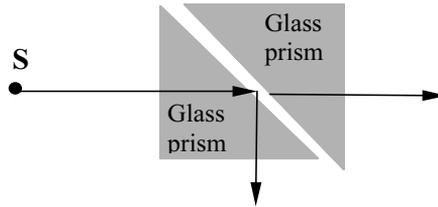
### 8.3. SOLUTION OF UKRAINE

#### Problem № 15: Optical tunneling

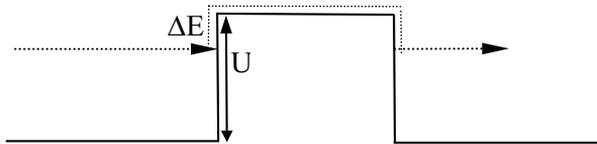
*Oleg Matveichuk, Lena Filatova, Grygoriy Fuchedzhy, Alyeksyey Kunitskiy,  
Valentin Munit  
Richelieu lycium, Shepkina Str 5, Odessa, Ukraine*

#### The problem:

*Take two glass prisms separated by a small gap. Investigate under what conditions light incident at angles greater than the critical angle is not totally reflected.*



At first we should understand what is tunneling. Tunneling is the phenomenon when a particle gets through the potential barrier if its energy is less than barrier's height.

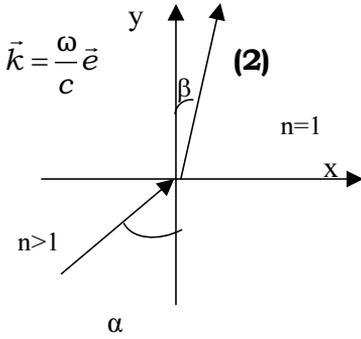


We will investigate this phenomenon from two different sides of view: optical and quantum mechanical.

Let's start with the optical approximation. The electromagnetic wave is traveling in the first prism. Then it comes into the air and starts to dampen. If there is no other prism then it disappears very fast but in our case it continues to go in the second glass prism and we see two rays: reflected and transmitted.

$$E(\vec{r}, t) = E_0 \cdot \exp(i(\omega t - \vec{k} \cdot \vec{r})) \quad (1)$$

At first, let's write the equation of the running wave:  
where  $k$  is the wave vector of the ray,  $r$  is the radius vector of the point at considered moment of time.



After the refraction of the light ray we get:

$$\vec{e} = (\sin \beta; \cos \beta) \quad \mathbf{(3)}$$

$$\vec{k} \cdot \vec{r} = \frac{\omega}{c} (x \sin \beta + y \cos \beta) \quad \mathbf{(4)}$$

By the Snellius law of refraction:

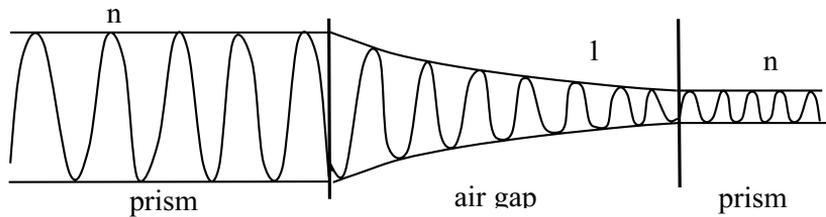
$$\frac{\sin \beta}{\sin \alpha} = n \quad \mathbf{(5)}$$

Condition for the critical angle:

$$n \sin \alpha > 1 \quad \mathbf{(6)}$$

$$E(\vec{r}, t) = E_0 \exp\left(-\frac{\omega}{c} y \sqrt{n^2 \sin^2 \alpha - 1}\right) \exp i\left(\omega t - \frac{\omega}{c} x \sin \alpha\right) \quad \mathbf{(8)}$$

A little manipulation yields:

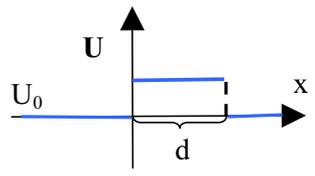


$$\frac{\omega}{c} L_{typical} \sqrt{n^2 \sin^2 \alpha - 1} = 1 \quad \mathbf{(9)}$$

$$\frac{\omega}{c} = \frac{2\pi}{\lambda} a \quad \Rightarrow \quad L_{typical} = \frac{\lambda}{2\pi \sqrt{n^2 \sin^2 \alpha - 1}} \quad \mathbf{(10)}$$

It is obvious that for  $n \sin \alpha < 1$  light will just reflect and we won't see transmitted ray.

Now we come to the quantum mechanical solution.  $U(x)$  is the function of the energy distribution,  $d$  is the length of the gap between two prisms.



$$U(\mathbf{x}) = \begin{cases} 0, & \mathbf{x} > 0 \\ U_0, & \mathbf{x} \in [0; d] \\ 0, & \mathbf{x} > d \end{cases}$$

Let's write Schrödinger's equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi + U\Psi \quad (1)$$

But in our case  $U$  doesn't depend on time so we are going to look for the stationary solution:  
where



$$\Psi(x, t) = a(x) \exp(-i\omega t) \quad (2)$$

$$\omega = \frac{E}{\hbar}$$

$$\Psi(x, t) = a(x) \exp\left(-\frac{iEt}{\hbar}\right) \quad (3)$$

$$\mathbf{U}(\mathbf{x}) = \begin{cases} 0, \mathbf{x} > 0 \\ \mathbf{U}_0, \mathbf{x} \in [0; \mathbf{d}] \\ 0, \mathbf{x} > \mathbf{d} \end{cases} \quad \left. \begin{aligned} a(x) &= A \exp\left(\pm \sqrt{\frac{2m(U-E)}{\hbar}} x\right) & (4) \\ a(x) &= A_1 \exp\left(-\sqrt{\frac{2mE}{\hbar}} x\right) & (5) \\ a(x) &= A_2 \exp\left(-\sqrt{\frac{2m(U-E)}{\hbar}} x\right) & (6) \\ a(x) &= A_3 \exp\left(-\sqrt{\frac{2mE}{\hbar}} x\right) & (7) \end{aligned} \right\}$$

$$A_1 = A_2 \quad (8)$$

$$A_3 = A_2 \exp\left(-\frac{\sqrt{2m(U-E)}}{\hbar} d\right) \exp\left(i \frac{\sqrt{2mE}}{\hbar} d\right) \quad (9)$$

$$\frac{\sqrt{2m(U-E)}}{\hbar} L_{\text{typical}} = 1 \quad (10)$$

$$L_{\text{typical}} = \frac{\hbar}{\sqrt{2m(U-E)}} \quad (11)$$

Now, when the problem is solved from two points of view we can bring to confrontation optical relevant parameters with mechanical.

$$\begin{array}{ccc}
 L_{\text{typical}} = \frac{\lambda}{2\pi\sqrt{n^2 \sin^2 \alpha - 1}} & \longleftrightarrow & L_{\text{typical}} = \frac{\hbar}{\sqrt{2m(U-E)}} \\
 m = \frac{E}{c^2} = \frac{h\nu}{c^2} & c = \lambda\nu & h = 2\pi\hbar \\
 L_{\text{typical}} = \frac{\lambda}{2\pi\sqrt{n^2 \sin^2 \alpha - 1}} & \longleftrightarrow & L_{\text{typical}} = \frac{\lambda}{2\pi\sqrt{2\left(\frac{U}{E} - 1\right)}} \\
 \boxed{n^2 \sin^2 \alpha} & \longleftrightarrow & \boxed{\frac{2U}{E} - 1}
 \end{array}$$

Maxvel's equations have the same mathematical sense as the stationary Schrödinger's equation so their solutions for the corresponding initial and final conditions have to be the same from the mathematical approach. In this problem we see such similarity between relative index of refraction with angle of incident ray and ratio of potential energy of the barrier and of the particle.

### Acknowledgments:

Special thanks to:  
Oleg Matveichuk, main author of the idea.

## 9. PROBLEM № 16: OBSTACLE IN A FUNNEL

SOLUTION OF HUNGARY

### Problem № 16: Obstacle in a funnel

Reporter: Nóra Horváth

#### The problem

*Granular material is flowing out of a vessel through a funnel. Investigate if it is possible to increase the outflow rate by putting an 'obstacle' above the outlet pipe.*

#### Introduction

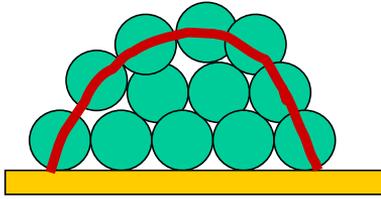
Granular materials are more common than one would guess; let us just think about agriculture, building-trade or plastic industry. For handling so many times with these in some aspects extraordinary materials, people have thoroughly studied their properties in order to find a more efficient way of storage and usage.

#### Background

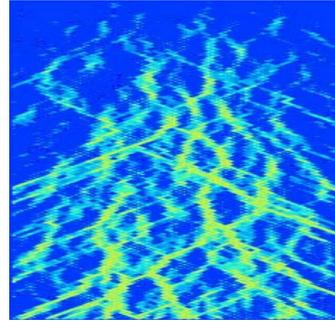
A granular material consists of many macroscopical (i.e. above 10  $\mu\text{m}$ ) particles. In this range of dimension there are three main forces acting inside the system: the gravity force, the repulsive force between touching particles and the friction force between particles at contact points.

Probably the most interesting property of granular systems is that if applying stress onto the aggregation, above a certain threshold of stress the particles may jam up. The cause of this jamming is that particles form so called force chains in compressional dimensions. These chains can be modeled as linear strings of rigid particles in point contact. Chains only support mass along their own axis so they are strictly collinear. They end on the walls of the container (if there is any, in other cases they end on the bottom of the aggregation), thus there is a significant pressure on the wall and/or on the bottom. The force applied to the granules is mediated to the walls by the force chains. If the force in a chain is too large or its direction changes, then the force chain is broken. This can happen when the granular material is stirred or moved, and afterwards a network of new force chains is formed.

We investigated the phenomenon referred to experimentally. For our measurements glass funnels of three different sizes were used (diameters 0.6 cm, 0.8 cm and 1.2 cm). As granular materials we chose sand, semolina, lead-balls (diameters 0.25 cm and 0.5 cm) and plastic 'balls' of an irregular shape. And finally, we used a wide range of obstacles. The funnels were attached to an



*Fig. 0 Simplified conception of a force chain building up in an aggregation*

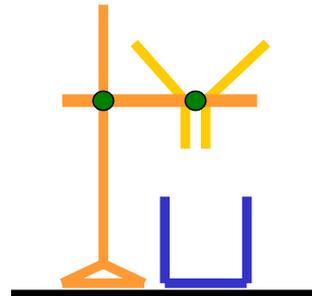


*Fig. 2 Experimental observation of force chains building up in a granular system (Ref. [1])*

eprouvette-stand. Material of given volume ( $125 \text{ cm}^3$ ) was poured into the funnel (while the outlet pipe was kept closed)



*Fig. 3 Granules, funnels and obstacles used*



*Fig. 4 Schematic figure of the way of attachment*

As our first experiment, we measured the flow time of each material in the funnels without an obstacle ( $t_0$ ). Afterwards as further experiments we measured the flow rate by placing an obstacle inside the funnel ( $t$ ). We varied some parameters: shape of the obstacle (blunt (dull), peaked and spherulitic), diameter of the obstacle (0.3 cm, 0.8 cm, 1.2 cm) and the distance of the obstacle from the mouth of the funnel ( $h$ ).

We assumed namely, that following parameters may be relevant:

- the size of the particles ( $d$ )
- the shape of the particles
- the material of the funnel – may influence friction
- the material of the obstacle – may also influence friction
- the diameter of the obstacle ( $d_{\text{obs}}$ )
- the diameter of the funnel mouth ( $\Phi$ )
- the distance between the obstacle and the mouth of the funnel ( $h$ )

**Our observations** Three different cases were analyzed. In the first case the obstacle acted really as an obstacle: the flow was hindered. In the second case the obstacle surprisingly increased the flow rate – this was the main point of our study. We also had cases in which the obstacle had no influence on the flow rate at all.

During experiments we noticed the fact that the outflow time reducing effect only appears if  $d_{\text{obs}} > 3d$ , so we had to take this into account in our investigations. This means that we did not go through all measurements in cases like pouring big lead-balls ( $d = 0.5 \text{ cm}$ ) into a medium-sized ( $\Phi = 0.8 \text{ cm}$ ) funnel.

1. Firstly, concerning the obstacles see Table 1.

Obstacle	Average flow time (s)
None	3.8
Blunt (all sizes)	3.8
Peaked (0.8 cm)	3.4
Peaked (1.2 cm)	3.6
<i>Spherulitic</i>	3.26

*Table 1 Comparison for one material (small lead-ball,  $d = 0.25 \text{ cm}$ ) in a given funnel ( $\Phi = 1.2 \text{ cm}$ ) at given mouth-obstacle distance ( $h = 1 \text{ cm}$ )*

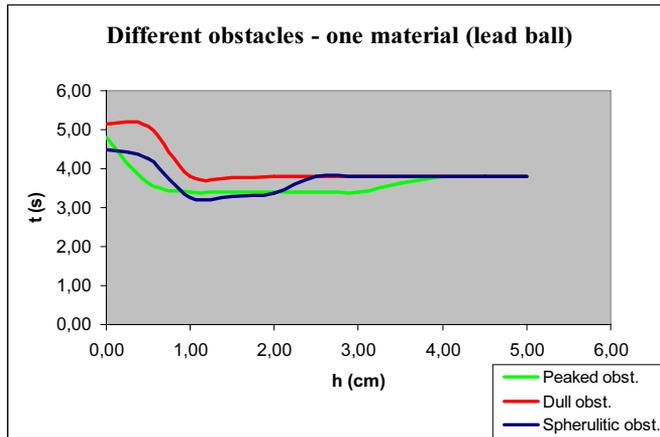
As it is clearly noticeable, the obstacle with a spherulitic end had the largest influence on the flow rate.

2. It is also important to observe, that there is a *certain region* ( $\Delta h = h_{\text{max}} - h_{\text{min}}$ ) in which the time of outflow is reduced. This value depends strongly on the granules investigated.

	$h_{\text{min}}$ (cm)	$h_{\text{max}}$ (cm)	$\Delta h$ (cm)
small lead-balls	1	2.3	1.3
big lead-balls	1	2	1
Semolina	0	1.6	1.6

*Table 2 The region ( $\Delta h$ ) in which reduced time of outflow can be found in case of spherulitic obstacle in a funnel ( $\Phi = 1.2 \text{ cm}$ )*

3. Let us now investigate what happens if we change the *mouth-obstacle distance*!

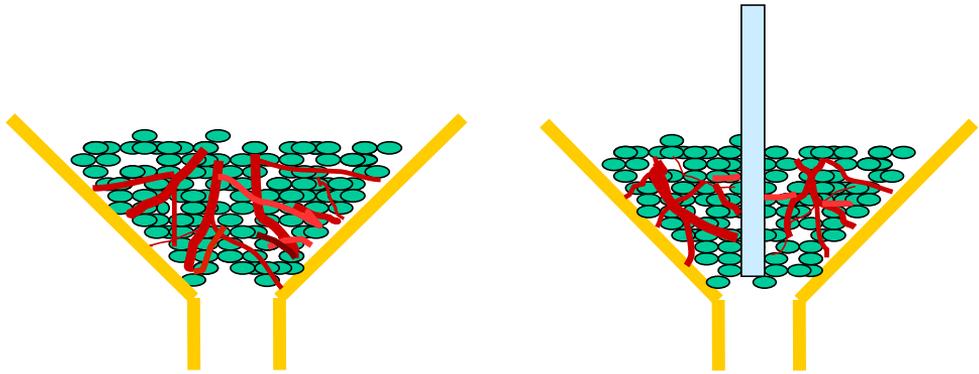


*Fig. 5 Comparison of the obstacles by changing the mouth-obstacle distance (material was given)*

As one can see there is a well-defined height at which the reduced time of outflow reaches its maximal value. It is also very remarkable that the previously recognized schema (see figure 5) remained: the spherulitic obstacle had the largest outflow time reducing influence on the granules, followed by the peaked obstacle and finally the blunt one. However the peaked obstacle is special in some way: the reducing effect appears even at relatively small mouth-obstacle distances. This may be explained with its conical shape

## Explanation

As already told, force chains building up in a granular aggregation end on the walls and bottom of the vessel (in our case there is no 'bottom' of the vessel, for we investigate funnels). This is not the case if there is some kind of obstacle inside the system: the obstacle hinders the formation of the 'basic' chains, a different force chain network forms where some chains will end on the obstacle itself. This means of course a smaller compression of the bottom particles, which allows them an easier motion or flow in optimal cases (see figure 8). However, what do we mean by optimal case? By optimal case we mean that if placing the obstacle too near the mouth, it will really act like a physical obstacle, and the rate of outflow is reduced. The other extreme case is when we place our obstacle too far from the mouth of the funnel. In this case force chains build up even in regions below the obstacle, so the compression on the bottom particles becomes significant again and jamming appears.

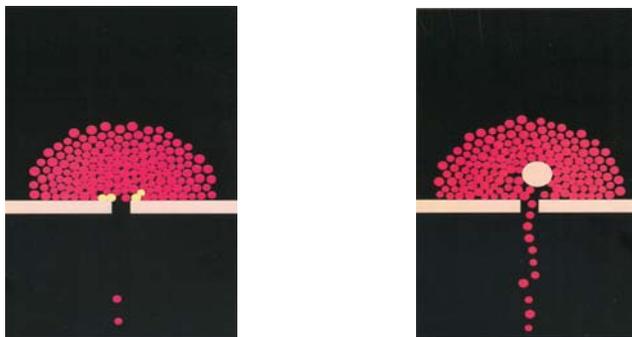


*Fig. 6 The schematic diagram of a granular system in a funnel without an obstacle and with an obstacle. The red lines indicate force chains*

### The model

Probably the most difficult part of our investigation was to set up an appropriate model. Granular systems are namely far more complicated than non-granular ones, and thus the physical description is very complex.

But before introducing our model, let us make a detour to another, in some aspects surprisingly similar phenomenon, called pedestrian escape panic. That means, if a room is crowded with people and somehow they are forced to leave the place (for example if flames come up for some reason), just like granular particles, people may jam up at the exits. However it is a known fact that columns placed near the gates help people to get out. This observation is very similar to our original topic. That means a crowd can be modeled as a self-driven many-particle system.

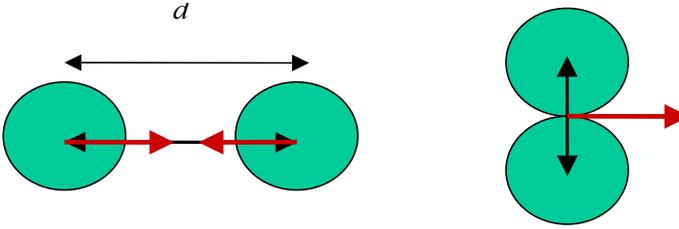


*Fig. 7 Influence of a column placed in front of an exit (Ref. [4])*

Escape panic can be correctly modeled. In Ref. [4] the following equation is suggested for the description:

$$m_i \cdot \vec{a}_i = m_i \cdot \frac{d\vec{v}_i}{dt} = m_i \cdot \frac{v_0 \cdot \vec{e}_i - \vec{v}_i}{\tau} = \sum_j F_{ij}^{\text{rad}} + \sum_j F_{ij}^{\text{tg}}$$

where  $\mathbf{a}$  stands for the acceleration of the particles,  $\mathbf{v}_0$  for the desired speed,  $\mathbf{e}_i$  the desired direction,  $\mathbf{v}_i$  the adapted actual velocity divided by a certain characteristic time  $\tau$ . The interaction forces shown in the equation mean the radial ( $F^{\text{rad}}$ ) and the tangential ( $F^{\text{tg}}$ ) components of the force rising between two particles (see figure 10).



**Fig. 8 Radial and tangential interaction force between two pedestrians**

We applied this model to our problem, as well.

Although the main idea was the same, there are some remarkable differences between the two models. Let us now compare granular flows and panicking crowds.

The main difference is that granular flows are always accelerated by gravity instead of varying factors like in crowds of people. It is also extremely important to emphasize that while the tangential force ( $F^{\text{tg}}$ , friction force) is proportional to the force compressing the surfaces in both systems, the radial force cannot be given by such an idea. Namely the repulsive force rising for keeping off of each other is a long-range interaction ( $d > r_1 + r_2$ ) which does not exist in granular systems. Additionally, the repulsive force rising when colliding is a short-range interaction ( $d = r_1 + r_2$ ) which does not appear in a panicking crowd.

Despite these remarks the difference between the granular system model and the escape panic model (Ref. [4]) is relatively small, so most of the calculations and simulations made for an escaping crowd are also valid for granular systems investigated here.

Finally, we got the following equation as our mathematical model for the problem:

$$m_i \cdot \vec{a}_i = m_i \cdot \frac{d\vec{v}_i}{dt} = m_i \cdot \frac{v_0 \cdot \vec{e}_i - \vec{v}_i}{\tau} = \sum_j F_{ij}^{\text{rad}} + \sum_j F_{ij}^{\text{tg}} \underbrace{\left( + m_i \cdot \vec{g} \right)}_{\text{gravity}}$$

## Conclusion

As you could see the movement of granular materials differs from the motion of liquids as well as the moving of single particles, and this was what made our job hard for it is very difficult to describe this extraordinary movement quantitatively. However with the help of another model we could draw our own one, as well. Finally, to answer the basic question, yes, we clearly found that it is possible to increase the outflow rate by using a kind of obstacle in the funnel. The cause of this phenomenon is that the obstacle influences the building up of force chains in our granules the stability of which thus decreases drastically.

## References:

1. <http://www.phy.duke.edu/~wambaugh/ForceNetworks/>
2. *Jamming, Force Chains and Fragile Matter* S. M. E. Cates, J. P. Wittmer, J.-P. Bouchaud, and P. Claudin, Phys. Rev. Vol. 81 Nr. 8 (1998)
3. *Mechanics of Granular Materials: New Models and Constitutive Relation* S. Alexander, Phys. Rep. 296, 65 (1998)
4. "Simulating dynamical features of escape panic" D. Helbing, I. J. Farkas, and T. Vicsek: Nature, 407, 487-490 (2000). <http://angel.elte.hu/panic>
5. „Granular Flow in Hoppers and Tubes”, K. J. Maloy, T. Le Pennec, E. G. Flekkoy, D. Bideau, M. Arruni, J. C. Messenger, X. L. Wu, A. Hansen: *Physics of Dry Granular Media*. Eds.: H.J.Herrmann, J.-P. Hovi, S. Luding (1997) 509-532.

## 10. PROBLEM № 17: OCEAN SOLARIS

### 10.1. SOLUTION OF CZECH REPUBLIC

#### Problem № 17: Ocean Solaris

*Janečka, A. – Kudela, J. – Lejnar, O. – Solny, P. – Roženkova, K. – Kluiber, Z.*

#### The problem

*A transparent vessel is half-filled with saturated salt water solution and then fresh water is added with caution. A distinct boundary between these liquids is formed. Investigate its behaviour if the lower liquid is heated*

Ocean Solaris is very unique problem that is mainly because of two reasons. The first is that the setting of this task is not absolutely clear; second no parameters under which you should work out the solution are mentioned, so you have to choose from wide spectrum of possible solutions.

Setting says: “A transparent vessel is half filled with saturated salt water solution and then fresh water is added with caution. A distinct boundary between these liquids is formed. Investigate its behaviour if the lower liquid is heated.”

You are given a hint how to prepare this experiment but you also have to solve how to put the fresh water on the saturated salt water solution (SSWS) first. And the mainly thing is that you have to work out the entire problem experimentally without any reasonable parameters given by the setting. So I had to do all the work with great diligence in order to do all necessary experiments.

(Used shortcuts: SSWS – saturated salt water solution, DW – distilled water, US – upper solution, LS – lower solution, OSE – Ocean Solaris effect)

I will try to describe my main findings in this article and I would like to mention the real Ocean Solaris effect (OSE) also.

#### Technique of the experiments

In order to prevent mixing fresh water with SSWS I used paint brush and burette fixed in a holder so that I could easily module flow and lifting force applied in the brush broke speed of the flowing water that means the boundary was not disturbed. I revised the clearness of the added water by measuring its electro conductivity before and after water addition. Difference between measured data using this procedure was insignificant (average difference was less than  $25\mu\text{s}$  micro siemens, for better measurements was used distilled water). In reality the OSE is not so influenced by the little gap between clearness of the added water but if the difference is big enough, than the OSE is worse observable. Most of the experiments I did, was made for principal temperature  $20^\circ\text{C}$  (US, LS, temperature of the air) but I also tried to change this so that I used warmer water. But nearly

nothing was changed and the higher the principal temperature was, the worse could be the experiment prepared because ions went through the boundary faster.

## How the boundary is formed

Boundary between SSWS and DW is like boundary between oil and water but oil doesn't do stable solution with water (standard conditions). Different densities prevent fast mixing so that the boundary appears. After about an hour you can see how the boundary is changed. Its sharpness has disappeared and now you can see SSWS gentle changing in to the upper solution which now contains hundred times more ions. After six hours you get "wary salt water" and here is no boundary any more.



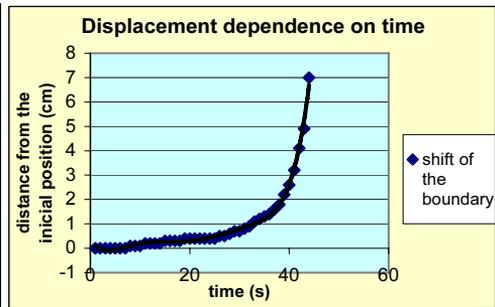
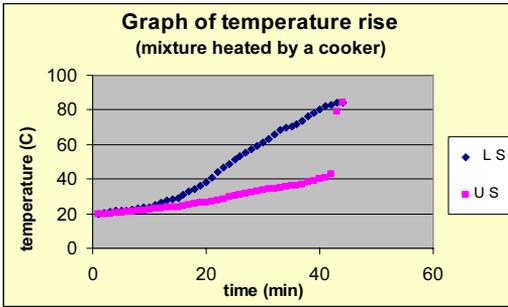
This mixing is caused by oscillating molecules of the water which disturb the boundary and allow more ions get through it. Boundary by itself is not able to prevent anything, which has higher density than the lower solution has, fall down but if you drop something with the right density (smaller than density of LS and bigger than density of US) it seems that this thing stand on the boundary. You can use it for demonstration of changing density during heating but it can influence the effects than.

During heating the boundary starts to wave and after some time it disappears. This is the point which nearly all the experiments have common but nothing else. Because of many varieties of the experiments I will mention only two most interesting.

## “Insulation” and shift of the boundary

(Main parameters of the experiments:  $t_{(SSWS)} = t_{(DW)} = 20^{\circ}\text{C}$ ,  $V_{(SSWS)} = V_{(DW)} = 180\text{ml}$  dw = distilled water) When prepared system is heated on a heater you can find really interesting thing if you compare measured temperatures from both solutions. The temperature of the lower (SSWS) liquid is higher and rise faster than the temperature in upper solution. After some time the temperature difference is stabilised on about  $40^{\circ}\text{C}$  than the lower solution reach about  $85^{\circ}\text{C}$  (it can even boil! But bubbles destroy the boundary than) and the boundary started to lift fast. In reality the boundary has been lifting since the temperature of the lower solution reach about  $60^{\circ}\text{C}$  but it hasn't been well observable before. With increasing temperature boundary moves faster and faster (due to the measured data shifting velocity rises exponentially in dependence on time, the power of the heater is constant).

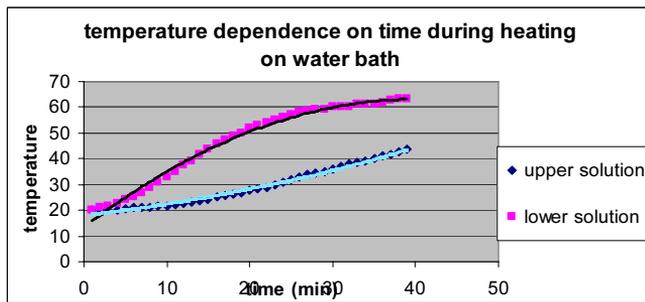
This phenomenon is caused by salinity of the solution which tries to keep the boundary and decreasing density which tries to destroy the boundary (density of the lower solution gets near to the density of the upper solution).



When densities (better say the solutions) are balanced than the only thing which keep them separated is the boundary. But it is not right to say that the densities are the same because density of the US is not constant in whole volume and density of the LS also is not constant in whole volume.

In the graphs you can see data from one of the measurements. Step temperature change in US means that the boundary went around the thermometer.

For heating on the water bath you don't reach high temperatures and the boundary doesn't shift but the temperature difference is also observable.



Main part of the heat added to the system is “store” in the lower solution so that the kinetic energy of the ions is bigger because of that they can go through the boundary faster and salt concentration in US rises. But the waving boundary is still well observable and the US still keeps its lower temperature. You can also see eddy currents in the LS because the added heat causes that the density is not the same in whole volume of the LS and they mixes LS in order to change the density step by step. Sometimes you can see small bubbles, which consist of salt water with different concentration of salt in them than the concentration in the mixture around them is. They appear on the bottom and go up where are stopped by the boundary (they cause bigger waves) and disappear after a while. I didn't study this effect much but I mention it in the part about “real” OSE at the end of this article because I explain OSE by it.

Shifting of the boundary means that molecules of water are transported trough the boundary down to the solution with higher salinity in order to keep the

boundary in system and they provide more “space” for the added heat (temperature in US is really much more lower than the LS).

## **Ocean Solaris**

Real Ocean Solaris effect is observable in quite big vessel with volume more than two liters and radius must be bigger than fifteen cm. It is very important how fast you are able to prepare the system for this experiment because is disturbed during the time. The absolutely same volume of the solutions is not necessary but volumes should get near each other. You also need quite large heater ensuring uniform heating of the bottom. The boundary starts to wave at first quite gently but then the waves become bigger and bigger and when they are big enough the boundary is destroyed and this effect is really gorgeous. I didn't overwhelmingly prove that the waving was caused by bubbles formed by less concentrated salt water solution but style of the destruction of the boundary indicated that something like huge bubbles arising at the bottom could really made this effect.

I think this is possible because the larger bottom provide more space and small bubbles which at first stay at the bottom can interweave and make themselves in one really huge bubble which still stay at the bottom until its density is so low that this bubble can easy break away from the bottom and it can hit the boundary with a great bump which is able to smash it. The main problem is the detection of these bubbles because it is hard to observe the smaller ones and I except densities aren't so different from the rest of the solution so it is really hard to observe them.

The second reason why I think this effect is caused by bubbles is that I didn't realise eddy currents in the lower solution as it was usual in the smaller vessels. Eddy currents mix the lower solution because temperature transport is not ideal so temperature at the bottom and near the boundary could be different. If the bottom is very small or quite large eddy currents don't appear or appear after longer time. If there are not eddy currents in (heated) LS than is more probable you can observe the “bubbles”. If there are not eddy currents than there have to be bubbles but you can't actually see them, the only thing you can observe is waving of the boundary and its destruction.

In fine I would like to say, that OSE how it was described in official solution of this task, appears to be less important if it is compared with the other effects which was observed during solving this task.

## **PROBLEM № 17: OCEAN SOLARIS**

### *10.2. SOLUTION OF UKRAINE*

#### **Problem № 17: Ocean Solaris**

*Oleg Matveichuk, Lena Filatova, Grygoriy Fuchedzhy, Alyeksyey Kunitskiy,  
Valentin Munitsa  
Richelieu lycium, Shepkina Str 5, Odessa, Ukraine*

#### **Problem**

*A transparent vessel is half-filled with saturated salt water solution and then fresh water is added with caution. A distinct boundary between these liquids is formed. Investigate its behaviour if the lower liquid is heated.*

As you see we have some conditions which we have to create in laboratory conditions to observe some specific and unexpected behaviour of the ‘distinct boundary’. Experiments made by our team in our laboratory conditions gave such results:

While heating process, the boundary was rising and at the same time, the waves that occur in it disturbing the surface of the boundary. Let’s explain the cause of this phenomenon.

The causes the waves to occur are the convection flows create as the heating process begun. These convection flows exist because of temperature gradient, inside the salt solution. But there’s one problem: these waves are impossible to be described using any method. This is because of the place of the convection flow pushes the boundary and other parameters are almost random, or factors that define these parameters are random, anyway they are impossible to be described in our model.

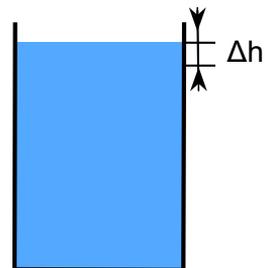
So let’s look at another effect – rising of the boundary’s height. Here everything is a bit easier. Here is the list of effects we consider in order to describe this raising:

1. Thermal expansion
2. Surface tension
3. Bubbles of gas
4. Convection flows

#### **Thermal expansion**

Thermal expansion is an effect of changing of the volume of the body (in our case it’s salt solution) with increasing of its temperature. Using some approximations

we got a result of changing height ( $\Delta h$ ) of the boundary due to thermal expansion about 1mm. The same value was on experiment – about 2mm



## Surface tension

Surface tension is not very important in our conditions, because of surface tension coefficient is very small. We have made both theoretical and experimental research on this question and had as a result surface tension coefficient is about  $0.01\text{N/m}^2$ . Comparing with water this coefficient is very small.



We used method of wire detachment for measuring surface tension coefficient (and this is the equipment we used for this).

## Bubbles of gas

Bubbles of gas have big influence on height of the boundary only when the liquid is near the boiling point. In this case big bubbles that can form in the case of narrow vessels like a piston that pushes water out. In wider vessels this effect is not worth considering (in our work we used quite wide one). So we consider case when bubbles of gas have no strong influence on behaviour of the boundary.

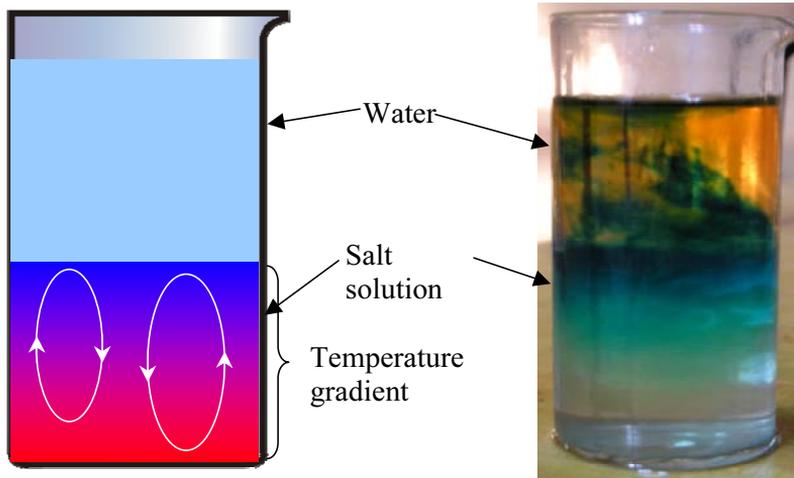
## Convection flows

In our opinion convection flows have the biggest influence on the raising of the boundary. But this is right only when the conditions for the convection flow to

occur are fulfilled, These conditions are: existence of the temperature gradient between different layers of liquid(in our case it is salt solution) and vessel shouldn't be very narrow(in this case bubbles and thermal expansion are important).Let's consider wide vessel(its cross section parameter is comparable with its height) and try to calculate some characteristics of the convection flow(this would be its speed).This will be done by dimension considerations:

Here's the list of values on which speed of the convection flow depends

- ✓  $\Delta T$ -temperature difference between upper and lower layers of the salt solution
- ✓  $\eta$ -dynamic viscosity of the salt solution
- ✓  $\rho$ -density of the salt solution
- ✓  $\beta$ -thermal expansion coefficient



White arrows-scheme of the convection flows

Here is the formula for the speed of the convection flow depending on the parameters, mentioned below:

$$V = \sqrt[3]{\frac{\eta \cdot g}{\rho} \cdot f(\beta \cdot \Delta T)},$$

where  $f(\beta \cdot \Delta T)$  – dimensionless function, which we approximate with linear function, because  $\beta \cdot \Delta T$  is quite small (for real values of  $\beta$  and  $\Delta T$ , their product is also  $\ll 1$ ), so in expansion into a Tailors series, we can neglect all terms, which power is more than 1. This means that  $f(\beta \cdot \Delta T) \approx f(0) + A \cdot \beta \cdot \Delta T$ , where  $A$  - is some coefficient and  $f(0) = 0$  – because if the temperature difference is zero convection flows doesn't occur.

At last we get  $f(\beta \cdot \Delta T) \approx A \cdot \beta \cdot \Delta T$ .

**Next step.** Consider a small volume of salt solution, moving up from the bottom. Its density is smaller than surrounding liquid. Writing the Second Newton's law for this small volume, we can get formula for its acceleration when it gets the boundary:

$$a = \frac{\Delta\rho}{\rho} \cdot g,$$

where  $\Delta\rho$  is the density difference between water and salt solution, it is negative. Then knowing acceleration and initial speed we can calculate the height on which this small volume can “jump” above the boundary, and thus, thus the whole boundary (because there are a lot of convection flows at the same time). This looks this way:

$$h = \sqrt[3]{\frac{\eta^2 \cdot \rho}{g}} \cdot \frac{(A \cdot \beta \cdot \Delta T)^2}{2\Delta\rho},$$

This formula isn't very precise, because we neglect the changing of viscosity, while liquid flows from salt water to fresh, and the velocity of the flow is estimated by the dimensions considerations.

Except of these problems, we have to remind that during the time all parameters are changing.

Also we have to remember that in truth to say – relevant parameters (such as density, temperature, viscosity) are continuously changing from point to point, so we have to find distribution of these values. This problem is very difficult from mathematical point of view.

So we preferred the other way. We made an experiment, where the boundary height measured on time, and using the results selected the possible dependencies for relevant parameters on time. Than we made experiments with other initial density difference (different salt concentrations) and determined that the found dependencies work.

We got the next empirical formula:

$$h(t) = \sqrt[3]{\frac{\eta^2 \cdot \rho}{g}} \cdot \frac{(A \cdot \beta \cdot \psi \cdot t)^2}{2 \cdot (\Delta\rho_0 - \xi \cdot \sqrt{t})},$$

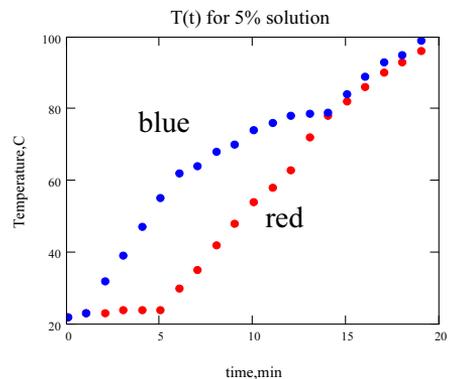
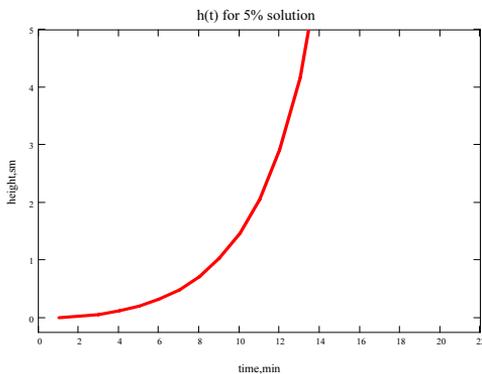
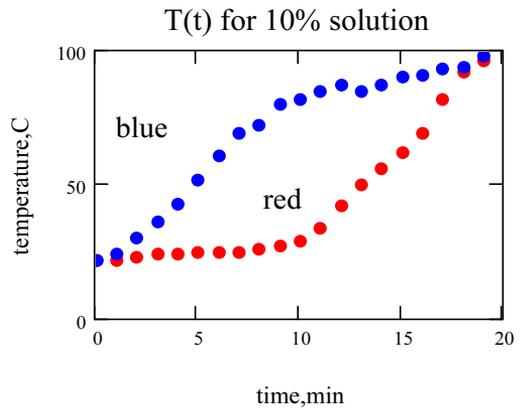
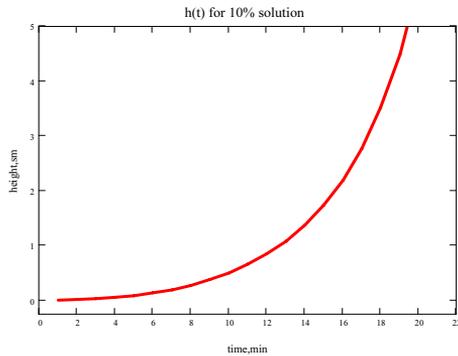
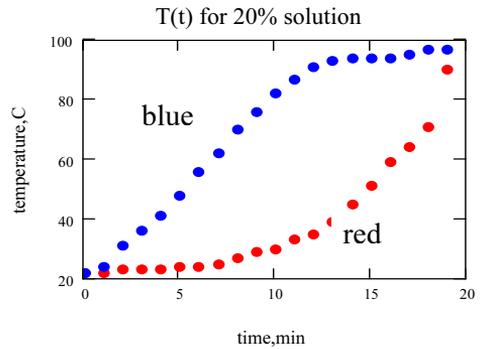
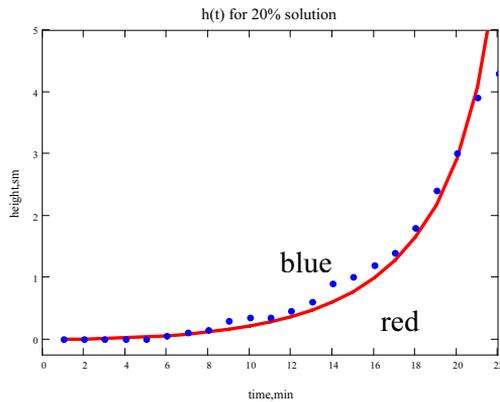
Coefficients  $\psi, \xi, A$  - are different for each kind of vessel and depend on the power of heating and quantity of fresh and salt water in the vessel. So we made experiments taking these requirements to regard, while stating previous formula.

Here are the graphs we got using this formula and comparison between them and our experimental results.

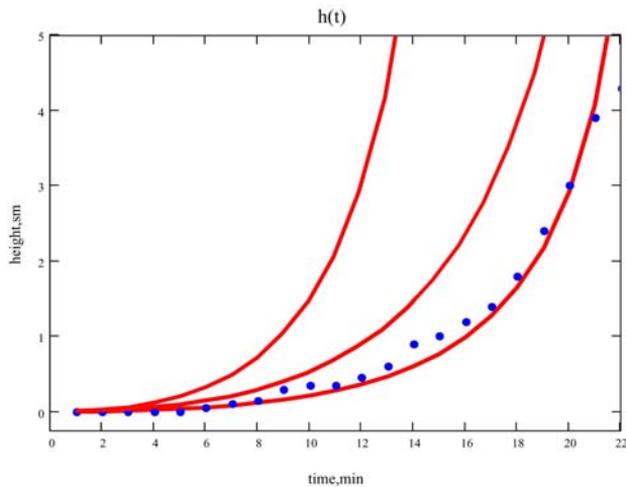
All graphs given for three different concentrations of salt in the solution-20%, 10%, 5%(the highest concentration that can be gained is 25% (for temperature  $20^{\circ}C$  ))

On T(t) graphs blue points-temperature of the water, red – of the salt solution

On h(t) for 20% solution blue points – our experimental results. The height is measured from initial position.



And here three graphs  $h(t)$  for three concentrations on one plot



As you see the less concentration, the bigger the acceleration of the boundary. Why you will ask. Because of density difference rely on acceleration of the boundary-the less concentration, the easier for convection to get to the water upwards, and further.

If you look at the graphs  $T(t)$  you see that at some moment temperature of the liquids becomes equal, this means that temperature at any point inside of the vessel is equal, convection stopped, but it is possible for the limit to exist at such conditions, if we heat all system very slowly. At this temperature boiling begins, so here bubbles begin to play more important part.

So we've made some theoretical research about the behaviour of the boundary between water and saturated salt solution when heating this system.

And we made experiments to compare them with our theoretical research. As you can see these results (I mean graphs  $h(t)$  and  $T(t)$ ) in experiments and theory are very near and can be comparable.

## Acknowledgement

Special thanks to:

Oleg Matveichuk, main author of the idea.

## **IV. Participating countries in the 18<sup>th</sup> IYPT**

### **1. PARTICIPATING COUNTRIES\* IN THE 18<sup>TH</sup> IYPT 2005**

**14<sup>TH</sup>-21<sup>ST</sup> JULY 2005, WINTERTHUR, SWITZERLAND**

Australia	Netherlands
Austria	New Zealand
Belarus	Poland
Brazil	Russia I
Bulgaria	Russia II
Croatia	Slovakia
Czech Republic	Sweden
Georgia	Switzerland I
Germany	Switzerland II
Hungary	Ukraine
Indonesia	United Kingdom
Kenya	United States of America
Korea	

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\* In lexicographic order.

## 2. FINAL RANKING\*

**First place:** Germany

**Second place:** Belarus  
United States of America

**Third place:** Australia  
Poland  
Bulgaria  
Brazil  
New Zealand  
Korea  
Slovakia  
United Kingdom

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\* Order within each group corresponds to obtained score.

## **V. Problems for the IYPT from previous years**

### **1. PROBLEMS FOR THE 17TH IYPT – BRISBANE, AUSTRALIA, 2005**

#### **1. Misty**

Invent and construct a device that would allow the size of a droplet of a mist to be determined using a sound generator.

#### **2. Stubborn Ice**

Put a piece of ice (e.g. an ice cube) into a container filled with vegetable oil. Observe its motion and make a quantitative description of its dynamics.

#### **3. Electric Pendulum**

Use a thread to suspend a ball between the plates of a capacitor. When the plates are charged the ball will start to oscillate. What does the period of the oscillations depend on?

#### **4. Dusty Blot**

Describe and explain the dynamics of the patterns you observe when some dry dust (e.g. coffee powder or flour) is poured onto a water surface. Study the dependence of the observed phenomena on the relevant parameters.

#### **5. Sea-Shell**

When you put a sea-shell to your ear you can hear ‘the sea’. Study the nature and the characteristics of the sound.

#### **6. Seebeck Effect**

Two long metal strips are bent into the form of an arc and are joined at both ends. One end is then heated. What are the conditions under which a magnetic needle placed between the strips shows maximum deviation?

#### **7. Coin**

Stand a coin on its edge upon a horizontal surface. Gently spin the coin and investigate the resulting motion as it settles.

#### **8. Pebble Skipping**

It is possible to throw a flat pebble in such a way that it can bounce across a water surface. What conditions must be satisfied for this phenomenon to occur?

#### **9. Flow**

Using a dc source, investigate how the resistance between two metallic wires dipped into flowing water (or water solution) depends upon the speed and direction of the flow.

### **10. Two Chimney**

Two chimneys stand on a box with one transparent side. Under each chimney there is a candle. A short period after the candles are lit one flame becomes unstable. Examine the case and present your own theory of what is happening.

### **11. String Telephone**

How do the intensity of sound transmitted along a string telephone, and the quality of communication between the transmitter and receiver, depend upon the distance, tension in the line and other parameters? Design an optimal system.

### **12. Kundt's Tube**

In a 'Kundt's Tube' type of experiment the standing waves produced can be made visible using a fine powder. A closer look at the experiment reveals that the regions of powder have a sub-structure. Investigate its nature.

### **13. Egg White**

White light appears red when it is transmitted through a slice of boiled egg white. Investigate and explain this phenomenon. Find other similar examples.

### **14. Fountain**

Construct a fountain with a 1m 'head of water'. Optimise the other parameters of the fountain to gain the maximum jet height by varying the parameters of the tube and by using different water solutions.

### **15. Brazil Nut Effect**

When a granular mixture is shaken the larger particles may end up above the smaller ones. Investigate and explain this phenomenon. Under what conditions can the opposite distribution be obtained?

### **16. Small Fields**

Construct a device based upon a compass needle and use your device to measure the Earth's magnetic field.

### **17. Didgeridoo**

The 'didgeridoo' is a simple wind instrument traditionally made by the Australian aborigines from a hollowed-out log. It is, however, a remarkable instrument because of the wide variety of timbres that it produces. Investigate the nature of the sounds that can be produced and how they are formed.

## 1.1. PROBLEM № 2: “STUBBORN ICE” – IYPT 2004

SOLUTION OF AUSTRIA

### Problem № 2: Stubborn Ice

*Harald Altinger, Bernhard Frena, Eva Hasenhütl, Christina Koller, Camilla Ladinig*

*Reporter: Harald Altinger  
(Power Point Presentation)*

#### The problem

*Put a piece of ice (e.g. an ice cube) into a container filled with vegetable oil. Observe its motion and make a quantitative description of its dynamics.*

#### Structure

- Supposition
- Experimental Setup
- Observation
- Interpretation
- Quantitative Estimation
- Conclusions
- Literature

#### Supposition

1. Dynamics dependent on the following parameters:

⇒ difference between the densities of ice, oil & water → buoyancy

⇒ temperature of the oil

⇒ friction

⇒ surface energy

⇒ height of the container

2. Not considered:

⇒ different temperatures/densities in the ice

⇒ layers of different densities in the oil

⇒ boundary effects at the walls of the vessel, etc.

#### Experimental Setup

- large container (height: 13cm, diameter: 7cm)
- ice (average density):  $\rho_{ice} = 917\text{kg/m}^3$
- water:  $\rho_{water} = 1000\text{kg/m}^3$
- vegetable oils (room temperature):  $\rho_1 = 925\text{kg/m}^3$   
 $\rho_2 = 923,3\text{kg/m}^3$  (olive oil)  
( $\rho$  of vegetable oils varies usually between  $910\text{kg/m}^3$  and  $928\text{kg/m}^3$ )
- ice cubes of different shape and volume

## Observation

$\rho_{oil} < \rho_{ice}$ :

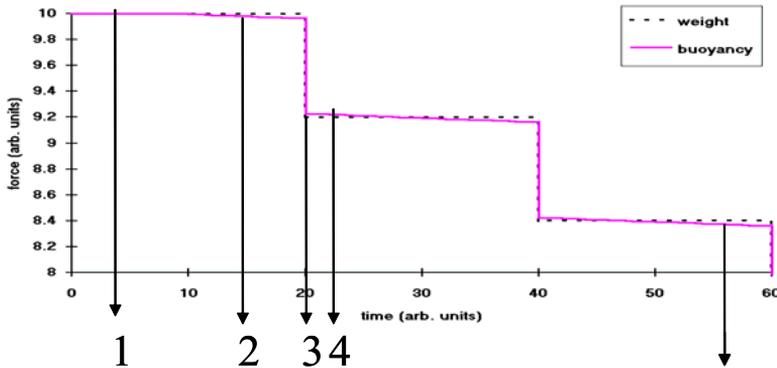
ice descends to the bottom accelerating until the accelerating force due the weight of the cube equals the retarding force due to the friction (Stokes friction)

$\rho_{ice} < \rho_{oil} < \rho_{water}$  :

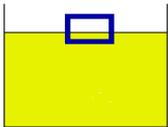
the cubes show vertical oscillation – several up and down motions may occur

No dependence on the shape and no qualitative dependence on the volume of the cubes can be observed.

## Dynamics

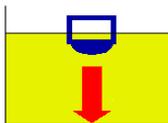


damped oscillation



### Interpretation

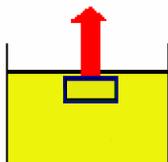
1 cube swims due to buoyancy – ice starts melting and forms water drops until weight is bigger than buoyancy



2 molten water sticks to the cube (surface energy) average density of the combined bodies increases and becomes larger than the density of the oil: the assembly moves down



3 surface tension does not hold the drop attached: the drop separates and moves downwards



4 the buoyancy increases (less water) – the cube slows down – the cube might rise again until damping stops the process

## Quantitative Estimation (1)

Newtons Law:  $M a = F - \beta \eta v$

$$F = M g - F_B$$

$M$  ... total mass

$a$  ... acceleration

$M g$  ... weight of the ice cube  
plus weight of the water

$F_B$  ... buoyancy force due to  
the dispelled oil

The buoyancy force results to:  $F_B = g \rho_{oil} (V_{ice} + V_{water})$  (1)

## Quantitative Estimation (2)

Friction force  $\beta \eta v$  (linearly depending on velocity) :

To good approximation: mass of the ice cube mice linearly decreasing with time  
due to melting

$$M_{ice} = M - \alpha t \quad \dot{Q} = k A \Delta T \quad \Delta T = T_{oil} - T_{icewater}$$

$$M_{water} = \alpha t$$

$A$  ... surface area of the cube



$$F = g \left[ -M \left( \frac{\rho_{oil}}{\rho_{ice}} - 1 \right) + \rho_{oil} \left( \frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}} \right) \alpha t \right] \quad (2)$$

The first term is the buoyancy reduced weight of the ice cube (negative) and second term is positive since  $\rho_{water}$  is bigger than that of oil and is linearly increasing with time.

## Quantitative Estimation (3)

The acceleration for the ice/water combination is

$$a \equiv \frac{dv}{dt} = g \left[ - \left( \frac{\rho_{oil}}{\rho_{ice}} - 1 \right) + \rho_{oil} \left( \frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}} \right) \alpha t \right] - \frac{\beta \eta}{M} v \quad (3)$$

and gives the velocity

$$v(t) = \frac{M}{\beta \eta} g \left[ \left( \frac{\rho_{oil}}{\rho_{ice}} - 1 \right) + \frac{\alpha}{\beta \eta} \rho_{oil} \left( \frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}} \right) \right] \left( e^{\frac{-\beta \eta}{M} t} - 1 \right) + g \frac{\alpha}{\beta \eta} \rho_{oil} \left( \frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}} \right) t \quad (4)$$

## Conclusions

- The drops may oscillate because of the interaction between the increasing density of the combined water/ice –system and the buoyancy force
- The phenomenon will stop when the ice cube is so small that its buoyancy force cannot withstand the weight of the new water drop formed.
- Such movement results from the theory and can be observed as shown in the movie.

## Literature:

- Density of cooking oil  
<http://hypertextbook.com/facts/2000/IngaDorfman.shtml>
- Density of ice  
<http://hypertextbook.com/facts/2000/AlexDallas.shtml>
- Density of water  
<http://www.ucdsb.on.ca/tiss/stretton/chem2/data19.htm>

### HANDOUT “STUBBORN ICE”

$$F_B = g \rho_{oil} (V_{ice} + V_{water}) \quad (1)$$

$$F = g \left[ -M \left( \frac{\rho_{oil}}{\rho_{ice}} - 1 \right) + \rho_{oil} \left( \frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}} \right) \alpha t \right] \quad (2)$$

$$a \equiv \frac{dv}{dt} = g \left[ - \left( \frac{\rho_{oil}}{\rho_{ice}} - 1 \right) + \rho_{oil} \left( \frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}} \right) \alpha t \right] - \frac{\beta \eta}{M} v \quad (3)$$

$$v(t) = \frac{M}{\beta \eta} g \left[ \left( \frac{\rho_{oil}}{\rho_{ice}} - 1 \right) + \frac{\alpha}{\beta \eta} \rho_{oil} \left( \frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}} \right) \right] \left( e^{\frac{-\beta \eta}{M} t} - 1 \right) + g \frac{\alpha}{\beta \eta} \rho_{oil} \left( \frac{1}{\rho_{ice}} - \frac{1}{\rho_{water}} \right) t \quad (4)$$

## 1.2. PROBLEM № 3: ELECTRIC PENDULUM – IYPT 2004

SOLUTION OF AUSTRIA

### Problem № 3: Electric Pendulum

*Harald Altinger, Bernhard Frena, Eva Hasenhütl, Christina Koller, Camilla Ladinig  
(Power Point Presentation)*

#### The problem

*Use a thread to suspend a ball between the plates of a capacitor. When the plates are charged the ball will start to oscillate. What does the period of the oscillations depend on?*

#### Structure

- Basic consideration
- Experimental Setup
- Observation (1)
- Assumption
- Quantitative Estimation
- Observation (2)
- Results
- Special Arrangement
- Conclusions
- Literature

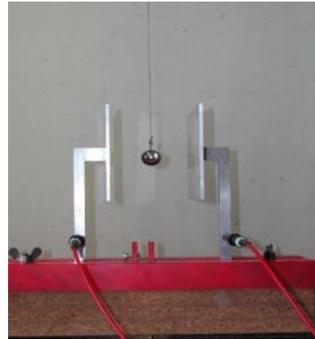
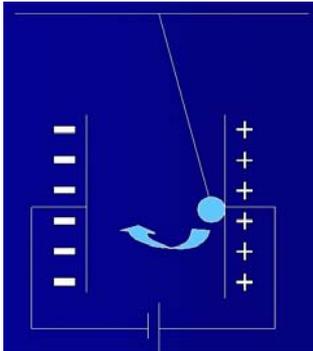
#### Basic consideration

The cause of the oscillation will be the fact that whenever the ball touches a capacitor plate it will be charged as the plate and therefore be repelled afterwards. Simultaneously it will be attracted by the oppositely charged plate and the process will carry on.

The issues of the investigation are:

- ⇒ Why does the oscillation start?
- ⇒ What are the parameters which influence the oscillation?

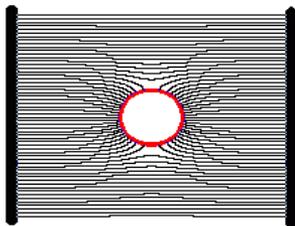
## Experimental Setup



- two capacitor plates in variable distance (in the order of 10-1m)
- thread of 2,5m
- balls of iron ( $d = 1,91\text{cm}$ ,  $m=28,7\text{g}$ ), wood ( $d_1= 2,02\text{cm}$ ,  $m_1= 3,8\text{g}$ ;  $d_2= 3,32\text{cm}$ ,  $m_2 = 15,1\text{g}$ ), aluminium (hollow;  $d = 2,52\text{cm}$ ,  $m = 12,3\text{g}$ ) and table tennis ( $d = 3,75\text{cm}$ ,  $m = 2,8\text{g}$ )

### Observation (1)

The ball symmetrically situated between the capacitor plates would not move:



( fixed homogeneous field assumed)

Little asymmetry starts the process:

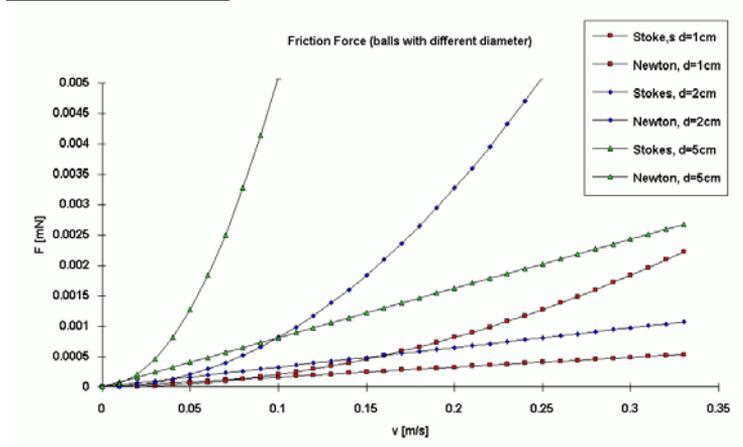
[2004\ElectricPendulum\Video\DSCN3273.MOV](#)

### Assumption

- infinite length of the suspending thread
- Newton's friction law  $F_{Newton} = c_D \rho A v^2 / 2$  (1)
- accelerated motion of the charged ball until the gained energy in the electric field equals the energy loss due to friction
- total elastic reflection at the plates

## Quantitative Estimation (1)

Newton's friction:



Newton's friction more likely than Stokes' friction since for air viscosity Newton's friction outweighs Stokes' friction for velocities > 0.05m/s.

## Quantitative Estimation (2)

Equation of motion:  $m \frac{dv}{dt} = F_{el} - F_{Newton}$  with  $F_{el} = QE = Q \frac{U}{D}$

$$Q = \alpha dU$$

D ... distance of the plates  
D ... diameter of the ball

$$(2) \quad m \frac{dv}{dt} = \alpha \frac{dU^2}{D} - \beta d^2 v^2$$

$\beta$  ... constant variables in Newton's friction law  
 $\alpha = 2\pi\epsilon_0$

Steady state:  $\frac{dv}{dt} = 0 \quad v = U \sqrt{\frac{\alpha}{\beta D d}}$  (3)

## Quantitative Estimation (3)

Period of the oscillation:  $T = 2 \frac{(D-d)}{v} \equiv \frac{2(D-d)}{U} \sqrt{\left(\frac{\beta D d}{\alpha}\right)}$  (4)

[2004\ElectricPendulum\Video\DSCN3224.MOV](#)

With a thread of finite length:  $T_{max} = 2\pi \sqrt{\frac{l}{g}}$  l ... length of the thread  
g ... gravitational acceleration

Thus

⇒ velocity proportional to the voltage

- ⇒ the period of the oscillation is inversely proportional to the voltage
- ⇒ the upper limit for the period is given by the length of the thread

### Quantitative Estimation (4)

Total reflection of the balls: 1 iron ball

[2004\ElectricPendulum\Video\DSCN3230.MOV](#)

2 table tennis ball

[2004\ElectricPendulum\Video\DSCN3242.MOV](#)

Energy loss:

$$\kappa \Delta E = \frac{mv^2}{2}$$

proportional to  $v^2$  as Newton's friction →  
no change of the dependence of  $v$  from  $U$

- ⇒ Dependence on the mass (total elastic) :
- no analytical solution of the equation of motion
- with evidence shown in the experiments

### Quantitative Estimation (5)

Fully inelastic case:

- predominant energy loss at the reflection
- uniformly accelerated motion
- full stop at hitting the electrodes

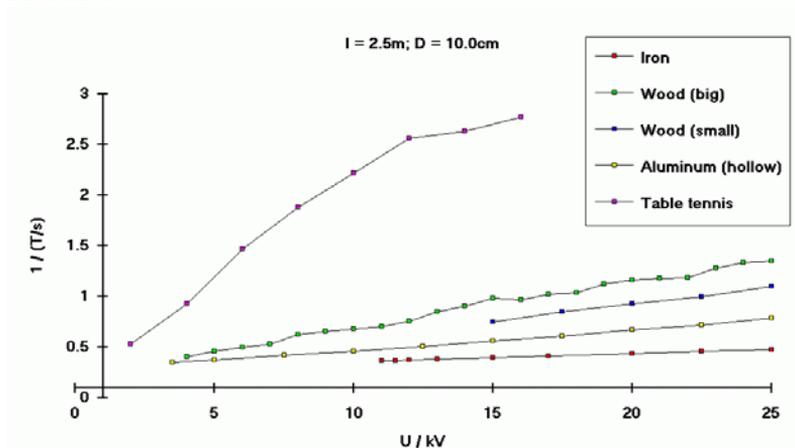
Then follows

$$m \frac{dv}{dt} = QE \quad \text{with the solution} \quad D = \frac{\alpha d U^2}{2mD} T_U^2$$

and the period

$$T_U = \frac{D}{U} \sqrt{\frac{2m}{\alpha d}} \quad (5)$$

### Results

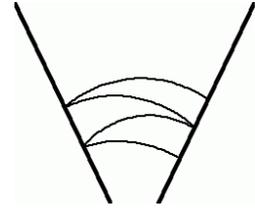


- inverse period of the oscillation  $1/(T/s)$  as a function of  $U \rightarrow$  all balls show a linear dependence on  $U$ ] less inelastic reflection  $\rightarrow$  more evidence of the dependence table tennis ball).

## Special Arrangement

In this case the ball is deflected at every bounce and moves outward until the outward motion which is caused at every reflection is compensated by the inward motion due to the torque build up.

[..IYPT 2004 Turnier\Electric Pendulum\Objekte](#)



## Conclusions

- Energy loss  $\sim v^2$  for all balls
- $1/U$  dependence for the period of oscillation for all balls
- $1/U$  dependence increases elasticity
- for low voltages the period of oscillation is limited by the free oscillation period of the ball

## HANDOUT “ELECTRIC PENDULUM”

$$F_{Newton} = c_D A \rho / 2 v^2 \quad (1)$$

$$m \frac{dv}{dt} = \alpha \frac{dU^2}{D} - \beta d^2 v^2 \quad (2)$$

$$\frac{dv}{dt} = 0 \quad v = U \sqrt{\frac{\alpha}{\beta D d}} \quad (3)$$

$$T_U = 2 \frac{(D-d)}{v} \equiv \frac{2(D-d)}{U} \sqrt{\left( \frac{\beta D d}{\alpha} \right)} \quad (4)$$

$$m \frac{dv}{dt} = QE \quad D = \frac{\alpha d U^2}{2mD} T_U^2 \quad T_U = \frac{D}{U} \sqrt{\frac{2m}{\alpha d}} \quad (5)$$

## 2. Problems for the IYPT from the 7th to the 16th IYPT

### 2.1. Problems for the 7th IYPT

#### NETHERLANDS, 1994

##### **Think up a problem yourself**

(problems 1, 2, 3) Invent yourself and solve a problem on the given theme.

##### **1. Optics**

Think up and solve a problem connected with employing a thin lens of a large focal length.

##### **2. Compass**

“In sledge trips we use liquid compasses, the most exact of the small ones. But you understand of course that due to proximity to the magnetic pole the arrow usually points downwards. To make it horizontal, its opposite end is balanced with a weight”. (From the letter of Cherry-Garrad, member of the last expedition of R. Scott.) Use the context of this quotation to formulate a problem.

##### **3. Magnetization**

A cylindrical permanent magnet falling inside a copper tube is found to move at an almost constant velocity, the slower the thicker and the walls of the tube. Use this fact to formulate a problem (see also 14).

##### **Gravitation machine**

(problems 4, 5, 6). A horizontal plate (a vibrator) oscillates harmonically up and down. A steel ball put on the surface of the plate starts jumping higher or lower. For the experimental device one may successfully use a ferrite core in a coil connected to an alternating current generator (a sound generator). The butt-end of the ferrite core will play the part of the vibrating horizontal plane. Steel balls of diameter 1 or 2 mm are suitable for the experiment. The glass tube approximately 1m long can also be very helpful.

##### **4. Upper boundary**

Measure experimentally the maximum height to which the ball rises to and explain the result.

##### **5. Distribution function**

Determine experimentally what part of a sufficiently large time interval the ball is in the range of heights  $H$ ,  $H \pm dH$  and explain the result.

## **6. Acceleration**

The mechanical energy of the ball changes after every impact. The mean mechanical energy (averaged overall successive impacts) increases at the beginning of the process and then tends to a constant value. Try to obtain experimentally the time dependence of the mean mechanical energy of the ball.

## **7. Aspen leaf**

Even in windless weather aspen leaves tremble slightly. Why does an aspen leaf tremble?

## **8. Superball**

A highly elastic ball (a superball) falls on a horizontal surface from a small height (4cm or less) and recoils several times. What is the number of impacts of the superball against a table?

## **9. Meteorite**

A meteorite of mass 1000 tons files directly to the Sun. Can modern instruments register the fact of its fall on the Sun?

## **10. Water dome**

A vertical water jet falls on the butt-end of a cylindrical bar and creates a bell-like water dome. Explain this phenomenon and evaluate the parameters of the dome.

## **11. Siphon**

A rubber tube is used as a siphon to flow water from one vessel into another. The vessels are separated by a high partition and the levels of water in them are different. If one withdraws the tube from one vessel, lets the pole of air enter it and then puts the tube into the water again, the action of the siphon may be resumed or not. Investigate this phenomenon.

## **12. Boiling**

Put a metallic ball heated to the temperature  $150^{\circ}\text{C} - 200^{\circ}\text{C}$  into hot water at the temperature close to  $100^{\circ}\text{C}$  and observe the process of intensive evaporation of the water. Explain the observed phenomenon.

## **13. Spirits**

A closed vessel (a bottle) contains spirits – pure or substantially diluted by water. Suggest a method of estimation of the concentration of spirits without opening the vessel.

## **14. Magnetic friction**

To investigate the phenomenon described in the problem 3 we suggest to create the device containing the following elements:

a) a copper plate (or a set of plates) 0.3 to 15 mm thick. The length and the width of the plate may be chosen according to one's convenience, but they should be large enough to avoid the effect of the boundaries;

- b) a cylindrical electromagnet with a flat butt-end;
- c) a device providing free motion of the flat butt-end of the electric magnet over the horizontal surface of the copper plate. It is very important that the gap between the magnet and the plate is small as possible and constant everywhere;
- d) the push providing the uniform motion of the magnet at a given velocity over the plate surface.

Introduce the following notation:  $T$  – the push (and the force of magnetic friction),  $v$  – the velocity of the magnet,  $h$  – the thickness of the plate. Investigate and determine experimentally the dependence of  $T$  on  $h$  at  $v = \text{constant}$  for several values of  $v$ .

### **15. Transmission of energy**

Transmit without wires to a distance of 3 meters the largest possible part of the energy stored in a capacitor having capacity of  $C = 10\mu\text{F}$  charged to voltage  $U = 100\text{ V}$ . Measure this energy.

Your device should not contain energy sources. Naturally the capacitor itself must not be transported.

### **16. The Moon and the Sun**

“ If you are asked what is more important, The Sun or the Moon, you should answer the Moon. For the Sun shines in daytime when there is enough light without it”, says a joke. When is it possible to see the Sun and the Moon at the same time? Calculate the schedule of the events for the European countries during 1994.

### **17. Straw**

The Russian proverb says “Had I known the place where I fell, I would have laid some straw there”. How much straw should be laid to guarantee a safe fall?

## 2.2. Problems for the 8th IYPT

### SPALA, POLAND, 1995

#### 1. Think up a problem yourself (paradox)

Try to puzzle your rivals by a paradoxical physical experiment.

#### 2. Boiling water

Some people say it is important to put a lid on the pot when you want to boil water for tea to save energy and time. Investigate this phenomenon and determine the energy and time saving.

#### 3. Drop

A drop of salted water drying on a smooth surface creates a system of rings. Investigate and explain this phenomenon.

#### 4. Gravitational spacecraft

A spacecraft (having a shape of a dumb-bell of variable length) can shift from the Earth orbit (300 km above the Earth surface) to the Moon orbit without the use of jets. Calculate the time taken by such a manoeuvre.

#### 5. Sound

Transfer the electric energy stored in a capacitor of 0.1 mF charged to the voltage of 30 V into the energy of the sound, with the highest efficiency possible. No external energy sources are allowed. Determine the fraction of energy converted into sound in the discharge.

#### 6. Curtain

A light curtain (light scatters on dust particles) is used in some theatres. Suggest the design of a light curtain, which allows its effective action with the minimum power supplied for one meter of stage width?

#### 7. Three discs

Investigate collisions of three homogeneous, rigid discs which can move in a plane. At first two discs are at rest. The third disc:

- a) collides at exactly the same time with two other discs,
- b) collides at first with one of the discs.

#### 8. Carpet

When a carpet is rolled into a cylinder it sometimes unrolls by itself or with the help of a gentle push. Determine the factors on which the speed of the rolling carpet depends.

### **9. Ice cream**

Obtain super-cooled water in an experimental setup. By how many degrees below 0 C did you manage to super-cool it? What can be the record in this experiment? Determine the freezing point of water.

### **10. Cathode –ray tube**

While a well-known physicist A. First watched a football match by TV, another well-known physicist B. Second made a hole of diameter 0.001 mm in the cathode-ray tube. Did A. First manage to see the football match up to the end?

### **11. Moon light**

It is possible to set paper on fire using a lens and solar radiation. Could it be possible using lunar instead of solar light? If yes – invent an optimal optical system for such a purpose. If not – what should the Moon be like, for being this possible?

### **12. Tinder box**

When someone strikes two pieces of flint rock, sparks are created. Investigate and explain this phenomenon.

### **13. Air lens**

Lenses are usually made of solids and sometimes made of liquids. Construct an optical lens made of air in such a way that light can travel through the lens without crossing any material but air. Determine on which factors the focal length of an air lens depends.

### **14. Frozen lake**

The water surface of a lake is in winter exposed to cold air at a fixed temperature below zero. There is no wind. Determine the thickness of the ice layer as a function of time.

### **15. Bottle**

A plastic bottle of a capacity between 1 and 2 litres completely filled with water is "accidentally" dropped on the floor from the height  $H = 1$  m. What maximum height can the spray reach and why? Determine the minimal height from which the bottle should be dropped to burst?

### **16. Oscillation of plates**

Water has been poured on a horizontal glass plate and a second glass plate placed on it. If the lower plate is oscillating in a horizontal plane, at certain amplitudes and frequencies, the upper plate begins to oscillate in vertical direction. Investigate and describe this phenomenon. Is there any difference when you use another liquid ?

### **17. Epic Hero**

An epic Russian hero Ilya Muromets had once thrown his mace weighing forty poods (1 pood = 16 kg) and in forty days this mace fell at the same place. Estimate the parameters of the throw of the hero.

## 2.3. Problems for the 9th IYPT

KUTAISSI, GEORGIA, 1996

### 1. Invent yourself.

Invent and solve yourself a problem concerning the ozone holes.

### 2. Paper clot.

Crumple arbitrarily a sheet of paper A4 in your hand. This clot can be approximated by a sphere. Making many of this clots and measuring their average diameters a histogram of distribution of diameters can be plotted. Try to explain the result obtained. Make more comprehensive investigation of the dependence of the average diameter of a clot on the parameters which you consider important.

### 3. Cycle racing

According to the forecast of specialists two very strong and "absolutely identical" sportsmen had to show equal time in a highway race for 100 km. But, alas, one sportsman lagged behind. Later it was found out that some malefactor adjusted a nut of mass 5 g to the rim of the rear wheel of his bicycle. For what time is the victim?

### 4. Self-formation of a pile.

A horizontal rigid plate vibrates vertically at a frequency of the order of 100 Hz. A cone-shaped pile of fine dispersed powder (e.g. Lycopodium or talc) which is heaped up on the plate remains stable at small amplitudes of the vibration. If the amplitude is increased the cone decays. Further increase of the amplitude yields a distribution confined by a sharp border and at still higher amplitudes a pile appears again. Investigate and explain this phenomenon.

### 5. Auto oscillations.

Produce and investigate auto oscillating system containing thermistor as a single non-linear element.

### 6. Water generator.

If some volume of water is frozen from one side, a potential difference appears across the ice-water frontier. Measure this potential difference and explain the phenomenon.

### 7. Sun.

In the centre of the Sun suddenly an extra quantity of energy is produced which is equal to the energy emitted by the Sun per year. How will the parameters of the Sun observed on the Earth change during one year?

### **8. Surface information.**

Develop a method for transferring information by the waves on the surface of water. Investigate the angular characteristics of the emitter and the receiver ( the antennas ) which you constructed.

### **9. Floor-polisher.**

A device stands on two identical disks lying flat on a horizontal surface. The disks can rotate in opposite directions at a given velocity. Investigate how the value of a force providing a uniform motion this device along a horizontal plane depends on the velocity of this motion and the velocity of rotation of these disks.

### **10. Soap bubbles.**

Dip the ring of a children's toy for blowing out soap bubbles into a soap solution and blow on the film formed in the ring. At what velocity of the air flux blown into the ring will the bubbles form ? How must the velocity of the air flux be adjusted to produce the bubble of maximum size ?

### **11. Candle**

Some candles twinkle before dying out. Investigate and explain this phenomenon.

### **12. Motor car.**

A car driven at constant power moves onto a wet section of a straight road. How will its speed change when the thickness of the water layer increases slightly and linearly with the distance ?

### **13. Grey light.**

Construct a source of light which would seem to be grey.

### **14. Coherer.**

It is known that a glass tube with two electrodes and metallic filings between them (coherer) has different resistance in d.c. and a.c. circuits. Investigate the frequency dependence of the coherer's resistance.

### **15. Salt water oscillator.**

A cup with a small hole in its bottom containing salt water is partially immersed in a big vessel with fresh water and fixed. Explain the mechanism of the observed periodical process and investigate the dependence of its period on different parameters. To visualize the process, the water in the cup should be coloured.

### **16. Hail**

Explain the mechanism of hail formation and propose your own method to prevent the hailing.

### **17. Gloves**

Some people refuse to wear gloves in winter because they suppose to feel colder than without gloves. Others prefer to wear mittens instead. What is your opinion?

## 2.4. Problems for the 10th IYPT

### CHEB, CZECH REPUBLIC, 1997

#### 1. Invent yourself

Construct and demonstrate a device which moves in a definite direction under chaotic influence.

#### 2. Coin

From what height must a coin with heads up be dropped, so that the probability of landing with heads or tails up is equal?

#### 3. Paper

How does the tensile strength of paper depend on its humidity?

#### 4. Electron Beam

An electron beam is cast upon a planparallel plate of known homogenous material. Some of the electrons get through it, some do not. Try to simulate processes taking place, e.g. using Monte Carlo method and compare your results with the ones described in literature.

#### 5. Blue Blood

Human blood is known to be red, but the veins seem to be blue. Explain this phenomenon and illustrate it by a model.

#### 6. Magic Tube

A compressor blows air into Ranque-Hilsch T-shaped tube at a pressure of 0,5 Mpa or higher so that the air begins to circulate. In such a case hot air is coming out from one end of the tube and cold air from the opposite one. Find out which end of the tube is the "hot" one and explain the difference of the temperatures obtained. Investigate the parameters this difference depends on.

#### 7. Water Jet

A water jet streaming vertically downwards from a tube is divided into drops at some distance from the tube. Choose the conditions under which the length of the unseparated jet is largest. What maximum length did you obtain?

#### 8. Floatation

A piece of chocolate, which is dropped into a glass of soda water, periodically sinks and goes back to the surface. Investigate the dependence of the period of these oscillations on various parameters.

### **9. Jet-spread**

A water jet falling onto a horizontal plane spreads out radially. At some distance from the center the thickness of the layer increases dramatically. Explain the phenomenon.

### **10. Cooling the Earth**

How would the temperature of the Earth change with time, if the Sun suddenly stopped radiating?

### **11. Candle Generator**

Construct a device for charging an electric capacitor ( $1000 \mu\text{F}/100\text{V}$ ) using the energy of a candle burning for a period of one minute.

### **12. Static Friction**

A force of motion friction is known to independent on the rubbing surface area of a body. How does the static friction depend on the rubbing surface area?

### **13. Tea Cup**

If one fills a cup with hot tea ( $60^\circ - 80^\circ\text{C}$ ), a thin layer of steam emerges above the surface. One can see that some parts of the steam layer disappear suddenly and reappear after a few seconds. Investigate and explain this phenomenon

### **14. Rain**

On a long-time exposure photograph of night rain taken in the light of a projector, the tracks of drops appear interrupted. Explain this phenomenon.

### **15. Cell and Accumulator**

How does the voltage-current characteristics of a cell and of an accumulator change during discharging?

### **16. Roghe Spiral**

The Roghe Spiral is a device where a source of current is connected to a vertically suspended spring, the lower end of which dipped mercury. Mercury is a highly dangerous chemical substance and thus the experiments with it are not permitted. Substitute the mercury with a less dangerous substance and investigate the functioning of this device.

### **17. Leap**

To make a leap it is necessary to squat. How does the height of a leap depend on the depth of the s

## 2.5. Problems for 11th IYPT

### DONAUESCHINGEN, GERMANY, 1997

#### 1. Invent yourself

Construct an aeroplane from a sheet of paper (A4, 80 g/mI). Make it fly as far and/or as long as possible. Explain why it was impossible to reach a greater distance or a longer time.

#### 2. Popping body

A body is submerged in water. After release it will pop out of the water. How does the height of the pop above the water surface depend on the initial conditions (depth and other parameters)?

#### 3. Spinning disc

Investigate and explain the phenomenon of a spinning annular disc as they progress down a straight, cylindrical rod. If the rod is moved upwards at a defined velocity, the disc spins at constant height. Investigate the mechanism.

#### 4. Water streams

A can with three holes in the side-wall at the same height slightly above the bottom is filled with water. The water will escape in three separate streams. By gently touching the streams with a finger they may unite. Investigate the conditions for this to happen.

#### 5. Water jet

If a vertical water jet falls down onto a horizontal plate, standing waves will develop on the surface of the jet. Investigate the dependence of this phenomenon on different parameters.

#### 6. Mount Everest

Can you see Mount Everest from Darjeeling?

#### 7. Air bubble

An air bubble rises in a water-filled, vertical tube with inner diameter 3 to 5 mm. How does the velocity of the rising bubble depend on its shape and size?

#### 8. Trick

It is known that a glass filled with water and covered with a sheet of paper may be turned upside down without any loss of water. Find the minimum amount of water to perform the trick successfully.

#### 9. Woven textiles

Look at a point-like light source through different woven textiles. Describe what you see. What is the explanation of the phenomenon?

### **10. Repeated freezing**

While a vessel filled with an aqueous solution of a volatile fluid, e.g., ammonia, ethanol or acetone, is being cooled, repeated freezing and melting may be observed near the surface. Describe and explain the phenomenon.

### **11. Current system**

In a Petri dish (shallow bowl), small metal balls, e.g., 2 mm in diameter, are immersed in a layer of castor oil. The inner rim of the dish contains an earthed metal ring. Above the centre of the dish there is a metal needle which does not touch the oil surface. Investigate what happens when the voltage between needle and earth is about 20 kV.

*Warning:* The high voltage should be obtained by means of a safe generator, e.g., an electrostatic generator!

### **12. Powder conductivity**

Measure and explain the conductivity of a mixture of metallic and dielectric powders with various proportions of the two components.

### **13. Rope**

How is it possible that a very long and strong rope can be produced from short fibers? Prepare a rope from fibers and investigate its tensile strength.

### **14. Water rise**

Immerse the end of a textile strip in water. How fast does the water rise in the strip and what height does it reach? In which way do these results depend on the properties of the textile?

### **15. Luminescent sugar**

Investigate and explain the light produced when sugar crystals are pulverized. Are there other substances with the same property?

### **16. Strange motion**

Make a mixture of ammonium nitrate and water, proportion 5 to 1. When the mixture is heated to about 100 °C it melts. When it cools, it crystallizes and you may observe a strange motion below the surface. Investigate and explain the phenomenon.

*Safety rules:* Do not heat the ammonium nitrate without water, preferably use a water bath! Use protection glasses during the experiment!

### **17. Icicles**

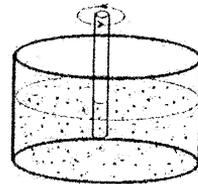
Investigate and explain the formation of icicles.

## 2.6. Problems for the 12th IYPT

VIENNA, AUSTRIA, 1999

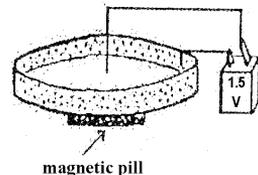
### 1. Rotation

A long rod, partially and vertically immersed in a liquid, rotates about its axis. For some liquids this will cause an upward motion of the liquid on the rod and for other liquids a downward motion



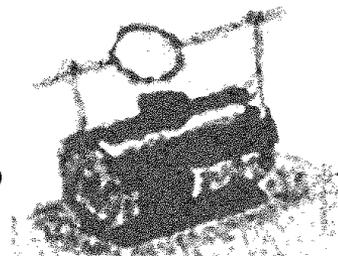
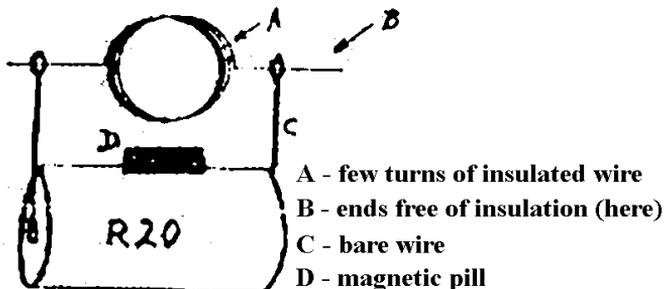
### 2. Ionic Motor

An electrolyte (an aqueous solution of  $\text{CuSO}_4$ ,  $\text{NaCl}$ , ...) in a shallow tray is made to rotate in the field of a permanent magnet (a small "pill" placed under the tray). The electric field is applied from a 1.5 V battery in such a way that one electrode is in the form of a conducting ring immersed in the electrolyte --- the other is a tip of wire placed vertically in the centre of the ring. Study the phenomenon and find possible relations between the variables.



### 3. Magic Motor

Construct a DC motor without a commutator, using a battery, permanent magnet and a coil. Explain how it functions



### 4. Soap Film

Explain the appearance and development of colours in a soap film, arranged in different geometries.

### 5. Dropped Paper

If a rectangular piece of paper is dropped from a height of a couple of meters, it will rotate around its long axis whilst sliding down at a certain angle. How does this angle depend on various parameters?

## **6. Singing Glass**

When rubbing the rim of a glass containing a liquid a tone can be heard. The same happens if the glass is immersed in a liquid. How does the pitch of the tone depend on different parameters.

## **7. Heated Needle**

A needle hangs on a thin wire. When approached by a magnet the needle will be attracted. When heated the needle will return to its original position. After a while the needle will be attracted again. Investigate this phenomenon, describe the characteristics and determine relevant parameters.

## **8. Energy Converter**

A body of mass 1 kg falls from a height of 1 m. Convert as much as possible of the released potential energy into electric energy and use that to charge a capacitor of 100 mikroF.

## **9. Air Dryer**

During 4 minutes collect as much water as possible from the air in the room. The mass of the equipment must not exceed 1 kg. The water should be collected in a glass test tube, provided by the jury.

## **10. Charged Balloon**

An air-filled balloon rubbed with wool or dry paper may stick to the ceiling and stay there. Investigate this phenomenon and measure the charge distribution on the surface of the balloon.

## **11. Billiard**

Before a snooker game starts, 15 balls form an equilateral triangle on the table. Under what conditions will the impact of the white ball (16th ball) produce the largest disorder of the balls.

## **12. Flour Craters**

If you drop a small object in flour, the impact will produce a surface structure which looks like moon crater. What information about the object can be deduced from the crater?

## **13. Gas Flow**

Measure the speed distribution of the gas flow in and around the flame of a candle. What conclusions can be drawn from the measurements?

## **14. Wheat Waves**

The wind blowing through a wheat field creates waves. Describe the mechanism of wave formation and discuss the parameters which determine the wavelength.

### **15. Bright Spots**

Bright spots can be seen on dew drops if you look at them from different angles. Discuss this phenomenon in terms of the number of spots, their location and angle of observation.

### **16. Liquid Diode**

Make an electrochemical diode and investigate its properties, in particular the frequency dependence.

### **17. Sound from Water**

When you heat water in a kettle you hear a sound from the kettle before the water starts to boil. Investigate and explain this phenomenon.

## 2.7. Problems for the 13th IYPT

**BUDAPEST, HUNGARY, 2000**

### 1. Invent for yourself

Suggest a contact-free method for the measurement of the surface tension coefficient of water. Make an estimate of the accuracy of the method.

### 2. Tuning fork

A tuning fork with resonant frequency of about 100 Hz is struck and held horizontally, so that its prongs oscillate up and down. A drop of water is placed on the surface of the upper prong. During the oscillation of the tuning fork standing waves appear on the surface of the drop and change with time. Explain the observed phenomena.

### 3. Plasma

Investigate the electrical conductivity of the flame of a candle. Examine the influence of relevant parameters, in particular, the shape and polarity of the electrodes. The experiments should be carried out with a voltage not exceeding 150V.

### 4. Splash of water

Measure the height reached by splashes of water when a spherical body is dropped into water. Find a relationship between the height of the splashes, the height from which the body is dropped, and other relevant parameters.

### 5. Sparkling water

Bubbles in a glass of sparkling water adhere to the walls of the glass at different heights. Find a relationship between the average size of the bubbles and their height on the side of the glass.

### 6. Transmission of signals

Using a bulb, construct the optimum transmitter of signals without any modulation of the light beam between transmitter and receiver. Investigate the parameters of your device. The quality of the device is defined by the product of the information rate (bits/sec) and the distance between transmitter and receiver.

### 7. Merry-go-round

A small, light, ball is kept at the bottom of a glass filled with an aqueous solution and then set free. Select the properties of the solution, so that a moving up time of several seconds is achieved. How will this time change if you put your glass on the surface of a rotating disk?

### **8. Freezing drop**

Drops of melted lead or tin fall from some height into a deep vessel filled with water. Describe and explain the shape of the frozen drops as a function of height of fall.

### **9. Radioactivity**

Use efficient methods to collect as much radioactive material as you can in a room. Measure the half-life of the material you have collect

### **10. Liquid fingers**

When a layer of hot salt solution lies above a layer of cold water, the interface between the two layers becomes unstable and a structure resembling fingers develops in the fluid. Investigate and explain this phenomenon.

### **11. Throwing stone**

A student wants to throw a stone so that it reaches the greatest distance possible. Find the optimum mass of the stone that should be used.

### **12. Tearing paper**

Tear a sheet of paper and investigate the path along which the paper tears

### **13. Rolling can**

A can partially filled with water rolls down an inclined plane. Investigate its motion.

### **14. Illumination**

Two bulbs, 100 and 40 watts, respectively, illuminate a table tennis ball placed between them. Find the position of the ball, when both sides of the ball appear to be equally lit. Explain the result.

### **15. Cooling water**

Two identical open glasses, filled with hot and warm water, respectively, begin to cool under normal room conditions. Is it possible that the glass filled with hot water will ever reach a lower temperature than the glass filled with warm water? Make an experiment to investigate this and explain the result.

### **16. Coloured sand**

Allow a mixture of differently coloured, granular materials to trickle into a transparent, narrow container. The materials build up in distinct bands. Investigate and explain this phenomenon.

### **17. A strange sound**

Pour hot water into a cup containing some cappuccino or chocolate powder. Stir slightly. If you then knock the bottom of the cup with a teaspoon you will hear a sound of low pitch. Study how the pitch changes when you continue knocking. Explain the phenomenon.

## 2.8. Problems for the 14th IYPT

ESPOO, FINLAND, 2001

### 1. Electrostatic motor

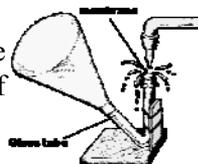
Is it possible to create a motor which works by means of an electrostatic field? If yes, suggest how it may be constructed and estimate its parameters.

### 2. Singing saw

Some people can play music on a handsaw. How do they get different pitches? Give a quantitative description of the phenomenon.

### 3. Tuning dropper

Make the music resonator shown in the [picture](#). Investigate the conditions that affect the pitch. Can you observe amplification of external sounds? If yes, how can you explain this?



### 4. Dancing sand clock

Investigate the trickling of sand when a sand clock (egg-timer) is placed on a vibrating base.

### 5. Rubber heat machine

Investigate the conversion of energy in the process of deformation of rubber. Construct a heat machine, which uses rubber as the working element and demonstrate how it works.

### 6. Fractal diffraction

Produce, demonstrate and analyse diffraction pictures of fractal structures of different orders.

### 7. Cracks

When drying a starch solution, you will see cracks forming. Investigate and explain this phenomenon.

### 8. Speedometer

Two electrodes of different metal are immersed in an electrolyte solution. Investigate the dependence of the measured potential difference on the relative motion of electrodes and their shapes.

### 9. Pouring out

Investigate how to empty a bottle filled with a liquid as fast as possible, without external technical devices.

### **10. Water stream pump**

Construct and demonstrate a water stream vacuum pump. What is your record value for the minimum pressure?

### **11. Rolling balls**

Place two equal balls in a horizontal, V-shaped channel, with the walls at 90 degrees to each other, and let the balls roll towards each other. Investigate and explain the motion of the balls after the collision. Make experiments with several different kinds of ball pairs and explain the results.

### **12. Reaction**

Make an aqueous solution of gelatine (10g gelatine in 90ml of water), heat it to 80 degrees C in a water bath and mix it with a solution of potassium iodide. Pour the solution in a test tube and cool it. Pour a solution of copper sulphate on the surface of the gel. Find a physical explanation to the observed phenomena.

### **13. Membrane electrolyser**

In an electrolyser, containing a membrane which completely divides the space between two inert electrodes, the pH-value of the diluted salt solution will change substantially after electrolysis. Investigate how this difference depends on the pore size of the membrane.

### **14. Thread dropper**

One end of a thread is immersed in a vessel filled with water. The other end hangs down outside without contact with the outer wall of the vessel. Under certain conditions, one can observe drops on that end of the thread. What are those conditions? Determine how the time of appearance of the first drop depends on relevant parameters.

### **15. Bubbles in magnetic field**

Observe the influence of an alternating magnetic field (50 or 60 Hz) on the kinetics of gas bubbles in a vessel filled with water. The bubbles can be generated by blowing air into the water.

### **16. Adhesive tape**

Investigate and explain the light produced, when adhesive tape is ripped from a smooth surface.

### **17. Seiches**

Seiching is a phenomenon known for long, narrow and deep lakes. For reasons of changes in the atmospheric pressure, the water of the lake can start moving in such a way that the water level at both ends of the lake makes periodic motions, which are identical, but out of phase. Make a model that predicts the period of seiching depending on the appropriate parameters and test its validity.

## 2.9. Problems for the 15th IYPT

### ODESSA, UKRAINE, 2002

#### Heat engine

A tall glass cylinder is half-filled with hot water and topped up with cold water. A small ampoule, containing a few drops of ether or alcohol (and closed off by a rubber pipette cap), is then put in. Describe the phenomena occurring in the system. How does the motion of the ampoule change with time?

#### 2. Spider's web

A spider's thread looks like a string of pearls. What is the reason for this? Make experiments to investigate the relevant parameters.

#### 3. Flying colours

Why do flags flutter in the wind? Investigate experimentally the airflow pattern around a flag. Describe this behaviour.

#### 4. Hazy

The colour of a distant forest appears not green, but hazy blue. What is the minimum distance at which this phenomenon is observed? How do weather conditions affect this? Is it possible that a forest can appear grey?

#### 5. Pond skater

It is known that unwettable small bodies can float on water due to the surface tension force. Construct a floating raft based on this principle and determine its static and dynamic parameters.

#### 6. Stop and start

Sometimes a flow of traffic can experience sudden stops and starts for no apparent reason. Build a physical model to explain why this occurs.

#### 7. Ohm's Law for a liquid

It is said that electric current “flows”. Is this the only analogy between electric current and the flow of a liquid? Investigate theoretically and experimentally other analogies between these two.

#### 8. Charged sand

Fine, well-dried quartz sand is poured out of a short thin tube into a conical metallic vessel connected to an electrometer. Investigate the behaviour of the sand stream as the vessel fills up. What changes if the stream is lit by a UV-lamp?

#### 9. Chromatography

Put a drop of coloured liquid on a piece of absorbant paper. Describe quantitatively the observed phenomena.

### **10. Sound cart**

Construct and demonstrate a device that can be propelled solely by sound. Investigate its properties.

### **11. Equilibrium**

Fill a glass with water up to the point where a convex meniscus is formed. Place a table tennis ball on the surface of the water. Investigate and explain the stability of its equilibrium. Repeat your experiment with other liquids.

### **12. Electroconductivity**

How can you measure the electroconductivity of salt solutions without using direct contact electrodes? Analyse the problem and demonstrate your device.

### **13. Spinning ball**

A steel ball of diameter 2-3 cm is put on a horizontal plate. Invent and construct a device, which allows you to spin the ball at high angular velocity around a vertical axis. The device should have no mechanical contact with the ball.

### **14. Torn sail**

Determine the dependence of the efficiency of a sail on its degree of perforation. What would be the effect of using a fishing net as a sail?

### **15. Pulsating air bubble**

Trap an air bubble of radius 1-2 cm under an inverted watch glass beneath a water surface. Introduce alcohol into the bubble through a thin tube, controlling and adjusting the rate of flow until the bubble pulsates rhythmically. Study the phenomenon and explain your observations.

### **16. Elastic pendulum**

Study and describe the behaviour of a pendulum where the bob is connected to a spring or an elastic cord rather than to a stiff rod.

### **17. Bottle battle**

Take two opened glass bottles of cola and knock one against the other. After a short while, the cola spurts out of one of the bottles. Investigate and explain the phenomenon.

## 2.10. Problems for the 16th IYPT

### UPPSALA, SWEDEN, 2003

#### 1. Motion of a kite

On windy days one can see kites flying in the wind. Often, one-string kites move on a stable track, which looks like a number 8. Why does a kite move in such a way? Are there other stable tracks?

#### 2. Water drops

Investigate and explain the movement of raindrops on a window pane.

#### 3. Transparent film

If you cover printed text with a piece of transparent polyethylene film you can still easily read it. As you gradually lift up the film, the text becomes increasingly blurred and may even disappear. Study the properties of the film. On what parameters of the film is the phenomenon based?

#### 4. Bright spots

Blow a soap bubble and allow it to rest on a liquid surface or a glass plate. When illuminated by sunlight, bright spots can be observed on the bubble. Investigate and explain the phenomenon.

#### 5. Bubbles at an interface

Certain liquids can be layered one above the other with a sharp interface between them. If the surface tensions of the liquids are different, then an interesting phenomenon can be observed. Blow bubbles of different sizes into the lower liquid and observe their behaviour near the interface. Investigate and explain the phenomenon.

#### 6. Freezing soft drinks

On opening a container of cold soft (carbonated) drink the liquid inside sometimes freezes. Study the relevant parameters and explain the phenomenon.

#### 7. Oscillating box

Take a box and divide it into a number of small cells with low walls. Distribute some small steel balls between the cells. When the box is made to oscillate vertically, the balls occasionally jump from one cell to another. Depending on the frequency and the amplitude of the oscillation, the distribution of the balls can become stable or unstable. Study this effect and use a model to explain it.

#### 8. Heat engine

Construct a heat engine from a U-tube partially filled with water (or another liquid), where one arm of the tube is connected to a heated gas reservoir by a

length of tubing, and the other arm is left open. Subsequently bringing the liquid out of equilibrium may cause it to oscillate. On what does the frequency of the oscillation depend? Determine the pV diagram of the working gas.

### **9. Falling chimney**

When a tall chimney falls it sometimes breaks into two parts before it hits the ground. Investigate and explain this.

### **10. Tungsten lamp**

The resistance of the tungsten filament in a light bulb shows a strong temperature dependence. Build and demonstrate a device based on this characteristic.

### **11. Light scattering**

Construct an optical device for measuring the concentration of non-soluble material in the 'viscous' properties of hens' eggs that have been boiled to different extents.

### **12. Boiled egg**

Construct a torsion viscometer. Use it to investigate and explain the difference in the 'viscous' properties of 'hens' eggs that have been boiled to different extents.

### **13. Electro-osmosis**

Develop a device that will drain wet sand, with the aid of an electrical voltage but without significant heating.

### **14. Rotating disk**

Find the optimum way of throwing a 'frisbee' as far as possible. Explain your findings.

### **15. Vortices**

Make a box that has a hole in its front wall and a membrane as its back wall. Hitting the membrane creates a vortex that propagates out from the hole. Investigate the phenomenon and explain what happens when two vortices interact.

### **16. Pot and ice**

It is sometimes argued that to cool a pot effectively one should put ice above it. Estimate to what extent this is more effective than if the ice is put under the pot.

### **17. Prometheus problem**

Describe and demonstrate the physical mechanism, based on friction, which allowed our ancestors to make fire. Estimate the time needed to make fire in this way.

## VI. Problems for the 19th IYPT

BRATISLAVA, SLOVAKIA, 2006

### 1. Froth

Investigate the nature of the decay in height of the “froth” or “foam” on a liquid. Under what conditions does the froth remain for the longest time?

### 2. Shades

If small non-transparent objects are illuminated with light, patterns in the shadows are observed. What information can be obtained about these objects using these patterns?

### 3. Duck's cone

If one looks at the wave pattern produced by a duck paddling across a pond, this reminds one of Mach's cone. On what parameters does the pattern depend?

### 4. Whispering Gallery

The Whispering Gallery at St Paul's Cathedral in London, for example, is famous for the fact that the construction of the circular gallery makes a whisper against its walls on the side of the gallery audible on the opposite side of the gallery. Investigate this phenomenon.

### 5. Probability

A coin is held above a horizontal surface. What initial conditions will ensure equal probability of heads and tails when the coin is dropped and has come to rest?

### 6. Wet cleaning

A wet rag is hard to drag when it is spread out and pulled across the floor. What does the resistive force depends on?

### 7. Airglider

A paper sheet is on the table. If one blows along the table the sheet begins to glide over it. Determine the flight characteristics of the paper.

### 8. Electrostatics

Propose and make a device for measuring the charge density on a plastic ruler after it has been rubbed with a cloth.

### 9. Sound and foam

Investigate the propagation of sound in foam.

### **10. Inverted pendulum**

It is possible to stabilize an inverted pendulum. It is even possible to stabilize an inverted multiple pendulum (one pendulum the top of the other). Demonstrate the stabilization and determine on which parameters this depends.

### **11. Singing tube**

A tube open at both ends is mounted vertically. Use a flame to generate sound from tube. Investigate the phenomenon.

### **12. Rolling magnets**

Investigate the motion of a magnet as it rolls down an inclined plane.

### **13. Sound**

Measure the speed of sound in liquids using light.

### **14. Cellular materials**

Investigate the behaviour of a stream of liquid when it strikes the surface of a sponge-like material.

### **15. Heat and temperature**

A tube passes steam from a container of boiling water into a saturated aqueous salt solution. Can it be heated by the steam to a temperature greater than 100°C? Investigate the phenomenon.

### **16. Hardness**

A steel ball falls onto a horizontal surface. If one places a sheet of paper onto the surface with a sheet of carbon paper on top of it, a round trace will be produced after the impact. Propose a hardness scale based on this method.

### **17. Magnetohydrodynamics**

A shallow vessel contains a liquid. When an electric and magnetic fields are applied, the liquid can start moving. Investigate this phenomenon and suggest a practical application.